
Algorithm 1: Multivariate Polynomial Division Algorithm

Input: $f, F = (f_1, \dots, f_s), \geq$ (monomial ordering)

Output: $(q_1, \dots, q_s), r$ such that $f = \sum_{i=1}^s q_i f_i + r$, $\text{LT}_{\geq}(r)$ is not divisible by any of $\text{LT}_{\geq}(f_i)$ or $r = 0$

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1  $q_1 \leftarrow \dots \leftarrow q_s \leftarrow r \leftarrow 0$ 
2  $p \leftarrow f$ 
3 while  $p \neq 0$  do
4    $i \leftarrow 1$ 
5   divisionoccured  $\leftarrow$  False
6   while  $i \leq s$  and not divisionoccured do
7     if  $\text{LT}_{\geq}(f_i)$  divides  $\text{LT}_{\geq}(p)$  then
8        $q_i \leftarrow q_i + \frac{\text{LT}_{\geq}(p)}{\text{LT}_{\geq}(f_i)}$ 
9        $p \leftarrow p - \frac{\text{LT}_{\geq}(p)}{\text{LT}_{\geq}(f_i)} \cdot f_i$ 
10      divisionoccured  $\leftarrow$  True
11     else
12        $i \leftarrow i + 1$ 
13   if not divisionoccured then
14      $r \leftarrow r + \text{LT}_{\geq}(p)$ 
15      $p \leftarrow p - \text{LT}_{\geq}(p)$ 
16 return  $(q_1, \dots, q_s), r$ 
```
