WINDOWING

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FEL CTU PRAGUE

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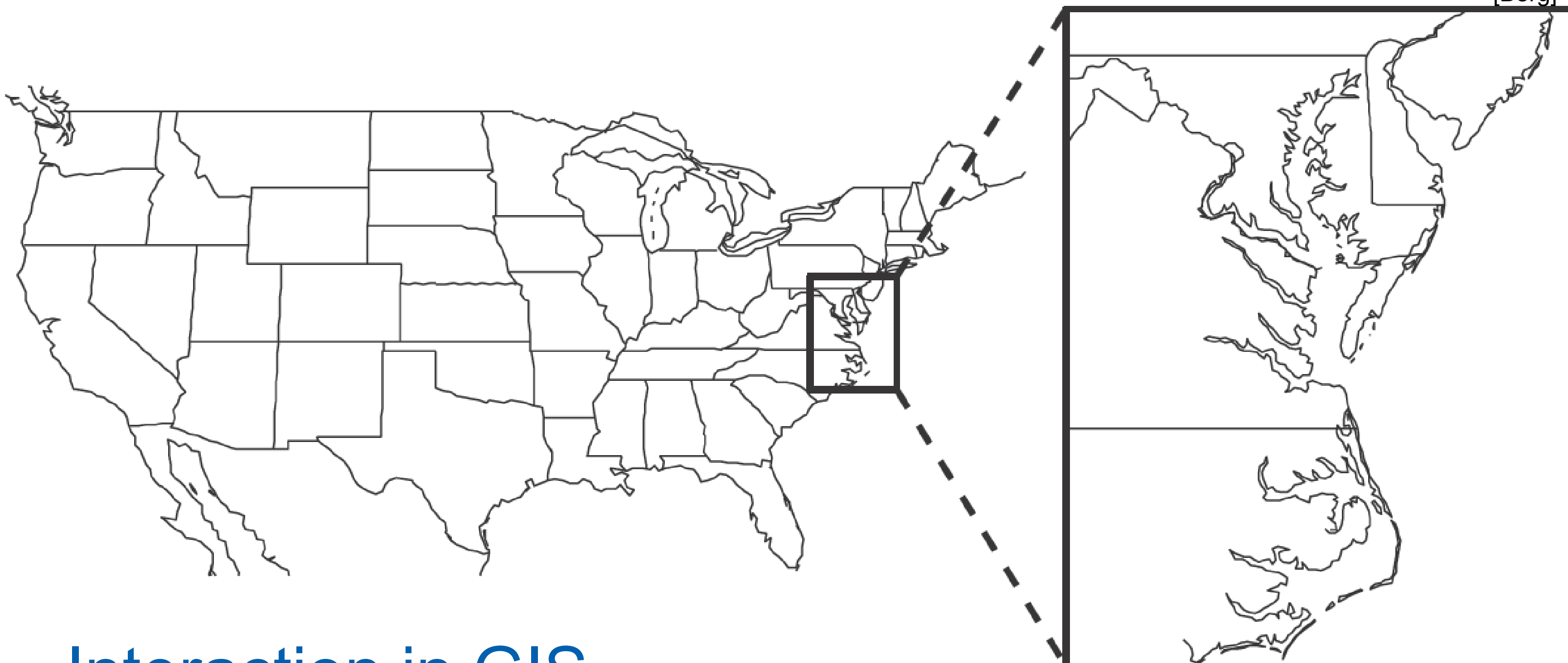
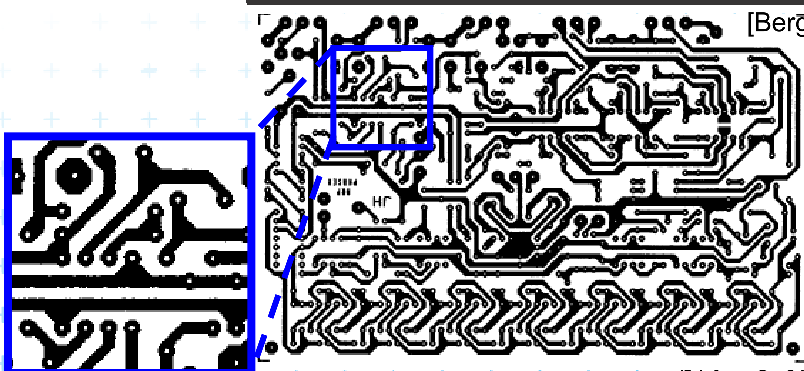
https://cw.felk.cvut.cz/doku.php/courses/a4m39vg/start

Based on [Berg], [Mount]

Version from 26.11.2024

Windowing queries - examples





- Interaction in GIS
  - Select subset by outlining
  - Zoom in and re-center
- Circuit board inspection,…

Windowing versus range queries

- Range queries (see range trees in Lecture 03)
  - Points
  - Often in higher dimensions
- Windowing queries
  - Line segments, curves, …
  - Usually in low dimension (2D, 3D)

- The goal for both:
  Preprocess the data into a data structure
  - so that the objects intersected by the query rectangle can be reported efficiently

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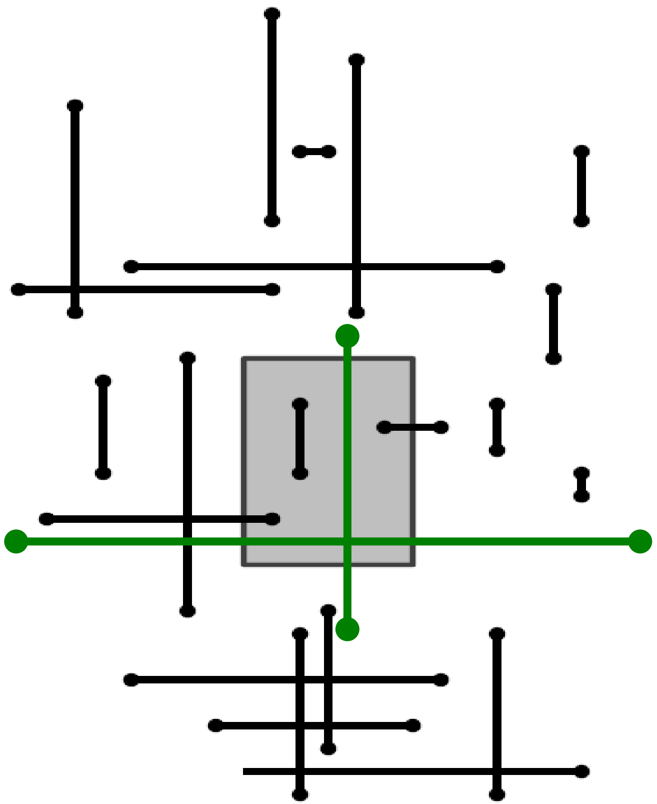
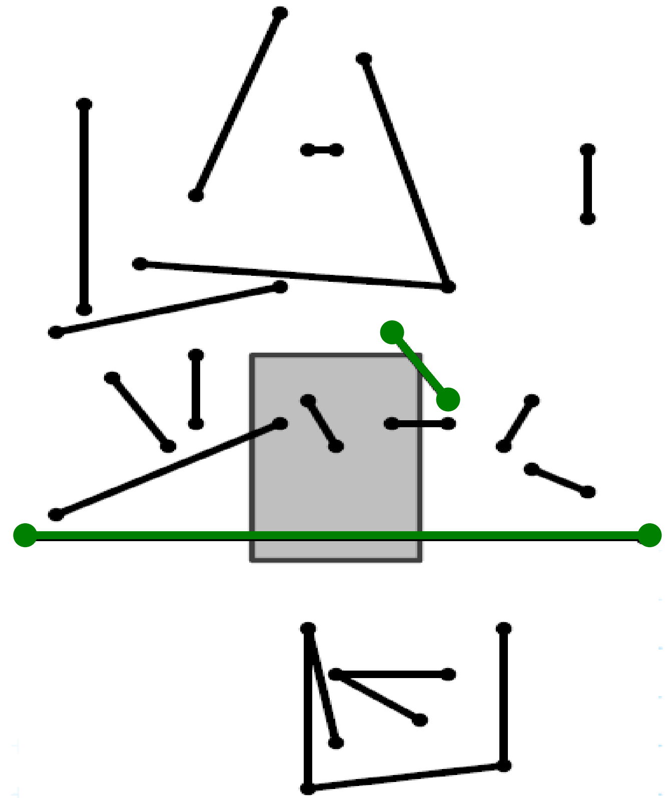
Windowing queries on line segments



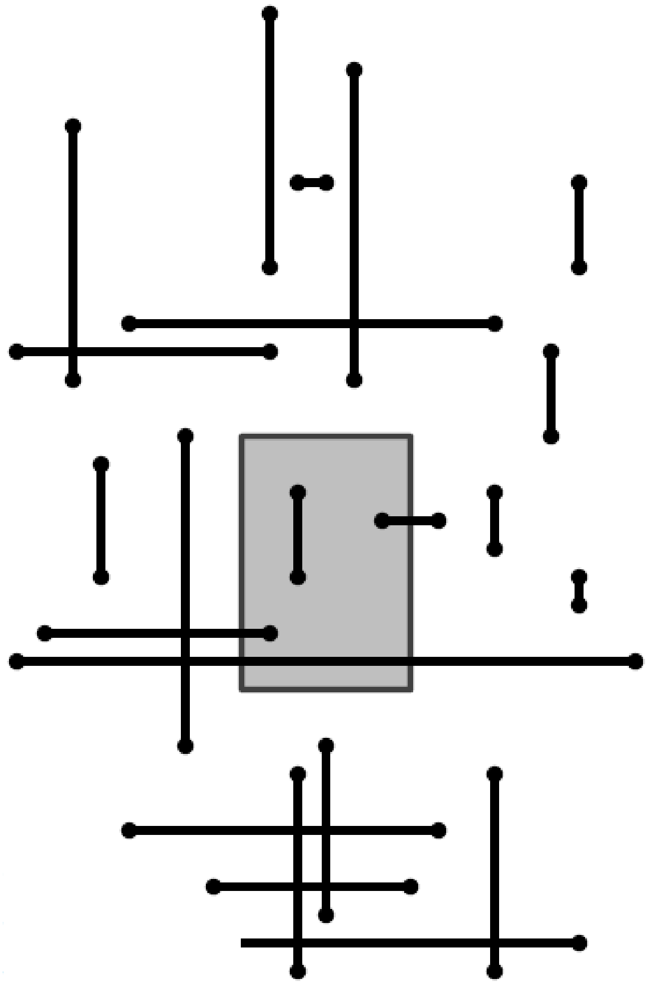
1. Axis parallel line segments



2. Arbitrary line segments (non-crossing)

[Vakken]

1. Windowing of axis parallel line segments

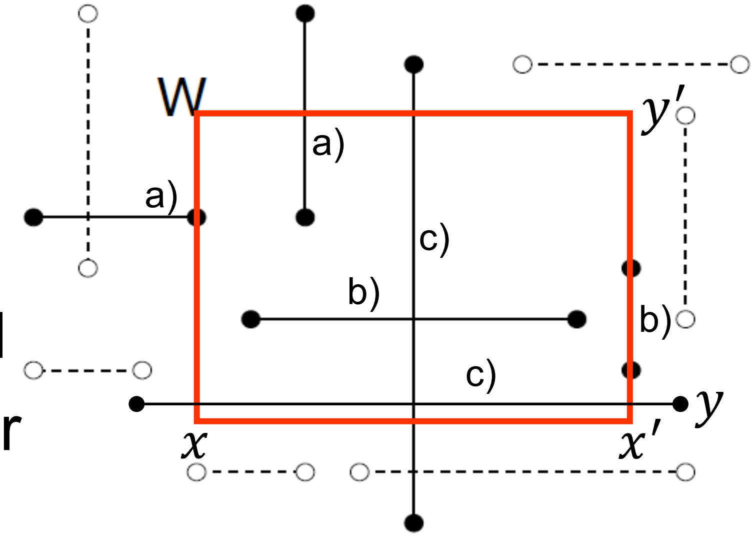


[Vakken]

1. Windowing of axis parallel line segments

Window query

- Given
  - a set of orthogonal line segments *S* (preprocessed),
  - and orthogonal query rectangle $W = [x : x'] \times [y : y']$
- Count or report all the line segments of *S* that intersect *W*
- Such segments have
  a) one endpoint in
  b) two end points in – included
  c) no end point in – cross over

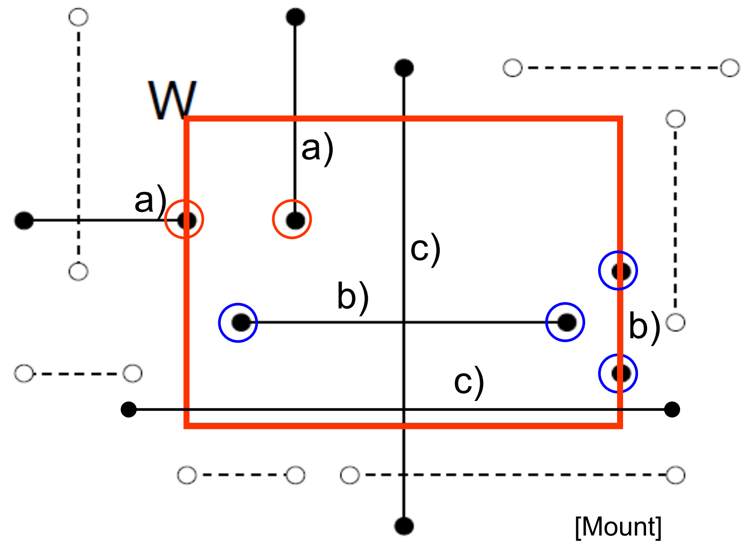


[Mount]

Line segments with 1 or 2 points inside

a) one point inside

- Use a 2D range tree (lesson 3)
- $O(n \log n)$ storage
- $O(\log^2 n + k)$ query time or
- $O(\log n + k)$ with fractional cascading



b) two points inside – as a) one point inside

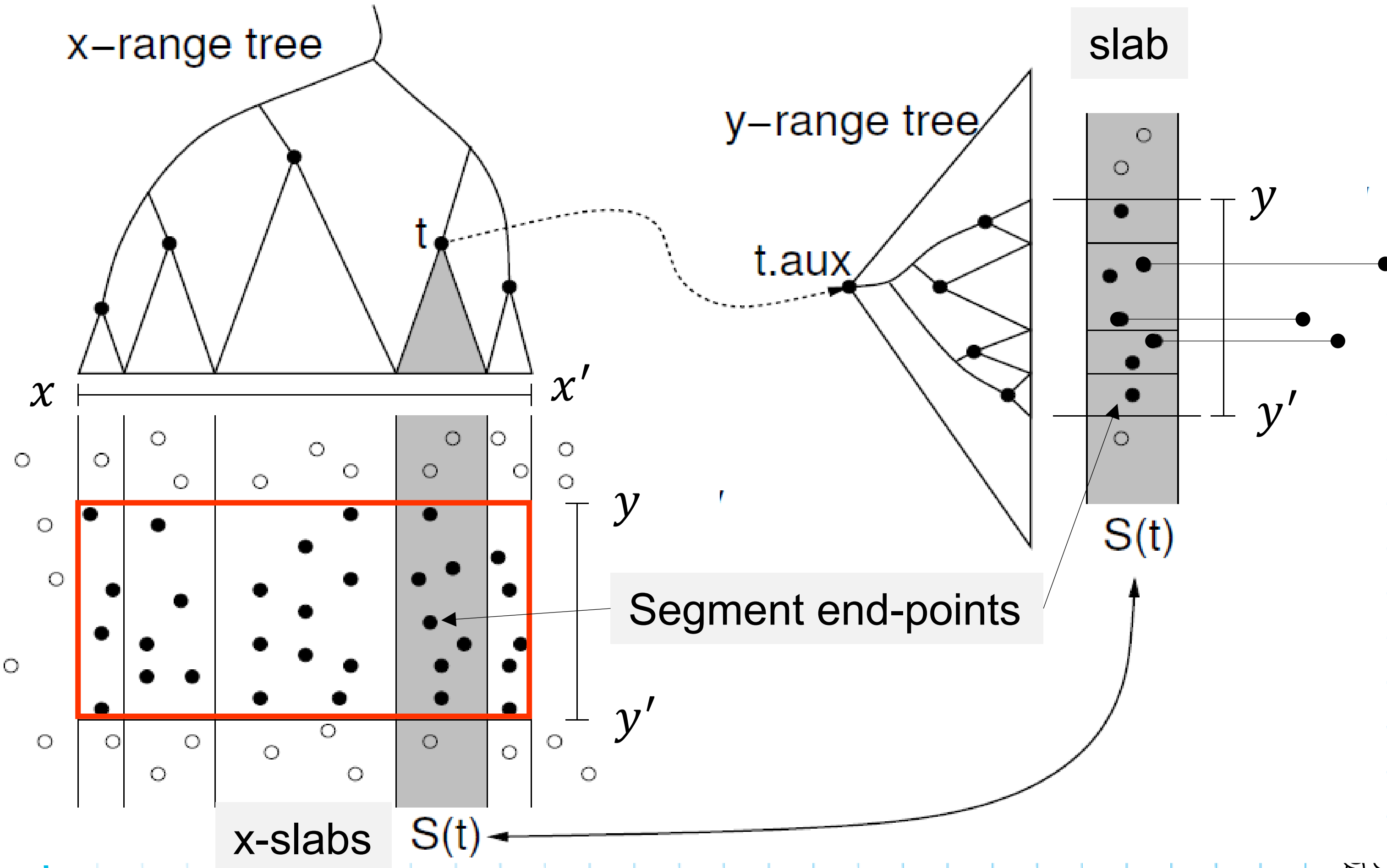
- Avoid reporting twice:
  - Mark segment when reported (clear after the query) and skip marked segments or
  - when end point found, check the other end-point and report only one of them (the leftmost or the bottom)

2D range tree (without fractional cascading-more in Lecture 3)

Search space: points

Query: Orthogonal intervals $[x : x'] \times [y : y']$

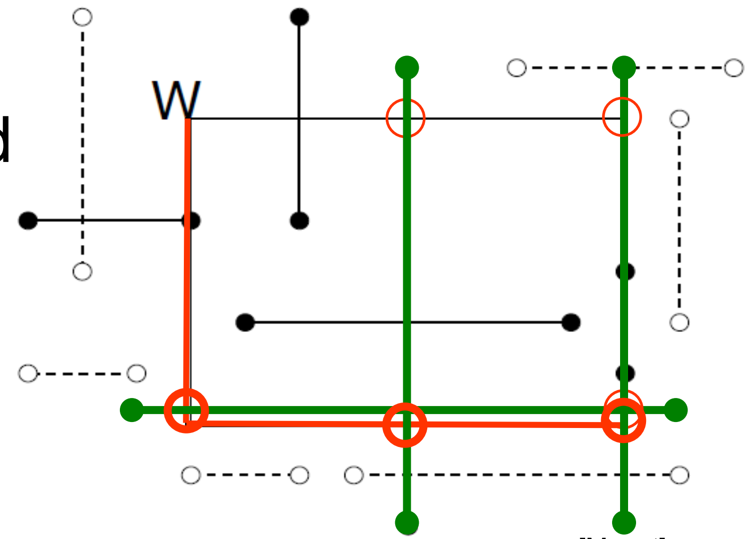
a), b)



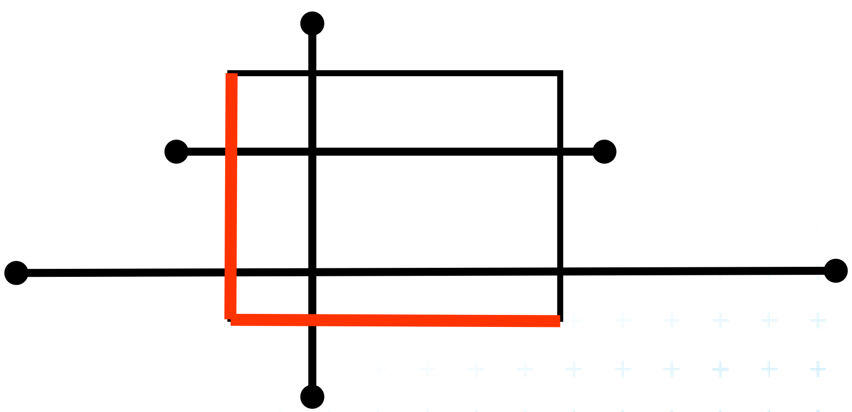
Line segments that cross over the window

c) No points inside

- Such segments not detected using end-point range tree
- Cross the boundary twice

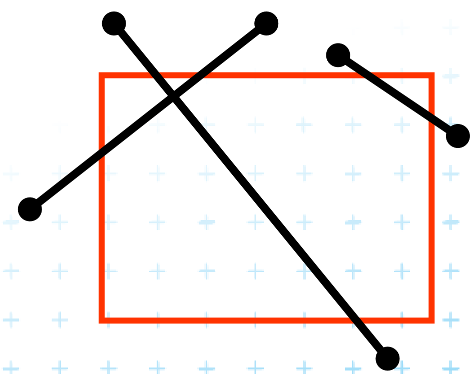


For axis parallel segments



Check left and bottom boundary

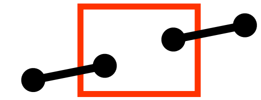
For non-parallel segments



Check all 4 boundaries

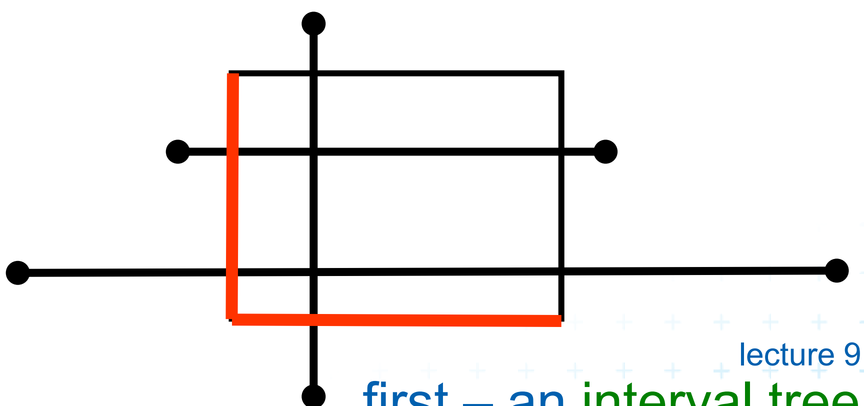
Windowing problem summary

Cases a) and b)



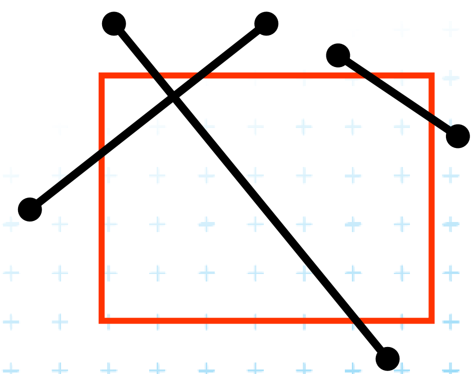
- – Segment end-point in the query rectangle (window)
- – Solved by 2D range trees (see lecture 3, $O(n \log n)$ time & memory)

- We will discuss only case c)
  - – Segment crosses the window



lecture 9

first – an interval tree (three variants)



later – a segment tree

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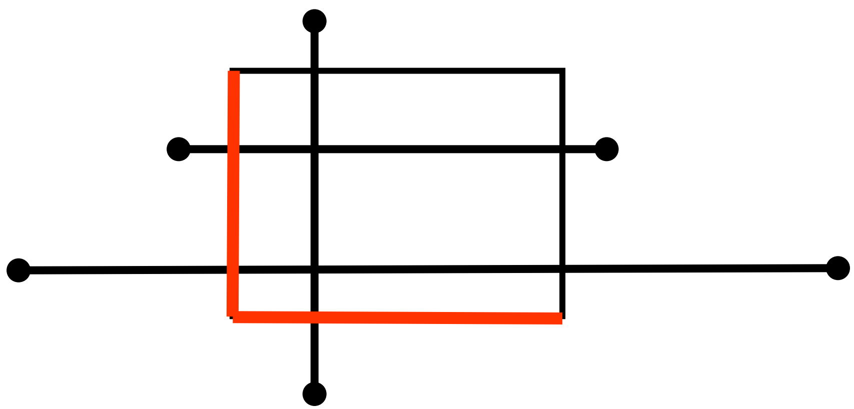
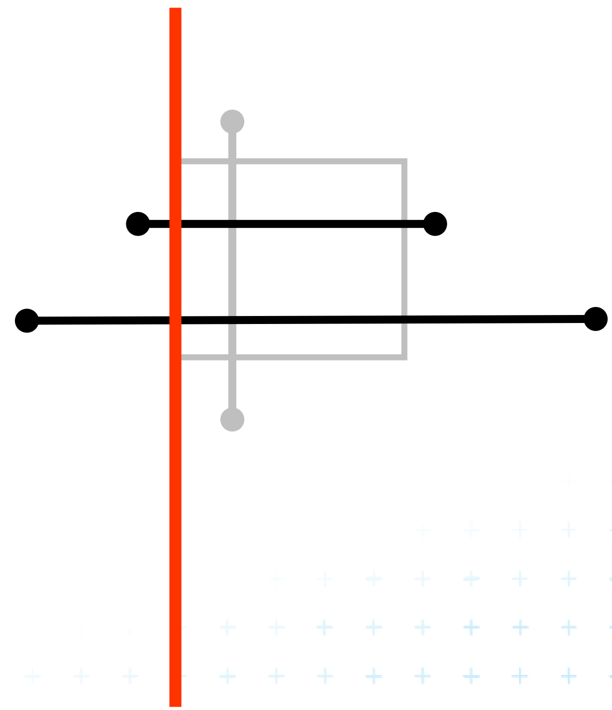
case c) principle





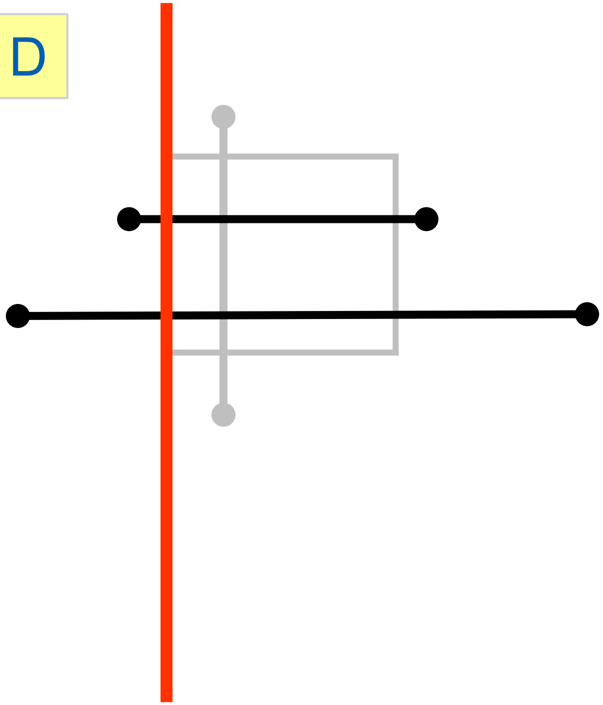
Segments cross the window

solved as

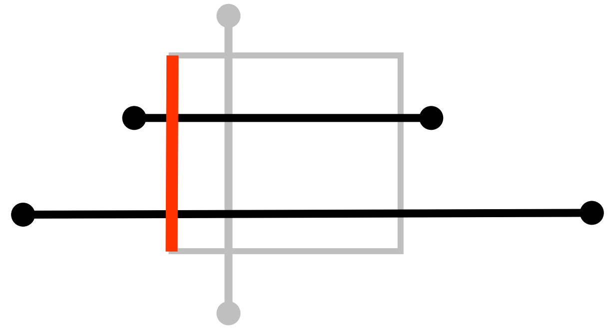
Line crosses the segments
(horizontal + vertical)

Talk Outline

1D



2D

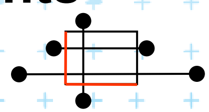


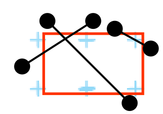
Line x line segments

interval tree

For heat-up

Line segment x line segments

2 variants of interval tree 

1 variant of segment tree 

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Data structures for case c)

Interval tree (1D IT)

- stores 1D intervals (end-points in sorted lists)
- computes intersections with query interval

  see intersection of axis angle rectangles – there is y-overlap used, here is x-overlap

We must extend Interval tree to 2D

- variants differ in storage of interval end-points M_L, M_R
  - 2D range trees
  - priority search trees

Segment tree

- splits the plane to slabs in x in elementary intervals

Talk overview

1. Windowing of axis parallel line segments in 2D
   - 3 variants of *interval tree – IT in x-direction*
   - Differ in storage of segment end points M_L and M_R

1D
   i. Line stabbing (standard *IT* with *sorted lists*) lecture 9 - intersections

2D
   ii. Line segment stabbing (*IT* with *range trees*)
   iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position

2D
   - *segment tree + BST*

i. Segment intersected by vertical line

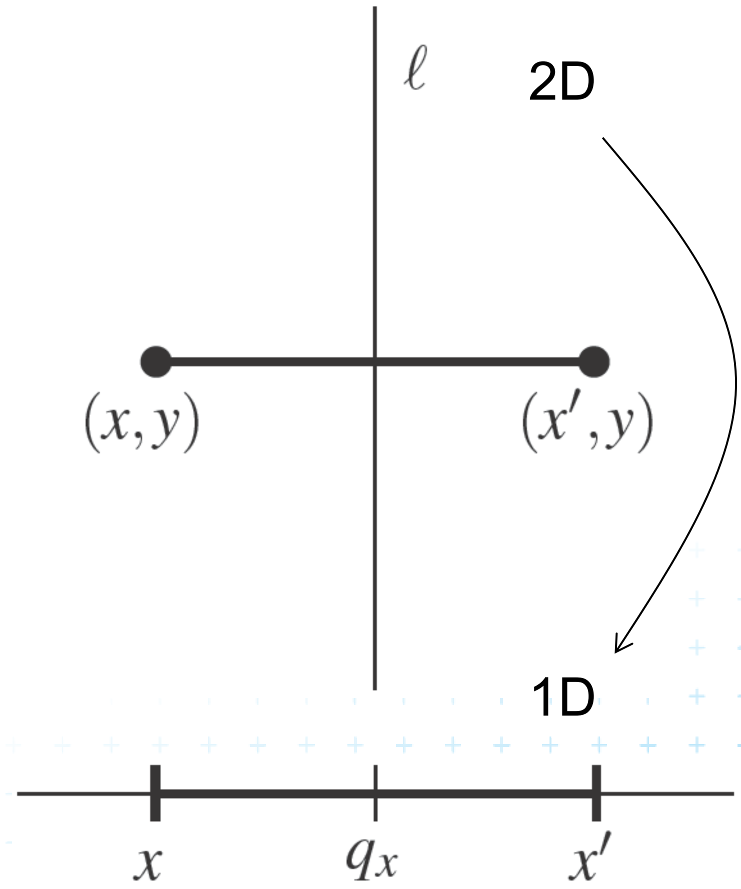
- Query line $\ell := (x = q_x)$

  Report the segments stabbed by a vertical line

  = 1 dimensional problem

  (ignore y coordinate)

$\Rightarrow$ Report the interval $[x : x']$ containing query point q_x

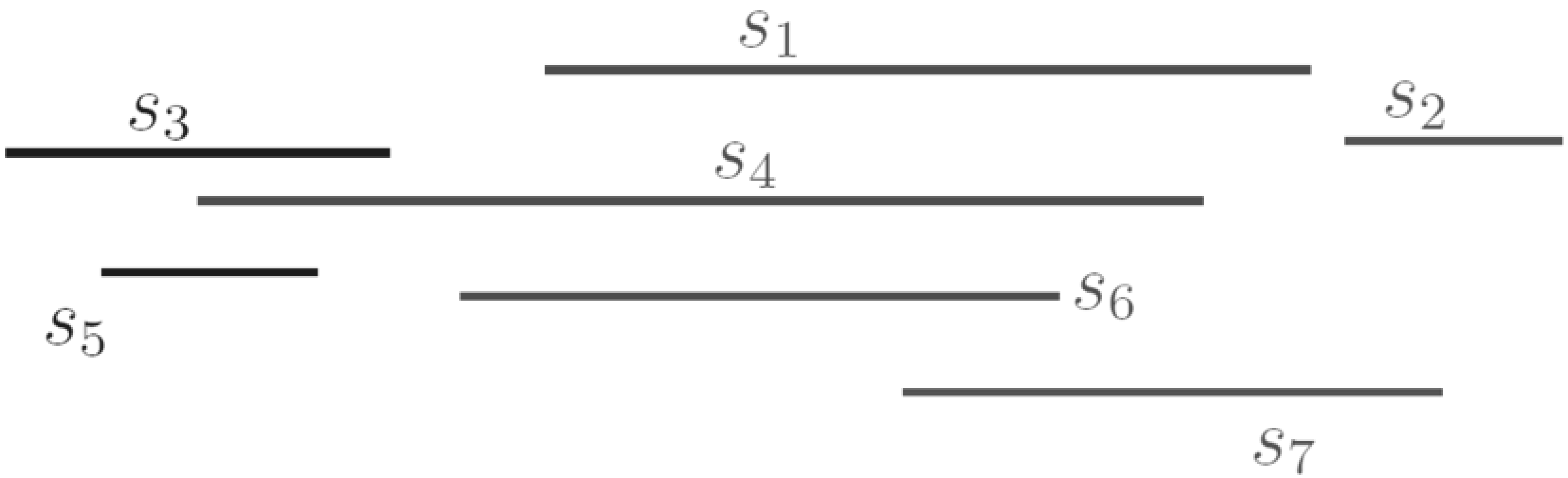


DS: Interval tree with sorted lists

[Mount]

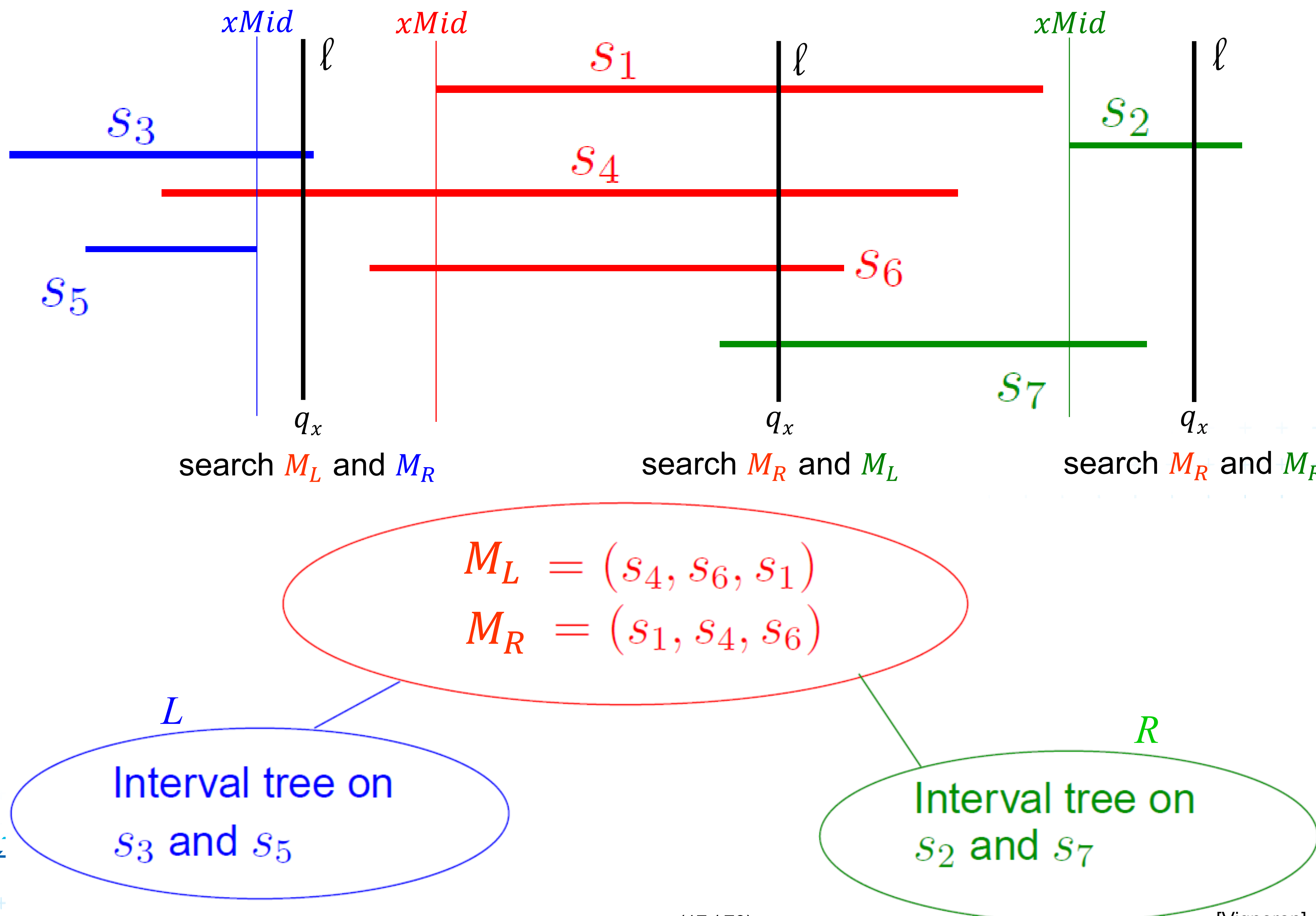
Interval tree principle

(see lecture 9 - intersections)



Interval tree principle

(see lecture 9 - intersections)



[Vigneron]

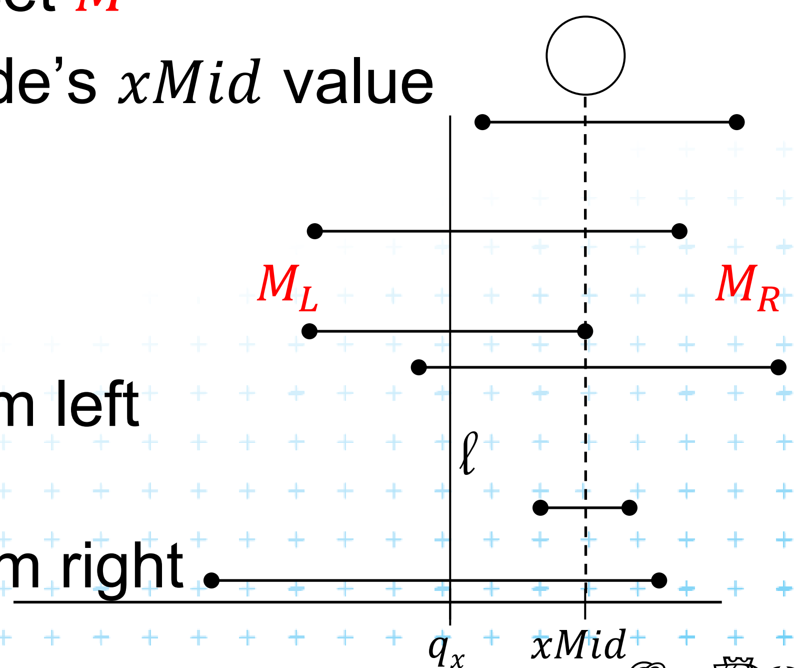
i. Segment intersected by vertical line

Principle

- Store input segments in static interval tree
- In each interval tree node
  - Check the segments in the set M
  - These segments contain node's $xMid$ value
    - M_L are left end-points
    - M_R are right end-points
  - q_x is the query value
  - If $(q_x < xMid)$ Sweep M_L from left

    p $\in M_L$: if $p_x \leq q_x \Rightarrow$ intersection
  - If $(q_x > xMid)$ Sweep M_R from right

    p $\in M_R$: if $p_x \geq q_x \Rightarrow$ intersection



Inspired by [Berg]

Segment intersection (left from $xMid$)

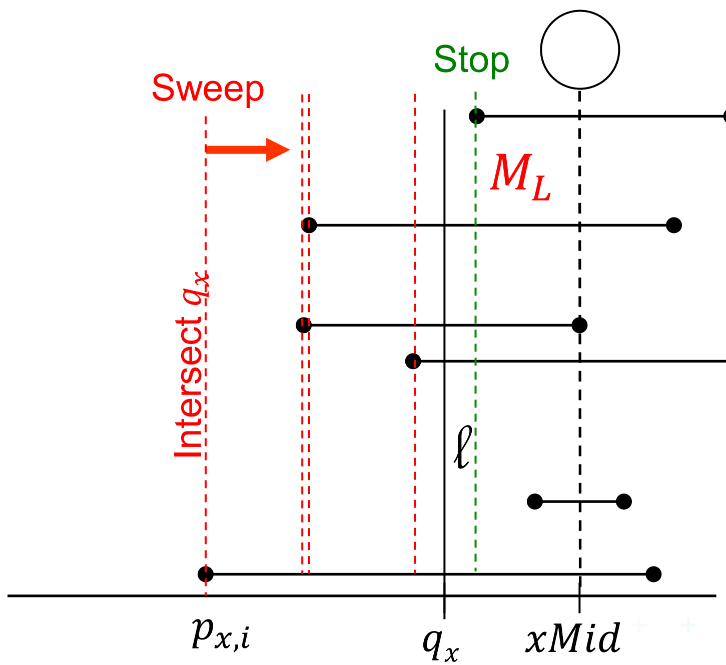
All line segments from M pass through $xMid$

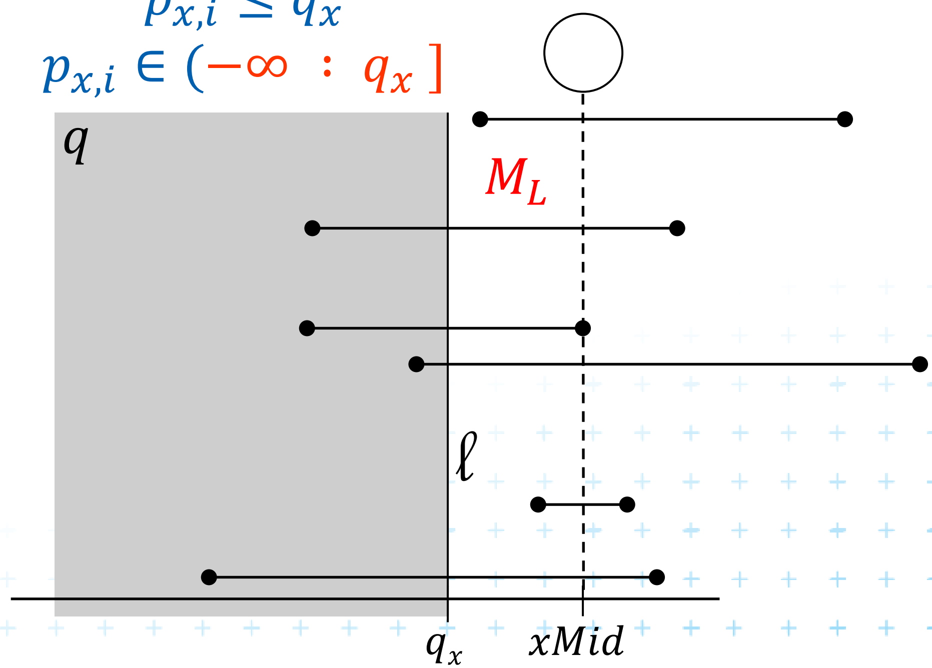
$\Rightarrow q_x$ must be between $p_{x,i}$ and $xMid$ to intersect the line segment i

$\Rightarrow$ left endpoints $p_{x,i} \le q_x \Rightarrow$ intersection

$p_{x,i} \le q_x$

$p_{x,i} \in (-\infty : q_x]$





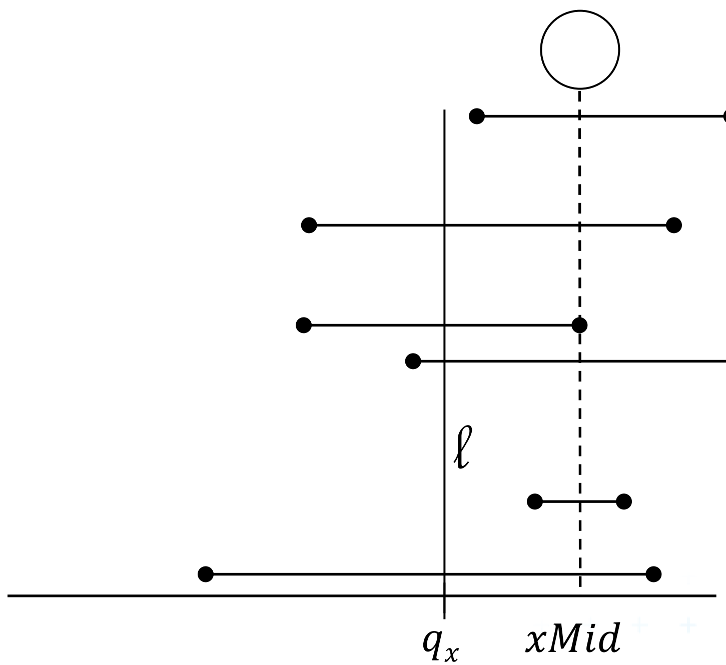
Intersection with line ℓ means Intersection with a half space q

$\ell := q_x \times [-\infty : \infty]$ $\quad q := (-\infty : q_x] \times [-\infty : \infty]$

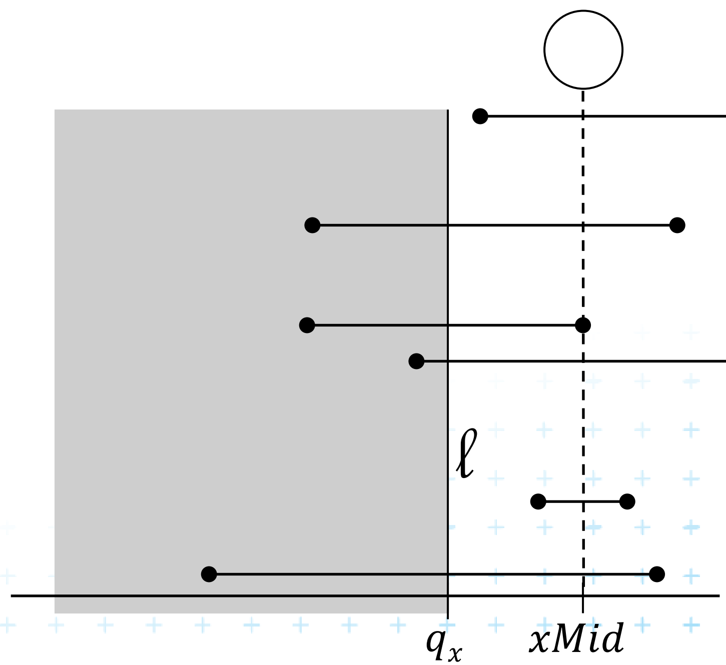
Inspired by [Berg]

Principle once more

Instead of intersecting edges by line



search end-points in half-space

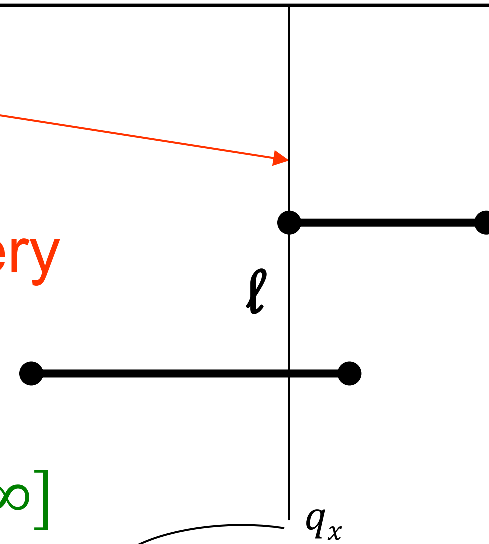


i. Segment intersected by vertical line

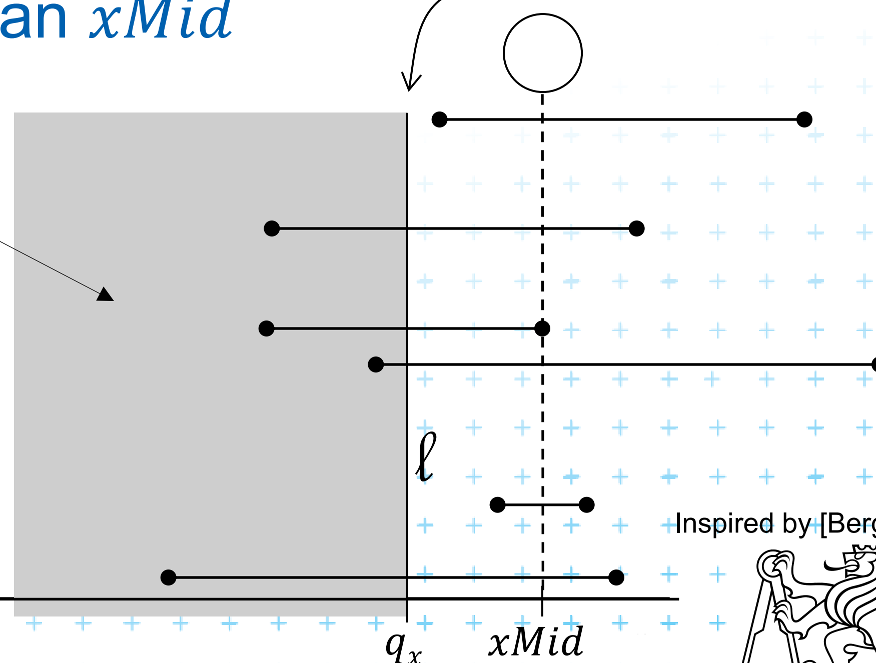
De facto a 1D problem

- Query line $\ell := q_x \times [-\infty : \infty]$
- Horizontal segment of M stabs the query line ℓ left of $xMid$ **iff** its (segment's) left endpoint lies in half-space

$$q := (-\infty : q_x\,] \times [-\infty : \infty]$$



- In IT node with stored median $xMid$ report all segments from M
  - M_L: whose left point lies in $(-\infty : q_x\,]$ if ℓ lies left from xMid
  - M_R: whose right point lies in $[q_x : +\infty)$ if ℓ lies right from xMid

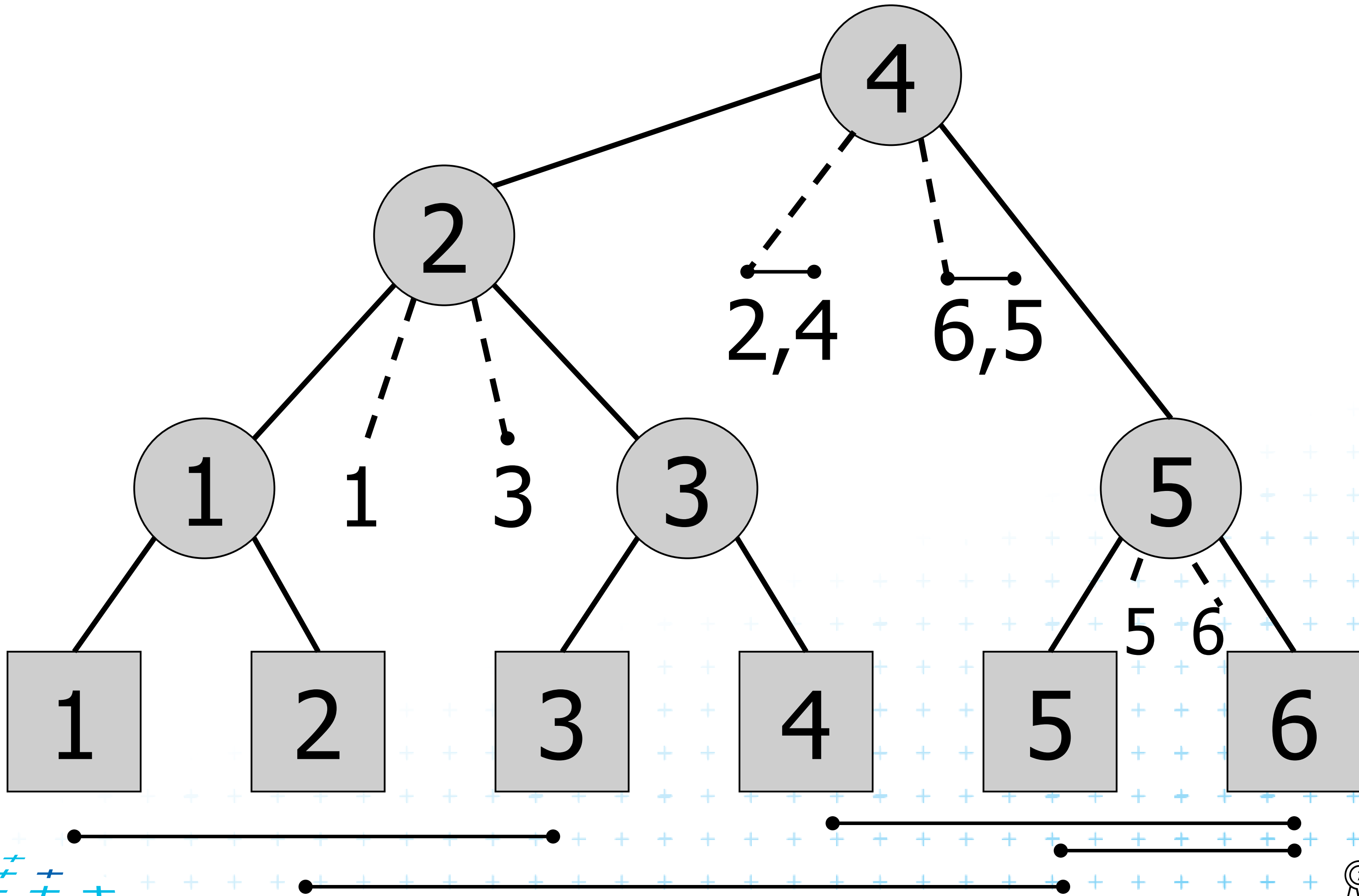


Inspired by [Berg]

DCGI

Static interval tree [Edelsbrunner80]

Tree over sorted segment end-points



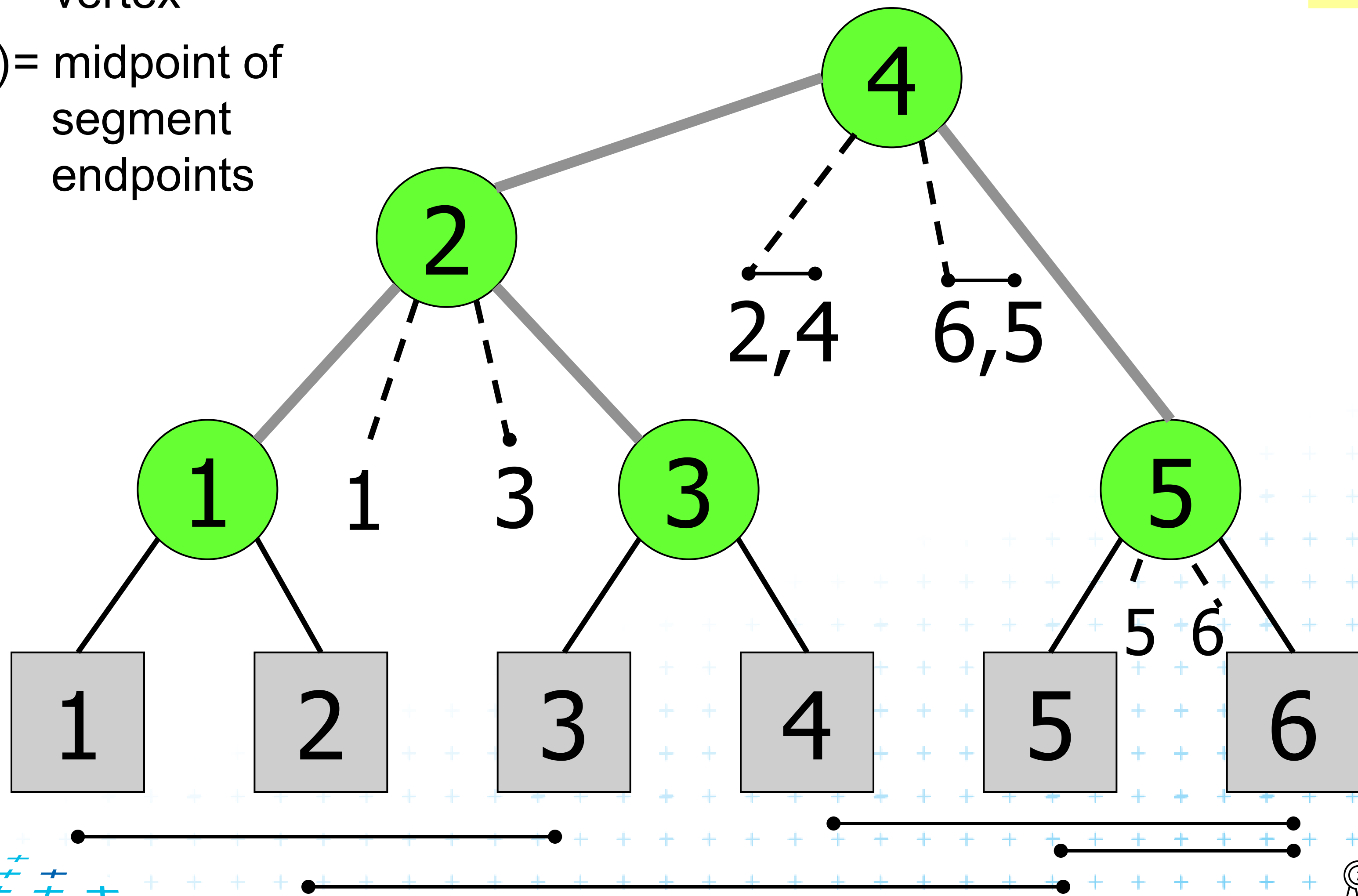
[Kukral]

Primary structure – static tree for endpoints

Static

v = vertex

d(v)= midpoint of segment endpoints



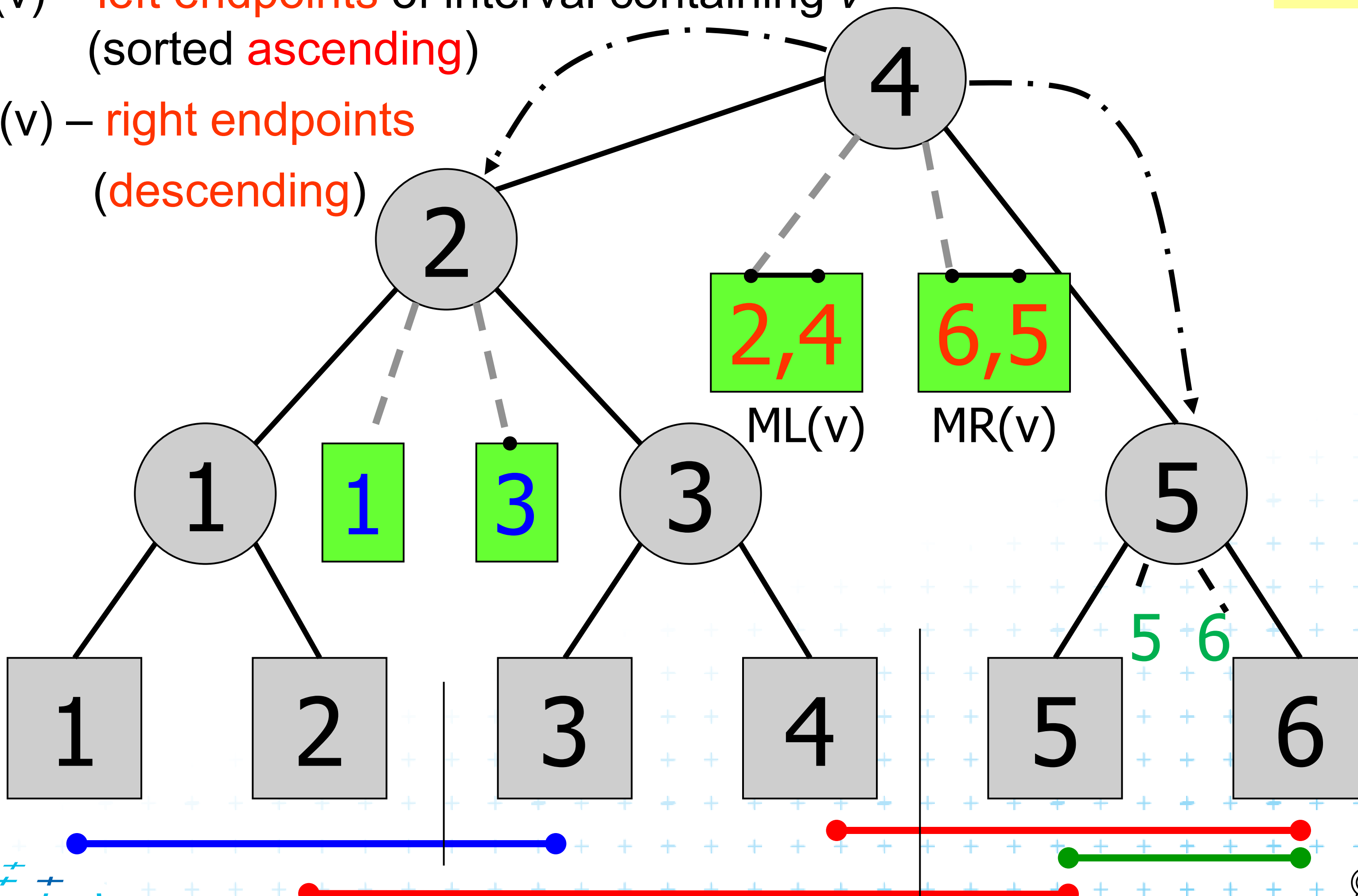
[Kukral]

Secondary lists of incident interval end-pts.

Dynamic

ML(v) – left endpoints of interval containing *v*
(sorted ascending)

MR(v) – right endpoints
(descending)



[Kukral]

Interval tree construction

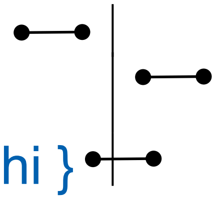
Merged procedures from in lecture 09
- PrimaryTree(*S*) on slide 33
- InsertInterval (*b, e, T*) on slide 35

ConstructIntervalTree(*S*) **// Intervals all active – no active lists**

Input: Set S of intervals on the real line – on *x-axis*

Output: The root of an interval tree for *S*

```
1.  if (|S| == 0) return null                              // no more intervals
2.  else
3.      xMed = median endpoint of intervals in S           // median endpoint
4.      L = { [xlo, xhi] in S | xhi < xMed }               // left of median
5.      R = { [xlo, xhi] in S | xlo > xMed }               // right of median
6.      M = { [xlo, xhi] in S | xlo <= xMed <= xhi }       // contains median
7.      ML = sort M in increasing order of xlo             // sort M
8.      MR = sort M in decreasing order of xhi
9.      t = new IntTreeNode(xMed, ML, MR)                  // this node
10.     t.left  = ConstructIntervalTree(L)                 // left subtree
11.     t.right = ConstructIntervalTree(R)                 // right subtree
12.     return t
```



steps 4.,5.,6. done in one step if presorted [Mount]

Line stabbing query for an interval tree

Less effective variant of QueryInterval (b, e, T)
on slide 34 in lecture 09
with merged parts: fork and search right

Stab(t, qx)

Input: IntTreeNode t, Scalar qx

Output: prints the intersected intervals

```
if (t == null) return                          // no leaf: fell out of the tree
if (qx < t.xMed)                               // left of median?
    for (i = 0; i < t.ML.length; i++)          // traverse M_L left end-points
        if (t.ML[i].lo ≤ qx) print (t.ML[i])   // ..report if in range
        else break                             // ..else done
    Stab (t.left, qx)                          // recurse on left subtree
else  // (qx ≥ t.xMed)                         // right of or equal to median
    for (i = 0; i < t.MR.length; i++) {        // traverse M_R right end-points
        if (t.MR[i].hi ≥ qx) print (t.MR[i])   // ..report if in range
        else break                             // ..else done
    Stab (t.right, qx)                         // recurse on right subtree
```

Note: Small inefficiency for qx == t.xMed – recurse on the right

[Mount]

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Complexity of line stabbing via interval tree

with *sorted lists*

- Construction - $O(n \log n)$ time
  - Each step divides at maximum into two halves or less (minus elements of M) => tree of height $h = O(\log n)$
  - If presorted endpoints in three lists L, R, and M then median in O(1) and copy to new L, R, M in $O(n)$
- Vertical line stabbing query - $O(k + \log n)$ time
  - One node processed in $O(1 + k')$, k' reported intervals
  - v visited nodes in $O(v + k)$, k total reported intervals
  - $v = h =$ tree height = $O(\log n)$ $k = \Sigma k'$
- Storage - $O(n)$
  - Tree has $O(n)$ nodes, each segment stored twice (two endpoints)

DCGI

Talk overview

1. Windowing of axis parallel line segments in 2D
   - 3 variants of *interval tree – IT in x-direction*
   - Differ in storage of segment end points M_L and M_R

1D i. Line stabbing (standard *IT* with *sorted lists*) lecture 9 - intersections

2D ii. Line segment stabbing (*IT* with *range trees*)

iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position

2D – *segment tree + BST*

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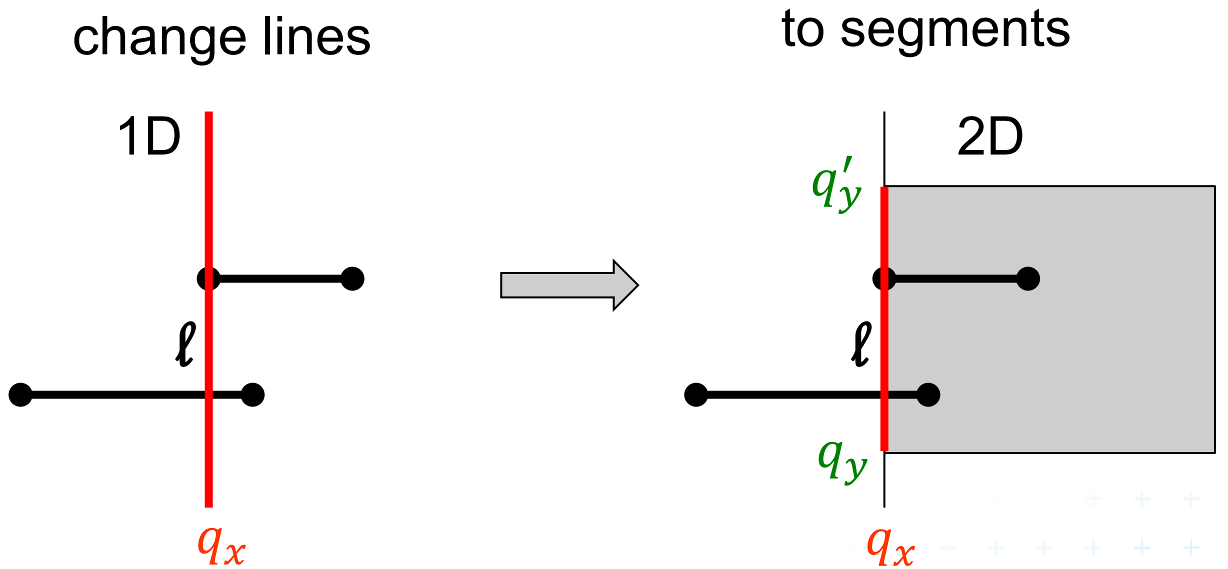
Line segment stabbing (*IT* with *range trees*)

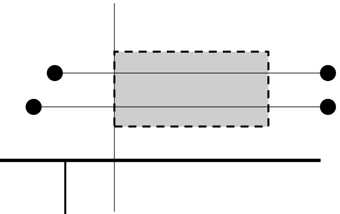
Enhance 1D interval trees to 2D



$q_x \times [-\infty : \infty]$ (no y-test) | $q_x \times [q_y : q'_y]$ (additional y-test)

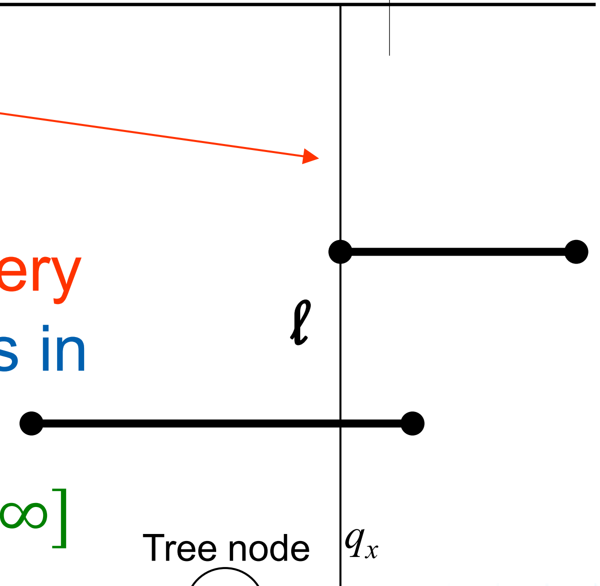
Sorted lists | Range trees

i. Segments × vertical line



De facto a 1D problem

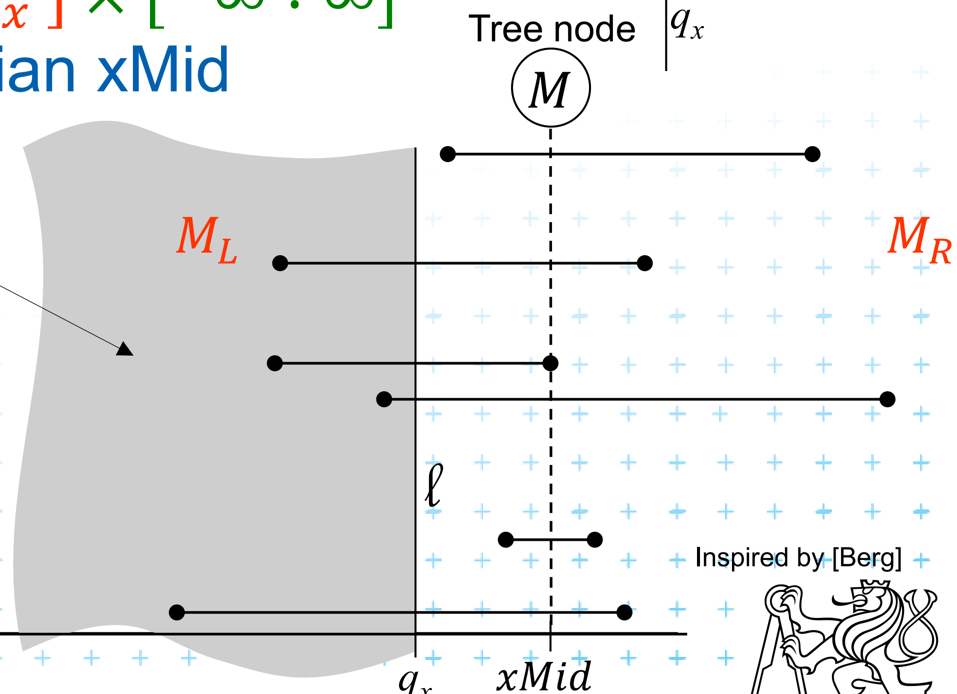
- Query line $\ell := q_x \times [-\infty : \infty]$



- Horizontal segment of M_L stabs the query line ℓ left of $xMid$ **iff** its left endpoint lies in half-space

$$q := (-\infty : q_x] \times [-\infty : \infty]$$

- In IT node with stored median xMid report all segments from M
  - M_L: whose left point lies in $(-\infty : q_x]$ if ℓ lies left from xMid
  - M_R: whose right point lies in $[q_x : +\infty)$ if ℓ lies right from xMid



Inspired by [Berg]

DCGI

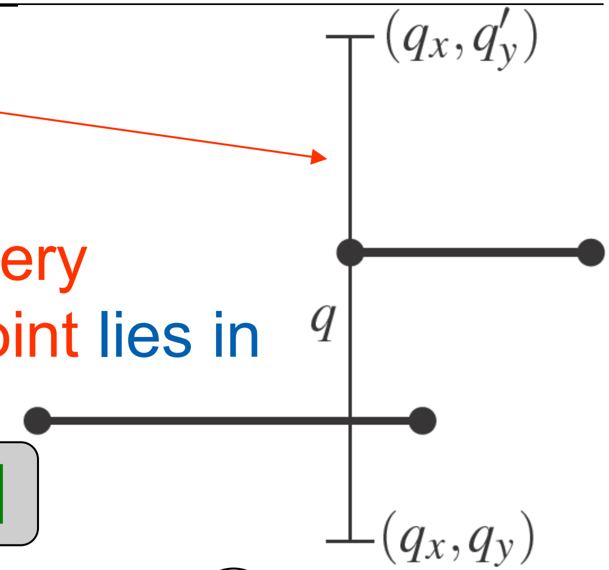
ii. Segments × vertical line segment

A 2D problem

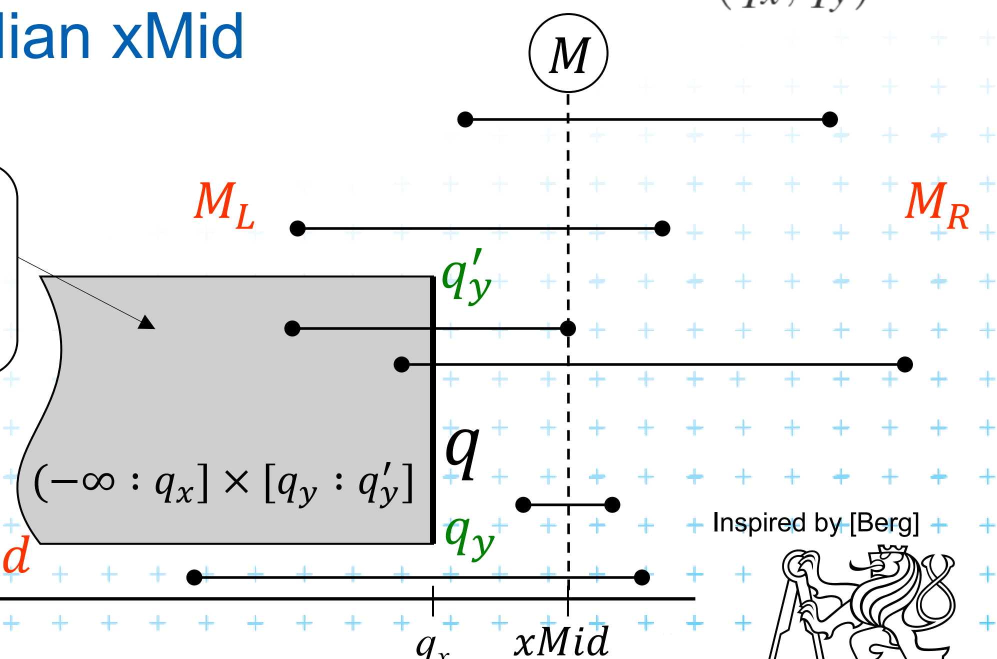
- Query segment $q := q_x \times [q_y : q'_y]$
- Horizontal segment of M_L stabs the query segment q left of $xMid$ **iff** its left endpoint lies in semi-infinite rectangular region

New test

$$q := (-\infty : q_x] \times [q_y : q'_y]$$



- In IT node with stored median xMid report all segments
  - M_L: whose left points lie in $(-\infty : q_x] \times [q_y : q'_y]$ where q_x lies left from $xMid$
  - M_R: whose right point lies in $[q_x : +\infty) \times [q_y : q'_y]$ where q_x lies right from $xMid$



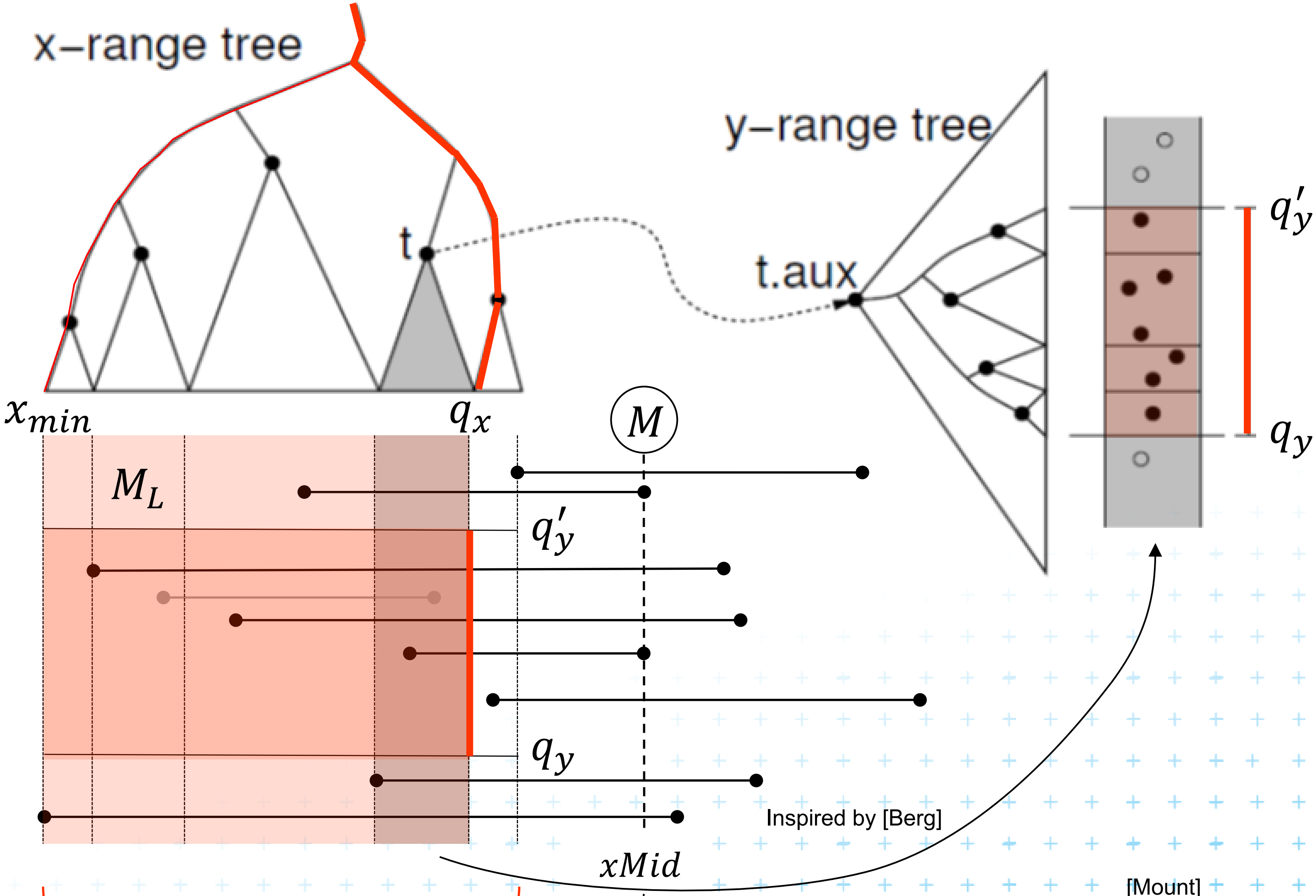
Inspired by [Berg]

DCGI

Data structure for endpoints

- Storage of M_L and M_R
  - 1D Sorted lists is not enough for line segments
  - We need to test in y too
  - Use 2D range trees
    (one for M_L and one for M_R in each node)
- Instead $O(n)$ sequential search in M_L and M_R
  perform $O(\log n)$ search
  in a 2D range tree with fractional cascading

M_L in 2D range tree without fractional cascading-more in Lect.3



Segment left end-points for M_L

Complexity of range tree line segment stabbing

- Construction - $O(n \log n)$ time
  - Each step divides at maximum into two halves L,R or less (minus elements of M) => int. tree height $O(\log n)$
  - If the range trees are efficiently build in $O(n)$ after points sorted
- Vertical line segment stab. q. - $O(k + \log^2 n)$ time
  - One node processed in $O(\log n + k')$, k' reported segm. (interval tree; 2D range tree search with Fractional Cascading)
  - v-visited nodes in $O(v \log n + k)$, k total reported segm. (interval tree)
  - v = interval tree height = $O(\log n)$ $\quad k = \sum k'$
  - $O(k + \log^2 n)$ time - range tree with fractional cascading
  - $O(k + \log^3 n)$ time - range tree without fractional casc.
- Storage - $O(n \log n)$ Can be done better?
  - Dominated by the range trees

Talk overview

1. Windowing of axis parallel line segments in 2D
   - 3 variants of *interval tree – IT in x-direction*
   - Differ in storage of segment end points M_L and M_R

1D i. Line stabbing (standard *IT* with *sorted lists*) lecture 9 - intersections

2D ii. Line segment stabbing (*IT* with *range trees*)

iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position

2D – *segment tree + BST*

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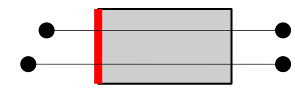
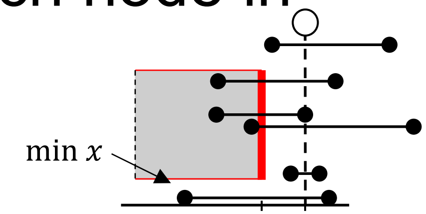
iii. Priority search trees [McCreight85]

- Another variant for case c) on slide 9
  
  - Exploit the fact that query rectangle in each node in interval tree is unbounded (in x direction)
  

- Priority search trees
  - as secondary data structure for both left and right endpoints (M_L and M_R) of segments in nodes of interval tree – one for ML, one for MR
  - Improve the storage to $O(n)$ for horizontal segment intersection with left window edge (2D range tree has $O(n \log n)$)
- For cases a) and b) - $O(n \log n)$ storage remains
  - we need range trees for windowing segment endpoints

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Rectangular range queries variants

- Let $P = \{p_1, p_2, \dots, p_n\}$ is set of points in plane
- Goal: rectangular range queries of the form
  $(-\infty : q_x] \times [q_y : q'_y]$ – unbounded (in x direction)
- In 1D: search for nodes v with $v_x \in (-\infty : q_x]$
  - range tree $O(\log n + k)$ time (search the end, report left)
  - ordered list $O(1 + k)$ time 1 is for the fail test
    (start in the leftmost, stop on v with $v_x > q_x$)
  - use heap $O(1 + k)$ time !
    (traverse all children, stop when $v_x > q_x$)
- In 2D – use heap for points with $x \in (-\infty : q_x]$
  + integrate information about y-coordinate

  = Priority search tree

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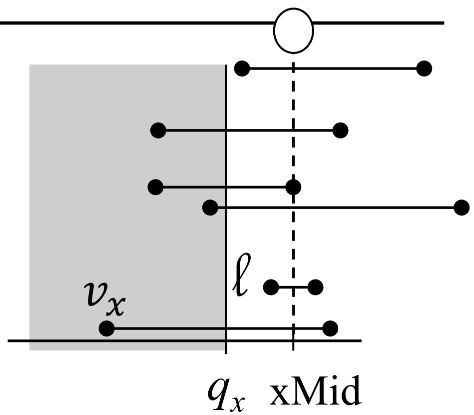
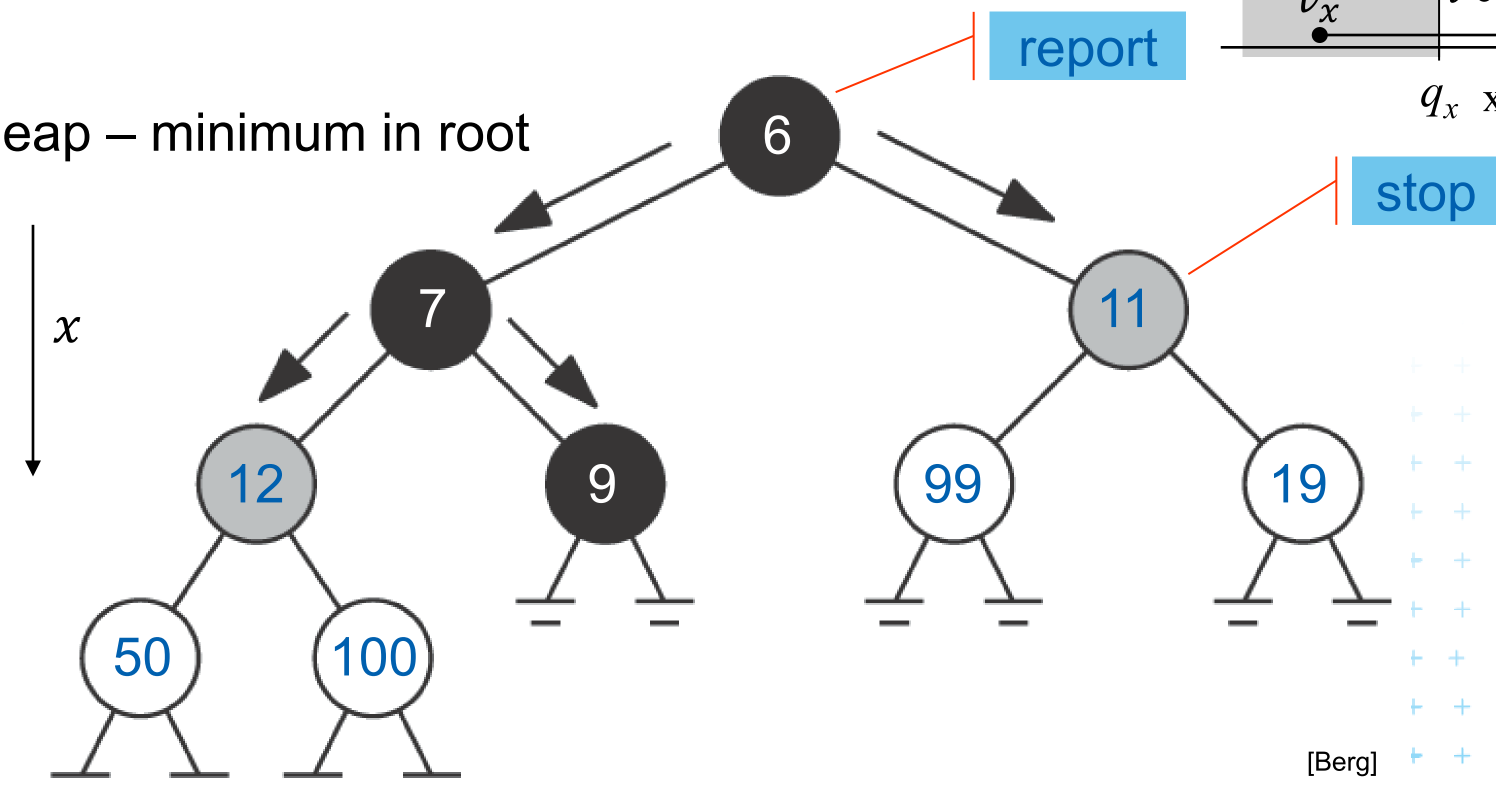
Heap for 1D unbounded range queries

- Traverse all children, stop if $v_x > q_x$
- Example: Query $(-\infty : 10]$, $q_x = 10$



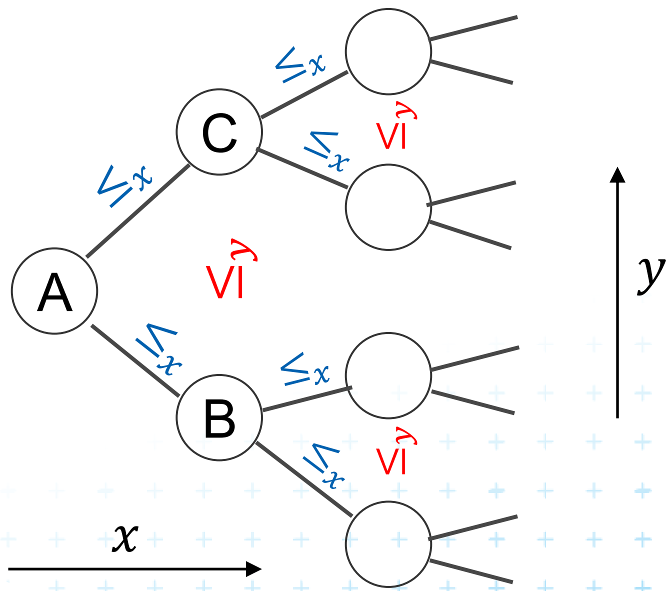
heap – minimum in root



[Berg]

Principle of priority search tree

- Heap $\leq_x$
  - relation between parent and its child nodes only
  - no relation between the child nodes themselves
- Priority search tree
  - relate the child nodes according to y $\leq_y$



x Heap

A $\leq_x$ B

A $\leq_x$ C

y BVS

B $\leq_y$ A $\leq_y$ C $\Rightarrow$ B $\leq_y$ C

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Priority search tree (PST)

= Heap in 2D that can incorporate info about both x, y

- BST on y-coordinate (horizontal slabs) ~ 1D range tree
- Heap on x-coordinate (minimum x from slab along x)

- If P is empty, PST is empty leaf
- else
  - p_{min} = point with smallest x-coordinate in P – a heap root
  - y_{med} = y-coord. median of points $P \setminus \{p_{min}\}$ – BST root
  - $P_{below} \coloneqq \{ p \in P \setminus \{p_{min}\} \;:\; p_y \leq y_{med} \}$
  - $P_{above} \coloneqq \{ p \in P \setminus \{p_{min}\} \;:\; p_y > y_{med} \}$
- Point p_{min} and scalar y_{med} are stored in the PST root
- The left subtree is PST of P_{below}
- The right subtree is PST of P_{above}

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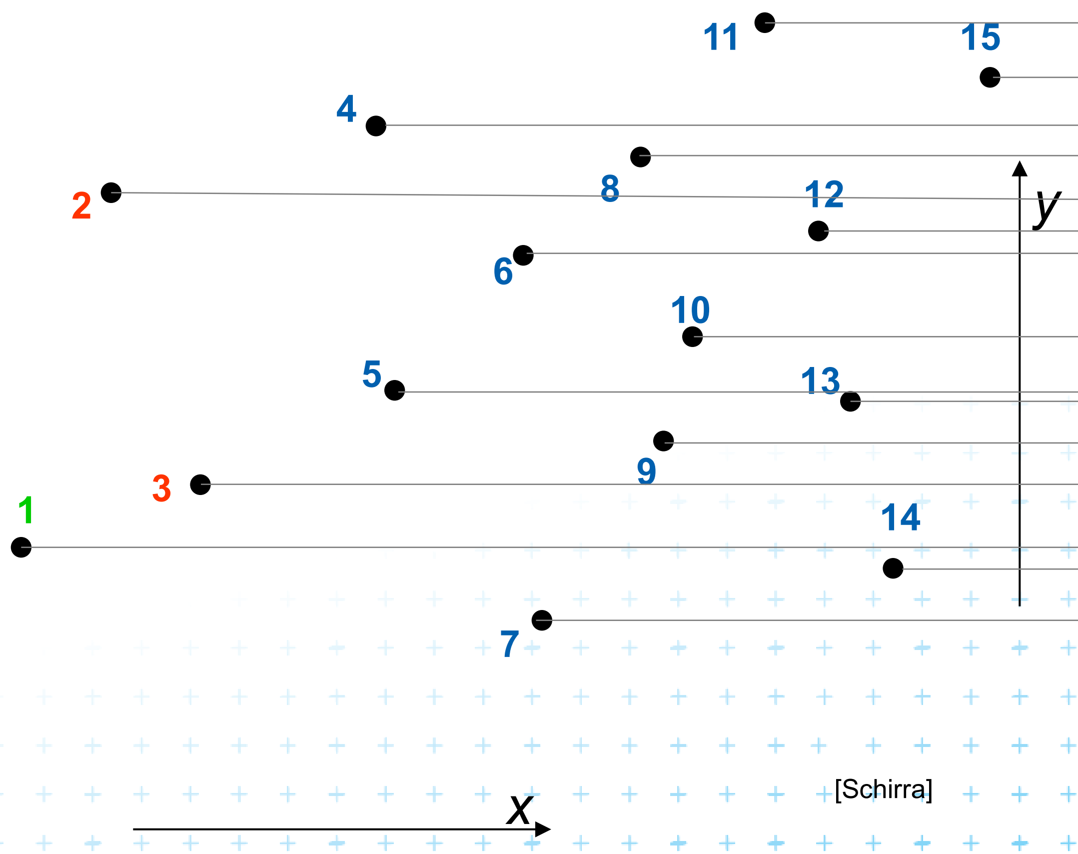
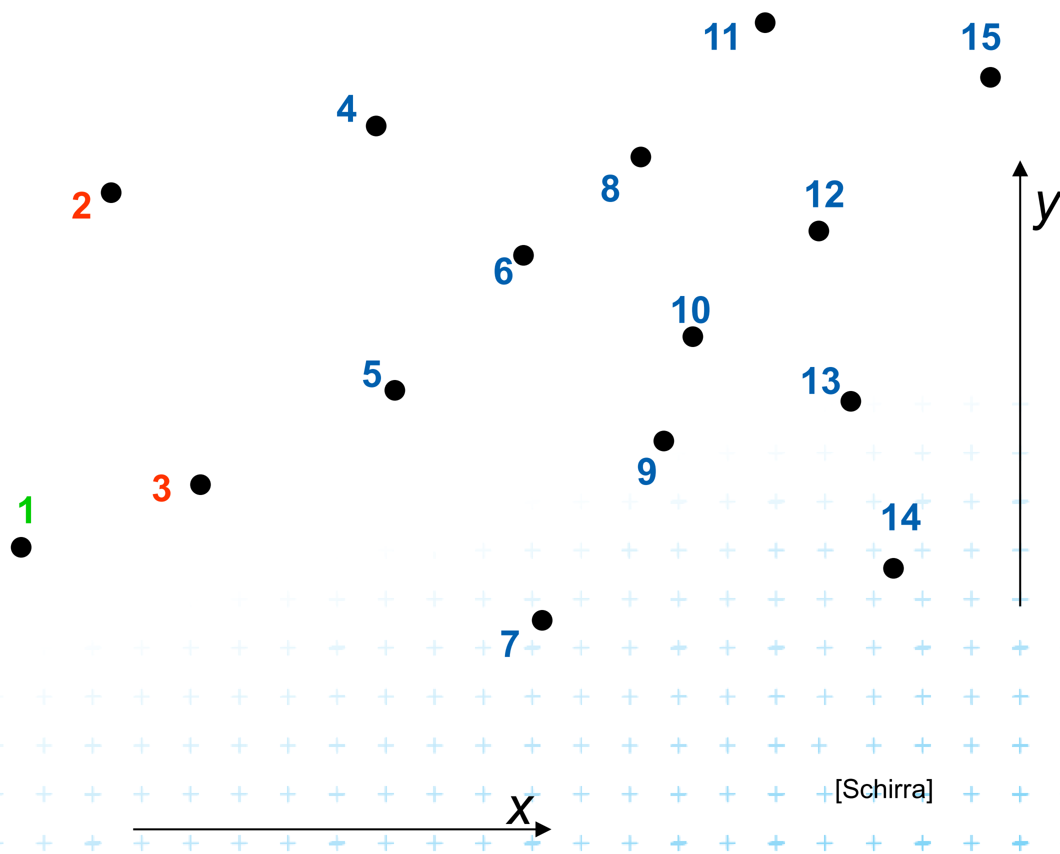
Priority search tree construction example



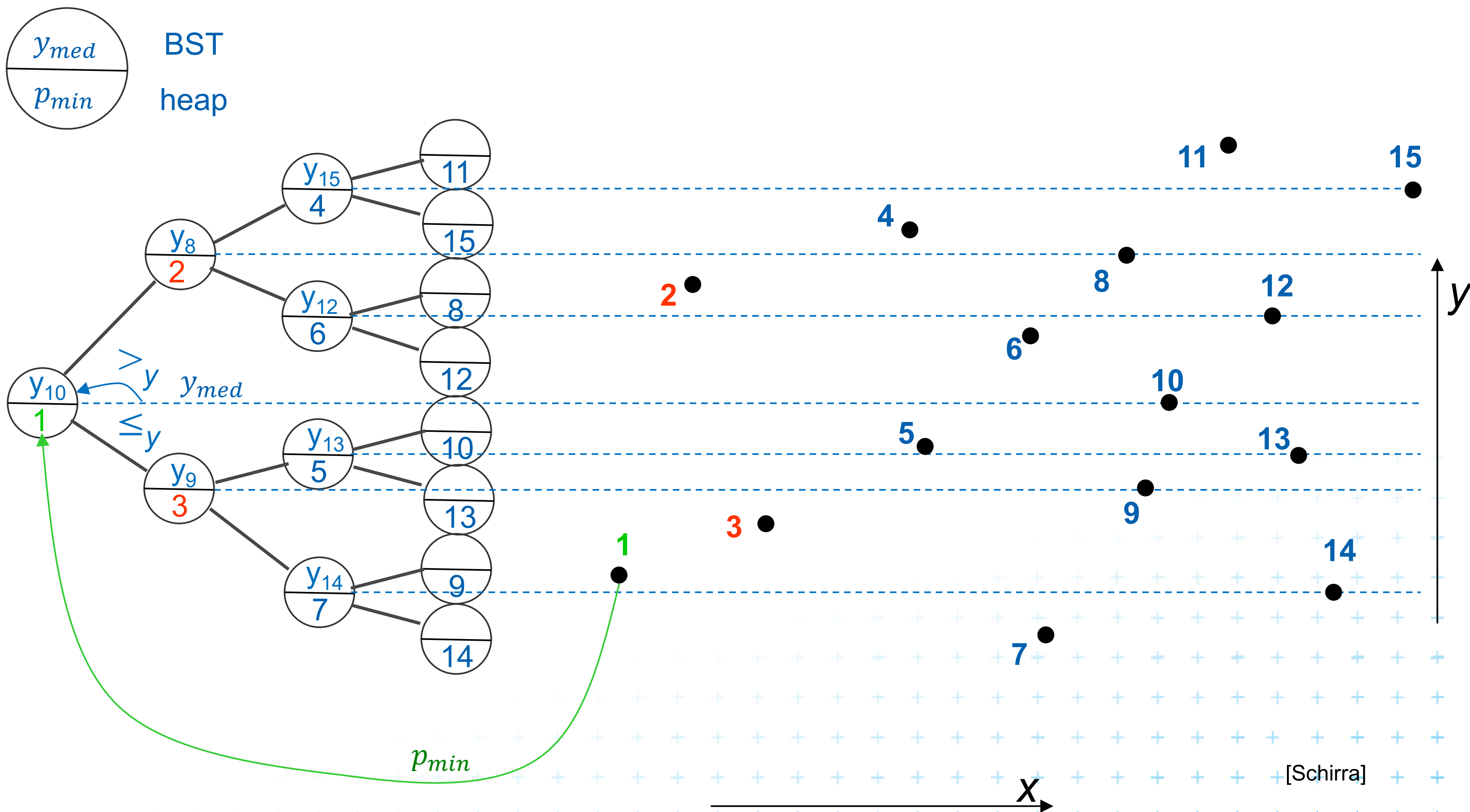
[Schirra]

Priority search tree construction example



[Schirra]

Priority search tree construction example



[Schirra]

Priority search tree construction

PrioritySearchTree(P)

Input: set P of points in plane

Output: priority search tree T

```
if P = ∅ then PST is an empty leaf
else
    p_min = point with smallest x-coordinate in P          // heap on x → root
    y_med = y-coord. median of points P \ {p_min}          // BST on y → root
    Split points P \ {p_min} into two subsets – according to y_med
        P_below := { p ∈ P \ {p_min} : p_y ≤ y_med }
        P_above := { p ∈ P \ {p_min} : p_y > y_med }
    T = newTreeNode()
    T.p = p_min        // point [ x, y ]
    T.y = y_med        // scalar
    T.left  = PrioritySearchTree( P_below )
    T.rigft = PrioritySearchTree( P_above )
```

… Notation on the next slide:

… $p(v)$, v = tree node

… $y(v)$

… $l(v)$

… $r(v)$

$O(n \log n)$, but $O(n)$ if presorted on y-coordinate and bottom up

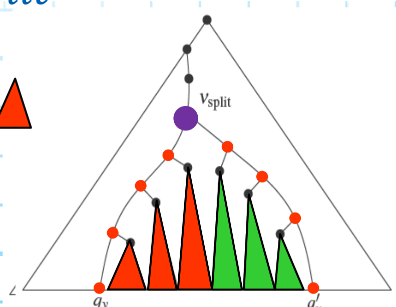
Query Priority Search Tree

QueryPrioritySearchTree(T, $(-\infty : q_x] \times [q_y : q'_y]$**)**

Input: A priority search tree and a range, unbounded to the left

Output: All points lying in the range

1. Search with q_y and q'_y in T // BST on y-coordinate – select y range
   Let v_{split} be the node where the two search paths split (split node)
2. for each node v on the search path of q_y or q'_y // points along the paths
3. if $p(v) \in (-\infty : q_x] \times [q_y : q'_y]$ then Report $p(v)$ // starting in tree root
4. for each node v on the path of q_y in the left subtree of v_{split} // inner trees
5. if the search path goes left at v
6. ReportInSubtree($r(v)$, q_x) // report right subtree
7. for each node v on the path of q'_y in right subtree of v_{split}
8. if the search path goes right at v
9. ReportInSubtree($l(v)$, q_x) // report left subtree



[Berg]

Reporting of subtrees between the y-paths

ReportInSubtree(v, q_x)

Input: The root v of a subtree of a priority search tree and a value q_x.

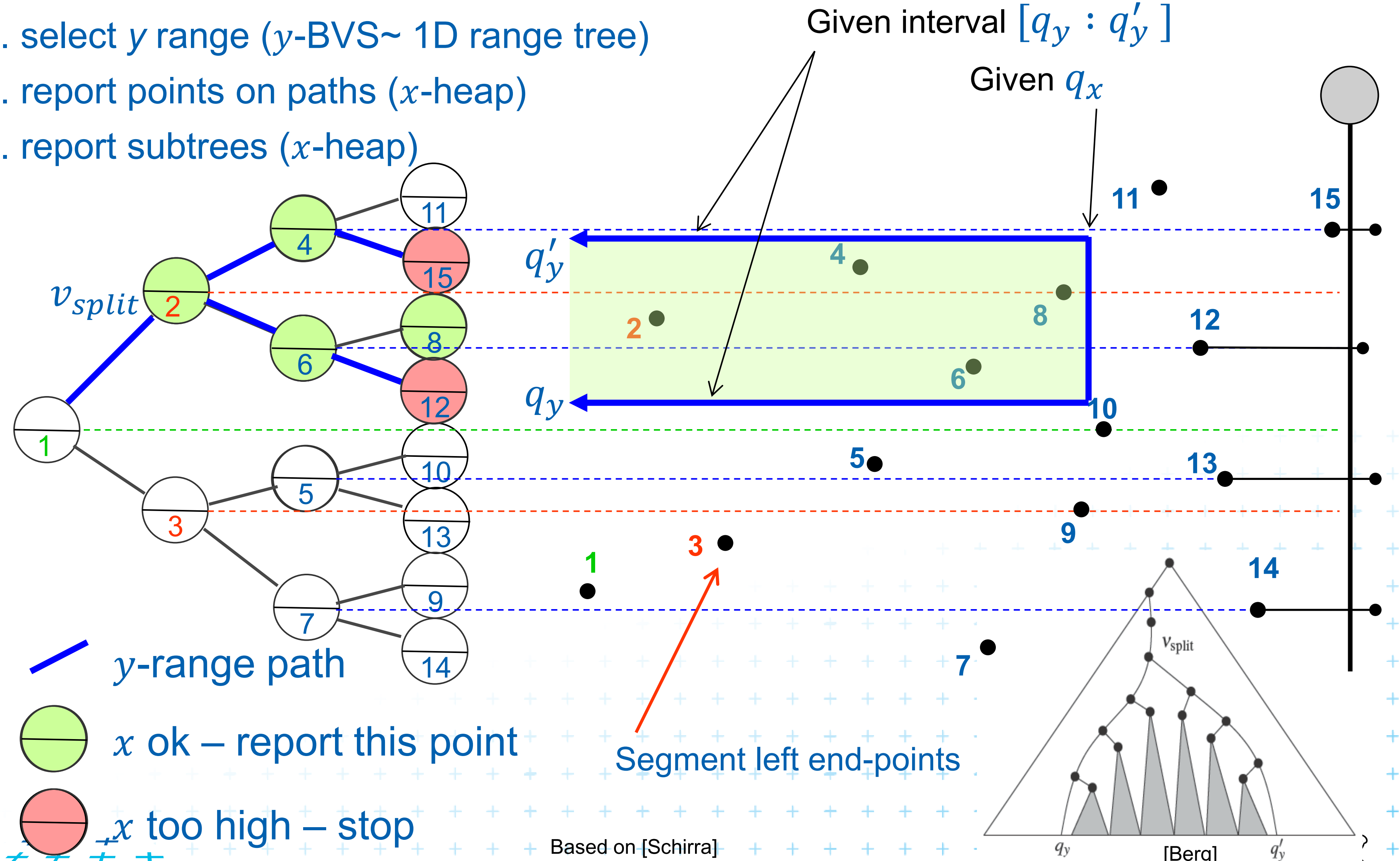
Output: All points p in the subtree with x-coordinate at most q_x.

1. if $x\big(p(v)\big) \le q_x$ // root, heap condition: $x \in (-\infty : q_x]$
2. Report point $p(v)$.
3. if v is not a leaf
4. ReportInSubtree($l(v), q_x$) 
5. ReportInSubtree($r(v), q_x$) 

Search according to x in the heap

Priority search tree query $(-\infty : q_x] \times [q_y : q_y']$

1. select y range (y-BVS~ 1D range tree)
2. report points on paths (x-heap)
3. report subtrees (x-heap)



Priority search tree complexity

For set of n points in the plane

- Build $O(n \log n)$
- Storage $O(n)$
- Query $O(k + \log n)$
  - points in query range $(-\infty : q_x] \times [q_y : q'_y]$
  - k is number of reported points

- Use Priority search tree as associated data structure for interval trees for storage of set M (one for M_L, one for M_R)

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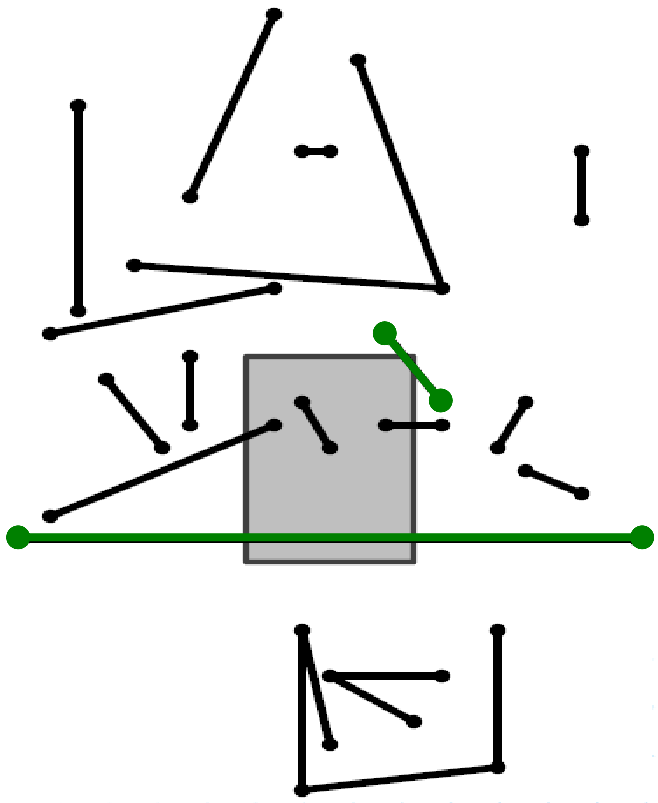
2D – *segment tree + BST*

2. Windowing of line segments in general position



[Vakken]

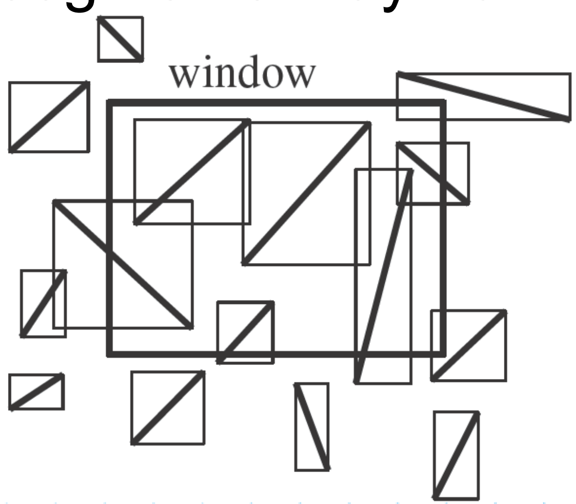
Windowing of arbitrary oriented line segments

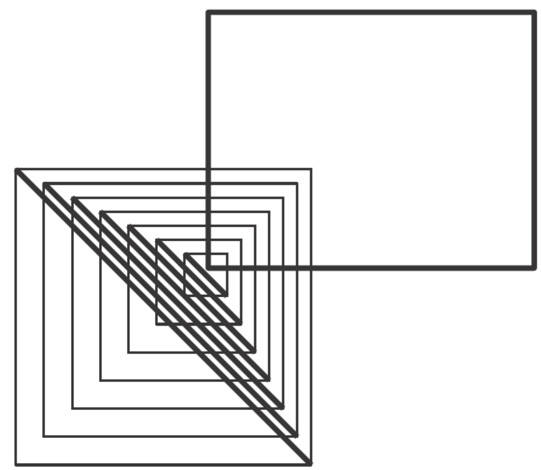
- Two cases of intersection

  a,b) Endpoint inside the query window => range tree

  c) Segment intersects side of query window => ???

- Intersection with BBOX (segment bounding box)?
  - Intersection with $4n$ sides of the segment BBOX?
  - But segments may not intersect the window –> query y





NOT

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2. Windowing of line segments in general position

2D – *segment tree*

Note: segment = interval

it consists of elementary intervals

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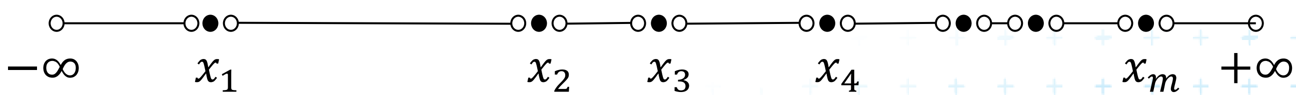
Segment tree

[Bentley, 1977]

- Exploits locus approach
  - Partition parameter space into regions of same answer
  - Localization of such region = knowing the answer
- For given set S of n intervals (segments) on real line
  - Finds m elementary intervals (induced by interval end-points)
  - Partitions 1D parameter space into these elementary intervals



$$(-\infty : x_1), [x_1 : x_1], (x_1 : x_2), [x_2 : x_2], \ldots,$$
$$(x_{m-1} : x_m), [x_m : x_m], (x_m : +\infty)$$

  - Stores line segments s_i with the elementary intervals
  - Reports the segments s_i containing query point q_x.

Plain is partitioned into vertical slabs

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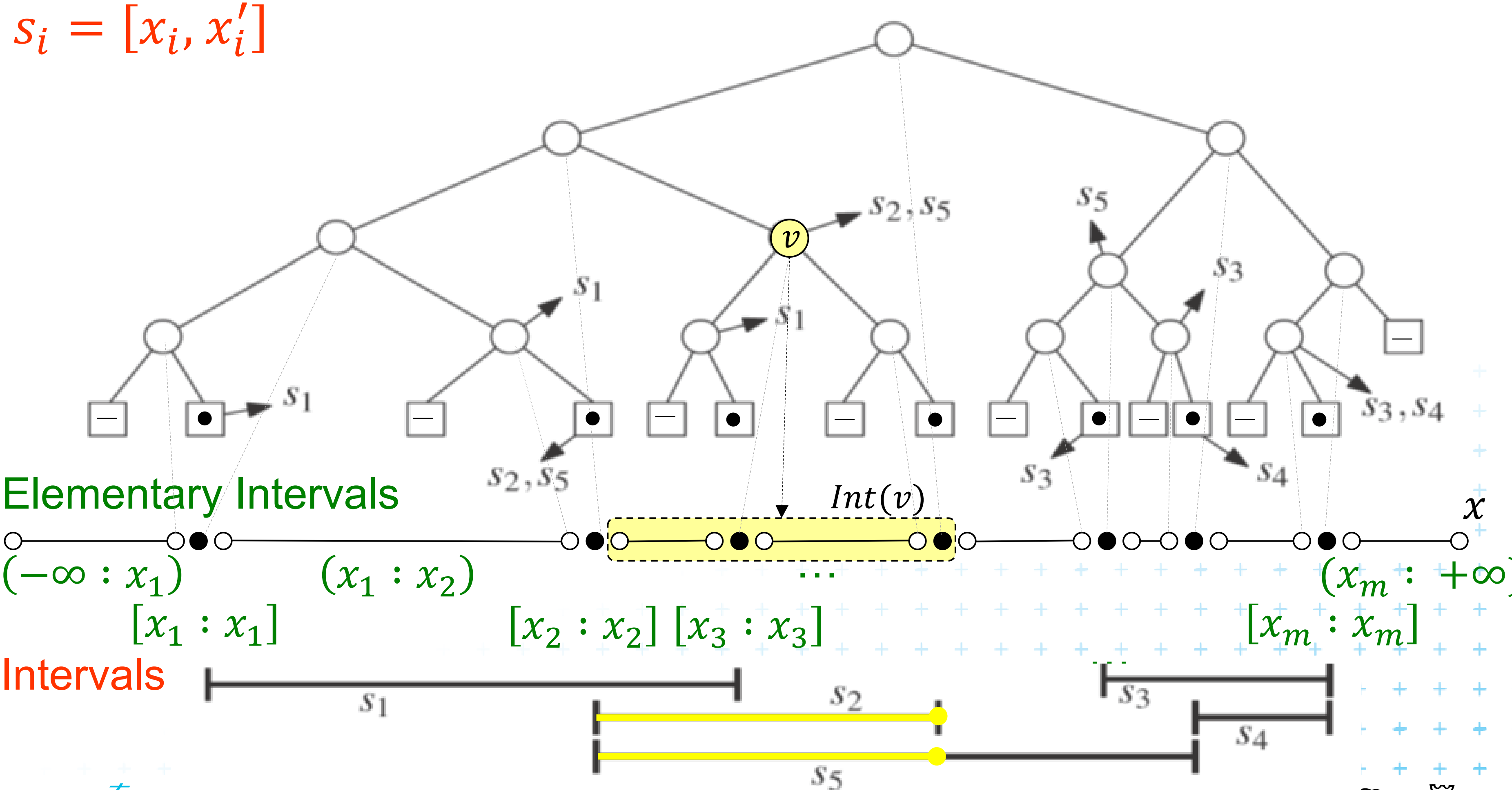
Segment tree example

Segments $S = \{s_1, s_2, \dots, s_n\}$

$s_i = [x_i, x_i']$



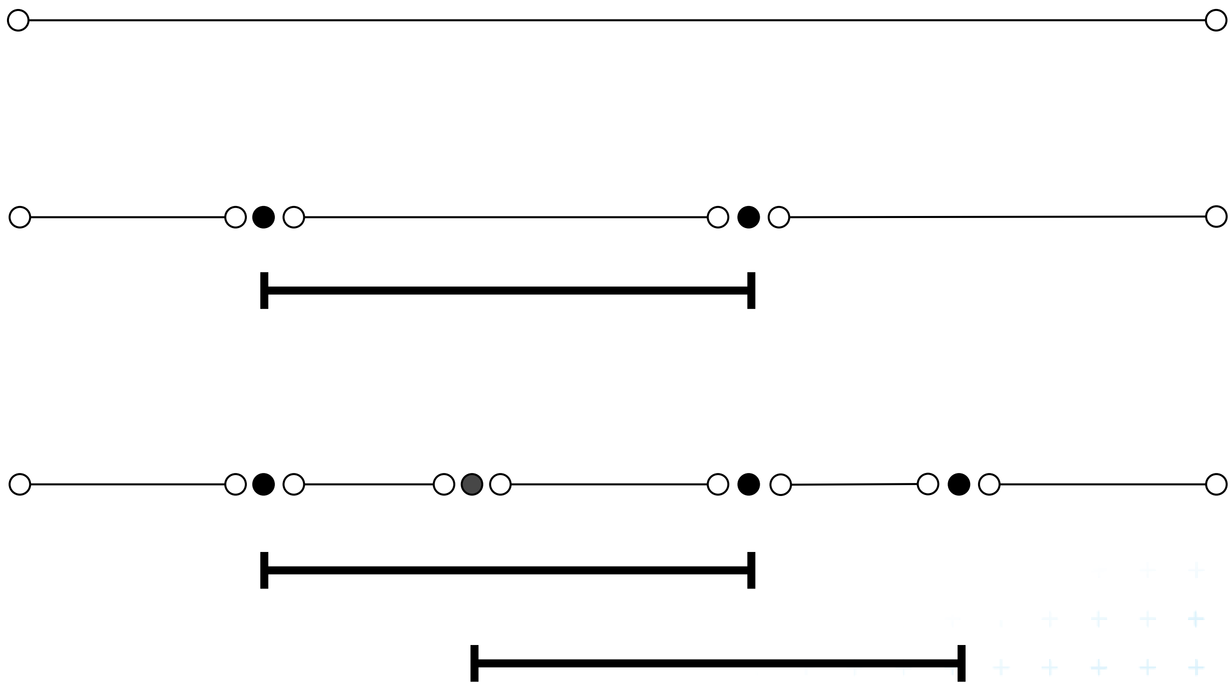
[Berg]

Number of elementary intervals for n segments

$n = 0$ $\quad \# = 1$

$n = 1$ $\quad \# = 4 + 1$

$n = 2$ $\quad \# = 4 * 2 + 1$



n Each end-point adds two elementary intervals $\quad \# = 4n + 1$

Each segment four…

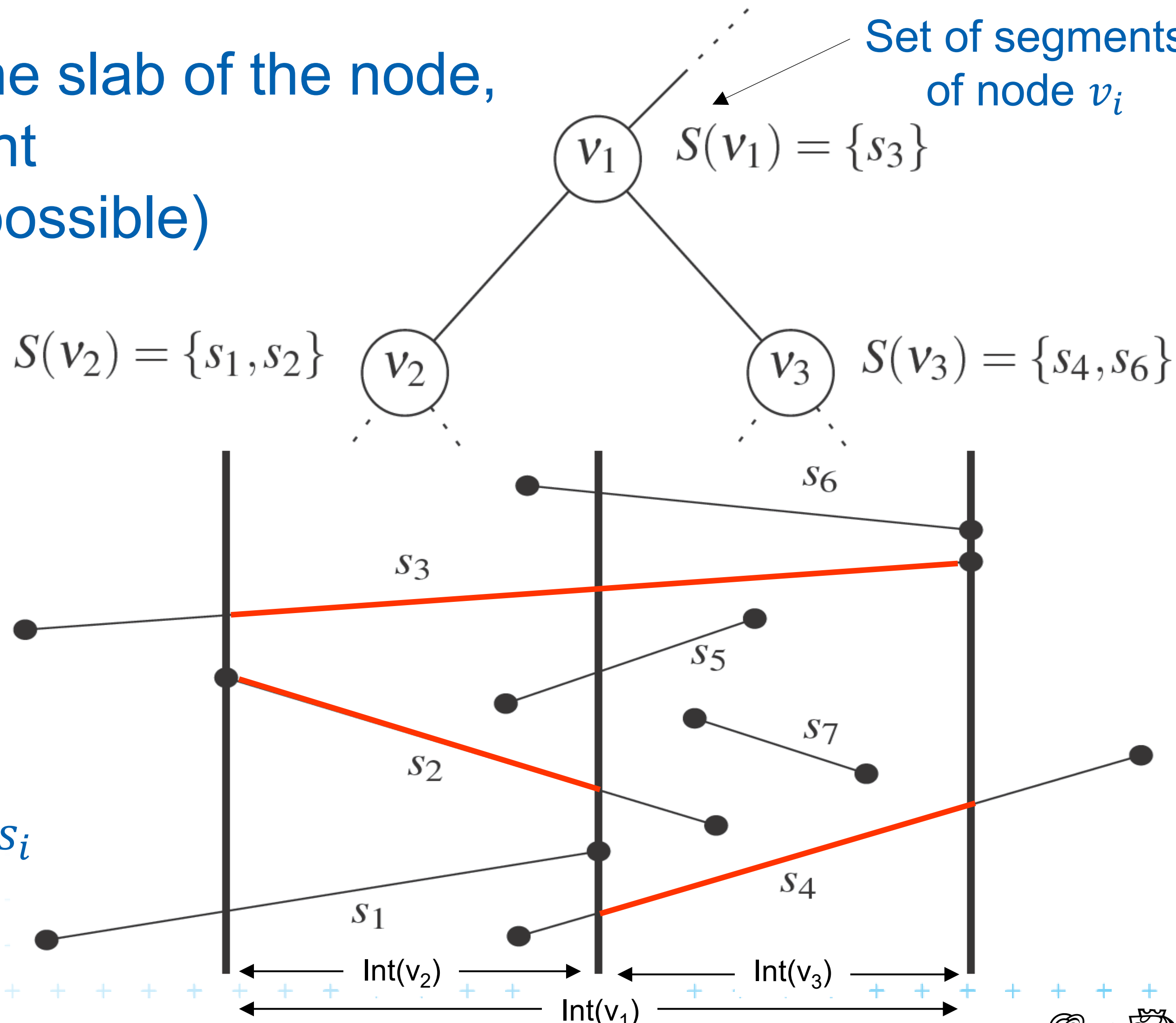
Segment tree definition

Segment tree

- Skeleton is a balanced binary tree T
- Leaves ~ elementary intervals
- Internal nodes v
  ~ union of elementary intervals of its children
  - Store: 1. interval $Int(v)$ se = union of elementary intervals of its children
    2. canonical set $S(v)$ of segments $[x_i : x_i']\in S$ segments s_i
  - Segments $[x_i : x_i']$ s_i are stored as high as possible, such that $Int(v)$ is completely contained in the segment
  - Holds $Int(v) \subseteq [x_i : x_i']$ and $Int(\text{parent}(v)) \not\subseteq [x_i : x_i']$ parent not
    (node interval is not larger than the segment)

Segments span the slab

Segments span the slab of the node, but not of its parent (stored as up as possible)



$Int(v_j) \subseteq s_i$

and

$Int(\text{parent}(v)) \nsubseteq s_i$

[Berg]

Query segment tree – stabbing query (1D)

QuerySegmentTree(v, q_x)
Input: The root v (of a subtree) of a segment tree and a query point q_x
Output: All intervals (=segments s_i) in the tree containing q_x.

1. Report all the intervals s_i in $S(v)$. // covered by the current node
2. **if** v is not a leaf // root covers "all"$(-\infty, +\infty)$
3. if $q_x \in$ Int($l(v)$) // go left
4. QuerySegmentTree($l(v)$, q_x)
5. else // or go right
6. QuerySegmentTree($r(v)$, q_x)

Query time $O(\log n + k)$, where k is the number of reported intervals

$O(1 + k_v)$ for one node

Height $O(\log n)$

Segment tree construction

ConstructSegmentTree(S)
Input: Set of intervals (segments) S
Output: segment tree

1. Sort endpoints of segments in S, get elementary intervals … $O(n \log n)$
2. Construct a binary search tree T on elementary intervals … $O(n)$
   (bottom up) and determine the interval $Int(v)$ it represents
3. Compute the canonical subsets of segments for the nodes
   (lists of their segments s_i):
4. v = root(T)
5. for all segments $s_i = [x_i : x_i'] \in S$
6. InsertSegmentTree(v, $[x_i : x_i']$)

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Segment tree construction – interval insertion

InsertSegmentTree(v, $[x : x']$)
Input: The root of (a sub-tree of) a segment tree and an interval.
Output: The interval will be stored in the sub-tree.

```
if Int(v) ⊆ [x : x']                        // Int(v) contains s_i = [x : x']
    store s_i = [x : x'] at v
else if Int( l(v) ) ∩ [x : x'] ≠ ∅           // part of s_i is to the left
        InsertSegmentTree( l(v), [x : x'] )
    if Int( r(v) ) ∩ [x : x'] ≠ ∅            // part of s_i is to the right
        InsertSegmentTree( r(v), [x : x'] )
```

One interval is stored at most twice in one level =>

- Single interval insert $O(\log n)$, insert n intervals $O(\not{2}n \log n)$
- Construction total $O(n \log n)$

Storage $O(n \log n)$

- Tree height $O(\log n)$, name s_i stored max 2 × in one level
- Storage total $O(n \log n)$ – see the next slide

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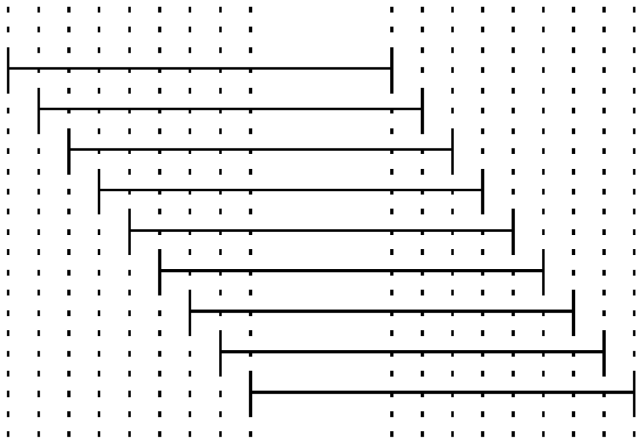
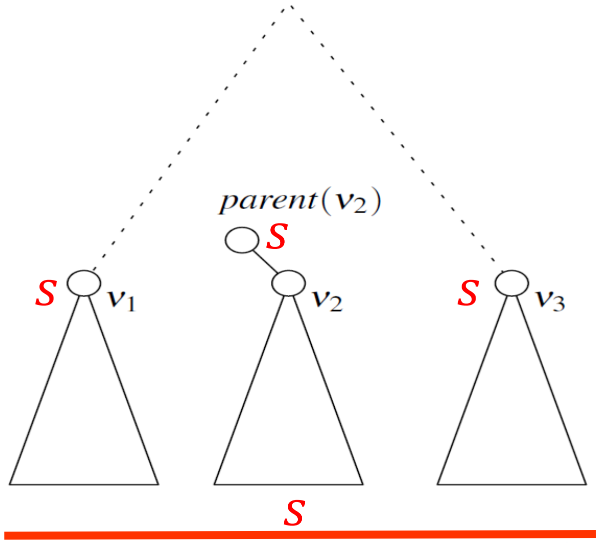
Space complexity - notes


[Berg]



[Berg]

Worst case – $O(n^2)$ segment names in leafs

But

Segments stored as high, as possible

Segment is max 2 times in one level ⇐

s covered by v_1 and v_3
⇒ v_2 covered, $Int(v_2) \in s$
As v_2 lies between v_1 and v_3
⇒ $Int(parent(v_2)) \in s$
⇒ s will be stored in parent
not stored in v_2

$\max 4n+1$ elementary intervals (leaves)
⇒ tree $O(n)$ space
$O(\log n)$ height

⇒ $O(n \log n)$ space for interval names

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Segment tree complexity

A segment tree for set S of n intervals in the plane,

- Build $O(n \log n)$
- Storage $O(n \log n)$
- Query $O(k + \log n)$
  - Report all intervals that contain a query point
  - k is number of reported intervals

Segment tree versus Interval tree

- Segment tree
  - $O(n \log n)$ storage versus $O(n)$ of Interval tree
  - But returns exactly the intersected segments s_i, interval tree must search the lists M_L and/or M_R
- Good for
  1. extensions (allows different structuring of intervals)
  2. stabbing counting queries
     - store number of intersected intervals in nodes
     - $O(n)$ storage and $O(\log n)$ query time = optimal
  3. higher dimensions – multilevel segment trees

     (Interval and priority search trees do not exist in ^dims)

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2D
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   iii. Line segment stabbing (*IT* with *priority search trees*)

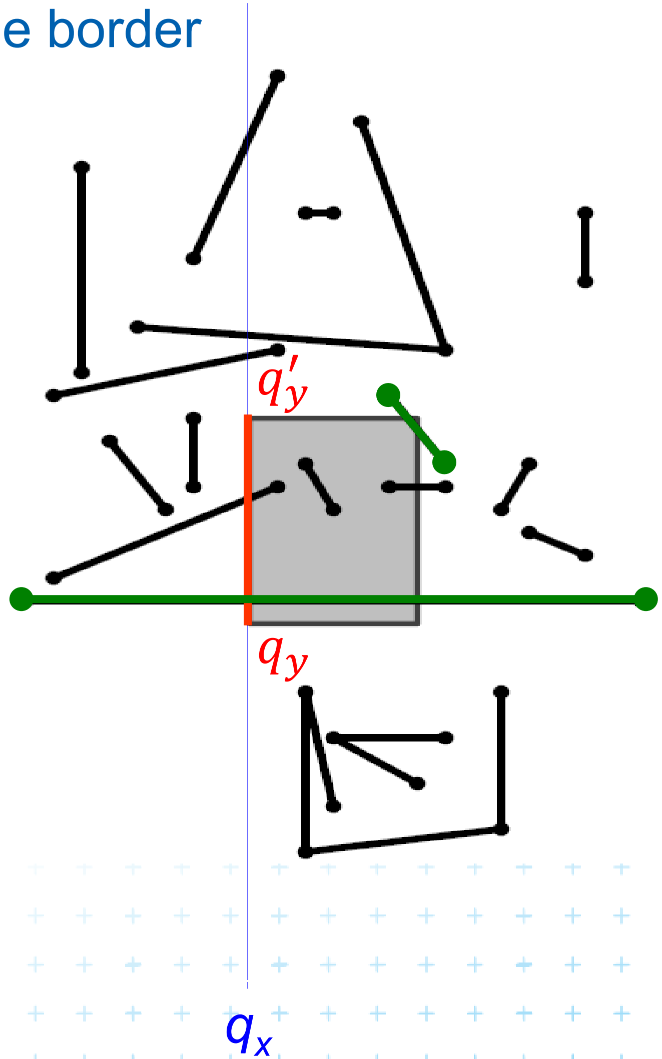
2. Windowing of line segments in general position

2D
   – *segment tree*
   – the windowing algorithm

2. Windowing of line segments in general position

Test intersection with the border

Done 4x (rectangle)

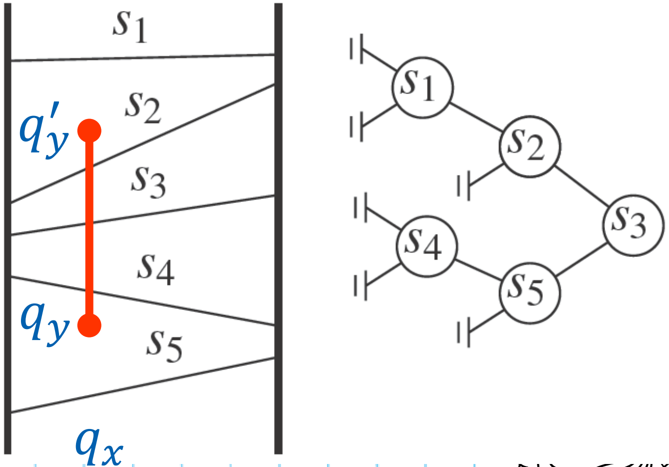


[Vakken]

Windowing of arbitrary oriented line segments

- Let S be a set of arbitrarily oriented line segments in the plane.
- Report the segments intersecting a vertical query segment $q := q_x \times [q_y : q'_y]$ – window border
- Segment tree T on x intervals of segments in S
  - node v of T corresponds to vertical slab $Int(v) \times (-\infty : \infty)$
  - segments span the slab of the node, but not of its parent
  - segments do not intersect

  => segments in the slab (node) can be vertically ordered – BST

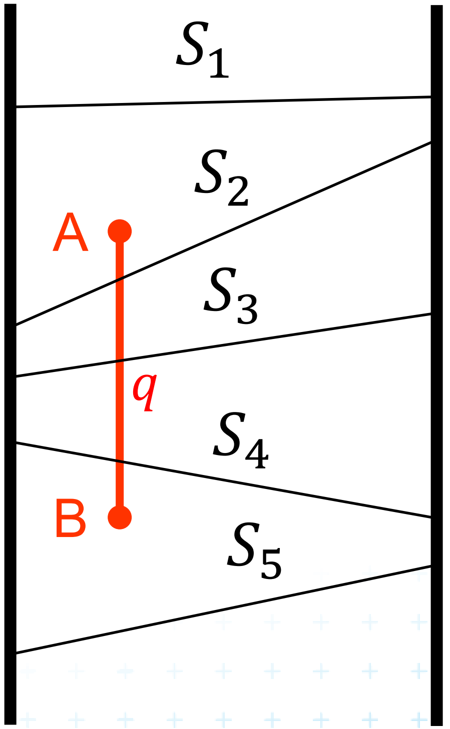


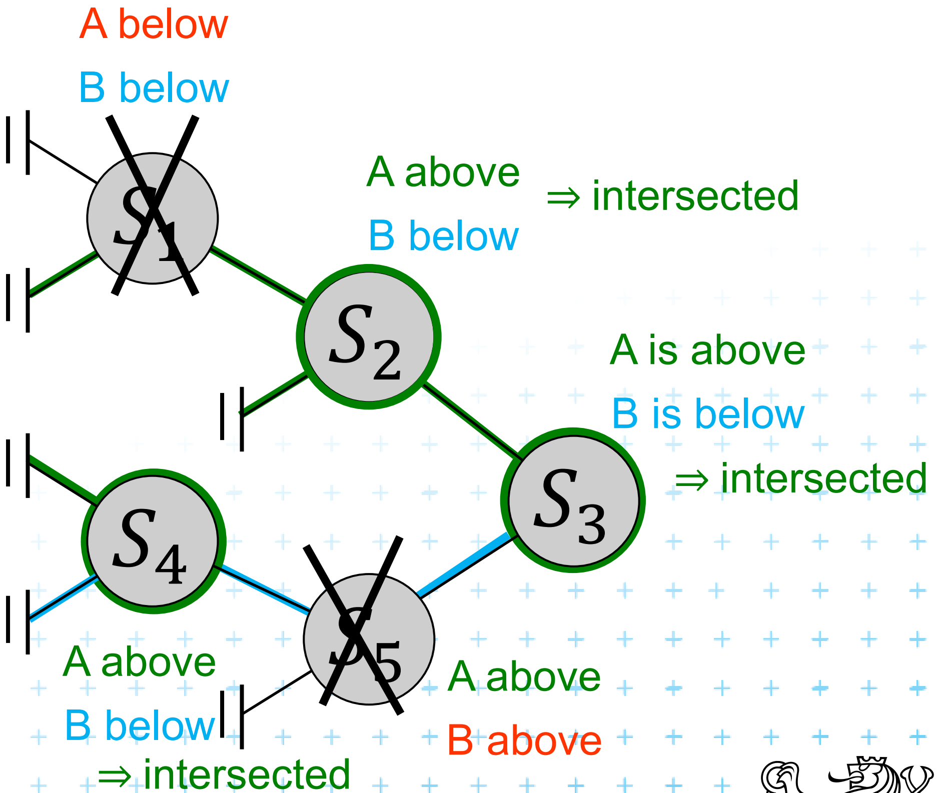
[Berg]

Segments between vertical segment endpoints

Segment s is intersected by vert.query segment q iff

- the lower endpoint (B) of q is below s and
- the upper endpoint (A) of q is above s





[Berg]

Segments between vertical segment endpoints

- Segments (in the slab) do not mutually intersect
  => segments can be vertically ordered and stored in BST
  - Each node v of the x segment tree (vertical slab) has an associated y-BST
  - y-BST $T(v)$ of node v stores the canonical subset $S(v)$ according to the vertical order
    - 2 paths – A above, B below
    - both $O(\log |S(v)|) = O(\log n)$
  - Intersected segments can be found by searching $T(v)$ in $O(k_v + \log n)$, k_v is the number of intersected segments

Windowing of arbitrary oriented line segments complexity

Structure associated to node (BST) uses storage linear in the size of $S(v)$

- Build $O(n \log n)$
- Storage $O(n \log n)$
- Query $O(k + \log^2 n)$ … $O(\log n)$ segm tree $+O(\log n)$ BST
  - Report all segments that contain a query point
  - k is number of reported segments = suma of k_v

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Windowing of line segments in 2D – conclusions

Construction: all interval tree variants $O(n \log n)$

		Search	Memory
1. Axis parallel			
1D	i. Line (*sorted lists*)	$O(k + \log n)$	$O(n)$
2D	ii. Segment (*range trees*)	$O(k + \log^2 n)$	$O(n \log n)$
2D	iii. Segment (*priority s. tr.*)	$O(k + \log n)$	$O(n)$
2. In general position			
2D	– *segment tree + BST*	$O(k + \log^2 n)$	$O(n \log n)$

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