

Write your solutions in this sheet below the problem statement. Mark clearly your solutions by corresponding letters A, B, C, and separate visually each solution from the other ones by empty space or by a line. Use the other side of the sheet or ask for an additional blank sheet if necessary.

Each problem 1. – 4. is worth 0 – 3 points, each answer to a particular question A, B, C, contributes at most 1 point to the total.

1. The score of a undirected graph is a sequence of all node degrees listed in non-ascending order.

The particular score $S = (3, 3, 3, 2, 1, 1, 1, 1, 1)$ is given in this problem.

A. Draw two mutually non-isomorphic trees T_1 and T_2 with 9 nodes, the score of both trees should be S .

B. In tree T_2 , choose one of the nodes with degree 3. At this node, write the part of the tree certificate which corresponds to this node.

C. Write the complete certificate of T_1 and mark in it the part which you wrote down in answer B.

2. An alphabet $C = \{a, b, c\}$ is given. Language L over alphabet C is specified by regular expression $(ac + bc)(aa+bb)^*$. (The asterisk after the $(aa + bb)$ term is the Kleene star which denotes the iteration of the term $(aa + bb)$.)

A. Calculate the number of all words in L which length is equal to n . Suppose n is even.

B. Write down two words w_1 and w_2 in L . The lengths of w_1 and w_2 should be different and the Levenshtein distance between w_1 and w_2 should be 3. Explain, why Levenshtein distance between w_1 and w_2 cannot be smaller.

C. Draw a transition diagram of a finite automaton A which detects words of L in a text over alphabet C .

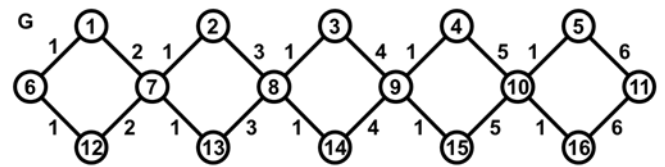
3. Construct a skip list L which contains the given keys. The number in the brackets at each key determines the level of the key. The level is also equal to the number of coin tosses associated with the key (the number of coin tosses includes the last toss when the coin came up tails). A(1), C(2), H(3), N(1), Q(2), R(4), W(1), Z(2).

A. Draw skip list L. Next, insert key P into skip list L. The level of P is 3. Determine how many comparisons between two key values will be performed during this operation. Explain your reasoning.

B. Delete key C from skip list L. Determine how many comparisons between two key values will be performed during this operation. Explain your reasoning.

C. Suppose a skip list K contains n keys. Another n^2 keys are to be inserted into K. What is the expected asymptotic complexity of this insertion process?

4. The image depicts a weighted undirected graph G which nodes are labeled 1 – 16.



A. Determine the weight of the minimum spanning tree of G and the number of different minimum spanning trees in G .

B. Prim's algorithm is run on G and it starts in node 11. Write a sequence of edges added one by one into the minimum spanning tree of G by Prim's algorithms. If there are more possible sequences, write any of them.

C. Explain why the Boruvka's algorithm for finding a MST of a graph contains a condition demanding that all edge weights are mutually different. Draw an example of a graph in which modified Boruvka's algorithm without the condition would fail.