

Electromagnetic Field Theory

Week 11

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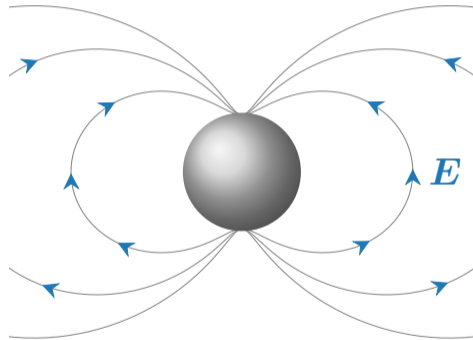
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1. Magnetic Field
2. Magnetization
3. Ampère's Law for Magnetic Field Intensity \mathbf{H}
4. Magnetic Susceptibility





Magnetic Flux

$$\Phi = \iint_S \mathbf{B} \cdot d\mathbf{S} \quad [\text{Wb}]$$

$$\Phi = \oint_l \mathbf{A} \cdot d\mathbf{l}$$



Comparison of Electrostatics and Magnetostatics

Electrostatic field

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho$$

$$\nabla \times \mathbf{E} = \mathbf{0}$$

BC $\mathbf{E} \rightarrow \mathbf{0}$ far from sources

$$\mathbf{E}(\mathbf{r}_2) = \frac{1}{4\pi\epsilon_0} \int_{V_1} \frac{\rho(\mathbf{r}_1)(\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^3} dV_1$$

$$\mathbf{F}_e = q\mathbf{E}$$

Magnetostatic field

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

BC $\mathbf{B} \rightarrow \mathbf{0}$ far from sources

$$\mathbf{B}(\mathbf{r}_2) = \frac{\mu_0}{4\pi} \int_{V_1} \frac{\mathbf{J}(\mathbf{r}_1) \times (\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^3} dV_1$$

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$$



Magnetic Dipole Moment

Multipole expansion for

$$\frac{1}{R} = \frac{1}{r} + \frac{\cos(\alpha)r'}{r^2} + \left(-\frac{1}{2} + \frac{3}{2}\cos^2(\alpha)(r')^2\right) \frac{1}{r^3} + \mathcal{O}((r')^3).$$

Magnetic dipole moment

$$\mathbf{A}_d(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2}, \quad [\text{Wb/m}]$$

with

$$\mathbf{m} = I \iint_{S'} d\mathbf{S}', \quad [\text{A m}^2].$$

The magnetic field of small loop of current is

$$\mathbf{B}_d = \frac{\mu_0 m}{4\pi r^3} \left(2 \cos(\theta) \hat{\mathbf{r}} + \sin(\theta) \hat{\boldsymbol{\theta}} \right).$$



Atomic Model and Material Classification

1. Electron in orbit.
2. Spin of the electron.
3. Nuclear spin.

The magnetic materials are classified as

- ▶ diamagnetic,
- ▶ paramagnetic,
- ▶ ferromagnetic,
- ▶ antiferromagnetic,
- ▶ ferrimagnetic,
- ▶ superparamagnetic.



Magnetization

Magnetization as the average of loop magnetic moments

$$\mathbf{M} = \lim_{\Delta V \rightarrow 0} \frac{\sum_i \mathbf{m}_i}{\Delta V} \quad [\text{A/m}].$$

Ampère's law including bound current I_b

$$\oint_{l'} \mathbf{B} \cdot d\mathbf{l}' = \mu_0 I_{\text{tot}} = \mu_0 (I_0 + I_b).$$

We have

$$\mathbf{m}_b = I_b \int d\mathbf{S} = I_b \mathbf{S},$$

and

$$I_b = hM.$$



Intensity of Magnetic Field

Ampère's law for free and bound current expressed as magnetization

$$\oint_{l'} \mathbf{B} \cdot d\mathbf{l}' = \mu_0 I_0 + \mu_0 \int_{l'} \mathbf{M} \cdot d\mathbf{l}'.$$

Definition of the intensity of magnetic field

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}, \quad [\text{A/m}].$$

The integral form of Ampère's law for free current only

$$\oint_{l'} \mathbf{H} \cdot d\mathbf{l} = I_0 = \iint_{S'} \mathbf{J} \cdot d\mathbf{S},$$

and its differential form

$$\nabla \times \mathbf{H} = \mathbf{J}_0.$$



Magnetic Susceptibility

Linear (linearized) materials

$$\mathbf{M} = \chi_m \mathbf{H},$$

where χ_m is magnetic susceptibility.

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \chi_m \mathbf{H} = \mu \mathbf{H},$$

Questions?

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