

Electromagnetic Field Theory

Week 10

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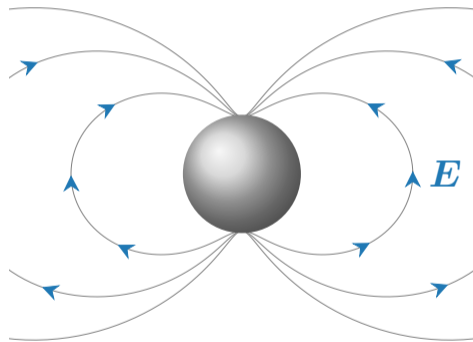
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1. Stationary Magnetic Field
2. Biot-Savart Law
3. Ampère's Law
4. Magnetic Flux
5. Vector Potential





Stationary Magnetic Field

Conditions

$$\frac{d\rho}{dt} = 0, \quad \frac{d\mathbf{J}}{dt} = 0.$$

The equivalent terms in the stationary magnetic field are

$$dQ \mathbf{v}, \quad I d\mathbf{l}, \quad \mathbf{J} dV, \quad [\text{A m}].$$



Biot-Savart Law

For linear current

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{l'} I(\mathbf{r}') \frac{d\mathbf{l}' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}, \quad [\text{T}],$$

or for the conductor's volume

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{V'} \frac{\mathbf{J}(\mathbf{r}') dV' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}, \quad [\text{T}].$$

where

μ_0 is vacuum permeability,

\mathbf{r} points to the observation region,

\mathbf{r}' points to the source region,

T is physical unit Tesla, equivalent to $\text{N}/(\text{A m})$.



Magnetic Field of a Straight Wire

Start from

$$\mathbf{B}(s) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{\mathbf{R}}}{R^2}$$

with $\mathbf{R} = \mathbf{r} - \mathbf{r}'$, $R = |\mathbf{R}|$, $\hat{\mathbf{R}} = \mathbf{R}/R$ to get

$$\mathbf{B}(s) = \frac{\mu_0 I}{2\pi s} \hat{\phi}.$$

Force between two parallel wires:

$$\frac{\mathbf{F}_m}{l} = -\frac{\mu_0 I_1 I_2}{2\pi d} \hat{x}.$$



Ampère's Law

Ampère's law dictates that

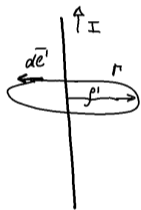
$$\oint_l \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{tot}},$$

or equivalently

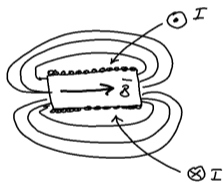
$$\oint_l \mathbf{B} \cdot d\mathbf{l} = \mu_0 \iint_S \mathbf{J} \cdot d\mathbf{S}.$$



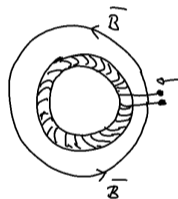
Coils: Basic Arrangements



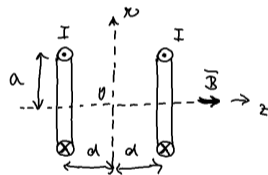
A loop.



A solenoid.



A toroid.



A loop.

Divergence of Magnetic Field



$$\nabla \cdot \mathbf{B}(\mathbf{r}) = 0.$$



Magnetic Flux

The flux of magnetic field through a closed surface

$$\oiint_S \mathbf{B} \cdot d\mathbf{S} = 0.$$

Magnetic flux

$$\Phi = \iint_S \mathbf{B} \cdot d\mathbf{S}, \quad [\text{Wb}].$$

(Wb is the physical unit Weber, equivalent to V s.)



Vector Potential

Use

$$\mathbf{B} = \nabla \times \mathbf{A}$$

where \mathbf{A} is the magnetic vector potential, with the choice

$$\nabla \cdot \mathbf{A} = 0$$

which gives

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J},$$

with

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{V'} \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV', \quad [\text{Wb/m}^2].$$

Questions?

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