

Electromagnetic Field Theory

Week 9

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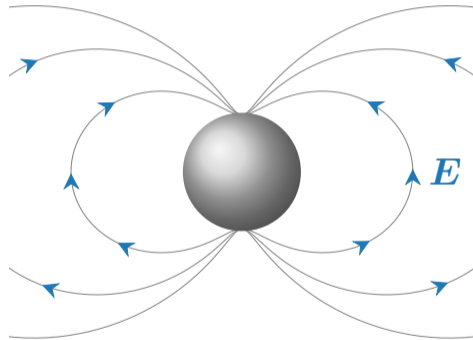
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1. Steady Currents
2. Ohm's Law
3. Resistors
4. Electromotive Force
5. Kirchhoff's Circuit Laws





Magnetostatic field

Magnetostatic field

$$\begin{aligned}\nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J}.\end{aligned}$$

Magnetic force

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B}), \quad [\text{N}].$$

Lorentz force

$$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad [\text{N}].$$



Steady current field

$$I = \iint_S Nq\mathbf{v} \cdot d\mathbf{S} = \iint_S \rho\mathbf{v} \cdot d\mathbf{S} = \iint_S \mathbf{J} \cdot d\mathbf{S}.$$

Different current types:

- ▶ conductive currents (in conductors and semiconductors),
- ▶ convective currents (e^- or ions in vacuum),
- ▶ in electrolyte (ions, *e.g.*, in battery).

Steady current condition:

$$\frac{\partial \mathbf{J}}{\partial t} = \mathbf{0}.$$



Current Continuity Equation – Steady Currents

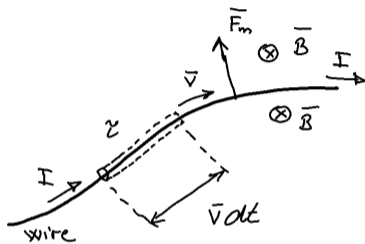
$$\oiint_S \mathbf{J} \cdot d\mathbf{S} = 0$$

$$\nabla \cdot \mathbf{J} = 0$$

(valid only in for steady currents, *i.e.*, $dd\rho/dt = 0$)



Magnetic Force Defined by Current



$$\mathbf{F}_m = \int_l I (\mathbf{dl} \times \mathbf{B}) = I \int_l (\mathbf{dl} \times \mathbf{B})$$



Ohm's Law in Differential Form

$$\mathbf{J} = \sigma \mathbf{E}, \quad [\text{A/m}^2]$$

with conductivity σ in [S/m]and resistivity

$$\rho = \frac{1}{\sigma}, \quad [\Omega \text{ m}].$$



Ohm's Law in Integral Form

$$R = \int_0^l \frac{dl}{\sigma S}, \quad [\Omega].$$

$$U = RI, \quad [\text{V}, \Omega, \text{A}],$$

with R being resistance and

$$G = \frac{1}{R}, \quad [\text{S}]$$

being conductance.



Connection of Resistors

Serial connection

$$R = \sum_i R$$

Parallel connection

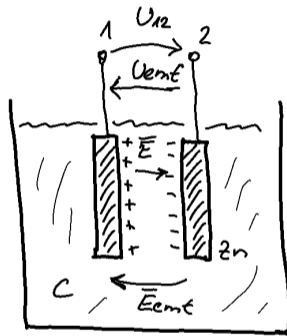
$$\frac{1}{R} = \sum_i \frac{1}{R_i}$$



Electromotive Force

Many possible mechanisms of emf

- ▶ chemical force,
- ▶ mechanical pressure,
- ▶ temperature gradient,
- ▶ light,
- ▶ running belt,
- ▶ moving charges in the magnetic field.



$$U_{\text{emf}} = \int_2^1 \mathbf{E}_{\text{emf}} \cdot d\mathbf{l} = - \int_2^1 \mathbf{E}_c \cdot d\mathbf{l} = \int_1^2 \mathbf{E}_c \cdot d\mathbf{l} = U_{12}$$



Boundary Conditions for Current Density

Normal Component

$$J_{1,\text{norm}} = J_{2,\text{norm}}$$

$$\sigma_1 E_{1,\text{norm}} - \sigma_2 E_{2,\text{norm}} = 0$$

$$\varepsilon_1 E_{1,\text{norm}} - \varepsilon_2 E_{2,\text{norm}} = \sigma_0$$

Tangential Component

$$\frac{J_{2,\text{tan}}}{\sigma_2} = \frac{J_{1,\text{tan}}}{\sigma_1}$$

$$E_{2,\text{tan}} = E_{1,\text{tan}}$$



Kirchhoff's Circuit Laws

Kirchhoff's Current Law

- ▶ The current entering a node escapes it:

$$\oiint_S \mathbf{J} \cdot d\mathbf{S} = 0$$

$$\sum_i I_i = 0$$

Kirchhoff's Voltage Law

- ▶ The closed-loop integral of the electric field is zero everywhere except EMF sources:

$$\oint_l \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\sum_i U_i = 0$$

Analogies Between Electrostatic and Steady Current Field



Electrostatics	\mathbf{E}	φ	U	\mathbf{D}	ϵ	Q	C
Steady current	\mathbf{E}	φ	U	\mathbf{J}	σ	I	G

What about differences?

Questions?

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