Electromagnetic Field Theory

Week 2

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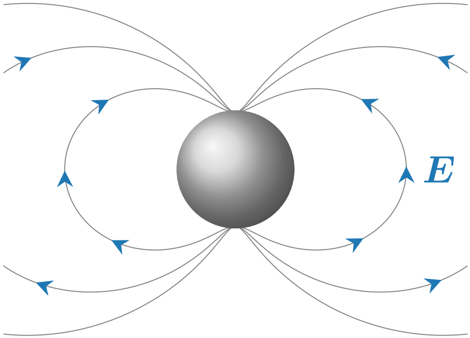
Outline

1. Definition of Electrostatic Field
2. Coulomb's Law
3. Gauss's Law



Electrostatic Field

Maxwell's equations (in differential form)

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{J}$$

Electrostatics ($\partial/\partial t = 0$)

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \boldsymbol{E} = \boldsymbol{0}$$

- Electric charge is displaced and fixed.
- All phenomena are time-independent.

Coulomb's Law

Coulomb's law

$$\boldsymbol{F}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q_1(\boldsymbol{r}) q_2(\boldsymbol{r}') (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3}, \quad [\mathrm{N}]$$

Permittivity of vacuum

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} \approx 8.854 \cdot 10^{-12}\,\mathrm{F m^{-1}}$$

Superposition

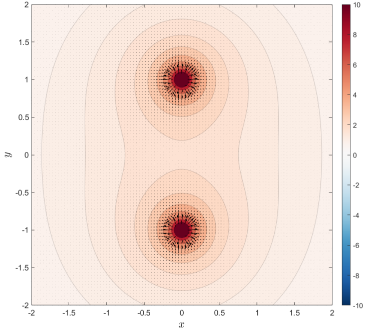
$$\boldsymbol{F}(\boldsymbol{r}) = \frac{q}{4\pi\varepsilon_0} \sum_{n=1}^{N} q_n \frac{\boldsymbol{r} - \boldsymbol{r}'_n}{|\boldsymbol{r}'_n - \boldsymbol{r}|^3}$$

Electric Field Intensity $\boldsymbol{E}$

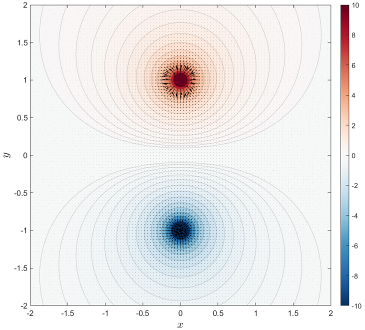
$$\boldsymbol{F}(\boldsymbol{r}) = q\boldsymbol{E}(\boldsymbol{r}), \quad [\mathrm{N}]$$

Electric field intensity $\boldsymbol{E}$ is a vector function in V/m.

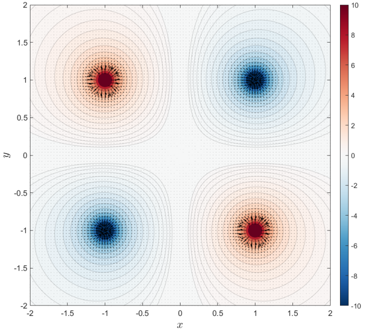
Electric Field of Several Point Charge Configurations



Two point charges of the same polarity.

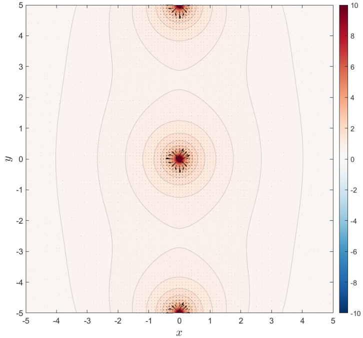


Two point charges of the opposite polarity.

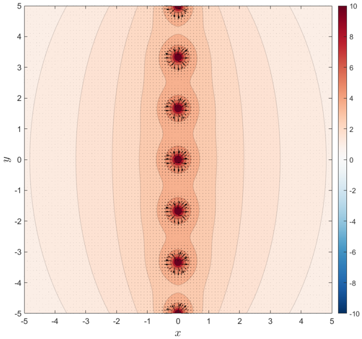


Four point charges – quadrupole configuration.

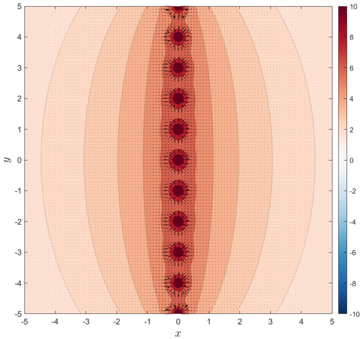
Electric Field of Several Point Charge Configurations



3 point charges along straight line.



7 point charges along straight line.



11 point charges along straight line.

Electric Field of Continuous Charge Distributions

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \int\limits_l \tau(\boldsymbol{r}') \frac{\boldsymbol{r} - \boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \, \mathrm{d}l$$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \iint\limits_S \sigma(\boldsymbol{r}') \frac{\boldsymbol{r} - \boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \, \mathrm{d}S$$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \iiint\limits_V \rho(\boldsymbol{r}') \frac{\boldsymbol{r} - \boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \, \mathrm{d}V$$

Gauss's Law

- integral form

$$\oiint_S \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S} = \frac{1}{\varepsilon_0} \int_{V'} \rho(\boldsymbol{r}') \, \mathrm{d}V' = \frac{Q}{\varepsilon_0}$$

- differential form

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

Questions?

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This document has been created as a part of BAB17EMP course.