Magnetic Resonance Imaging (MRI) based on Nuclear Magnetic Resonance (NMR)

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2008-2025

¹http://www.cis.rit.edu/htbooks/mri/

Introduction

MRI physics

Nuclear spin

Spectroscopy

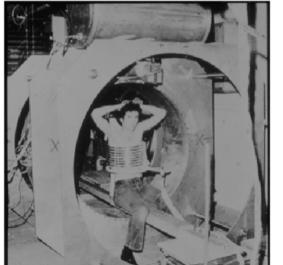
Excitation

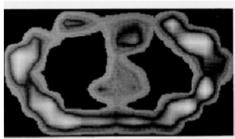
Relaxation

Bloch equation

First human MRI

První obraz člověka (1977)



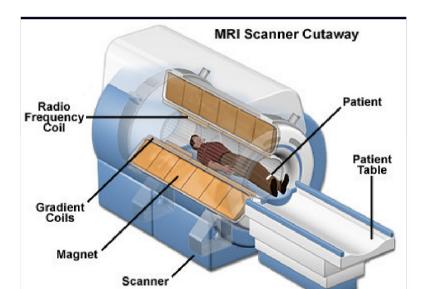


MRI scanner

selenoid, closed-bore magnet



MRI Scanner

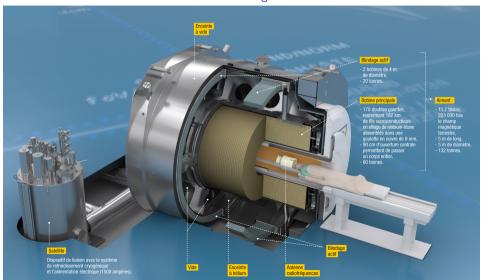


Permanentní magnety - architektura "OPEN"

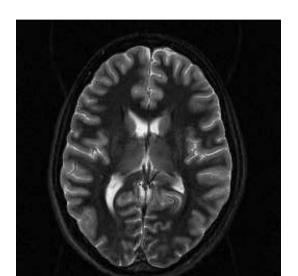


MRI with 11.7T

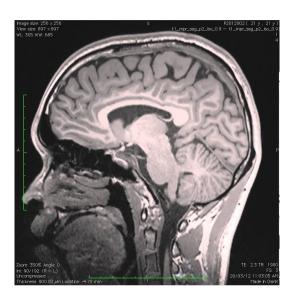
223000× earth magnetic field



Brain slice:



Brain slice:



Spine:





MRI principles

- 1. Insert object (subject) into a strong magnetic field.
- 2. (Repeatedly) send a radio-frequency impuls.
- 3. Spins are excited and then relax to equilibrium.
- 4. Receive and record emitted radio-frequency waves.
- 5. Reconstruct image from data.
- 6. Remove the object (subject) from the magnetic field.

Brief history of MRI

- 1946 Felix Bloch, Edward Purcell, independent discovery
- 1950–1970 NMR, spectroscopic analysis
- 1971 Raymond Damadian, tissue relaxation times differ
- 1973 Hounsfield, CT (showed demand for medical imaging)
- 1973 Paul Lauterbur, tomographic MRI (backprojection)
- 1975 Richard Ernst, Fourier MRI
- 1977 Peter Mansfield, echo-planar imaging (EPI), later 30 ms/slice

Brief history of MRI (2)

- 1980 Edelstein, whole-body MRI (3D), 5 min/slice
- 1986 whole-body MRI, 5 s/slice
- 1986 MRI microscopy, resolution 10 μ m
- 1987 beating heart imaging
- 1987 MRA angiography without contrast agents, blood flow
- 1992 functional MRI, brain mapping

Nobel prizes

- 1952 Felix Bloch, Edward Purcell, physics, discovery
- 1991 Richard Ernst, chemistry, Fourier MRI
- 2003 Paul Lauterbur, Peter Mansfield, medicine, MRI in medicine

Numbers related to MRI

- About 40000 MRI scanners worldwide
- About 20 examination per day per scanner
- 110 scanners in Czech Republic in 2018
- ullet One scanner costs $10\sim 100$ mil. CZK (millions of EUR)
- One examination 5 \sim 20 thousands CZK (hundreds of EUR)

Units review

- Time (s)
- Angle (degree) or (rad) 2π rad in 360°
- Temperature (Kelvin, K) 0 K = -273.15 degree Celsius
- Magnetic field strength (Tesla, T). Earth magnetic field $5 \cdot 10^{-5}$ T
- Energy (Joule, J)
- Frequency 1 Hz = 1/s
- Power (Watt, W)
- pico,p (10^{-12}) , nano,n (10^{-9}) micro, μ (10^{-6}) , milli,m (10^{-3}) , kilo,k (10^3) , mega,M (10^6) , giga,G (10^9)

Introduction

MRI physics

Nuclear spin

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Bloch equation

Spin

- Human body: fat and water. 63 % hydrogen (10 % by mass)
- Hydrogen nucleus = proton.
- Proton has a property called a **spin** (besides a mass and charge), related to angular momentum.
- Non-zero spin particles behave like small magnets \rightarrow MRI signal

- Free electrons, protons, and neutrons have a spin of 1/2
- Spins may pair and compensate
- Nuclear spin I is a multiple of 1/2
- $I \neq 0 \Rightarrow$ (small) magnet,

$$\gamma$$
 —gyromagnetic constant $\left[\frac{\text{rad}}{\text{s} \cdot \text{T}}\right]$ or $\left[\frac{\text{MHz}}{\text{T}}\right]$ $\hbar = \frac{h}{2\pi}$ — reduced Planck constant $h \approx 6.626 \cdot 10^{-34} \text{ J} \cdot \text{s}$

- Only unpaired spins $(I \neq 0)$ are useful for MRI
- Total nuclear spin no easy rule
- Even atomic number Z (number of protons) and even mass number A (total number of protons and neutrons) $\Rightarrow I = 0$ ($^{12}C, ^{16}O$)
- Most abundant isotopes for even Z have I = 0
- Isotopes with $I \neq 0$ are often
 - rare isotopes (1.11% for ¹³C)
 - biologically rare elements
 - give small signal

Biological abundance of elements (by count)

Element	Abundance [%]	
Н	63	
Ο	26	main isotope ¹⁶ O with zero spin
C	9.4	main isotope $^{12}\mathrm{C}$ with zero spin
Ν	1.5	
Р	0.24	
Ca	0.22	
Na	0.041	

Abundance of MRI active isotopes

by count

Isotope	Abundance	[%]
¹ H	99.985	
^{2}H	0.015	
¹³ C	1.11	
^{14}N	99.63	
^{15}N	0.37	
²³ Na	100	
^{31}P	100	
³⁹ K ⁴³ Ca	93.1	
⁴³ Ca	0.145	

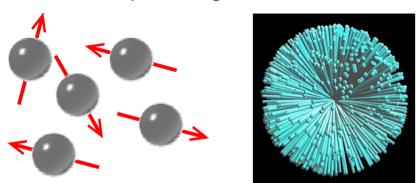
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Practical MRI exists for

- ¹H most often used, strongest signal, best quality
- ¹⁹F, ²³Na, ³¹P... mostly research

Spins in magnetic field

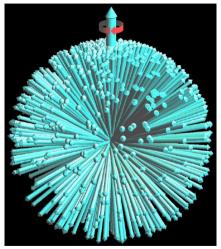


magnetic induction $B_0 = 0$, random orientation

Nuclear spin Spectroscopy Excitation Relaxation Bloch equation

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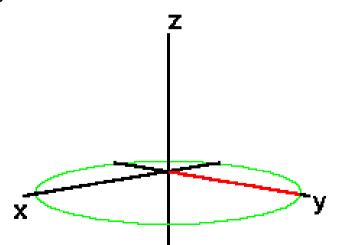
Spins in magnetic field



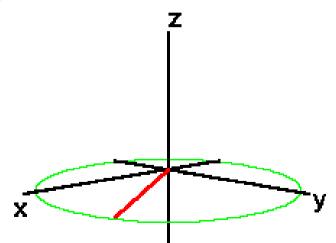
 $B_0 \neq 0$ (around $0.1 \sim 10\,\mathrm{T}$ needed)

Spin packet

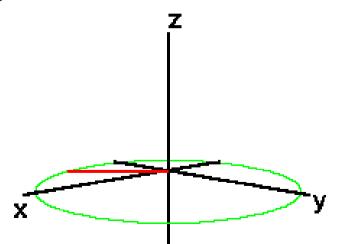
- For B_0 along z axis
- M rotates around z axis
- $au = \mu \times \mathsf{B}$



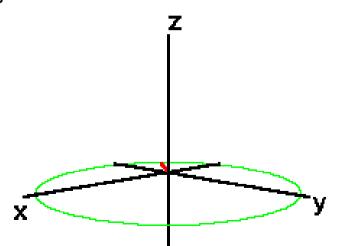
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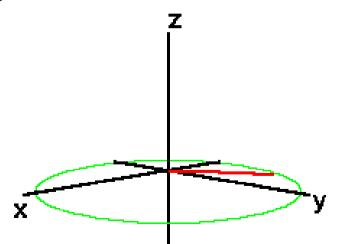
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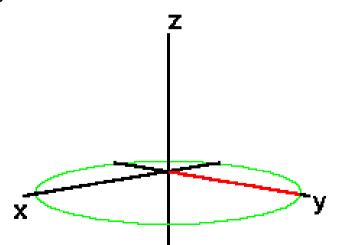


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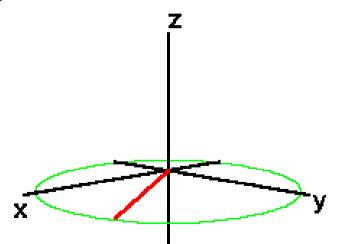
Precession

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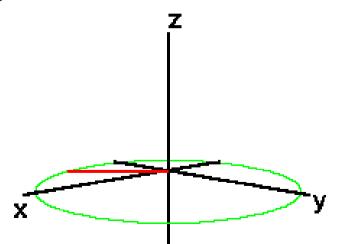


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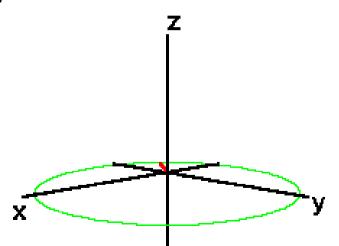


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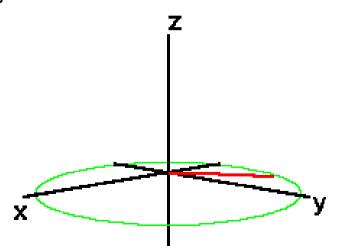


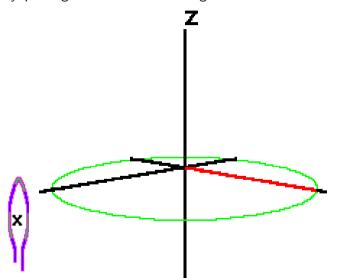
Precession

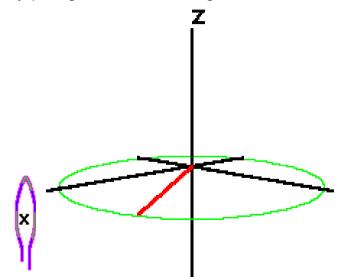
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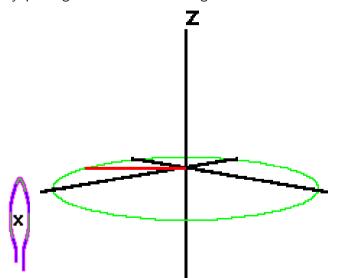


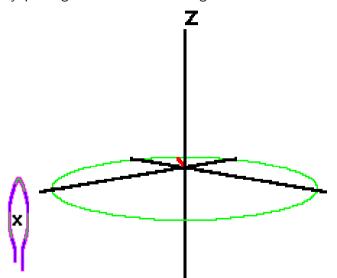
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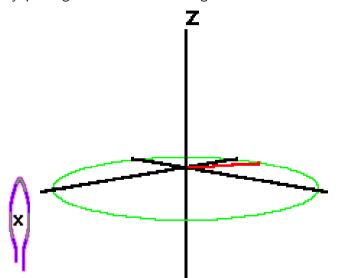


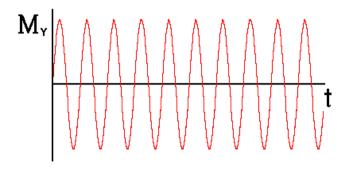




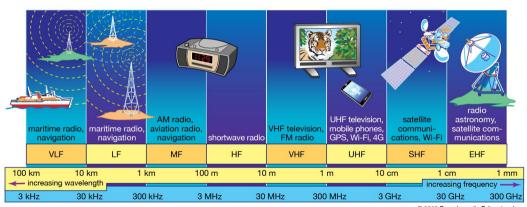








Radio Frequency

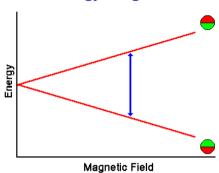


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$$f = \gamma B$$

- B [Tesla] magnetic field intensity
- γ gyromagnetic constant
- For hydrogen 1 H, $\gamma = 42.58 \, \text{MHz/T}$
- f is a frequency of:
 - the precession
 - the received signal
 - the excitation signal

Energy diagram



$$f = \gamma B$$
, $E_p = hf$, $\Delta E \propto \gamma B$

- low (parallel) and high (antiparallel) orientations
- For H, typically $f = 15 \sim 80 \, \text{MHz}$.
- Energy difference \sim signal amplitude

For a closed (non-quantum) system in thermal equilibrium:

- Number of low-energy spins N⁻
- Number of high-energy spins N⁺

$$\frac{N^+}{N^-} = e^{-\frac{\Delta E}{kT}}$$

Boltzmann constant $k = 1.3805 \cdot 10^{-23}$ Temperature T [K]

Boltzmann statistics

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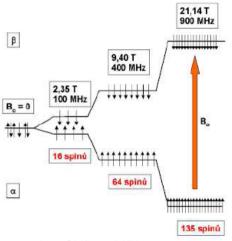
$$\frac{N^+}{N^-} = e^{-\frac{\Delta E}{kT}}$$

Boltzmann constant $k = 1.3805 \cdot 10^{-23}$ Temperature T [K]

- MRI signal $\propto \|\mathbf{M}\| \propto N^- N^+$
- low T, high $B \rightarrow \text{high signal}$
- high T, low $B \rightarrow low signal$

Boltzmann statistics example

Je-li stav β obsazen 10° spinů, stav α obsahuje 10° přebytek.



Example ¹H:

$$f = 400 \, \mathrm{MHz}$$

 $B = 9.5 \, \mathrm{T}$
 $\Delta E = 3.8 \cdot 10^{-5} \, \frac{\mathrm{Kcal}}{\mathrm{mol}}$

$$\frac{N^-}{N^+} = 1.000064 \sim 64 \, \mathrm{ppm}$$

 \rightarrow very low SNR in MRI.

Introduction

MRI physics

Nuclear spin

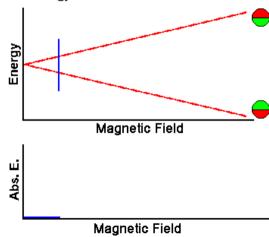
Spectroscopy

Relaxation

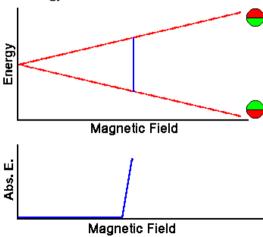
Bloch equation

Continous wave NMR (1)

- Constant frequency
- Variable magnetic field
- Measuring absorbed energy

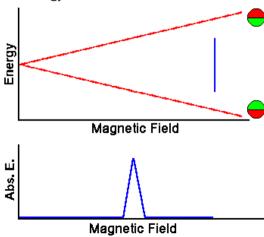


- Constant frequency
- Variable magnetic field
- Measuring absorbed energy

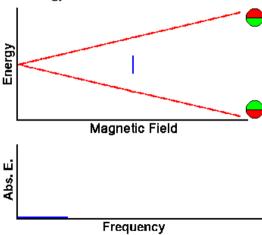


Continous wave NMR (1)

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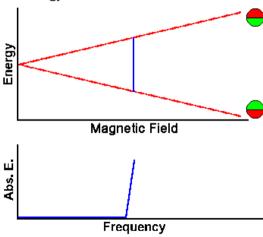


- Constant magnetic field
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- Measuring absorbed energy



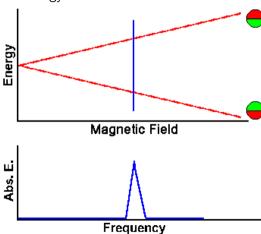
Continous wave NMR (2)

- Constant magnetic field
- Variable frequency
- Measuring absorbed energy

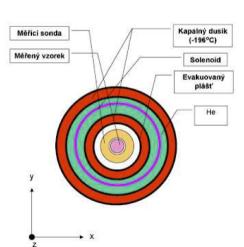


Continous wave NMR (2)

- Constant magnetic field
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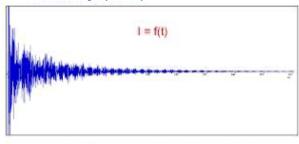
NMR spectroscopy



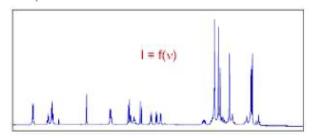


Free Induction Decay (FID)

- In a real sample there are many spin systems whose frequencies are different from the carrier frequency B
- Since we have effectively excited all these spins, we get a combination of signals and different frequencies



Po zpracování Fourierovou transformací dostaneme:



Introduction

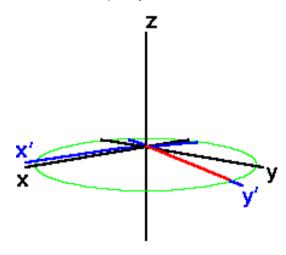
MRI physics

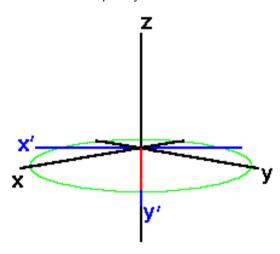
Nuclear spin Spectroscopy

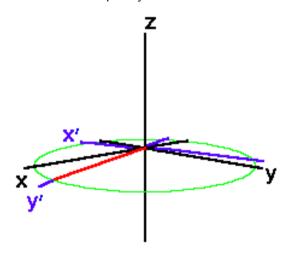
Excitation

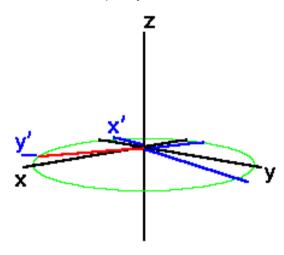
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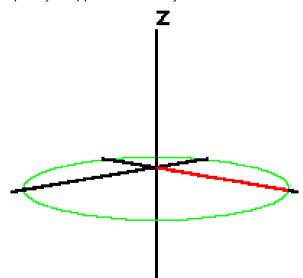




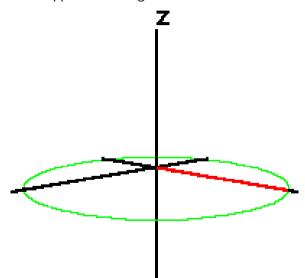




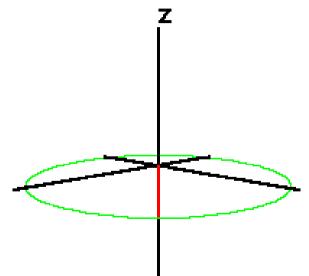
 μ rotating with frequency f appears stationary



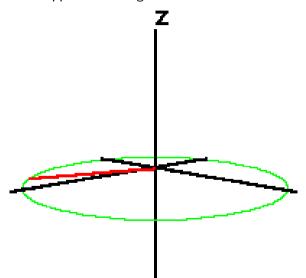
 μ rotating faster than f appears rotating in the same direction

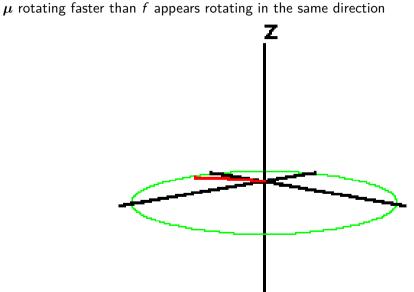


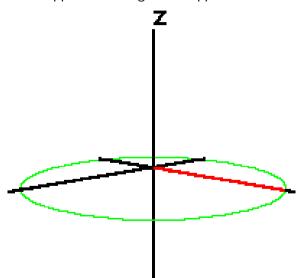
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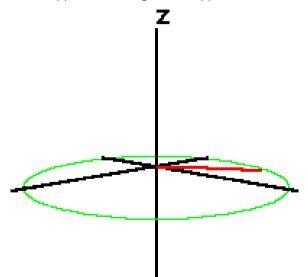


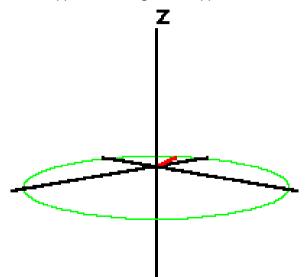
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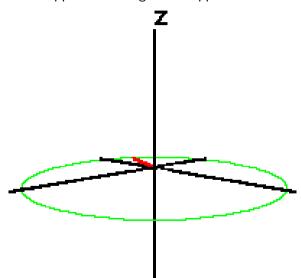








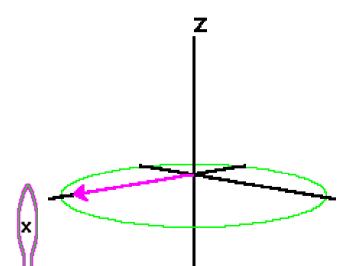




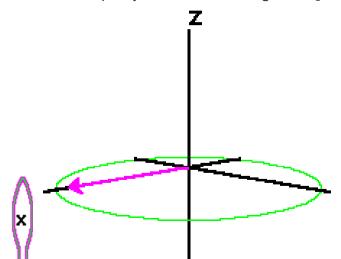
Introduction MRI physics Nuclear spin Spectroscopy Excitation Relaxation Bloch equation

Spin Precession in Radio Frequency (RF) Field

• Coil around x axis creates magnetic field along x



- Coil around x axis creates magnetic field along x
- Alternating current with frequency f creates alternating field \mathbf{B}_1



- Coil around x axis creates magnetic field along x
- Alternating current with frequency f creates alternating field \mathbf{B}_1
- \mathbf{B}_1 can be decomposed to constant amplitude $\mathbf{B}_1^+ + \mathbf{B}_1^-$, rotating around z with frequency $\pm f$

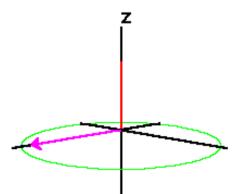
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- **B**₁⁺ is stationary in the rotating frame of reference.

- Coil around x axis creates magnetic field along x
- Alternating current with frequency f creates alternating field \mathbf{B}_1
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- \mathbf{B}_1^+ is stationary in the rotating frame of reference.
- \mathbf{B}_1^- has frequency 2f there, far from the resonance, can be neglected.

- Coil around x axis creates magnetic field along x
- Alternating current with frequency f creates alternating field \mathbf{B}_1
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- **B**₁⁺ is stationary in the rotating frame of reference.
- \mathbf{B}_1^- has frequency 2f there, far from the resonance, can be neglected.
- \rightarrow alternating field ${\bf B_1}$ will appear stationary along x' in the rotating frame of reference.

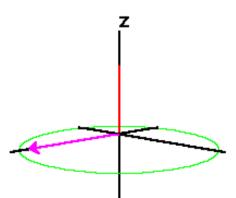
- RF impuls at Larmor frequency f (resonance), with amplitude B_1 and duration τ
- \rightarrow magnetization **M** will turn around B_1 (= x') by angle

$$lpha = 2\pi \gamma au B_1$$
 flip angle



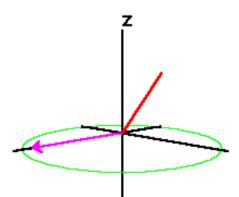
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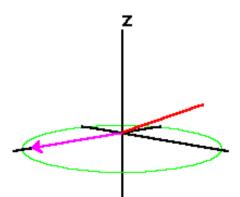
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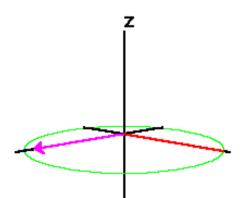
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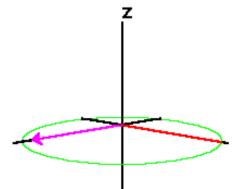
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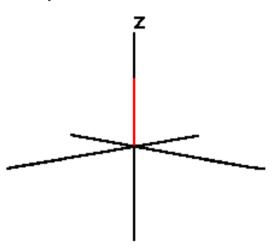


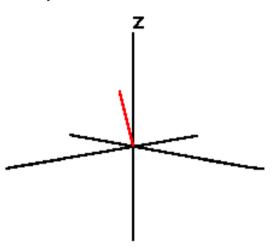
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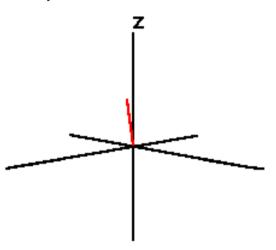
$$\alpha = 2\pi \gamma \tau B_1$$
 flip angle

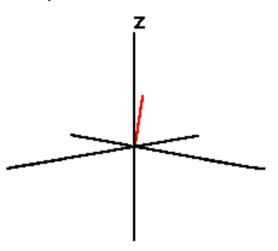
- 90° impuls turns M from z to y'
- 180° impuls turns M from z to -z'

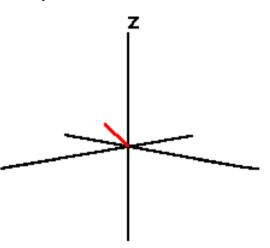


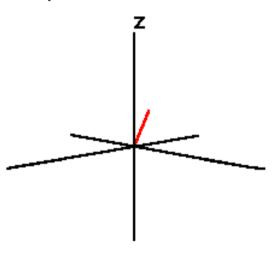


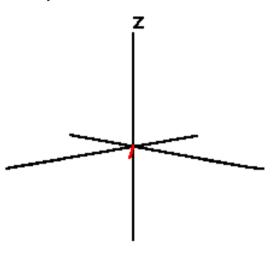


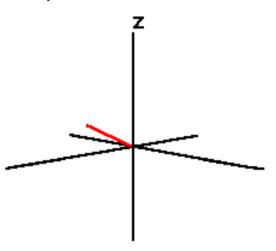


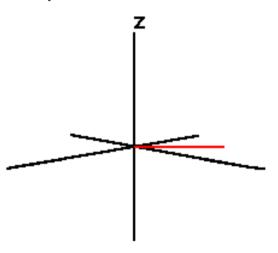




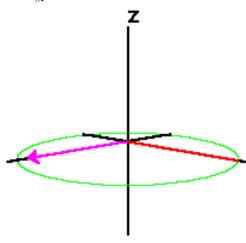






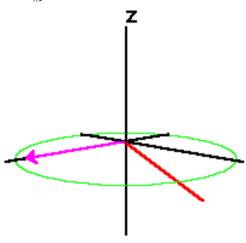


- Magnetization is turned by angle α from any initial position
- e.g. 180° impuls for $\mathbf{M} || y'$



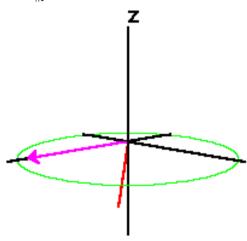
Magnetization vector flip (2)

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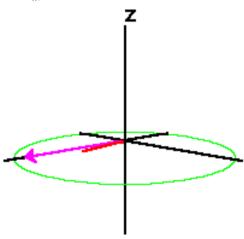


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Introduction

MRI physics

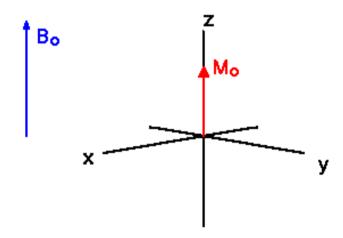
Nuclear spin

Spectroscopy Excitation

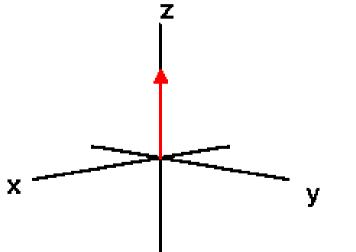
Relaxation

Bloch equation

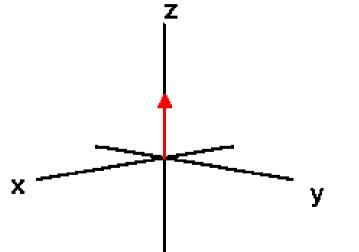
• In equilibrium, $\mathbf{M} = M_0 \mathbf{e}_z$, $M_z = M_0$



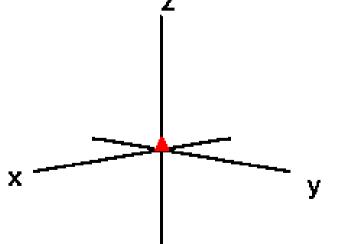
- In equilibrium, $\mathbf{M} = M_0 \mathbf{e}_z$, $M_z = M_0$
- Send an electromagnetic (RF) pulse. For a suitable duration $M_z = 0$



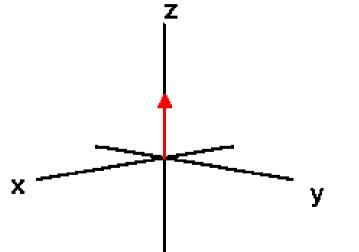
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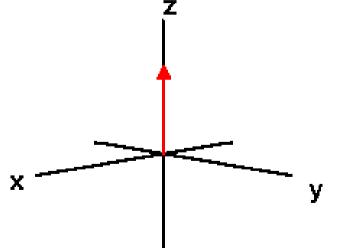
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T_1 relaxation (2)

After the pulse M_z returns to equilibrium.

$$M_z = M_0 \left(1 - e^{-\frac{t}{T_1}}\right)$$
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 $T_1 = t$

 T_1 — spin-lattice relaxation time the energy is dissipated into the lattice as heat

T_1 relaxation (3)

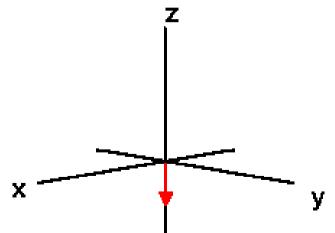
A stronger/longer pulse may cause e.g. $M_z = -M_0$.

$$M_z = M_0 \left(1 - 2\mathrm{e}^{-\frac{t}{T_1}} \right)$$

T_1 relaxation (3)

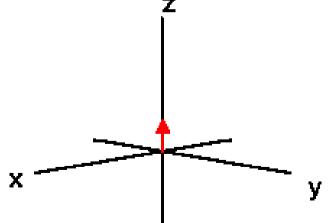
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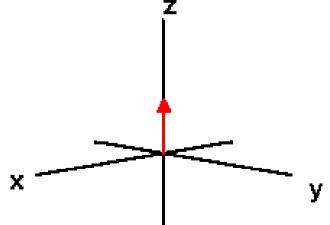


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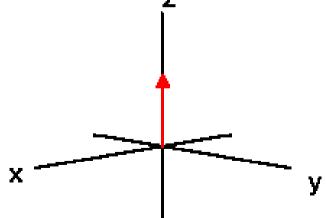
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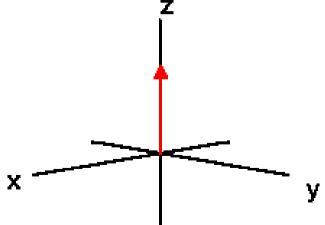
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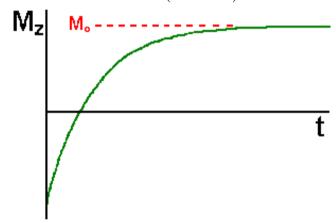
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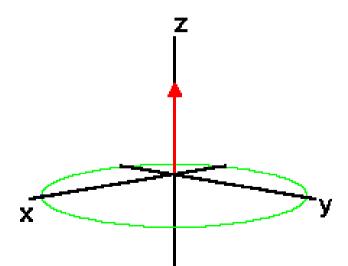


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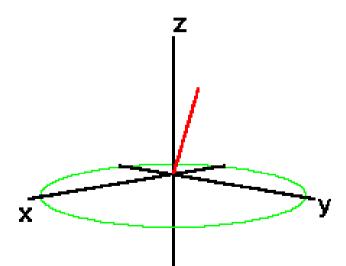
Precession (2)

• Once **M** turns away from the z axis...

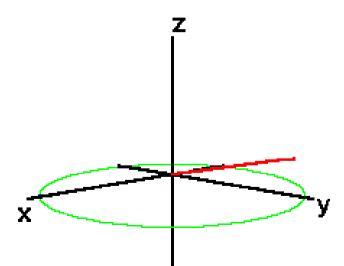


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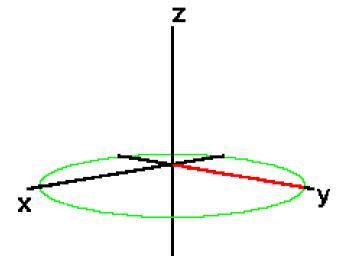
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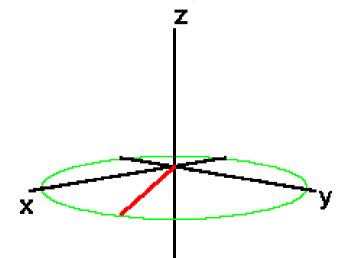
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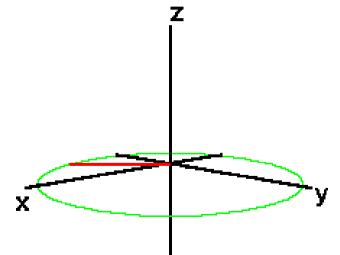
- Once **M** turns away from the z axis...
- ... it starts to rotate around z with $f = \gamma B$



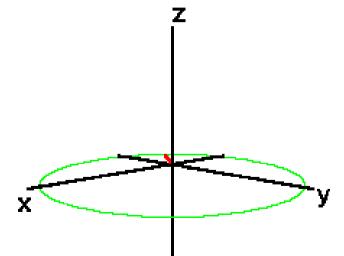
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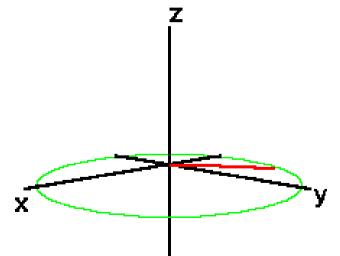


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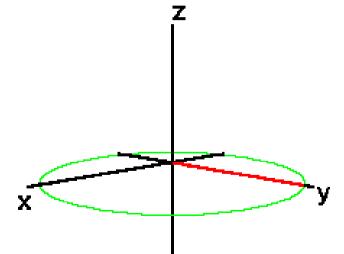


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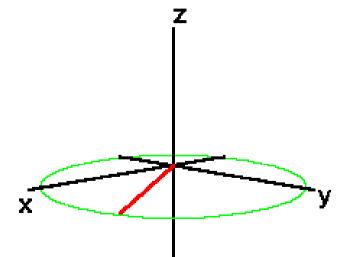
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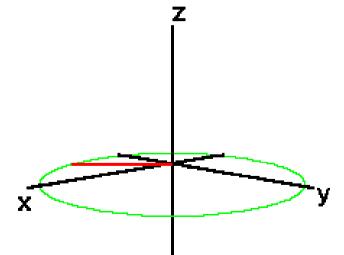
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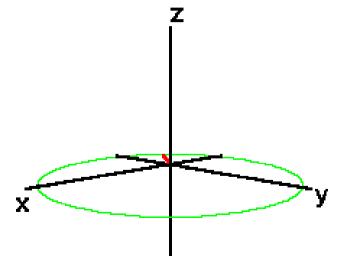
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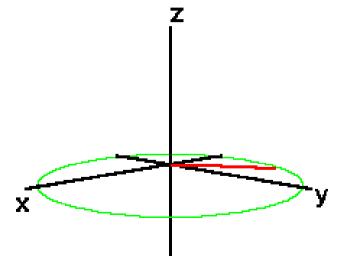
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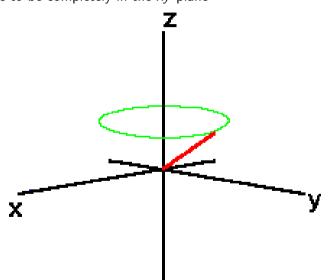
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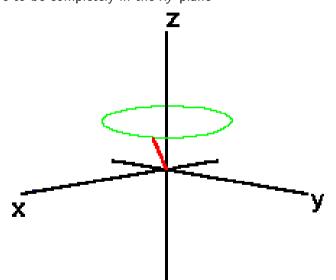
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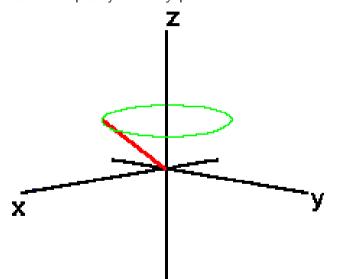


Precession (3)

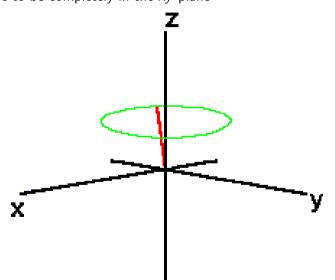


Precession (3)

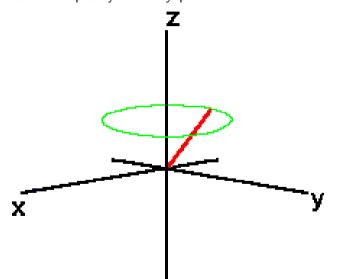


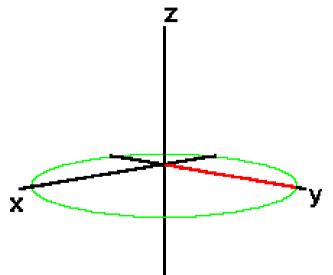


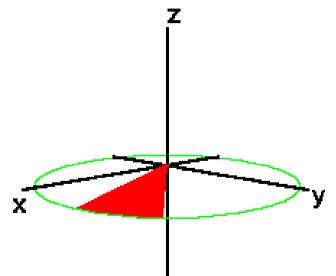
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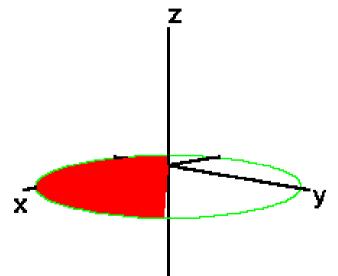


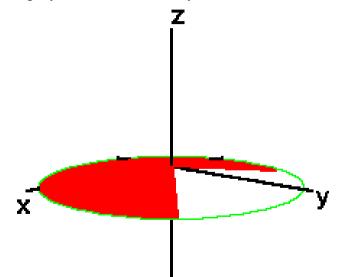
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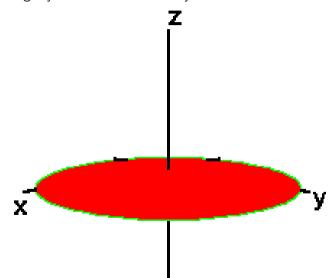




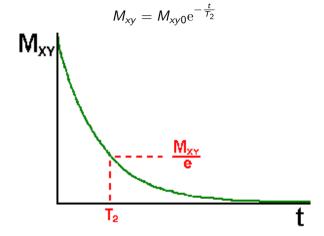






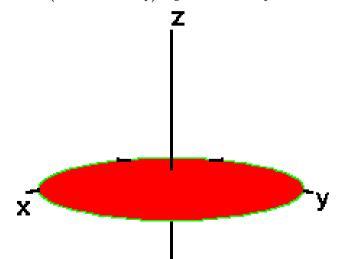


Transversal magnetization M_{xy} decreases

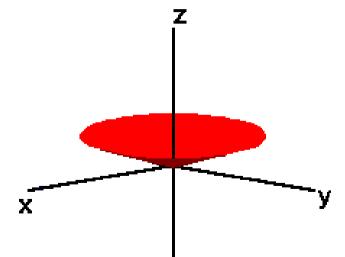


 T_2 — spin-spin relaxation time, $T_2 < T_1$

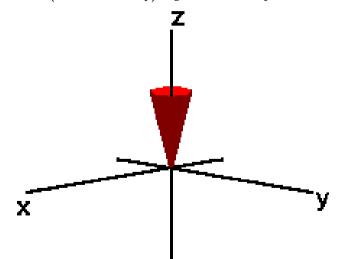
- Transversal magnetization M_{xy} decreases
- At the same time (but more slowly) M_z returns to M_0 .



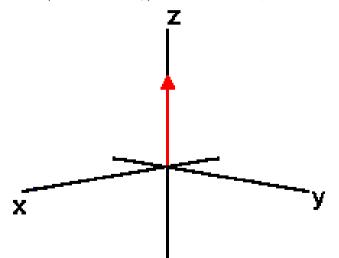
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Reasons for the T_2 relaxation

- Molecular interation (T₂)
- Inhomogeneity of the magnetic field (T_2^{inhom})

Combined time constant T_2^* :

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2^{\mathsf{inhom}}}$$

Molecular movement (because of the magnetic field inhomogeneity)

Introduction MRI physics

- Temperature
- Viscosity

Typical relaxation times

	1.5 T		3 T	
tissue	T_1 [ms]	T_2 [ms]	T_1 [ms]	T_2 [ms]
fat	260	80	420	100
muscle	870	45	1300	40
brain (gray matter)	900	100	1600	100
brain (white matter)	780	90	900	70
liver	500	40	800	34
cerebrospinal fluid	2400	160	4100	500

Reported values differ significantly.

Introduction

MRI physics

Nuclear spin

Excitation

Relaxation

Bloch equation

Bloch equation

$$rac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} = \gamma\mathbf{M} imes \mathbf{B}$$

where **B** is the total magnetic field $(\mathbf{B}_0 + \mathbf{B}_1)$.

$$\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} = \gamma \mathbf{M} \times \mathbf{B}$$

substituting for B, add losses and use the rotating frame of reference

$$\frac{\mathrm{d}M_{x'}}{\mathrm{d}t} = (\omega_0 - \omega)M_{y'} - \frac{M_{x'}}{T_2}$$

$$\frac{\mathrm{d}M_{y'}}{\mathrm{d}t} = -(\omega_0 - \omega)M_{x'} + 2\pi\gamma B_1 M_z - \frac{M_{y'}}{T_2}$$

$$\frac{\mathrm{d}M_z}{\mathrm{d}t} = -2\pi\gamma B_1 M_{y'} - \frac{M_z - M_{z0}}{T_1}$$

where $\omega_0 = 2\pi f_0 = 2\pi \gamma B_0$, ω is the spin rotation frequency.

Bloch equation diagram

