# Humanoid robots - Walking (& Balancing)

Introduction to the guest lecture by Prof. Sergej Čelikovský

doc. Mgr. Matěj Hoffmann, Ph.D.

## **Motivation**



A Compilation of Robots Falling Down at the DARPA Robotics Challenge <a href="https://youtu.be/g0TaYhipOfo">https://youtu.be/g0TaYhipOfo</a>



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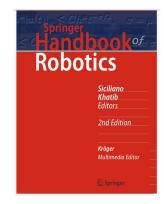
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Schomos

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The promise of legged robots over wheeled robots is to provide improved mobility over rough terrain. Unfortunately, this promise comes at the cost of a significant increase in complexity. We now have a good understanding of how to make legged robots walk and run dynamically, but further research is still necessary to make them walk and run efficiently in terms of energy, speed, reactivity, versatility, and robustness. In this chapter, we will discuss how legged robots are usually modeled, how their stability analysis is approached, how dynamic motions are generated and controlled, and finally summarize the current trends in trying to improve their performanproblem is avoiding to fall. This can cult since legged robots have to rely available contact forces to do so. The ity of leg motions appears to be a k this respect, as current control soluti continuous anticipation of future me some form of model predictive control ing more specifically on limit cycles stability.

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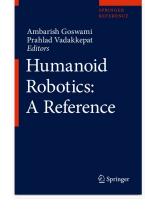
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## Honda Asimo - Fully actuated walking



All New Honda Asimo 2018 at the USA Science and Engineering Festival <a href="https://youtu.be/1urL\_X\_vp7w">https://youtu.be/1urL\_X\_vp7w</a>

# Passive dynamic walking

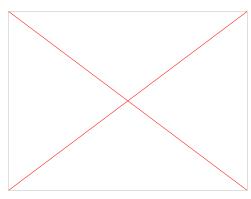


McGeer and Passive Dynamic Bipedal Walking https://youtu.be/WOPED7I5Lac

McGeer, T. (1990). Passive dynamic walking. *Int. J. Robotics Res.*, 9(2), 62-82.







Collins, S., Ruina, A., Tedrake, R., & Wisse, M. (2005). Efficient bipedal robots based on passive-dynamic walkers. *Science*, *307*(5712), 1082-1085.

## Passive dynamic walker

#### Tad McGeer

School of Engineering Science Simon Fraser University Burnaby, British Columbia, Canada V5A 1S6

#### **Passive Dynamic** Walking

Fig. 2. General arrangement of a 2D biped. It includes legs of arbitrary mass and inertia, semicircular feet, and a point mass at the hip.

#### Abstract

There exists a class of two-legged machines for which walking is a natural dynamic mode. Once started on a shallow slope, a machine of this class will settle into a steady gait quite comparable to human walking, without active control or enerey input. Interpretation and analysis of the physics are straightforward: the walking cycle, its stability, and its sensitivity to parameter variations are easily calculated. Experiments with a test machine verify that the passive walking effect can be readily exploited in practice. The dynamics are most clearly demonstrated by a machine powered only by gravity, but they can be combined easily with active energy input to produce efficient and dextrous walking over a broad range of terrain.

#### 1. Static vs. Dynamic Walking

Research on legged locomotion is motivated partly by fundamental curiousity about its mechanics, and partly by the practical utility of machines capable of traversing uneven surfaces. Increasing general interest in robotics over recent years has coincided with the appearance of a wide variety of legged machines. A brief classification will indicate where our own work fits in. First one should distinguish between static and dynamic machines. The former maintain static equilibrium throughout their motion. This requires at least tory), Instead the "feedforward" step is treated as a four legs and, more commonly, six. It also imposes a speed restriction, since cyclic accelerations must be limited in order to minimize inertial effects. Outstanding examples of static walkers are the Odex series (Russell 1983) and the Adaptive Suspension Vehicle (Waldron 1986). Dynamic machines, on the other hand, are more like people; they can have fewer legs than static machines, and are potentially faster.

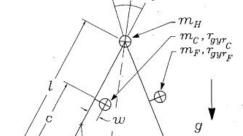
The International Journal of Robotics Research. Vol. 9, No. 2, April 1990. © 1990 Massachusetts Institute of Technology.

#### 2. Dynamics vs. Control

Our interest is in dynamic walking machines, which for our purposes can be classified according to the role of active control in generating the gait. At one end of the spectrum is the biped of Mita et al. (1984), whose motion is generated entirely by linear feedback control. At the end of one step, joint angles are commanded corresponding to the end of the next step. and the controller attempts to null the errors. There is no explicit specification of the trajectory between these end conditions. Yamada, Furusho, and Sano (1985) took an approach that also relies on feedback. but in their machine it is used to track a fully specified trajectory rather than just to close the gap between start and end positions. Meanwhile the stance leg is left free to rotate as an inverted pendulum, which, as we shall discuss, is a key element of passive walking. Similar techniques are used in biped walkers by Takanishi et al. (1985), Lee and Liao (1988), and Zheng, Shen, and Sias (1988).

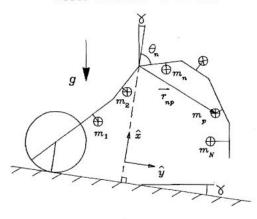
By contrast the bipeds of Miura and Shimoyama (1984) generate their gait by feedforward rather than feedback; joint torque schedules are precalculated and played back on command. Again the stance leg is left free. However, the "feedforward" gait is unstable, so small feedback corrections are added to maintain the walking cycle. Most significantly, these are not applied continuously (i.e., for tracking of the nominal trajecprocessor whose output (the end-of-step state) varies with the input (the start-of-step state). Thus the feedback controller responds to an error in tracking by modifying initial conditions for subsequent steps, and so over several steps the error is eliminated. In this paper you will see analysis of a similar process. Raibert (1986) has developed comparable concepts but with a more pure implementation, and applied them with great success to running machines having from one to

All of these machines use active control in some form to generate the locomotion pattern. They can be



- DOP DOC

N-LINK 2-D CHAIN WITH ROLLING SUPPORT

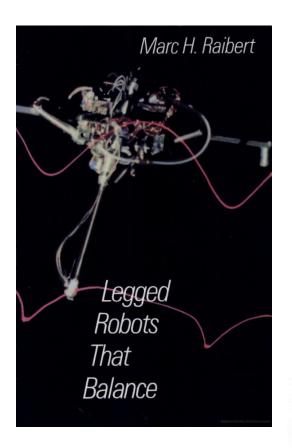


# **Boston dynamics - Atlas**



Atlas Gets a Grip | Boston Dynamics - 2023 https://youtu.be/-e1\_QhJ1EhQ

## Where it started....



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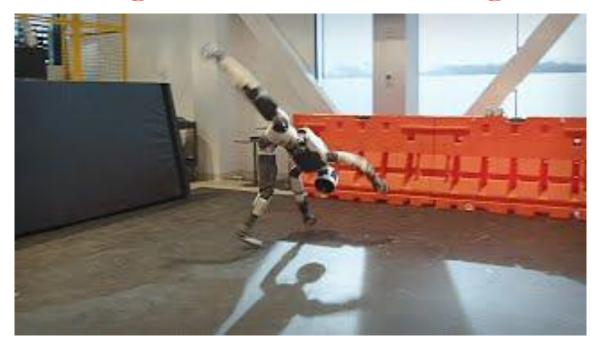


The MIT Press, Cambridge, Massachusetts Copyright © 1986 by Marc H. Raibert All rights reserved. Published 1986 Printed in the United States of America Robots from MIT's Leg Lab <a href="https://youtu.be/XFXj81mvInc">https://youtu.be/XFXj81mvInc</a>

# Do we need modeling?

- Or can we do with machine learning / deep learning like in grasping?
- Marc Raibert, CEO Boston Dynamics, IROS, Kyoto, October 2022:
  - In everything you have seen from Boston Dynamics till now, there is zero machine learning / deep learning.
  - Whenever we had to choose whether to put machine learning or a bunch of engineers on the problem, so far we always went for the engineers.
- How are Boston Dynamics robots controlled?
  - Principles originate in the early Raibert's work modeling and engineering.
  - Heavy use of Model Predictive Control (MPC).

## Learning humanoids walking

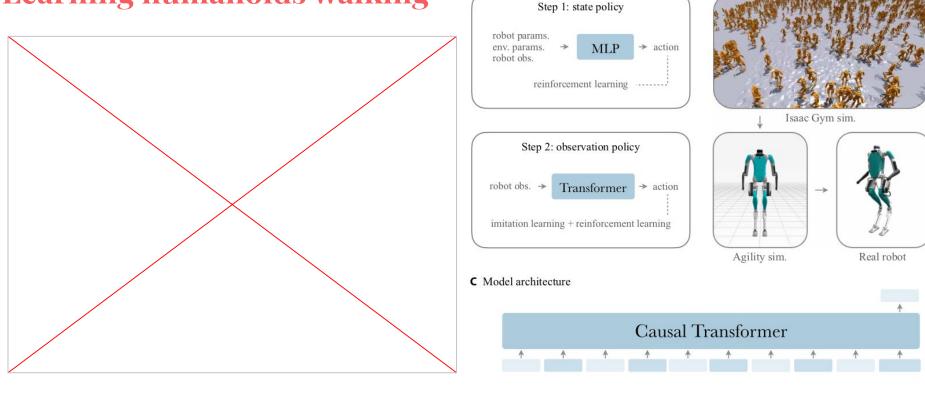


Walk, Run, Crawl, RL Fun |
Boston Dynamics | Atlas;
March 2025
<a href="https://youtu.be/l44\_zbEwz\_w">https://youtu.be/l44\_zbEwz\_w</a>
?si=JgoBHCKNwXC5IUVS

Atlas is demonstrating policies developed using reinforcement learning with references from human motion capture and animation.

Research partnership between Boston Dynamics and the Robotics and Al Institute (RAI Institute).

## Learning humanoids walking



A Model training

B Sim-to-real transfer

Radosavovic, I., Xiao, T., Zhang, B., Darrell, T., Malik, J., & Sreenath, K. (2024). Real-world humanoid locomotion with reinforcement learning. Science Robotics, 9(89), eadi9579.

### Resources

- Books / book sections
  - [Chapter 5 Balance control in Nenchev, D. N., Konno, A., & Tsujita, T. (2018). Humanoid robots:
     Modeling and control. Butterworth-Heinemann.]
- Articles
  - o McGeer, T. (1990). Passive dynamic walking. Int. J. Robotics Res., 9(2), 62-82.
  - Radosavovic, I., Xiao, T., Zhang, B., Darrell, T., Malik, J., & Sreenath, K. (2024). Real-world humanoid locomotion with reinforcement learning. Science Robotics, 9(89), eadi9579.