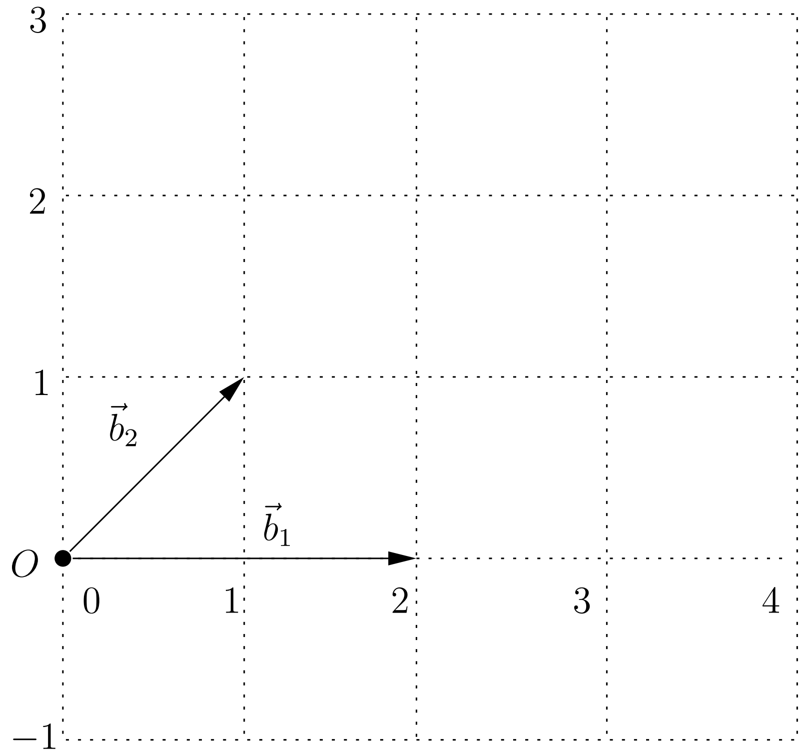
GVG Lab-05 Solution

Task 1. *The following picture shows a coordinate system $\sigma = (O, \beta)$ and a basis $\beta = (\vec{b}_1, \vec{b}_2)$.*



1. (a) *Find a coordinate system $\sigma' = (O', \beta')$, $\beta' = (\vec{b}'_1, \vec{b}'_2)$, whose basis vector $\vec{b}'_1$ has in basis β coordinates*

$$\vec{b}'_{1\beta} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

and its origin O' is in the coordinate system σ described by vector

$$\vec{O}'_\beta = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

and there exists point X described by vector $\vec{X}$ in σ and vector $\vec{X}'$ in σ' with coordinates

$$\vec{X}_\beta = \begin{bmatrix} 3/2 \\ 1 \end{bmatrix}, \quad \vec{X}'_{\beta'} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and draw it on the picture.

(b) *Write the coordinates of vector $\vec{b}'_2$ in basis β.*

(c) *Write the coordinates of the point O in coordinate system σ'.*

(d) *Write the coordinates of basis vectors of β in basis β'.*

2. (a) *Find a coordinate system $\sigma' = (O', \beta')$, $\beta' = (\vec{b}'_1, \vec{b}'_2)$, when you know that the basis vectors of basis β have in basis β' coordinates*

$$\vec{b}_{1\beta'} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}, \quad \vec{b}_{2\beta'} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

and there exists point X described by vector $\vec{X}$ in σ and vector $\vec{X}'$ in σ' with coordinates

$$\vec{X}_\beta = \begin{bmatrix} 2 \\ -1/2 \end{bmatrix}, \quad \vec{X}'_{\beta'} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

draw the coordinate system on the picture.

(b) *Write the coordinates of basis vectors of β' in basis β.*

(c) *Write the coordinates of point O in the coordinate system σ' and point O' in the coordinate system σ.*

Solution: By applying the same ideas as in the solution of Task 5 from Test-α we can write

$$\vec{X} = \vec{X}' + \vec{O}'$$

After passing to the coordinates of the above vectors in basis β we get

$$\vec{X}_\beta = \vec{X}'_\beta + \vec{O}'_\beta$$

$$\vec{X}_\beta = \mathtt{A}_{\beta' \to \beta} \vec{X}'_{\beta'} + \vec{O}'_\beta$$

$$\vec{X}_\beta = \begin{bmatrix} \vec{b}'_{1\beta} & \vec{b}'_{2\beta} \end{bmatrix} \vec{X}'_{\beta'} + \vec{O}'_\beta$$

$$\begin{bmatrix} 3/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & a \\ -1 & b \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

Rewriting the above matricial equation in terms of individual equations we obtain

$$\frac{3}{2} = 1 + a + \frac{1}{2}, \quad 1 = -1 + b + 1$$

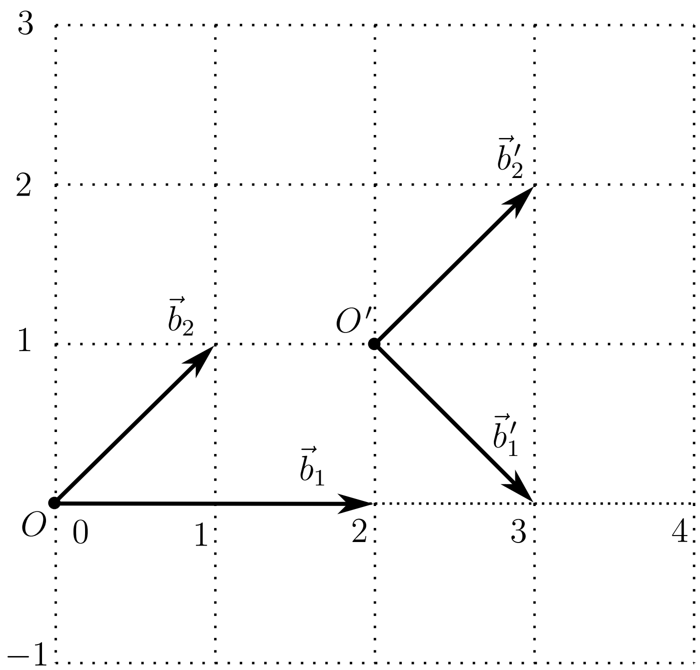
and hence

$$a = 0, \quad b = 1$$

which means that

$$\vec{b}'_{2\beta} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

In the picture the desired coordinate system σ' looks as follows:



□

Task 2. *Find coordinates of the image point which is the projection of point* $[1, 1, 1]^\top$ *by the camera with the following camera projection matrix*

$$\mathtt{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Solution: See the methodology in the solution to Lab 02, Task 1. □

Answer: $[u, v] = \begin{bmatrix} \frac{1}{2} & 1 \end{bmatrix}^\top$.

Task 3. *Find the camera calibration matrix* $\mathtt{K}$, *rotation* $\mathtt{R}$, *and the projection center* $\vec{C}_\delta$ *of a camera with the camera projection matrix*

$$\mathtt{P} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Solution: See the methodology in the solution to Lab 03, Task 3. The only difference is that in this task $\mathtt{KR} = \mathtt{P}_{1:3,1:3}$. □

Answer:

$$\mathtt{K} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathtt{R} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad \vec{C}_\delta = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

Task 4. *Denote the image coordinates by* $[u, v]^\top$. *Write down coordinates of all points in the three-dimensional space that projects on the line* $v = 0$ *by a camera with the following scaled image projection matrix*

$$\mathtt{Q} = \xi \mathtt{P}_\beta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Solution: See the solution to Lab 02, Task 4. □