

GVG Lab-04 Solution

Task 1. Create companion matrix for polynomial $2x^3 - 6x^2 + 11x - 6$.

Solution: The companion matrix M_f for a general univariate polynomial $f = a_n x^n + \dots + a_1 x + a_0$, $a_n \neq 0$ is defined to be

$$M_f = \begin{bmatrix} 0 & \dots & 0 & -\frac{a_0}{a_n} \\ 1 & \dots & 0 & -\frac{a_1}{a_n} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & -\frac{a_{n-1}}{a_n} \end{bmatrix}$$

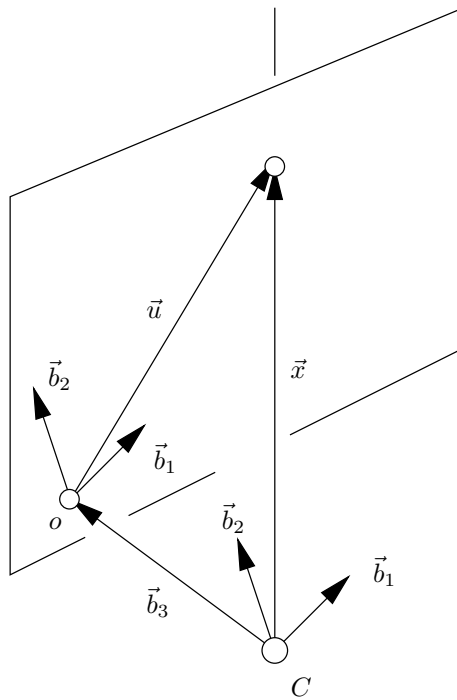
It can be verified by direct computation that $\det(xI - M_f) = \frac{1}{a_n} \cdot f$, which means that the roots of f can be obtained as the eigenvalues of M_f .

For the polynomial given in the task the companion matrix equals

$$M_f = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -\frac{11}{2} \\ 0 & 1 & 3 \end{bmatrix}$$

□

Task 2. Find a basis $\alpha = (\vec{a}_1, \vec{a}_2, \vec{a}_3)$ such that vector \vec{x} , which is obtained as $\vec{u} = 2\vec{b}_1 + 3\vec{b}_2$ as shown in the following figure, would have coordinates in α equal to $[2, 3, 2]^T$. Write down the coordinates of the vectors of α in basis $\beta = (\vec{b}_1, \vec{b}_2, \vec{b}_3)$.



Solution: We can see that

$$\vec{x} = \vec{u} + \vec{b}_3$$

By the task, we need to find linearly independent free vectors \vec{a}_1 , \vec{a}_2 and \vec{a}_3 such that

$$2\vec{a}_1 + 3\vec{a}_2 + 2\vec{a}_3 = 2\vec{b}_1 + 3\vec{b}_2 + \vec{b}_3$$

There are, obviously, infinitely many choices for $\vec{a}_1, \vec{a}_2, \vec{a}_3$, since \vec{a}_1 and \vec{a}_2 can be chosen to be arbitrary linearly independent vectors and \vec{a}_3 is defined then by

$$\vec{a}_3 = \frac{1}{2} \left(2\vec{b}_1 + 3\vec{b}_2 + \vec{b}_3 - 2\vec{a}_1 - 3\vec{a}_2 \right)$$

The simplest choice is to take

$$\vec{a}_1 = \vec{b}_1, \vec{a}_2 = \vec{b}_2, \vec{a}_3 = \frac{1}{2}\vec{b}_3.$$

The coordinates of the vectors of α in basis β are

$$\vec{a}_{1\beta} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{a}_{2\beta} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{a}_{3\beta} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

□

Task 3. Let us have a camera with projection matrix

$$P = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Write the cosine of the angle between rays passing through image points $[0, 0]^T$ and $[1, 1]^T$.

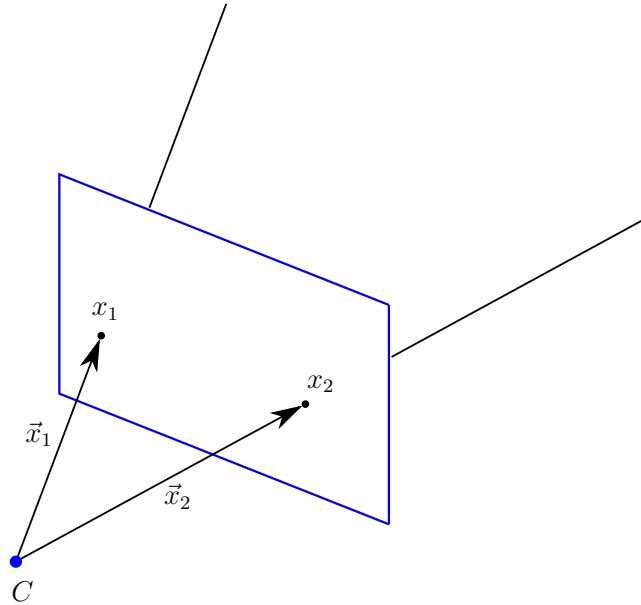


Figure 1: Two projection rays passing through the image points x_1 and x_2

Solution: We first compute the camera calibration matrix of the given camera projection matrix. For this we decompose the left 3×3 block B of P :

$$k_{23} = \mathbf{b}_2^\top \mathbf{b}_3 = [0 \quad 1 \quad 1] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 1,$$

$$k_{13} = \mathbf{b}_1^\top \mathbf{b}_3 = [1 \quad 0 \quad 1] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 1,$$

$$k_{22}^2 + 1^2 = \mathbf{b}_2^\top \mathbf{b}_2 = [0 \quad 1 \quad 1] \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 2 \Rightarrow k_{22} = 1,$$

$$k_{12} \cdot 1 + 1 \cdot 1 = \mathbf{b}_1^\top \mathbf{b}_2 = [1 \quad 0 \quad 1] \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 1 \Rightarrow k_{12} = 0,$$

$$k_{11}^2 + 0^2 + 1^2 = \mathbf{b}_1^\top \mathbf{b}_1 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 2 \Rightarrow k_{11} = 1.$$

Hence

$$\mathbf{K} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Remark. If the left 3×3 block \mathbf{B} of \mathbf{P} is an upper triangular matrix, then $\mathbf{K} = \mathbf{B}$ or $\mathbf{K} = \mathbf{BR}_x$ or $\mathbf{K} = \mathbf{BR}_y$ or $\mathbf{K} = \mathbf{BR}_z$, where \mathbf{R}_a is a rotation about axis a by 180° . In other words, the only upper triangular rotations are $\mathbf{I}, \mathbf{R}_x, \mathbf{R}_y, \mathbf{R}_z$. To prove this notice that $\mathbf{B} = \mathbf{KR}$ and since \mathbf{K} must also be upper triangular, then so is \mathbf{R} . This is because $\mathbf{R} = \mathbf{K}^{-1}\mathbf{B}$ and the inverse of an upper triangular matrix is upper triangular. The only upper triangular rotations are $\mathbf{I}, \mathbf{R}_x, \mathbf{R}_y, \mathbf{R}_z$. To show this notice that the last row must be equal to $\begin{bmatrix} 0 & 0 & \pm 1 \end{bmatrix}$ since the norms of rows must be equal to 1. Further, the first column must be equal to $\begin{bmatrix} \pm 1 & 0 & 0 \end{bmatrix}^\top$ for the same reason. Since $r_{11} = 1$, then $r_{12} = r_{13} = 0$. Since $r_{33} = 1$, then $r_{13} = r_{23} = 0$. Since $r_{21} = r_{23} = 0$ and $\det \mathbf{R} = r_{11}r_{22}r_{33} = 1$, then $r_{22} = \frac{1}{r_{11}r_{33}}$. Thus, there are 4 possibilities how to choose signs of r_{11} and r_{33} which gives rise to 4 rotations $\mathbf{R} = \mathbf{I}, \mathbf{R}_x, \mathbf{R}_y, \mathbf{R}_z$.

The direction vectors of the rays passing through the given image points are given by

$$\vec{x}_{1\beta} = \begin{bmatrix} \vec{u}_{1\alpha} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{x}_{2\beta} = \begin{bmatrix} \vec{u}_{2\alpha} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

To obtain the angle between the direction vectors by evaluating the scalar product of the vectors, we need to pass to an orthogonal basis (e.g. γ):

$$\begin{aligned} \vec{x}_{1\gamma} = \mathbf{K}^{-1}\vec{x}_{1\beta} &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \quad \vec{x}_{2\gamma} = \mathbf{K}^{-1}\vec{x}_{2\beta} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ \cos \angle(\vec{x}_1, \vec{x}_2) &= \frac{\vec{x}_{1\gamma}^\top \vec{x}_{2\gamma}}{\|\vec{x}_{1\gamma}\| \|\vec{x}_{2\gamma}\|} = \frac{\begin{bmatrix} -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}{\sqrt{3} \cdot 1} = \frac{1}{\sqrt{3}} \end{aligned}$$

Remark. Actually, we could use another orthogonal basis, namely κ (see [1, Figure 7.2 (d)]). The transition matrix $\mathbf{T}_{\beta \rightarrow \kappa}$ equals $(\mathbf{KR})^{-1} = \mathbf{P}_{1:3,1:3}^{-1}$. However, since $\mathbf{P}_{1:3,1:3} = \mathbf{K}$ in this task, then

$$\begin{aligned} \vec{x}_{1\kappa} &= \vec{x}_{1\gamma}, \quad \vec{x}_{2\kappa} = \vec{x}_{2\gamma} \\ \cos \angle(\vec{x}_1, \vec{x}_2) &= \frac{\vec{x}_{1\kappa}^\top \vec{x}_{2\kappa}}{\|\vec{x}_{1\kappa}\| \|\vec{x}_{2\kappa}\|} = \frac{1}{\sqrt{3}} \end{aligned}$$

Hence computing $\cos \angle(\vec{x}_1, \vec{x}_2)$ using this method requires less computations. □

Task 4 (P3P Problem).

Compute the calibrated camera pose $(\mathbf{R}, \vec{C}_\delta)$ of the camera with camera calibration matrix

$$\mathbf{K} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

if you know that 3 world points

$$\vec{X}_{1\delta} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{X}_{2\delta} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{X}_{3\delta} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

project to the following image points

$$\vec{u}_{1\alpha} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad \vec{u}_{2\alpha} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \quad \vec{u}_{3\alpha} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

respectively.

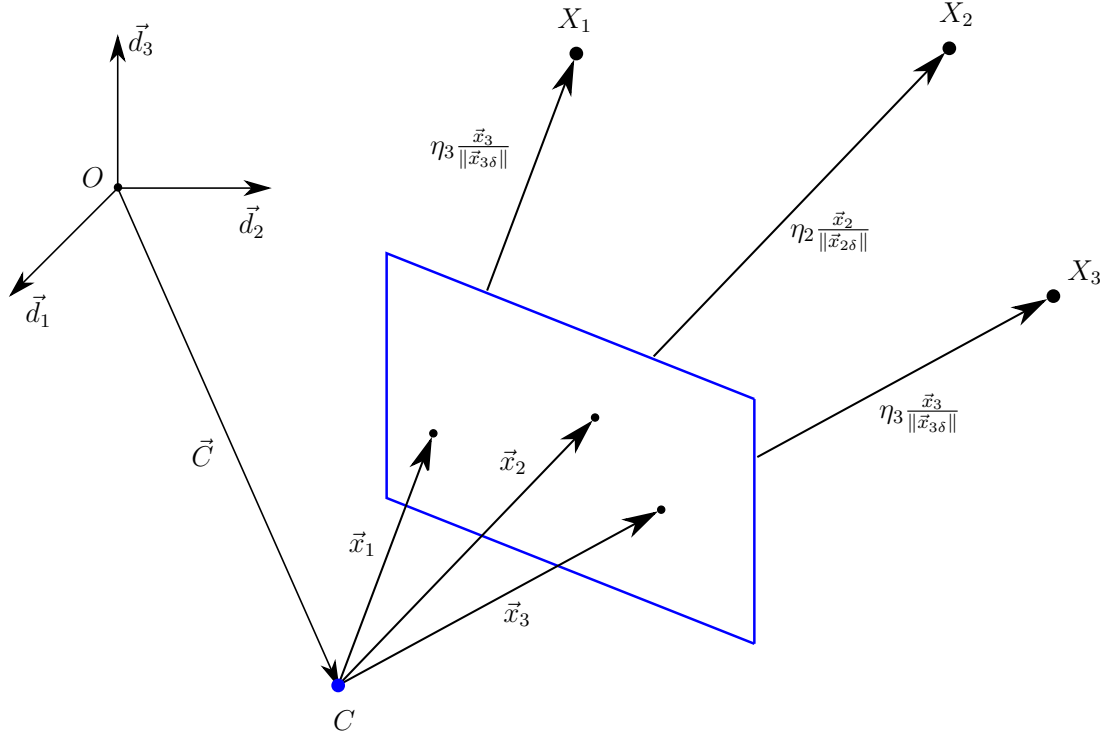


Figure 2: P3P Problem

Solution: We first obtain the coordinates of the vectors representing the image points in the camera coordinate system (C, β) :

$$\vec{x}_{1\beta} = \begin{bmatrix} \vec{u}_{1\alpha} \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}, \quad \vec{x}_{2\beta} = \begin{bmatrix} \vec{u}_{2\alpha} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}, \quad \vec{x}_{3\beta} = \begin{bmatrix} \vec{u}_{3\alpha} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

To obtain the angle between the direction vectors by evaluating the scalar product of the vectors, we need to pass to an orthogonal basis (e.g. γ):

$$\vec{x}_{1\gamma} = K^{-1} \vec{x}_{1\beta} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{x}_{2\gamma} = K^{-1} \vec{x}_{2\beta} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\vec{x}_{3\gamma} = K^{-1} \vec{x}_{3\beta} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

The cosines of the angles between the rays are then given by

$$c_{12} = \cos \angle(\vec{x}_1, \vec{x}_2) = \frac{\vec{x}_{1\gamma}^\top \vec{x}_{2\gamma}}{\|\vec{x}_{1\gamma}\| \|\vec{x}_{2\gamma}\|} = \frac{\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}{\sqrt{2} \cdot \sqrt{3}} = 0$$

$$c_{23} = \cos \angle(\vec{x}_2, \vec{x}_3) = \frac{\vec{x}_{2\gamma}^\top \vec{x}_{3\gamma}}{\|\vec{x}_{2\gamma}\| \|\vec{x}_{3\gamma}\|} = \frac{\begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}}{\sqrt{3} \cdot \sqrt{6}} = 0$$

$$c_{31} = \cos \angle(\vec{x}_3, \vec{x}_1) = \frac{\vec{x}_{3\gamma}^\top \vec{x}_{1\gamma}}{\|\vec{x}_{3\gamma}\| \|\vec{x}_{1\gamma}\|} = \frac{[-1 \quad -2 \quad 1] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}{\sqrt{6} \cdot \sqrt{2}} = 0$$

If we denote by η_1, η_2, η_3 the lengths of vectors $\overrightarrow{CX_1}, \overrightarrow{CX_2}, \overrightarrow{CX_3}$ in the world units and by d_{12}, d_{23}, d_{31} the lengths of vectors $\overrightarrow{X_1X_2}, \overrightarrow{X_2X_3}, \overrightarrow{X_3X_1}$ in the world units, then by looking at the triangles $\triangle CX_1X_2, \triangle CX_2X_3, \triangle CX_3X_1$ we can write the equations coming from the cosine rule ([1, Equations 7.60-7.62]):

$$d_{12}^2 = \eta_1^2 + \eta_2^2 \quad (1)$$

$$d_{23}^2 = \eta_2^2 + \eta_3^2 \quad (2)$$

$$d_{31}^2 = \eta_3^2 + \eta_1^2 \quad (3)$$

We have used the fact that all the cosines c_{12}, c_{23}, c_{31} are zero. We compute the distances between the world points:

$$d_{12} = \|\vec{X}_{1\delta} - \vec{X}_{2\delta}\| = \left\| \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\| = \sqrt{2} \Rightarrow d_{12}^2 = 2$$

$$d_{23} = \|\vec{X}_{2\delta} - \vec{X}_{3\delta}\| = \left\| \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\| = \sqrt{2} \Rightarrow d_{23}^2 = 2$$

$$d_{31} = \|\vec{X}_{3\delta} - \vec{X}_{1\delta}\| = \left\| \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\| = \left\| \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\| = \sqrt{2} \Rightarrow d_{31}^2 = 2$$

We can rewrite Equations (1), (2), (3) in a matrix form:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_1^2 \\ \eta_2^2 \\ \eta_3^2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} \eta_1^2 \\ \eta_2^2 \\ \eta_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Taking into account that the depths η_1, η_2, η_3 must be positive, we get

$$\eta_1 = 1, \quad \eta_2 = 1, \quad \eta_3 = 1.$$

Finally, we compute the camera pose $(\mathbf{R}, \vec{C}_\delta)$ using [1, Equations 7.122-7.124]:

$$\eta_1 \frac{\vec{x}_{1\gamma}}{\|\vec{x}_{1\gamma}\|} = \mathbf{R}(\vec{X}_{1\delta} - \vec{C}_\delta)$$

$$\eta_2 \frac{\vec{x}_{2\gamma}}{\|\vec{x}_{2\gamma}\|} = \mathbf{R}(\vec{X}_{2\delta} - \vec{C}_\delta)$$

$$\eta_3 \frac{\vec{x}_{3\gamma}}{\|\vec{x}_{3\gamma}\|} = \mathbf{R}(\vec{X}_{3\delta} - \vec{C}_\delta)$$

Eliminating \vec{C}_δ and using the properties of the rotation matrix we get [1, Equations 7.125, 7.126, 7.129]:

$$\underbrace{\frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{\vec{Z}_{2e}} = \mathbf{R} \left(\underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{\vec{Z}_{2\delta}} \right)$$

$$\underbrace{\frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} - \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{\vec{Z}_{3\epsilon}} = \mathbf{R} \left(\underbrace{\begin{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{pmatrix}}_{\vec{Z}_{3\delta}} \right)$$

$$\underbrace{\left(\frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right)}_{\vec{Z}_{1\epsilon}} \times \underbrace{\left(\frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} - \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right)}_{\vec{Z}_{1\delta}} = \mathbf{R} \left(\underbrace{\left(\begin{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{pmatrix} \times \begin{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{pmatrix} \right)}_{\vec{Z}_{1\delta}} \right)$$

The rotation matrix \mathbf{R} can be computed using [1, Equation 7.134]:

$$\begin{aligned} \mathbf{R} &= [\vec{Z}_{1\epsilon} \quad \vec{Z}_{2\epsilon} \quad \vec{Z}_{3\epsilon}] [\vec{Z}_{1\delta} \quad \vec{Z}_{2\delta} \quad \vec{Z}_{3\delta}]^{-1} = \\ &= \begin{bmatrix} \frac{3\sqrt{2}-2\sqrt{3}-\sqrt{6}}{6} & -\frac{2\sqrt{3}+3\sqrt{2}}{6} & -\frac{\sqrt{6}+3\sqrt{2}}{6} \\ \frac{\sqrt{3}-\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{6}}{3} \\ \frac{3\sqrt{2}+2\sqrt{3}+\sqrt{6}}{6} & \frac{2\sqrt{3}-3\sqrt{2}}{6} & \frac{\sqrt{6}-3\sqrt{2}}{6} \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}^{-1} = \\ &= \begin{bmatrix} \frac{3\sqrt{2}-2\sqrt{3}-\sqrt{6}}{6} & -\frac{2\sqrt{3}+3\sqrt{2}}{6} & -\frac{\sqrt{6}+3\sqrt{2}}{6} \\ \frac{\sqrt{3}-\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{6}}{3} \\ \frac{3\sqrt{2}+2\sqrt{3}+\sqrt{6}}{6} & \frac{2\sqrt{3}-3\sqrt{2}}{6} & \frac{\sqrt{6}-3\sqrt{2}}{6} \end{bmatrix} \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{6}}{6} \\ 0 & \frac{\sqrt{3}}{3} & -\frac{\sqrt{6}}{3} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{6} \end{bmatrix} \end{aligned}$$

The camera projection center \vec{C}_δ can be computed using [1, Equation 7.135]:

$$\vec{C}_\delta = \vec{X}_{1\delta} - \mathbf{R}^\top \eta_1 \frac{\vec{x}_{1\gamma}}{\|\vec{x}_{1\gamma}\|} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

□

References

- [1] Tomas Pajdla, *Elements of geometry for computer vision*, https://cw.fel.cvut.cz/wiki/_media/courses/gvg/pajdla-gvg-lecture-2021.pdf.