

# Learning by Approximation

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Today two examples:

1. Approximation in least square sense
2. Approximative Q-learning

# Least square approximation

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We have:

- ▶ given tuples  $(x_i, f(x_i)) : (0, 2.1), (1, 3.6), (2, 4.9), (3, 6.6), \dots$
- ▶ approximation of function  $\hat{f}(x, w) = w_1x + w_0$

Task: determine/compute parameters  $w_0, w_1$  with lowest error

How?:

- A: minimize difference in coordinates
- B: maximize error
- C: minimize sum of squared errors
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How? - minimize sum of squared errors.

Define:

$$A: \sum_i (f(x_i) - x_i)^2$$

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How?:

A: find solution of  $E = 0$

B: find maximum of  $E$

C: find minimum of  $E$

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How? Solve:

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B:  $\partial E = 0$

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Derive by:

A:  $x$

B:  $w$

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Evaluate  $\frac{\partial E}{\partial w_0}$ :

A:  $\frac{\partial E}{\partial w_0} = 2 \sum_i (w_1x_i - f(x_i))$

B:  $\frac{\partial E}{\partial w_0} = 2 \sum_i (w_1x_i + 1 - f(x_i))$

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Evaluate:

A:  $\frac{\partial E}{\partial w_0} = w_1 - w_0 + 5$

B:  $\frac{\partial E}{\partial w_0} = 2w_1 + w_0 - 4.2$

C:  $\frac{\partial E}{\partial w_0} = 3w_1 + 3w_0 - 10.6$

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B:  $\frac{\partial E}{\partial w_0} = 2w_1 + w_0 - 4.2$

C:  $\frac{\partial E}{\partial w_0} = (w_1 \cdot 0 + w_0 - 2.1) + (w_1 \cdot 1 + w_0 - 3.6) + (w_1 \cdot 2 + w_0 - 4.9) = 3w_1 + 3w_0 - 10.6$

D:  $\frac{\partial E}{\partial w_0} = w_1 - 2w_0 - 3.1$

## Least square approximation

We have:

- ▶ given tuples  $(x_i, f(x_i)) : (0, 2.1), (1, 3.6), (2, 4.9), (3, 6.6), \dots$
- ▶ approximation of function  $\hat{f}(x, w) = w_1x + w_0$

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Evaluate:

A:  $\frac{\partial E}{\partial w_1} = 5w_1 + 3w_0 - 13.4$

B:  $\frac{\partial E}{\partial w_1} = 2w_1 + 6.2$

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$$-2w_1 + 2.8 = 0 \rightarrow w_1 = 1.4$$

$$w_0 = 1/3(10.6 - 3w_1) = \frac{6.4}{3} \approx 2.133$$

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We have:

- ▶ an unknown grid world
- ▶ a few episodes the robot tried

Today:

- ▶ we approximate Q-function
- ▶  $\hat{q}(s, a, w) = \alpha w_1 + (1 - \alpha)w_0$
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Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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each field in the table is an n-tuple  $(s_t, a_t, s_{t+1}, r_{t+1})$

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SGD briefly:

- ▶ Find  $w$  that minimize  $\sum_t (\text{trial}_t - \hat{q}(s_t, a_t, w))^2$
- ▶ How to do it online?
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Define  $w_1$  update:

$$\text{A: } w_1^{t+1} = w_1^t + \alpha(\hat{q}(s_t, a_t, w^t))s_t a_t$$

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## Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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each field in the table is an n-tuple  $(s_t, a_t, s_{t+1}, r_{t+1})$

$$S = \{-1, 0, 1\}$$

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Task: compute Q-function - from each tuple refine  $w_0, w_1$

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For simplicity:  $\gamma = 1, \alpha = 1$

# Approximative Q-learning

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Initialize w:

A:  $w = (w_1, w_0) = (1, 1)$

B:  $w = (w_1, w_0) = (0, 1)$

C:  $w = (w_1, w_0) = (0, 0)$

D: arbitrarily

## Approximative Q-learning

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# Approximative Q-learning

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$$\blacktriangleright \text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$$

$$t = 0 \quad w = (w_1, w_0) = (0, 0)$$

Transition  $(s_t = 0, a_t = 1, s_{t+1} = 1, r_{t+1} = -2), t = 1:$

Compute:

A: trial = -2

B: trial = 0

C: trial = -1

D: trial = 1

# Approximative Q-learning

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$$t = 0 \quad w = (w_1, w_0) = (0, 0)$$

Transition  $(s_t = 0, a_t = 1, s_{t+1} = 1, r_{t+1} = -2), t = 1$ :

Compute:

$$\text{A: trial} = -2 + \max\{\hat{q}(s_{t+1} = 1, a = 0, w^t), \hat{q}(s_{t+1} = 1, a = 1, w^t)\} = -2 + \max\{0, 0\} = -2$$

$$\text{B: trial} = 0$$

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# Approximative Q-learning

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►  $w \leftarrow w + \alpha(\text{diff})\nabla\hat{q}(s_t, a_t, w)$ ;  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$

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$t = 0$   $w = (w_1, w_0) = (0, 0)$

Transition  $(s_t = 0, a_t = 1, s_{t+1} = 1, r_{t+1} = -2)$ ,  $t = 1$ :  $\text{trial} = -2$

Compute  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$ :

A:  $\text{diff} = 0$

B:  $\text{diff} = 1$

C:  $\text{diff} = -1$

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$t = 0$   $w = (w_1, w_0) = (0, 0)$

Transition  $(s_t = 0, a_t = 1, s_{t+1} = 1, r_{t+1} = -2)$ ,  $t = 1$ :  $\text{trial} = -2$

Compute  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$ :

A:  $\text{diff} = 0$

B:  $\text{diff} = 1$

C:  $\text{diff} = -1$

D:  $\text{diff} = -2 - 0 = -2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
(1, 1, exit, 2)	(-1, 0, exit, -1)	

each field in the table is an n-tuple  $(s_t, a_t, s_{t+1}, r_{t+1})$

$$S = \{-1, 0, 1\}$$

$$A = \{0, 1\}$$

$$\gamma = 1, \alpha = 1$$

$$\hat{q}(s, a, w) = asw_1 + (1 - a)w_0$$

Task: compute Q-function - from each tuple refine  $w_0, w_1$

$$\blacktriangleright w \leftarrow w + \alpha(\text{diff})\nabla\hat{q}(s_t, a_t, w); \text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$$

$$w_1^{t+1} = w_1^t + \alpha(\text{trial} - \hat{q}(s_t, a_t, w^t))s_t a_t$$

$$w_0^{t+1} = w_0^t + \alpha(\text{trial} - \hat{q}(s_t, a_t, w^t))(1 - a_t)$$

$$\blacktriangleright \text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$$

$$t = 0 \quad w = (w_1, w_0) = (0, 0)$$

Transition  $(s_t = 0, a_t = 1, s_{t+1} = 1, r_{t+1} = -2)$ ,  $t = 1$ : trial = -2, diff = -2

Compute :

$$\text{A: } w_1^{t+1} = 2$$

$$\text{B: } w_1^{t+1} = 0$$

$$\text{C: } w_1^{t+1} = 1$$

$$\text{D: } w_1^{t+1} = -2$$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
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Transition  $(s_t = 0, a_t = 1, s_{t+1} = 1, r_{t+1} = -2)$ ,  $t = 1$ : trial = -2, diff = -2

Compute :

$$\text{A: } w_1^{t+1} = 2$$

$$\text{B: } w_1^{t+1} = w_1^t + [\text{diff}]s_t a_t = 0 + (-2) \cdot 1 \cdot 0 = 0$$

$$\text{C: } w_1^{t+1} = 1$$

$$\text{D: } w_1^{t+1} = -2$$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
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$$t = 0 \quad w = (w_1, w_0) = (0, 0)$$

Transition  $(s_t = 0, a_t = 1, s_{t+1} = 1, r_{t+1} = -2)$ ,  $t = 1$ : trial = -2, diff = -2  $\Rightarrow w_1^{t+1} = 0$

Compute :

$$A: w_0^{t+1} = 2$$

$$B: w_0^{t+1} = 1$$

$$C: w_0^{t+1} = 0$$

$$D: w_0^{t+1} = -2$$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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Compute :

A:  $w_0^{t+1} = 2$

B:  $w_0^{t+1} = 1$

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$t = 0$   $w = (w_1, w_0) = (0, 0)$

Transition  $(s_t = 0, a_t = 1, s_{t+1} = 1, r_{t+1} = -2)$ ,  $t = 1$ :  $\text{trial} = -2$ ,  $\text{diff} = -2 \Rightarrow w_1^{t+1} = 0$

Compute :

A:  $w_0^{t+1} = 2$

B:  $w_0^{t+1} = 1$

C:  $w_0^{t+1} = w_0^t + [\text{diff}](1 - a_t) = 0 + -2(1 - 1) = 0$

D:  $w_0^{t+1} = -2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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$t = 1$   $w = (w_1, w_0) = (0, 0)$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2)$ ,  $t = 2$ :

Compute:

A: trial = -2

B: trial = 0

C: trial = -1

D: trial = 2

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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►  $\text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$

$t = 1$   $w = (w_1, w_0) = (0, 0)$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2)$ ,  $t = 2$ :

Compute:

A: trial = -2

B: trial = 0

C: trial = -1

D: trial = 2

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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$$\blacktriangleright \text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$$

$$t = 1 \quad w = (w_1, w_0) = (0, 0)$$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2), t = 2$ :

Compute:

$$A: \text{trial} = -2$$

$$B: \text{trial} = 0$$

$$C: \text{trial} = -1$$

$$D: \text{trial} = 2$$

## Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
(1, 1, exit, 2)	(-1, 0, exit, -1)	

each field in the table is an n-tuple  $(s_t, a_t, s_{t+1}, r_{t+1})$

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►  $w \leftarrow w + \alpha(\text{diff})\nabla\hat{q}(s_t, a_t, w)$ ;  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$

$$w_1^{t+1} = w_1^t + \alpha(\text{trial} - \hat{q}(s_t, a_t, w^t))s_t a_t$$

$$w_0^{t+1} = w_0^t + \alpha(\text{trial} - \hat{q}(s_t, a_t, w^t))(1 - a_t)$$

►  $\text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$

$t = 1$   $w = (w_1, w_0) = (0, 0)$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2)$ ,  $t = 2$ :

Compute:

A:  $\text{trial} = -2$

B:  $\text{trial} = 0$

C:  $\text{trial} = -1$

D:  $\text{trial} = 2 + \max\{0, 0\} = 2$

## Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
(1, 1, exit, 2)	(-1, 0, exit, -1)	

each field in the table is an n-tuple  $(s_t, a_t, s_{t+1}, r_{t+1})$

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$$\blacktriangleright \text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$$

$$t = 1 \quad w = (w_1, w_0) = (0, 0)$$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2), t = 2$ : trial = 2

Compute diff = trial -  $\hat{q}(s_t, a_t, w)$ :

A: diff = 0

B: diff = 2

C: diff = -1

D: diff = -2

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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$$w_0^{t+1} = w_0^t + \alpha(\text{trial} - \hat{q}(s_t, a_t, w^t))(1 - a_t)$$

►  $\text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$

$t = 1$   $w = (w_1, w_0) = (0, 0)$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2)$ ,  $t = 2$ :  $\text{trial} = 2$

Compute  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$ :

A:  $\text{diff} = 0$

B:  $\text{diff} = 2 - 0 = 2$

C:  $\text{diff} = -1$

D:  $\text{diff} = -2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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$$\blacktriangleright \text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$$

$$t = 1 \quad w = (w_1, w_0) = (0, 0)$$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2), t = 2$ : trial = 2, diff = 2

Compute :

$$\text{A: } w_1^{t+1} = 2$$

$$\text{B: } w_1^{t+1} = 0$$

$$\text{C: } w_1^{t+1} = 1$$

$$\text{D: } w_1^{t+1} = -2$$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
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$t = 1$   $w = (w_1, w_0) = (0, 0)$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2)$ ,  $t = 2$ :  $\text{trial} = 2$ ,  $\text{diff} = 2$

Compute :

A:  $w_1^{t+1} = 0 + 2 \cdot 1 \cdot 1 = 2$

B:  $w_1^{t+1} = 0$

C:  $w_1^{t+1} = 1$

D:  $w_1^{t+1} = -2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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$t = 1$   $w = (w_1, w_0) = (0, 0)$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2), t = 1$ :  $\text{trial} = 2, \text{diff} = 2 \Rightarrow w_1^{t+1} = 2$

Compute :

A:  $w_0^{t+1} = 2$

B:  $w_0^{t+1} = 1$

C:  $w_0^{t+1} = 0$

D:  $w_0^{t+1} = -2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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Task: compute Q-function - from each tuple refine  $w_0, w_1$

$$\blacktriangleright w \leftarrow w + \alpha(\text{diff})\nabla\hat{q}(s_t, a_t, w); \text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$$

$$w_1^{t+1} = w_1^t + \alpha(\text{trial} - \hat{q}(s_t, a_t, w^t))s_t a_t$$

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$$\blacktriangleright \text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$$

$$t = 1 \quad w = (w_1, w_0) = (0, 0)$$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2), t = 1$ : trial = 2, diff = 2  $\Rightarrow w_1^{t+1} = 2$

Compute :

$$\text{A: } w_0^{t+1} = 2$$

$$\text{B: } w_0^{t+1} = 1$$

$$\text{C: } w_0^{t+1} = 0$$

$$\text{D: } w_0^{t+1} = -2$$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
(1, 1, exit, 2)	(-1, 0, exit, -1)	

each field in the table is an n-tuple  $(s_t, a_t, s_{t+1}, r_{t+1})$

$$S = \{-1, 0, 1\}$$

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$$\gamma = 1, \alpha = 1$$

$$\hat{q}(s, a, w) = asw_1 + (1 - a)w_0$$

Task: compute Q-function - from each tuple refine  $w_0, w_1$

►  $w \leftarrow w + \alpha(\text{diff})\nabla\hat{q}(s_t, a_t, w)$ ;  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$

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►  $\text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$

$t = 1$   $w = (w_1, w_0) = (0, 0)$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2)$ ,  $t = 2$ :  $\text{trial} = 2$ ,  $\text{diff} = 2 \Rightarrow w_1^{t+1} = 2$

Compute :

A:  $w_0^{t+1} = 2$

B:  $w_0^{t+1} = 1$

C:  $w_0^{t+1} = 0 + 2(1 - 1) = 0$

D:  $w_0^{t+1} = -2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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►  $\text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$

$t = 2$   $w = (w_1, w_0) = (2, 0)$

Transition  $(s_t = 0, a_t = 0, s_{t+1} = -1, r_{t+1} = 0)$ ,  $t = 3$ :

Compute:

A: trial = -2

B: trial = 0

C: trial = -1

D: trial = 2

## Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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►  $\text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$

$t = 2$   $w = (w_1, w_0) = (2, 0)$

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## Approximative Q-learning

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$$t = 2 \quad w = (w_1, w_0) = (2, 0)$$

Transition  $(s_t = 0, a_t = 0, s_{t+1} = -1, r_{t+1} = 0), t = 3$ :

Compute:

$$A: \text{trial} = -2$$

$$B: \text{trial} = 0$$

$$C: \text{trial} = -1$$

$$D: \text{trial} = 2$$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
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►  $\text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$

$t = 2$   $w = (w_1, w_0) = (2, 0)$

Transition  $(s_t = 0, a_t = 0, s_{t+1} = -1, r_{t+1} = 0)$ ,  $t = 3$ :

Compute:

A:  $\text{trial} = -2$

B:  $\text{trial} = 0 + \max\{(2 \cdot (-1) \cdot 0 + 0(1 - 0)), (2(-1)1 + 0(1 - 1))\} = 0 + \max\{-2, 0\} = 0$

C:  $\text{trial} = -1$

D:  $\text{trial} = 2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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Transition  $(s_t = 0, a_t = 0, s_{t+1} = -1, r_{t+1} = 0)$ ,  $t = 3$ :  $\text{trial} = 0$

Compute  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$ :

A:  $\text{diff} = 0$

B:  $\text{diff} = 2$

C:  $\text{diff} = -1$

D:  $\text{diff} = -2$

## Approximative Q-learning

Episode 1	Episode 2	Episode 3
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$t = 2$   $w = (w_1, w_0) = (2, 0)$

Transition  $(s_t = 0, a_t = 0, s_{t+1} = -1, r_{t+1} = 0)$ ,  $t = 3$ :  $\text{trial} = 0$

Compute  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$ :

A:  $\text{diff} = 0 - (2 \cdot 0 \cdot 0 + 0(1 - 0)) = 0$

B:  $\text{diff} = 2$

C:  $\text{diff} = -1$

D:  $\text{diff} = -2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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$$\blacktriangleright \text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$$

$$t = 2 \quad w = (w_1, w_0) = (2, 0)$$

Transition  $(s_t = 0, a_t = 0, s_{t+1} = -1, r_{t+1} = 0)$ ,  $t = 3$ : trial = 0, diff = 0

Since [diff]= 0:

$\Rightarrow$  no change in  $(w_1, w_0)$

## Approximative Q-learning

Episode 1	Episode 2	Episode 3
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$$w_0^{t+1} = w_0^t + \alpha(\text{trial} - \hat{q}(s_t, a_t, w^t))(1 - a_t)$$

►  $\text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$

$t = 3$   $w = (w_1, w_0) = (2, 0)$

Transition  $(s_t = -1, a_t = 0, s_{t+1} = \text{exit}, r_{t+1} = -1), t = 4$ :

Compute:

A: trial = -2

B: trial = 0

C: trial = -1

D: trial = 2

## Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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$$\blacktriangleright \text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$$

$$t = 3 \quad w = (w_1, w_0) = (2, 0)$$

Transition  $(s_t = -1, a_t = 0, s_{t+1} = \text{exit}, r_{t+1} = -1), t = 4$ :

Compute:

A: trial = -2

B: trial = 0

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## Approximative Q-learning

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Transition  $(s_t = -1, a_t = 0, s_{t+1} = \text{exit}, r_{t+1} = -1), t = 4$ :

Compute:

$$\text{A: trial} = -2$$

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►  $\text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$

$t = 3$   $w = (w_1, w_0) = (2, 0)$

Transition  $(s_t = -1, a_t = 0, s_{t+1} = \text{exit}, r_{t+1} = -1), t = 4$ :

Compute:

A:  $\text{trial} = -2$

B:  $\text{trial} = 0$

C:  $\text{trial} = -1 + 0 = -1$

D:  $\text{trial} = 2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
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$t = 3$   $w = (w_1, w_0) = (2, 0)$

Transition  $(s_t = -1, a_t = 0, s_{t+1} = \text{exit}, r_{t+1} = -1)$ ,  $t = 4$ :  $\text{trial} = -1$

Compute  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$ :

A:  $\text{diff} = 0$

B:  $\text{diff} = 2$

C:  $\text{diff} = -1$

D:  $\text{diff} = -2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
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$t = 3$   $w = (w_1, w_0) = (2, 0)$

Transition  $(s_t = -1, a_t = 0, s_{t+1} = \text{exit}, r_{t+1} = -1)$ ,  $t = 4$ :  $\text{trial} = -1$

Compute  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$ :

A:  $\text{diff} = 0$

B:  $\text{diff} = 2$

C:  $\text{diff} = -1 - 0 = -1$

D:  $\text{diff} = -2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
(1, 1, exit, 2)	(-1, 0, exit, -1)	

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$$\gamma = 1, \alpha = 1$$

$$\hat{q}(s, a, w) = asw_1 + (1 - a)w_0$$

Task: compute Q-function - from each tuple refine  $w_0, w_1$

►  $w \leftarrow w + \alpha(\text{diff})\nabla\hat{q}(s_t, a_t, w)$ ;  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$

$$w_1^{t+1} = w_1^t + \alpha(\text{trial} - \hat{q}(s_t, a_t, w^t))s_t a_t$$

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►  $\text{trial} = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, w)$

$t = 3$   $w = (w_1, w_0) = (2, 0)$

Transition  $(s_t = -1, a_t = 0, s_{t+1} = \text{exit}, r_{t+1} = -1)$ ,  $t = 4$ :  $\text{trial} = -1$ ,  $\text{diff} = -1$

Compute :

A:  $w_1^{t+1} = 2$

B:  $w_1^{t+1} = 0$

C:  $w_1^{t+1} = 1$

D:  $w_1^{t+1} = -2$

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
(0, 1, 1, -2)	(0, 0, -1, 0)	(1, 1, exit, 2)
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$$t = 3 \quad w = (w_1, w_0) = (2, 0)$$

Transition  $(s_t = -1, a_t = 0, s_{t+1} = \text{exit}, r_{t+1} = -1), t = 4$ : trial = -1, diff = -1

Compute :

$$A: w_1^{t+1} = 2 + (-1) \cdot (-1) \cdot 0 = 2$$

$$B: w_1^{t+1} = 0$$

$$C: w_1^{t+1} = 1$$

$$D: w_1^{t+1} = -2$$

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Compute :

$$A: w_0^{t+1} = 2$$

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# Approximative Q-learning

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$$t = 3 \quad w = (w_1, w_0) = (2, 0)$$

Transition  $(s_t = -1, a_t = 0, s_{t+1} = \text{exit}, r_{t+1} = -1)$ ,  $t = 4$ : trial = -1, diff = -1  $\Rightarrow w_1^{t+1} = 2$

Compute :

$$\text{A: } w_0^{t+1} = 2$$

$$\text{B: } w_0^{t+1} = -1$$

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Compute :

A:  $w_0^{t+1} = 2$

B:  $w_0^{t+1} = 0 + (-1) \cdot (1 - 0) = -1$

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$t = 4$   $w = (w_1, w_0) = (2, -1)$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2)$ ,  $t = 5$ :

Compute:

A: trial = -2

B: trial = 0

C: trial = -1

D: trial = 2

# Approximative Q-learning

Episode 1	Episode 2	Episode 3
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# Approximative Q-learning

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A: trial = -2

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## Approximative Q-learning

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Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2)$ ,  $t = 5$ :  $\text{trial} = 2$

Compute  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$ :

A:  $\text{diff} = 0$

B:  $\text{diff} = 2$

C:  $\text{diff} = -1$

D:  $\text{diff} = -2$

# Approximative Q-learning

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Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2)$ ,  $t = 5$ :  $\text{trial} = 2$

Compute  $\text{diff} = \text{trial} - \hat{q}(s_t, a_t, w)$ :

A:  $\text{diff} = 2 - (2 \cdot 1 \cdot 1 + (-1)(1 - 1)) = 2 - 2 = 0$

B:  $\text{diff} = 2 - 0 = 2$

C:  $\text{diff} = -1$

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Since  $[\text{diff}] = 0$ :

$\Rightarrow$  no change in  $(w_1, w_0)$

Final solution:  $w = (w_1, w_0) = (2, -1)$

# Approximative Q-learning

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$$t = 4 \quad w = (w_1, w_0) = (2, -1)$$

Transition  $(s_t = 1, a_t = 1, s_{t+1} = \text{exit}, r_{t+1} = 2)$ ,  $t = 5$ : trial = 2, diff = 0

Since [diff]= 0:

$\Rightarrow$  no change in  $(w_1, w_0)$

Final solution:  $w = (w_1, w_0) = (2, -1)$