

# Reinforcement learning

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April 15, 2025

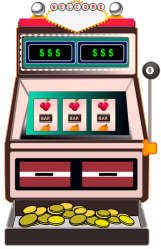
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## Notes

In Czech: Posilované učení

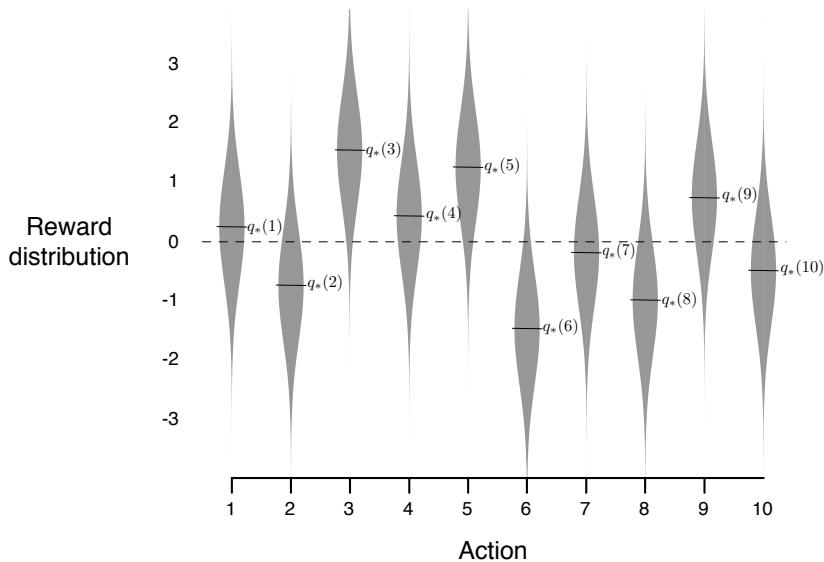
# (Multi-armed) Bandits



Think about not one but 10 arms you may choose to pull.

$p(s'|s, a)$  and  $r(s, a, s')$  not known!

# 10 armed bandit, what arm to pull?



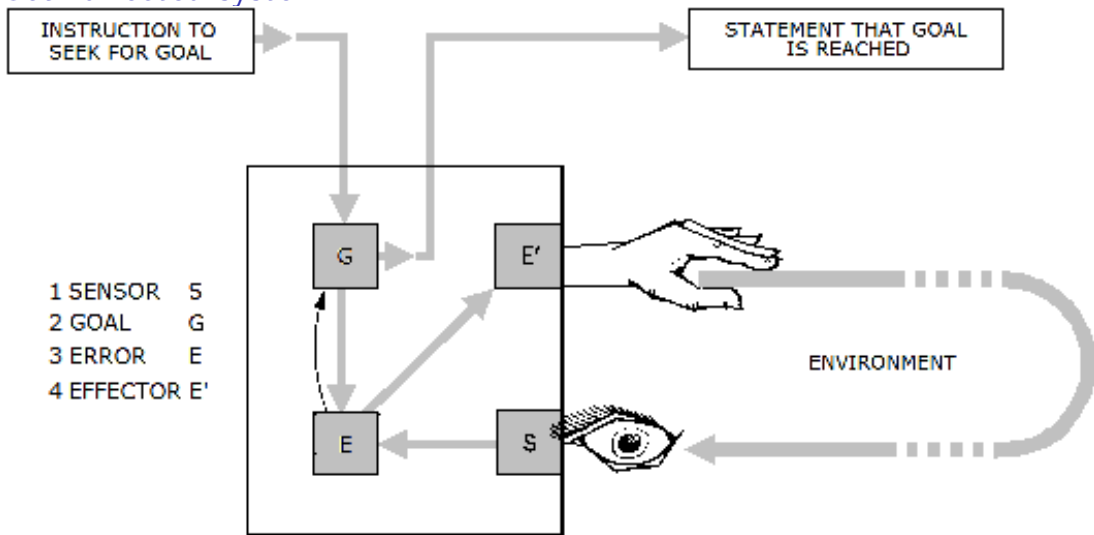
See chapters 2.2 and 2.3 in [4] for more detailed discussion

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## Notes

- 10 different arms
- action pulling  $k$ -th arm
- value of the action, i.e.  $q(a)$  is stochastic (Gaussian around  $q^*(a)$ ) and unknown!
- Think about expectimax tree. How would it look like?
- Playing (pulling) many times, what is the policy? Think also that each sample costs something.

# Goal-directed system



**A SIMPLE GOAL-DIRECTED SYSTEM**

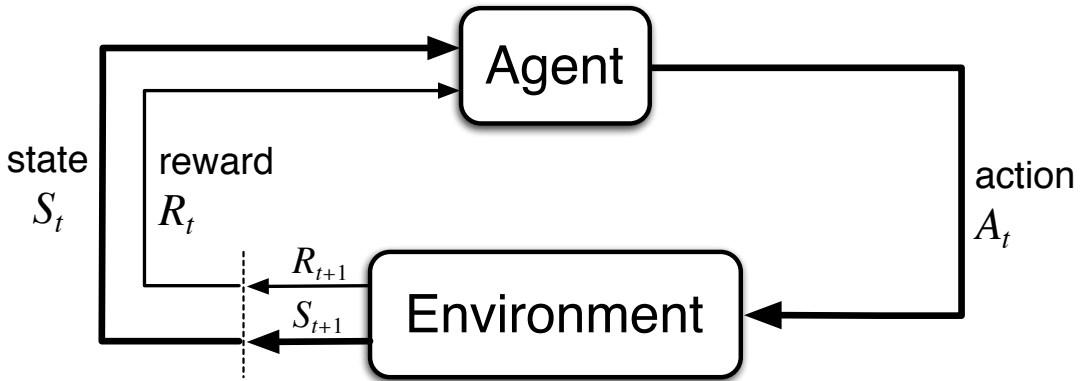
<sup>1</sup>Figure from <http://www.cybsoc.org/gcyb.htm>

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Notes

# Reinforcement Learning - performing actions, learning from rewards



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- ▶ Feedback in form of Rewards
- ▶ Learn to act so as to maximize expected rewards.

<sup>2</sup>Scheme from [4]

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## Reinforcement Learning - performing actions, learning from rewards



## Autonomous Flipper Control with Safety Constraints

Martin Pecka, Vojtěch Šalanský,  
Karel Zimmermann, Tomáš Svoboda

experiments utilizing  
Constrained Relative Entropy Policy Search

**Video: Learning safe policies<sup>3</sup>**

<sup>3</sup>M. Pecka, V. Salansky, K. Zimmermann, T. Svoboda. Autonomous flipper control with safety constraints. In Intelligent Robots and Systems (IROS), 2016, [https://youtu.be/\\_oUMbBtoRcs](https://youtu.be/_oUMbBtoRcs)

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### Notes

Policy search is a more advanced topic, only touched by this course. Later in master programme. Reinforcement learning beating humans in playing Atari games: <https://deepmind.google/discover/blog/agent57-outperforming-the-human-atari-benchmark/>

# From off-line (MDPs) to on-line (RL)

Markov decision process – MDPs. Off-line search, we know:

- ▶ A set of states  $s \in \mathcal{S}$  (map)
- ▶ A set of actions per state.  $a \in \mathcal{A}$
- ▶ A transition model  $T(s, a, s')$  or  $p(s'|s, a)$  (robot)
- ▶ A reward function  $r(s, a, s')$  (map, robot)

Looking for the optimal policy  $\pi(s)$ . We can plan/search before the robot enters the environment.

On-line problem:

- ▶ Transition model  $p$  and reward function  $r$  not known.
- ▶ Agent/robot must act and learn from experience.

For MDPs, we know  $p, r$  for all possible states and actions.

# (Transition) Model-based learning

The main idea: Do something, and:

- ▶ Learn an approximate model from experiences.
- ▶ Solve as if the model was correct.

Learning MDP model:

- ▶ In  $s$  try  $a$ , observe  $s'$ , count  $(s, a, s')$ .
- ▶ Normalize to get and estimate of  $p(s' | s, a)$ .
- ▶ Discover (by observation) each  $r(s, a, s')$  when experienced.

Solve the learned MDP.

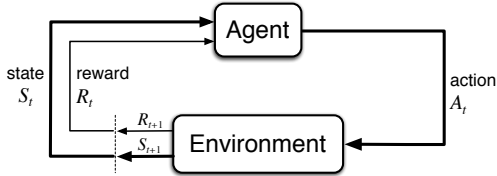
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## Notes

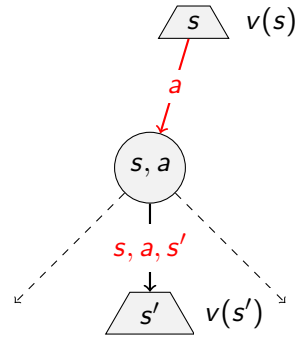
- Where to start?
- When does it end?
- How long does it take?
- When to stop (the learning phase)?

# Reward function $r(s, a, s')$

- ▶  $r(s, a, s')$  - reward for taking  $a$  in  $s$  and landing in  $s'$ .
- ▶ In Grid world, we assumed  $r(s, a, s')$  to be the same everywhere.
- ▶ In the real world, it is different (going up, down, ...)



In ai-gym `env.step(action)` returns  $s', r(s, \text{action}, s')$ .

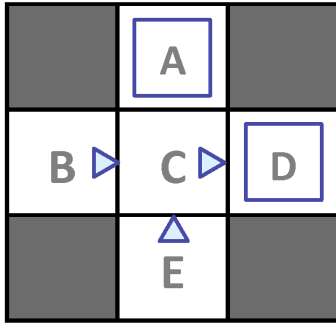


## Notes

In ai-gym `env.step(action)` returns  $s', r(s, \text{action}, s'), \dots$ . It is defined by the environment (robot simulator, system, ...) not by the (algorithms)

# Model-based learning: Grid example

## Input Policy $\pi$



Assume:  $\gamma = 1$

## Observed Episodes (Training)

### Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 2

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 3

E, north, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

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<sup>4</sup>Figure from [1]

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## Notes

## Learned Model

$$\hat{T}(s, a, s')$$

T(B, east, C) = 1.00  
T(C, east, D) = 0.75  
T(C, east, A) = 0.25  
...

$$\hat{R}(s, a, s')$$

R(B, east, C) = -1  
R(C, east, D) = -1  
R(D, exit, x) = +10  
...

# Learning transition model

$$\hat{p}(D \mid C, \text{east}) = ?$$

## Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

## Episode 2

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

## Episode 3

E, north, C, -1  
C, east, D, -1  
D, exit, x, +10

## Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

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### Notes

(C, east) combination performed 4 times, 3 times landed in D, once in A. Hence,  $\hat{p}(D \mid C, \text{east}) = 0.75$ .

## Learning reward function

$\hat{r}(C, \text{east}, D) = ?$

### Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 2

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 3

E, north, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

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### Notes

Whenever (C, east, D) performed, received reward was  $-1$ . Hence,  $\hat{r}(C, \text{east}, D) = -1$ .

# Model based vs model-free: Expected age $E[A]$

Random variable age  $A$ .

$$E[A] = \sum_a P(A = a)a$$

We do not know  $P(A = a)$ . Instead, we collect  $N$  samples  $[a_1, a_2, \dots, a_N]$ .

Model based

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a)a$$

Model free

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

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## Notes

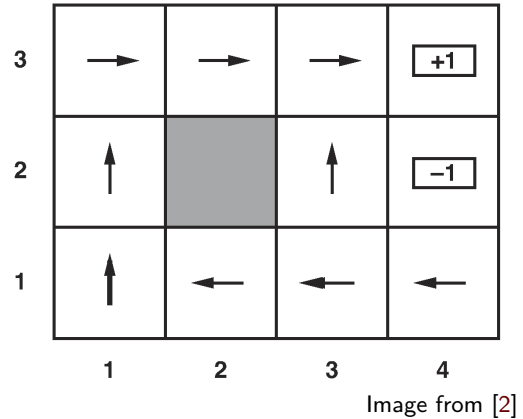
Just to avoid confusion. There are many more samples than possible ages (positive integer). Think about  $N \gg 100$ .

- Model based – eventually, we learn the correct model.
- Model free – no need for weighting; this is achieved through the frequencies of different ages within the samples (most frequent and hence most probable ages simply come up many times).

# Model-free learning

# Passive learning (evaluating given policy)

- ▶ **Input:** a fixed policy  $\pi(s)$
- ▶ We want to know how good it is.
- ▶  $r, p$  not known.
- ▶ Execute policy ...
- ▶ and learn on the way.
- ▶ **Goal:** learn the state values  $v^\pi(s)$



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## Notes

Executing policies - training, then learning from the observations. We want to do the policy evaluation but the necessary model is not known.

The word passive means we just follow a prescribed policy  $\pi(s)$ .

# Direct evaluation from episodes

Value of  $s$  for  $\pi$  – expected sum of discounted rewards – expected return

$$v^\pi(S_t) = \mathbb{E} \left[ \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \right]$$

$$v^\pi(S_t) = \mathbb{E}[G_t]$$

|   |   |   |   |    |
|---|---|---|---|----|
| 3 | → | → | → | +1 |
| 2 | ↑ |   | ↑ | -1 |
| 1 | ↑ | ← | ← | ←  |
|   | 1 | 2 | 3 | 4  |

$(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) \xrightarrow{+1}$   
 $(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (3, 2) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) \xrightarrow{+1}$   
 $(1, 1) \xrightarrow{-0.04} (2, 1) \xrightarrow{-0.04} (3, 1) \xrightarrow{-0.04} (3, 2) \xrightarrow{-0.04} (4, 2) \xrightarrow{-1}$

## Notes

- Act according to the policy.
- When visiting a state, remember what the sum of discounted rewards (returns) turned out to be.
- Compute average of the returns.
- Each trial episode provides a sample of  $v^\pi$ .

What is  $v^\pi(3, 2)$  after these episodes?

## Direct evaluation from episodes, $v^\pi(S_t) = E[G_t]$ , $\gamma = 1$

$(1, 1) \rightsquigarrow (1, 2) \rightsquigarrow (1, 3) \rightsquigarrow (1, 2) \rightsquigarrow (1, 3) \rightsquigarrow (2, 3) \rightsquigarrow (3, 3) \rightsquigarrow (4, 3) +1$   
 $(1, 1) \rightsquigarrow (1, 2) \rightsquigarrow (1, 3) \rightsquigarrow (2, 3) \rightsquigarrow (3, 3) \rightsquigarrow (3, 2) \rightsquigarrow (3, 3) \rightsquigarrow (4, 3) +1$   
 $(1, 1) \rightsquigarrow (2, 1) \rightsquigarrow (3, 1) \rightsquigarrow (3, 2) \rightsquigarrow (4, 2) -1$  .

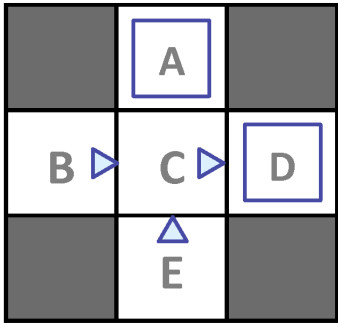
What is  $v(3, 2)$  after these episodes?

### Notes

- Not visited during the first episode.
- Visited once in the second, gathered return  $G = -0.04 - 0.04 + 1 = 0.92$ .
- Visited once in the third, return  $G = -0.04 - 1 = -1.04$ .
- Value, average return is  $(0.92 - 1.04)/2 = -0.06$ .

Direct evaluation: Grid world example

Input Policy  $\pi$



Assume:  $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 2

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 3

E, north, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

Notes

|         |          |          |
|---------|----------|----------|
|         | -10<br>A |          |
| +8<br>B | +4<br>C  | +10<br>D |
|         | -2<br>E  |          |

## Direct evaluation: Grid example, $\gamma = 1$

What is  $v(C)$  after the 4 episodes?

Let  $M$  be the number of recorded episodes.

Let  $N$  be the number of samples used to compute the averages.

What is the relation of  $M$  and  $N$ ?

- A**  $N = M$
- B**  $N \leq M$
- C**  $N \geq M$
- D**  $N$  has no relation to  $M$

### Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 2

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 3

E, north, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

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### Notes

- Episode 1,  $G = -1 + 10 = 9$
- Episode 2,  $G = -1 + 10 = 9$
- Episode 3,  $G = -1 + 10 = 9$
- Episode 4,  $G = -1 - 10 = -11$
- Average return  $v(C) = (9 + 9 + 9 - 11)/4 = 4$

For first-visit variant, B is correct. For every-visit variant, D is correct.

$N$  can be lower than  $M$  (state does not have to be attended in every episode). For every-visit variant,  $N$  can be higher than  $M$  (a state can be visited several times in one episode).

## Direct evaluation algorithm (every-visitfirst-visit version)

$(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) + 1$   
 $(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (3, 2) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) + 1$   
 $(1, 1) \xrightarrow{-0.04} (2, 1) \xrightarrow{-0.04} (3, 1) \xrightarrow{-0.04} (3, 2) \xrightarrow{-0.04} (4, 2) - 1$  .

Input: a policy  $\pi$  to be evaluated

Initialize:

$V(s) \in \mathbb{R}$ , arbitrarily, for all  $s \in \mathcal{S}$

$Returns(s) \leftarrow$  an empty list, for all  $s \in \mathcal{S}$

Loop forever (for each episode):

Generate an episode following  $\pi$ :  $S_0, A_0, R_1, S_1, A_1, R_2, \dots, S_{T-1}, A_{T-1}, R_T$

$G \leftarrow 0$

Loop backwards for each step of episode,  $t = T - 1, T - 2, \dots, 0$ :

$G \leftarrow R_{t+1} + \gamma G$

Append  $G$  to  $Returns(S_t)$

$V(S_t) \leftarrow \text{average}(Returns(S_t))$

If  $S_t$  does not appear in  $S_0, S_1, \dots, S_{t-1}$ : *// Use the return for the first visit only*

Append  $G$  to  $Returns(S_t)$

$V(S_t) \leftarrow \text{average}(Returns(S_t))$

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### Notes

The algorithm can be easily expanded to  $Q(S_t, A_t)$ . Instead of visiting  $S_t$  we consider visiting of a pair  $S_t, A_t$ .

# Direct evaluation: analysis

## The good:

- ▶ Simple, easy to understand and implement.
- ▶ Does not need  $p, r$  and eventually it computes the true  $v^\pi$ .

## The bad:

$(1, 1) \xrightarrow{.04} (1, 2) \xrightarrow{.04} (1, 3) \xrightarrow{.04} (1, 2) \xrightarrow{.04} (1, 3) \xrightarrow{.04} (2, 3) \xrightarrow{.04} (3, 3) \xrightarrow{.04} (4, 3) + 1$   
 $(1, 1) \xrightarrow{.04} (1, 2) \xrightarrow{.04} (1, 3) \xrightarrow{.04} (2, 3) \xrightarrow{.04} (3, 3) \xrightarrow{.04} (3, 2) \xrightarrow{.04} (3, 3) \xrightarrow{.04} (4, 3) + 1$   
 $(1, 1) \xrightarrow{.04} (2, 1) \xrightarrow{.04} (3, 1) \xrightarrow{.04} (3, 2) \xrightarrow{.04} (4, 2) - 1$  .

- ▶ Each state value learned in isolation.
- ▶ State values are not independent
- ▶ 
$$v^\pi(s) = \sum_{s'} p(s' | s, \pi(s)) [r(s, \pi(s), s') + \gamma v^\pi(s')]$$

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## Notes

In second trial, we visit  $(3, 2)$  for the first time. We already know that the successor  $(3, 3)$  has probably a high value but the method does not use until the end of the trial episode.

Before updating  $V(s)$  we have to wait until the training episode ends.

## (on-line) Policy evaluation?

In MDP, we did:

- ▶ Initialize the values:  $V_0^\pi(s) = 0$
- ▶ In each iteration, replace  $V$  with a one-step-look-ahead:  
$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} p(s' | s, \pi(s)) [r(s, \pi(s), s') + \gamma V_k^\pi(s')]$$

Problem: both  $p(s' | s, \pi(s))$  and  $r(s, \pi(s), s')$  unknown!

## Use samples for evaluating policy?

MDP ( $p, r$  known) : Update  $V$  estimate by a weighted average:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} p(s' | s, \pi(s)) [r(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

What about stop, try, try, ..., and average?

Samples at time  $t$ .  $\pi(S_t) \rightarrow A_t$ , repeat  $A_t$ .

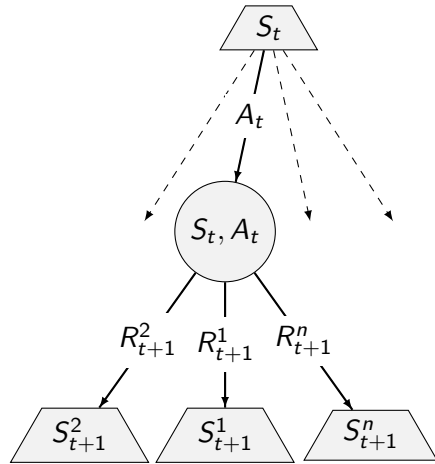
$$\text{sample}^1 = R_{t+1}^1 + \gamma V(S_{t+1}^1)$$

$$\text{sample}^2 = R_{t+1}^2 + \gamma V(S_{t+1}^2)$$

$$\vdots = \vdots$$

$$\text{sample}^n = R_{t+1}^n + \gamma V(S_{t+1}^n)$$

$$V(S_t) \leftarrow \frac{1}{n} \sum_i \text{sample}^i$$



Problem: We cannot re-set to  $S_t$  easily.

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### Notes

It looks promising. Unfortunately, we cannot do it that way. After an action, the robot is in a next state and cannot go back to the very same state where it was before. Energy was consumed and some actions may be irreversible; think about falling into a hole. We have to utilize the  $s, a, s'$  experience anytime when performed/visited.

# Temporal-difference value learning

$(1, 1) \xrightarrow{-.04} (1, 2) \xrightarrow{-.04} (1, 3) \xrightarrow{-.04} (1, 2) \xrightarrow{-.04} (1, 3) \xrightarrow{-.04} (2, 3) \xrightarrow{-.04} (3, 3) \xrightarrow{-.04} (4, 3) \xrightarrow{+1}$   
 $(1, 1) \xrightarrow{-.04} (1, 2) \xrightarrow{-.04} (1, 3) \xrightarrow{-.04} (2, 3) \xrightarrow{-.04} (3, 3) \xrightarrow{-.04} (3, 2) \xrightarrow{-.04} (3, 3) \xrightarrow{-.04} (4, 3) \xrightarrow{+1}$   
 $(1, 1) \xrightarrow{-.04} (2, 1) \xrightarrow{-.04} (3, 1) \xrightarrow{-.04} (3, 2) \xrightarrow{-.04} (4, 2) \xrightarrow{-1}$

$\gamma = 1$

From first trial (episode):  $V(2, 3) = 0.92$ ,  $V(1, 3) = 0.84, \dots$

In second episode, going from  $S_t = (1, 3)$  to  $S_{t+1} = (2, 3)$  with reward  $R_{t+1} = -0.04$ , hence:

$$V(1, 3) = R_{t+1} + V(2, 3) = -0.04 + 0.92 = 0.88$$

- ▶ First estimate 0.84 is a bit lower than 0.88.  $V(S_t)$  is different than  $R_{t+1} + \gamma V(S_{t+1})$
- ▶ Update ( $\alpha \times$  difference):  $V(S_t) \leftarrow V(S_t) + \alpha \left( [R_{t+1} + \gamma V(S_{t+1})] - V(S_t) \right)$
- ▶  $\alpha$  is the learning rate.
- ▶  $V(S_t) \leftarrow (1 - \alpha)V(S_t) + \alpha(\text{new sample})$

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## Notes

Trial episode: acting, observing, until it stops (in a terminal state or by a limit).

We visit  $S(1, 3)$  twice during the first episode. Its value estimate is the average of two returns.

Note the main difference. In *Direct evaluation*, we had to wait until the end of the episode, compute  $G_t$  for each  $t$  on the way, and then we update  $V(S_t)$ . We can do it  $\alpha$  incrementally

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t - V(S_t))$$

In *TD learning*, we update as we go.

# Exponential moving average

$$\bar{x}_n = (1 - \alpha)\bar{x}_{n-1} + \alpha x_n$$

What does it remember about the past? Try to derive:

$$\bar{x}_n = f(\alpha, x_n, x_{n-1}, x_{n-2}, x_{n-3}, \dots)$$

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## Notes

Recursively inserting we end up with

$$\bar{x}_n = \alpha \left[ x_n + (1 - \alpha)x_{n-1} + (1 - \alpha)^2 x_{n-2} + \dots \right]$$

We already know the sum of geometric series for  $r < 1$

$$1 + r + r^2 + r^3 + \dots = \frac{1}{1 - r}$$

Putting  $r = 1 - \alpha$ , we see that

$$\frac{1}{\alpha} = 1 + (1 - \alpha) + (1 - \alpha)^2 + \dots$$

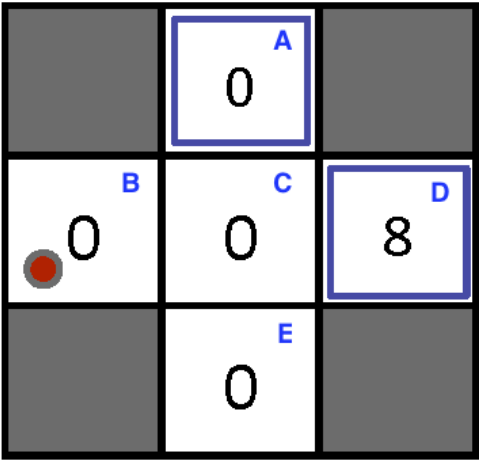
And hence:

$$\bar{x}_n = \frac{x_n + (1 - \alpha)x_{n-1} + (1 - \alpha)^2 x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + (1 - \alpha)^3 + \dots}$$

a weighted average that exponentially forgets about the past.

# Example: TD Value learning

$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$

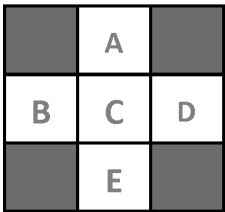


- ▶ Values represent initial  $V(s)$
- ▶ Assume:  $\gamma = 1, \alpha = 0.5, \pi(s) \Rightarrow$
- ▶  $(B, \rightarrow, C), -2, \Rightarrow V(B)?$
- ▶  $(C, \rightarrow, D), -2, \Rightarrow V(C)?$

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## Notes

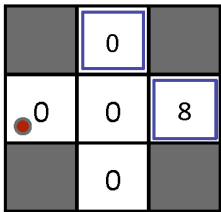
### States



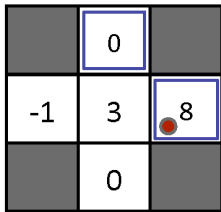
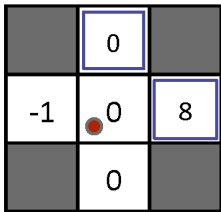
Assume:  $\gamma = 1, \alpha = 1/2$

### Observed Transitions

B, east, C, -2



C, east, D, -2



$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

## Temporal difference value learning: algorithm

Input: the policy  $\pi$  to be evaluated

Algorithm parameter: step size  $\alpha \in (0, 1]$

Initialize  $V(s)$ , for all  $s \in \mathcal{S}^+$ , arbitrarily except that  $V(\text{terminal}) = 0$

Loop for each episode:

    Initialize  $S$

    Loop for each step of episode:

$A \leftarrow$  action given by  $\pi$  for  $S$

        Take action  $A$ , observe  $R, S'$

$V(S) \leftarrow V(S) + \alpha[R + \gamma V(S') - V(S)]$

$S \leftarrow S'$

    until  $S$  is terminal

# What is wrong with the temporal difference Value learning?

**The Good:** Model-free value learning by mimicking Bellman updates.

**The Bad:** How to turn values into a (new) policy?

$$\blacktriangleright \pi(s) = \arg \max_a \sum_{s'} p(s' | s, a) [r(s, a, s') + \gamma V(s')]$$

$$\blacktriangleright \pi(s) = \arg \max_a Q(s, a)$$

---

## Notes

Learn Q-values, not V-values, and make the action selection model-free too!

# Q-learning

---

## Notes

So far we walked as prescribed by a  $\pi(s)$  because we did not know how to act better.

## Reminder: $V$ , $Q$ -value iteration for MDPs

Value/Utility iteration (depth limited evaluation):

- ▶ Start:  $V_0(s) = 0$
- ▶ In each step update  $V$  by looking one step ahead:  
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} p(s' | s, a) [r(s, a, s') + \gamma V_k(s')]$$

$Q$  values more useful (think about updating  $\pi$ )

- ▶ Start:  $Q_0(s, a) = 0$
- ▶ In each step update  $Q$  by looking one step ahead:  
$$Q_{k+1}(s, a) \leftarrow \sum_{s'} p(s' | s, a) \left[ r(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

---

### Notes

Draw the (s)-(s,a)-(s')-(s',a') tree. It will be also handy when discussing exploration vs. exploitation – where to drive next.

## Q-learning (one episode of)

MDP update:  $Q_{k+1}(s, a) \leftarrow \sum_{s'} p(s' | s, a) \left[ r(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$

Learn Q values as the robot/agent goes (temporal difference)

- ▶ Drive the robot (take action  $a$  in state  $s$ ) and fetch rewards  $(s, a, s', R)$
- ▶ We know old estimates  $Q(s, a)$  (and  $Q(s', a')$ ), if not, initialize.
- ▶ A new sample estimate (of  $Q(s, a)$ ) at time  $t$

$$\text{sample} = R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$$

- ▶  $\alpha$  update  
 $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(\text{sample} - Q(S_t, A_t))$   
or (the same)  
 $Q(S_t, A_t) \leftarrow (1 - \alpha)Q(S_t, A_t) + \alpha \text{sample}$

In each step  $Q$  approximates the optimal  $q^*$  function.

---

### Notes

There are alternatives how to compute the sample value. SARSA method takes  $Q(S_{t+1}, A_{t+1})$  directly, not the max. More next week.

# Q-learning: algorithm (repeating episodes, until terminal or exhausted)

step size  $0 < \alpha \leq 1$

initialize  $Q(s, a)$  for all  $s \in \mathcal{S}, a \in \mathcal{A}(s)$

**repeat** episodes:

    initialize  $S$

**for** for each step of episode: **do**

        choose  $A$  from  $\mathcal{A}(S)$

        take action  $A$ , observe  $R, S'$

$Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_a Q(S', a) - Q(S, A)]$

$S \leftarrow S'$

    until  $S$  is terminal

**until** Time is up, ...

# From Q-learning to Q-learning agent

- ▶ Drive the robot and fetch rewards.  $(s, a, s', R)$
- ▶ We know old estimates  $Q(s, a)$  (and  $Q(s', a')$ ), if not, initialize.
- ▶ A new sample estimate:  $\text{sample} = R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$
- ▶  $\alpha$  update:  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(\text{sample} - Q(S_t, A_t))$

## Technicalities for the Q-learning agent

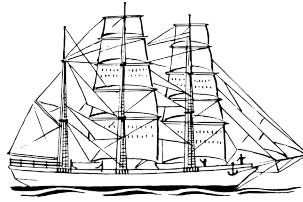
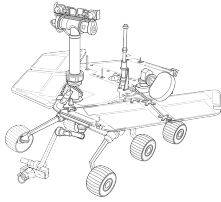
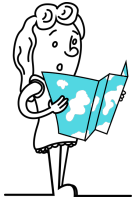
- ▶ How to represent the  $Q$ -function?
- ▶ What is the value for terminal?  $Q(s, \text{Exit})$  or  $Q(s, \text{None})$
- ▶ How to drive? Where to drive next? Does it change over the course?

---

### Notes

Q-function for a discrete, finite problem? But what about continuous space or discrete but a very large one?  
Use the  $(s)-(s,a)-(s')-(s',a')$  tree to discuss the next-action selection.

# Exploration vs. Exploitation



- ▶ Drive the known road or try a new one?
- ▶ Go to the university menza or try a nearby restaurant?
- ▶ Use the SW (operating system) I know or try a new one?
- ▶ Go to bussiness or study a demanding program?
- ▶ ...

# How to explore?

## Random ( $\epsilon$ -greedy):

- ▶ Flip a coin every step.
- ▶ With probability  $\epsilon$ , act randomly.
- ▶ With probability  $1 - \epsilon$ , use the policy.

## Problems with randomness?

- ▶ Keeps exploring forever.
- ▶ Should we keep  $\epsilon$  fixed (over learning)?
- ▶  $\epsilon$  same everywhere?

---

## Notes

- We can think about lowering  $\epsilon$  as the learning progresses.
- Favor unexplored states - be optimistic - exploration functions -  $f(u, n) = u + k/n$ , where  $u$  is the value estimated, and  $n$  is the visit count, and  $k$  is the training/simulation episode.

# What we have learned

- ▶ Agent/robot may learn by acting and getting rewards
- ▶ Model based vs. model-free methods
- ▶ Direct learning vs. temporal-difference learning
- ▶ From learning state values to Q-learning

# References I

Further reading: Chapter 21 of [2] (chapter 23 of [3]). More detailed discussion in [4], chapters 2, 5, and 6.

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[2] Stuart Russell and Peter Norvig.

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## References II

- [4] Richard S. Sutton and Andrew G. Barto.  
*Reinforcement Learning; an Introduction*.  
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<http://www.incompleteideas.net/book/the-book-2nd.html>.