Quantum Computing

Exercises 7: Quantum Fourier Transforms

The Quantum Fourier Transform acting on some state $|j\rangle$ is given by

$$|j\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi ijk/N} |k\rangle$$

.
Or in the tensor-product representation

$$|j\rangle \rightarrow \frac{1}{\sqrt{N}} (|0\rangle + e^{2\pi i \frac{j}{2^1}} |1\rangle) \otimes \cdots \otimes (|0\rangle + e^{2\pi i \frac{j}{2^n}} |1\rangle)$$

1. *Show that the Quantum Fourier Transform acting on the n-qubit $|0\rangle^{\otimes n}$ state is equivalent to applying a Hadamard transform to each qubit.*

2. *Directly proof that the general Quantum Fourier Transform is a unitary transformation.*

Hint: You may need to use the formula for a finite geometric series

$$\sum_{k=0}^{N-1} ar^k = a \left(\frac{1 - r^N}{1 - r} \right)$$

3. *Using both representations compute the output of applying the Quantum Fourier Transform on the state $|5\rangle_3$ ($n = 3$ qubits).*