Quantum Computing

Exercises 1: Intro to Quantum Physics

1. a) Show that the left and right states defined as:

$$|l\rangle = \frac{1}{\sqrt{2}}(|u\rangle + |d\rangle)$$

$$|r\rangle = \frac{1}{\sqrt{2}}(|u\rangle - |d\rangle)$$

are orthogonal:
b) Calculate the expectation values of σ_x in the states $|d\rangle$ and $|l\rangle$, and of σ_z in the state $|r\rangle$.

2. a) Normalise the state

$$|\psi\rangle = (1-i)|u\rangle + 2i|d\rangle.$$

b) For this (normalised) state, calculate the probability of getting both positive (+1) and negative (−1) spin eigenvalues by measuring σ_z.

3. [Nielsen & Chuang Ex. 2.17] (Eigendecomposition of a Pauli matrix) Find the eigenvectors, eigenvalues and diagonal representations of σ_y.

4. Show that the eigenvalues of hermitian matrices, $\mathbf{A} = \mathbf{A}^\dagger$, are real: $\lambda \in \mathbb{R}$.

5. [Susskind & Friedman Ex. 5.2] For any observables **A** and **B**, and state $|\psi\rangle$, derive Heisenberg's uncertainty relation: $\Delta\mathbf{A} \cdot \Delta\mathbf{B} \geq \frac{1}{2}|\langle\psi|[A,B]|\psi\rangle|$, where $(\Delta\mathbf{A})^2 = \sum_a (a - \langle\mathbf{A}\rangle)^2 P(a)$, is the standard deviation of the operator **A**.

6. Derive the evolution operator: $U(t) = e^{-\frac{i}{\hbar}Ht}$, by solving the Schrödinger equation: $i\hbar\frac{d|\psi(t)\rangle}{dt} = H|\psi(t)\rangle$.