Uncertainty estimation in factorgraph

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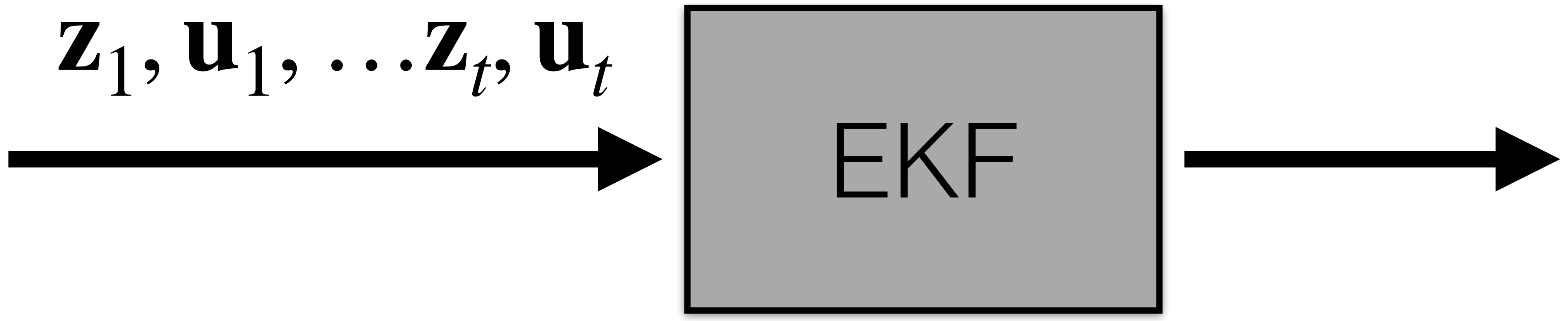
Comparison EKF vs FG

Assumptions: **Input:** **Outputs:** **Properties:**

complete states

$\mathbf{z}_1, \mathbf{u}_1, \ldots \mathbf{z}_t, \mathbf{u}_t$

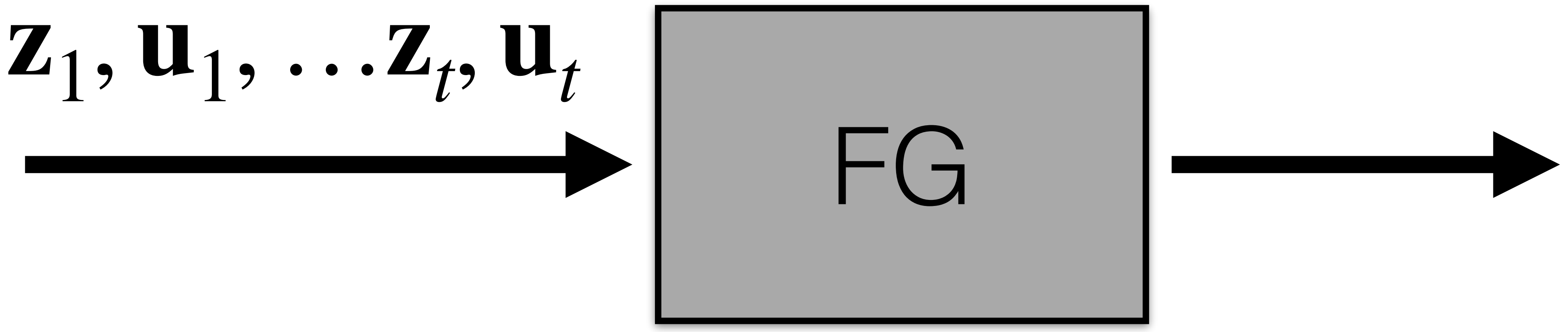


$\mathbf{x}_t, \mathbf{\Sigma}_t$

- fast recurrent estimation
- suffers from lineariz. => inconsistent loops

some conditional independences

$\mathbf{z}_1, \mathbf{u}_1, \ldots \mathbf{z}_t, \mathbf{u}_t$



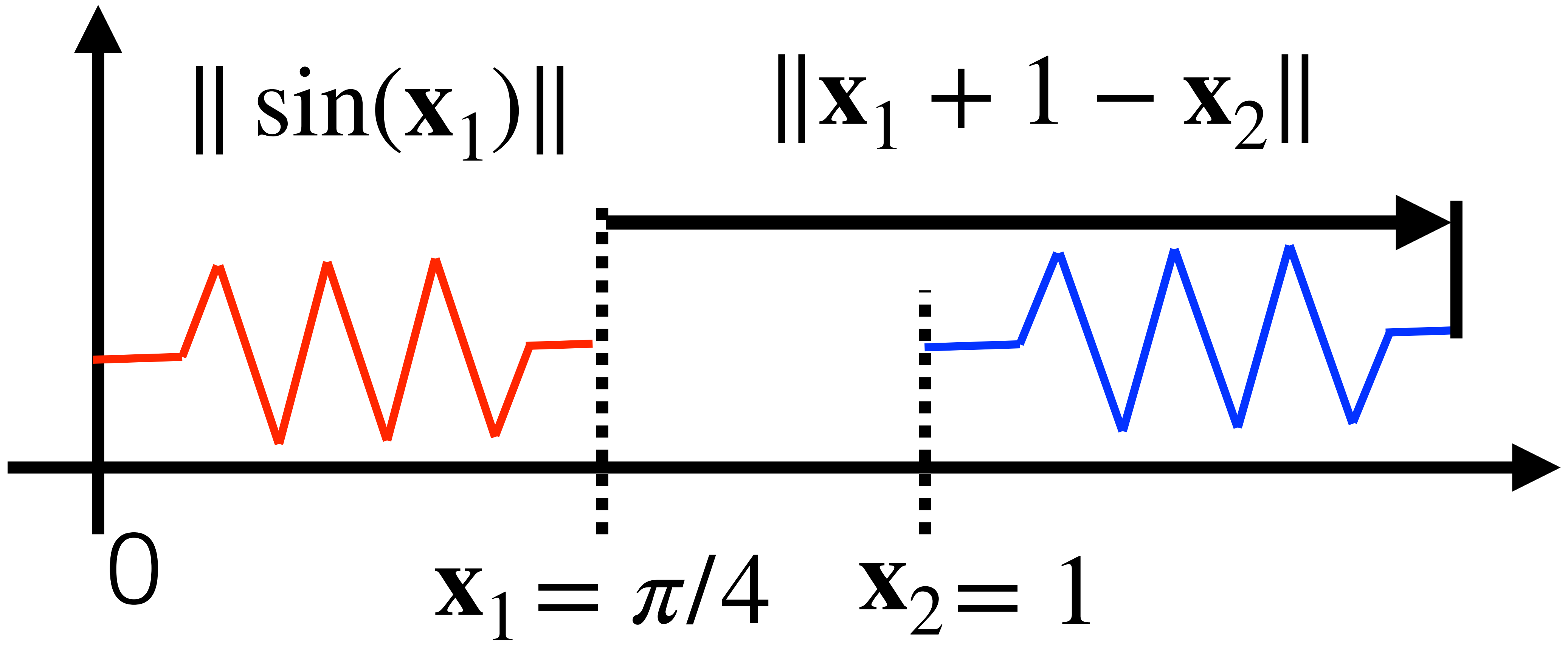
$\mathbf{x}_1, \ldots \mathbf{x}_t$

$\mathbf{\Sigma} = ?$

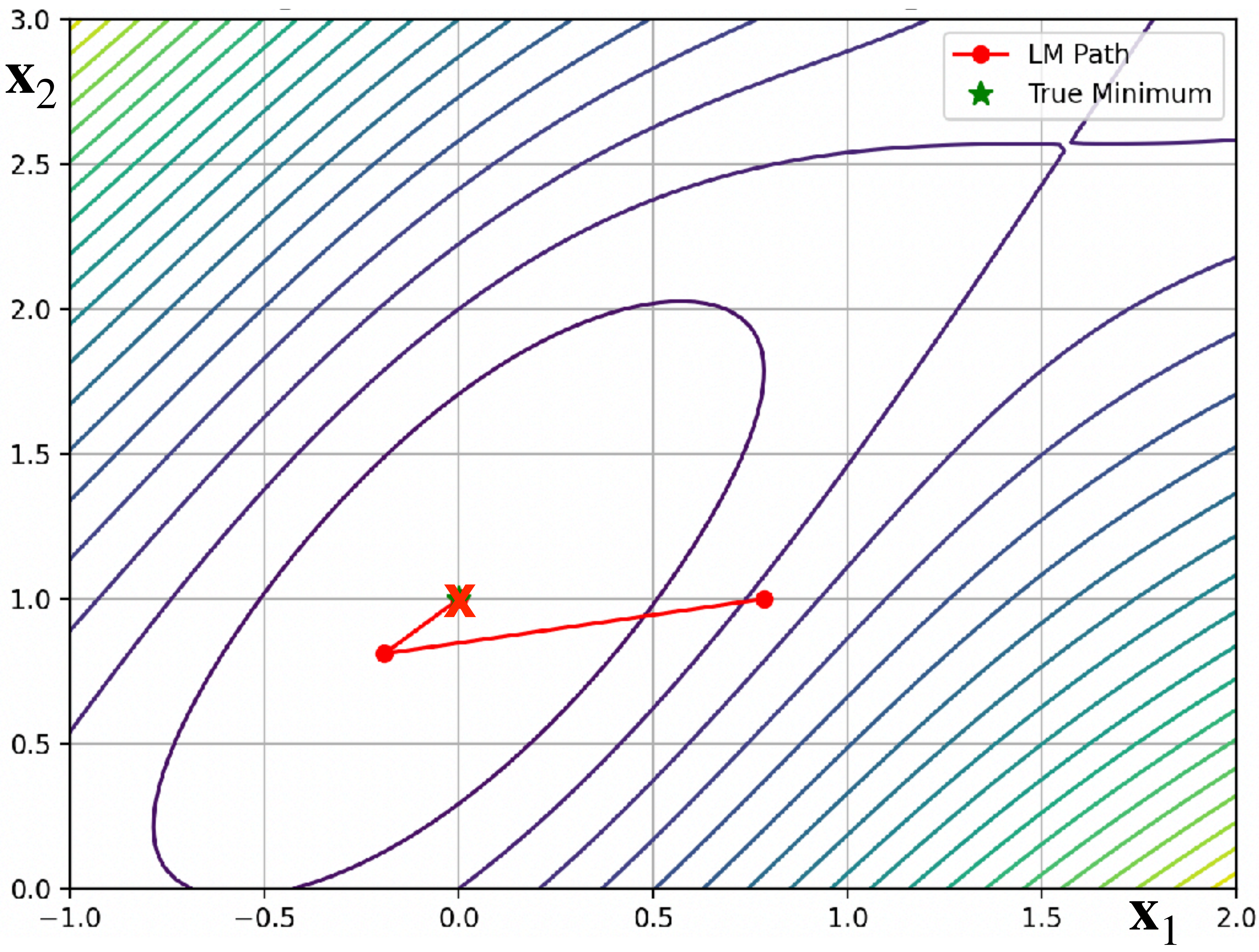
- time-consuming estimation
- relinearization => consistent loops

- So far we only estimated **expected mean** of unknown variables
- Can we get also **covariance** similarly to EKF?

Let's try it on 2D factorgraph problem



$$\Delta \mathbf{x} = \arg\min_{\mathbf{x}_1, \mathbf{x}_2} \left(\sin \mathbf{x}_1\right)^2 + \left(\mathbf{x}_1 + 1 - \mathbf{x}_2\right)^2$$



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- So far we only estimated **expected mean** of unknown variables
- Can we get also **covariance** similarly to EKF?
- **Assume** that we can **locally approximate** posterior distribution around estimated optimum **x*** by **Gaussian**.

  $p(\mathbf{x}\,|\,\mathbf{z},\mathbf{u}) \approx \mathcal{N}(\mathbf{x};\ \mathbf{x}^\star, \boldsymbol{\Sigma})$

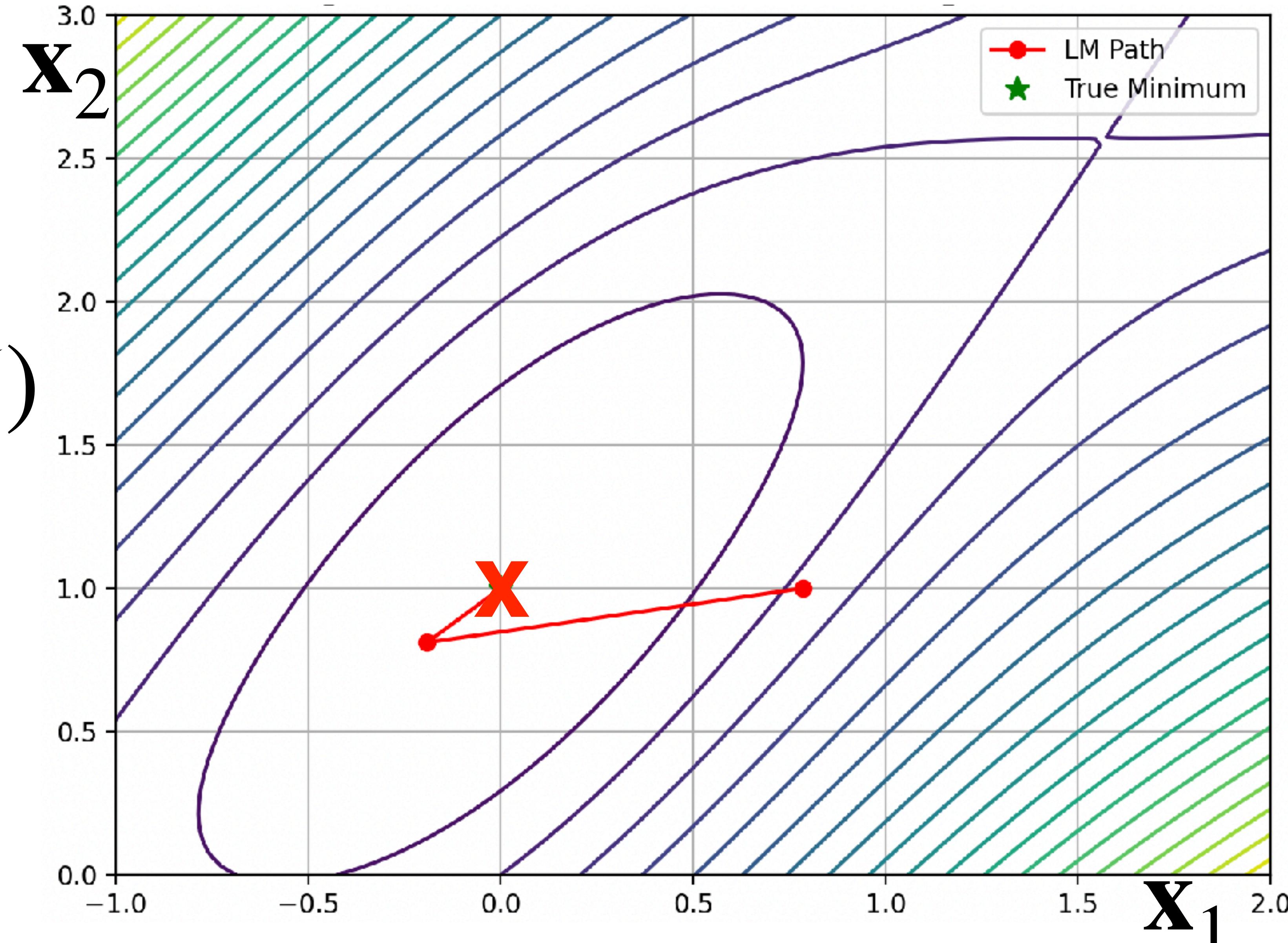
- Then factorgraph **criterion** (posterior in negative log-likelihood space) is **quadratic** form

$$\mathscr{L}(\mathbf{x}) = -\log p(\mathbf{x}\,|\,\mathbf{z},\mathbf{u}) \approx c + 0.5\,(\mathbf{x}-\mathbf{x}^\star)^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\mathbf{x}^\star)$$

- **Second order Taylor** approximation of $\mathscr{L}$ around **x***

$$\mathscr{L}(\mathbf{x}) \approx \mathscr{L}(\mathbf{x}^\star) + \underbrace{\nabla\mathscr{L}(\mathbf{x}^\star)^\top(\mathbf{x}-\mathbf{x}^\star)}_{\text{in optimum } \approx\, \mathbf{0}} + 0.5\,(\mathbf{x}-\mathbf{x}^\star)\,\mathbf{H}\,(\mathbf{x}-\mathbf{x}^\star)$$

- **Comparison** yields: $\boldsymbol{\Sigma} = \mathbf{H}^{-1}$



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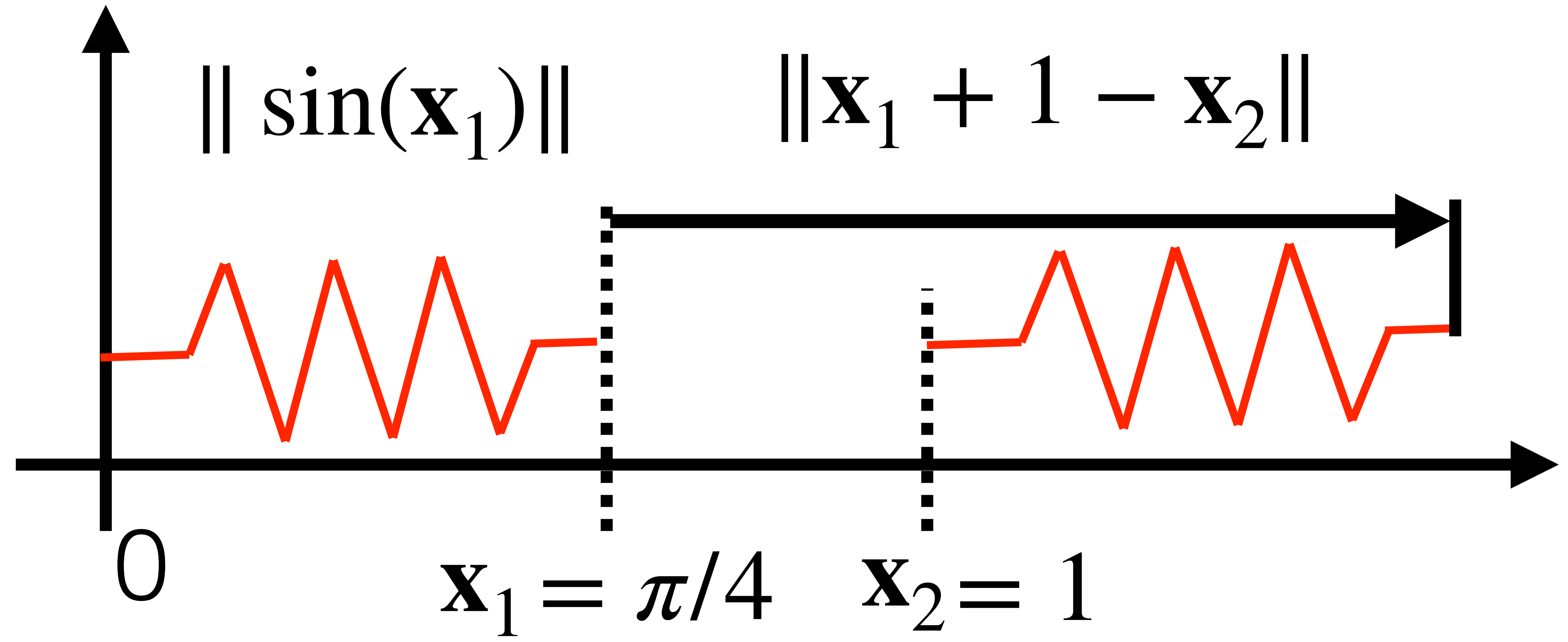

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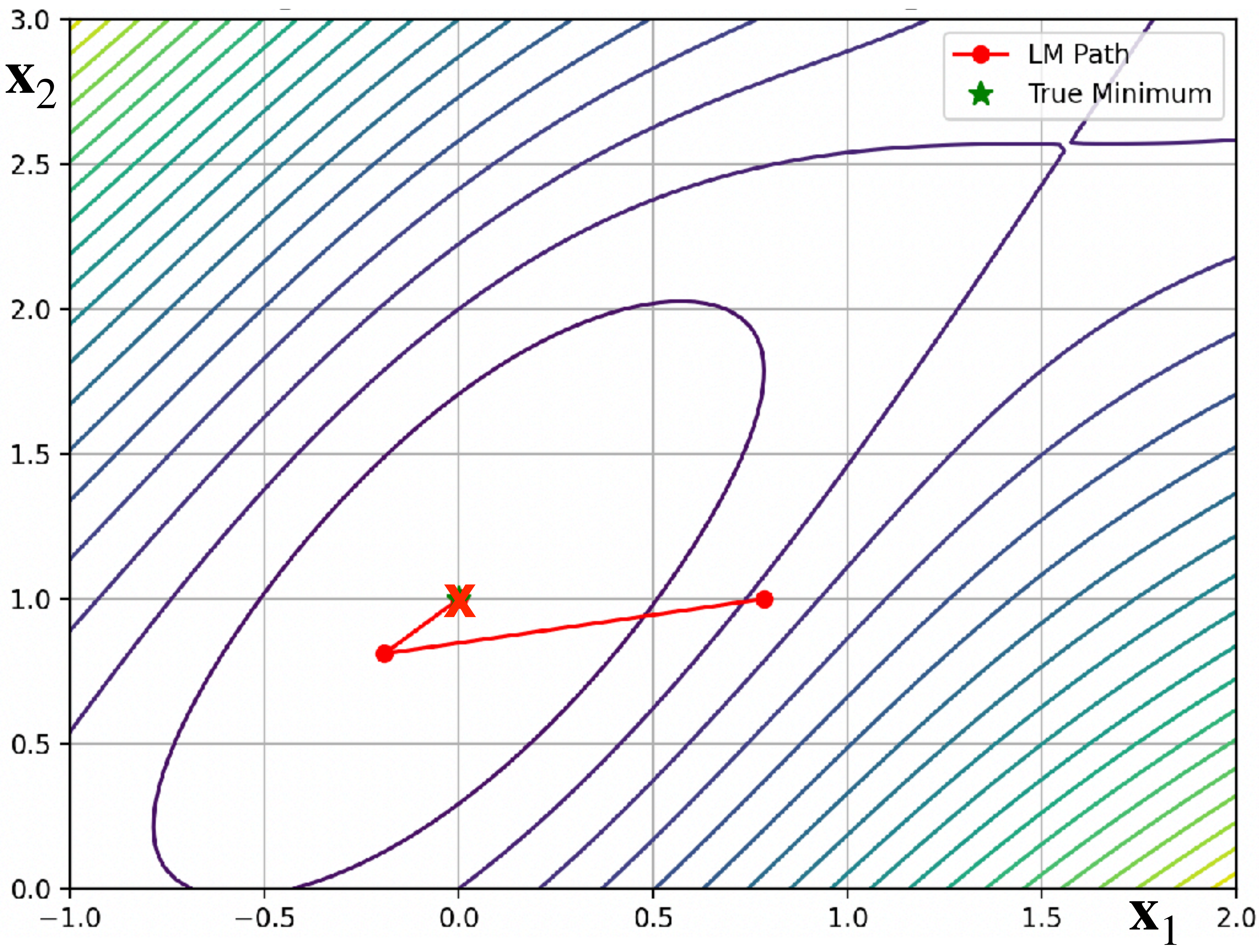

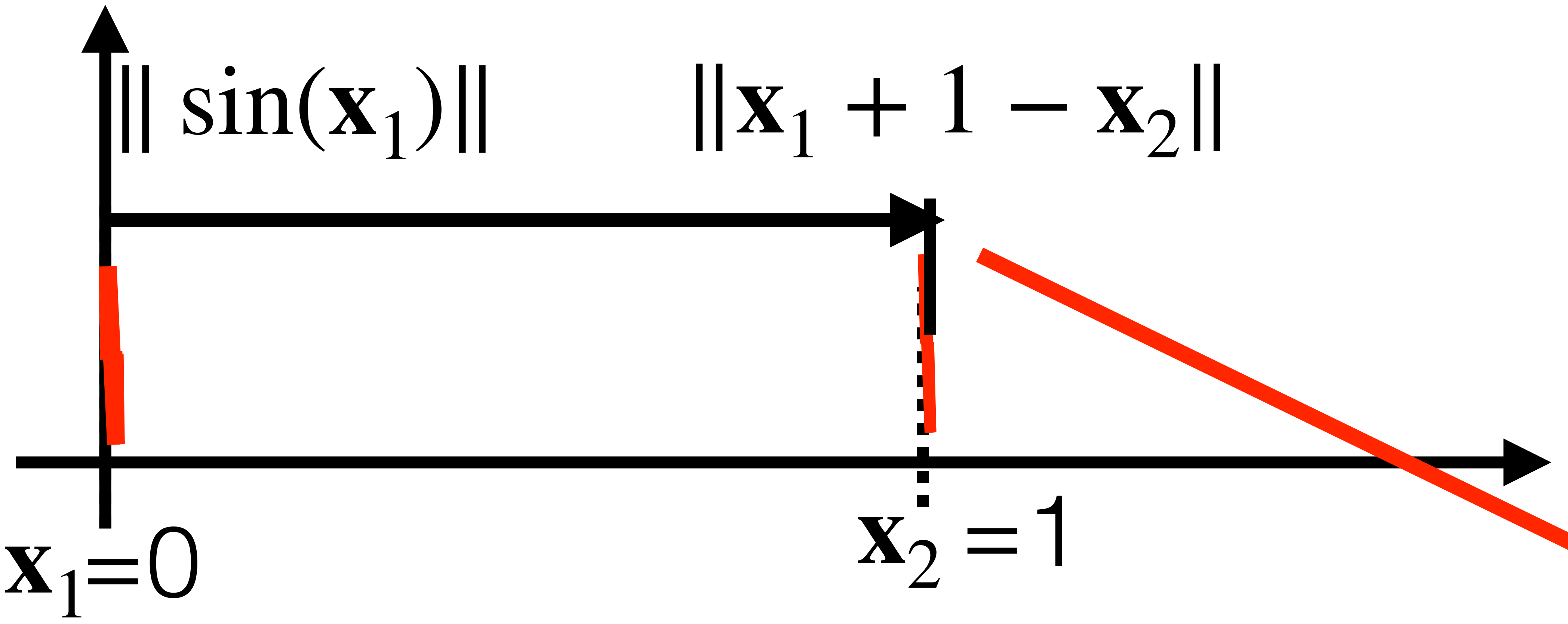

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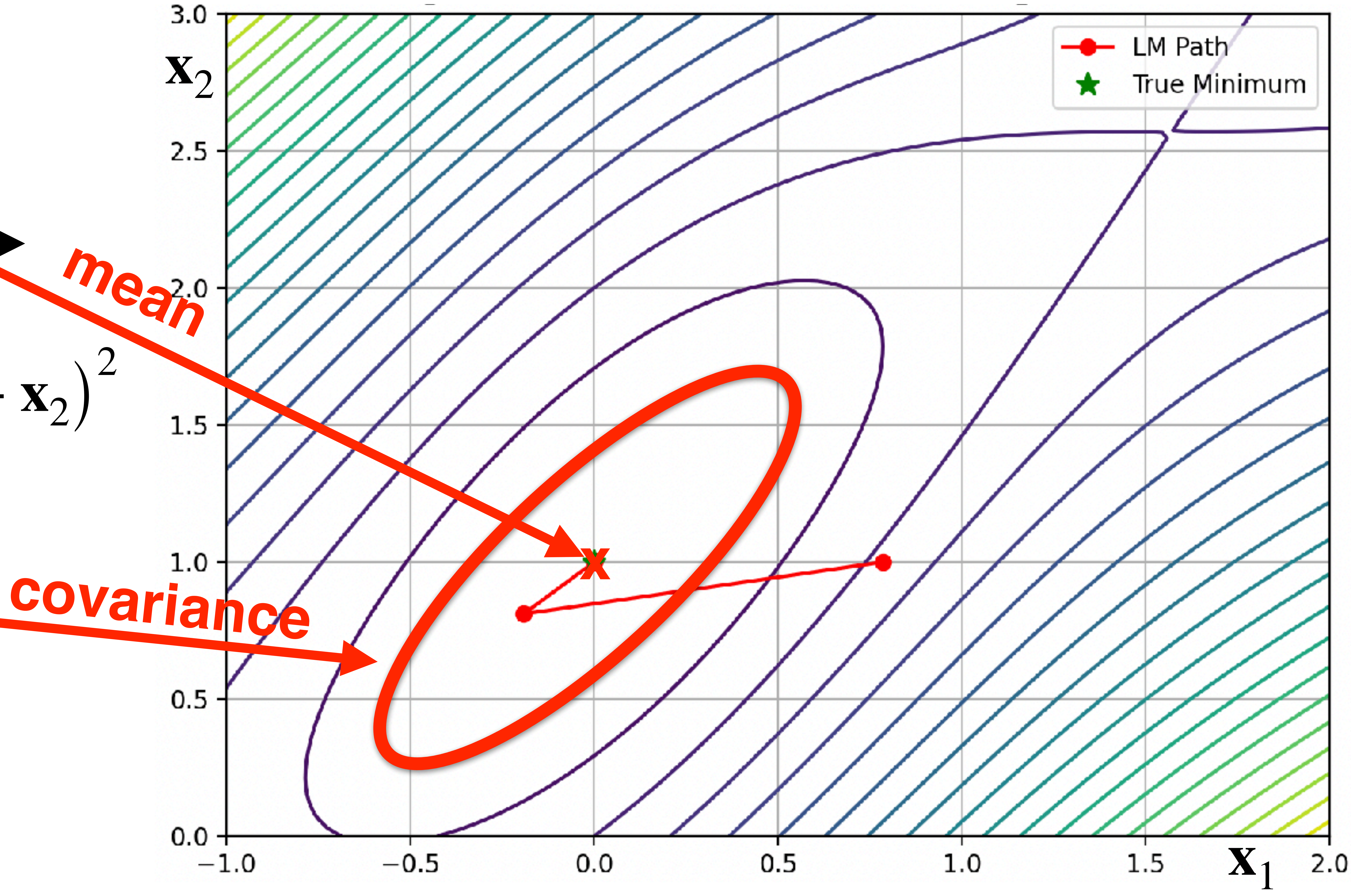
$$\mathbf{H} = \begin{bmatrix} 2\cos 2\mathbf{x}_1 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix}$$



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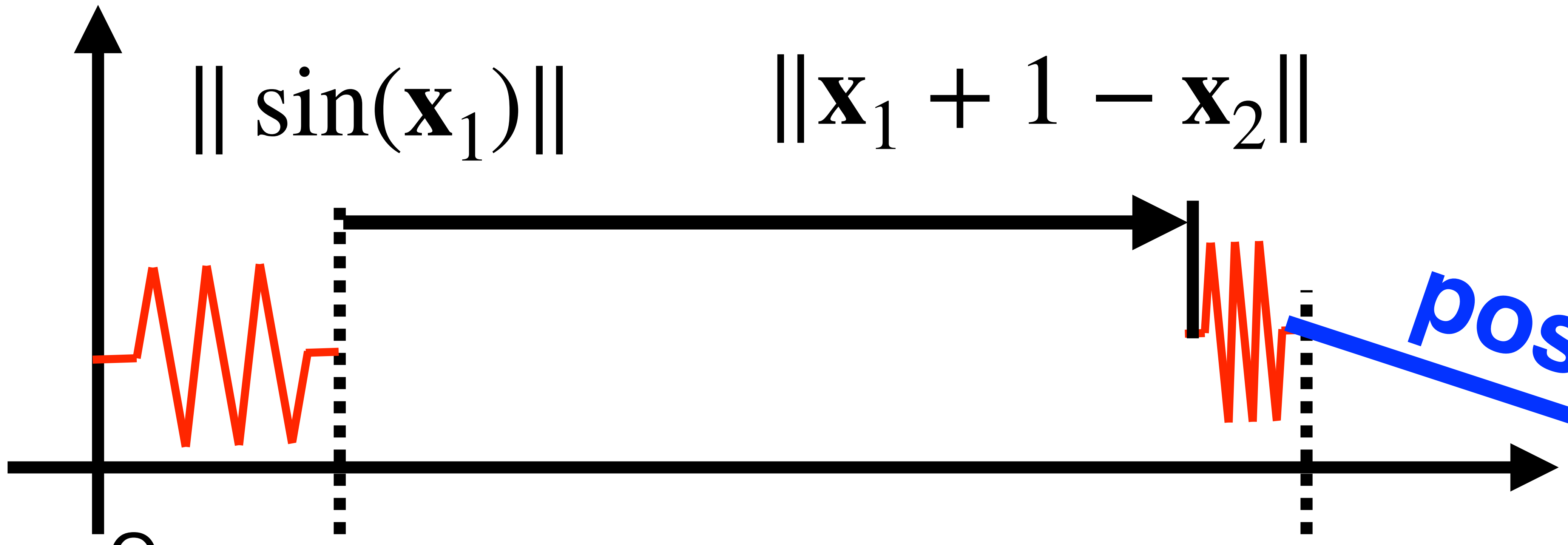

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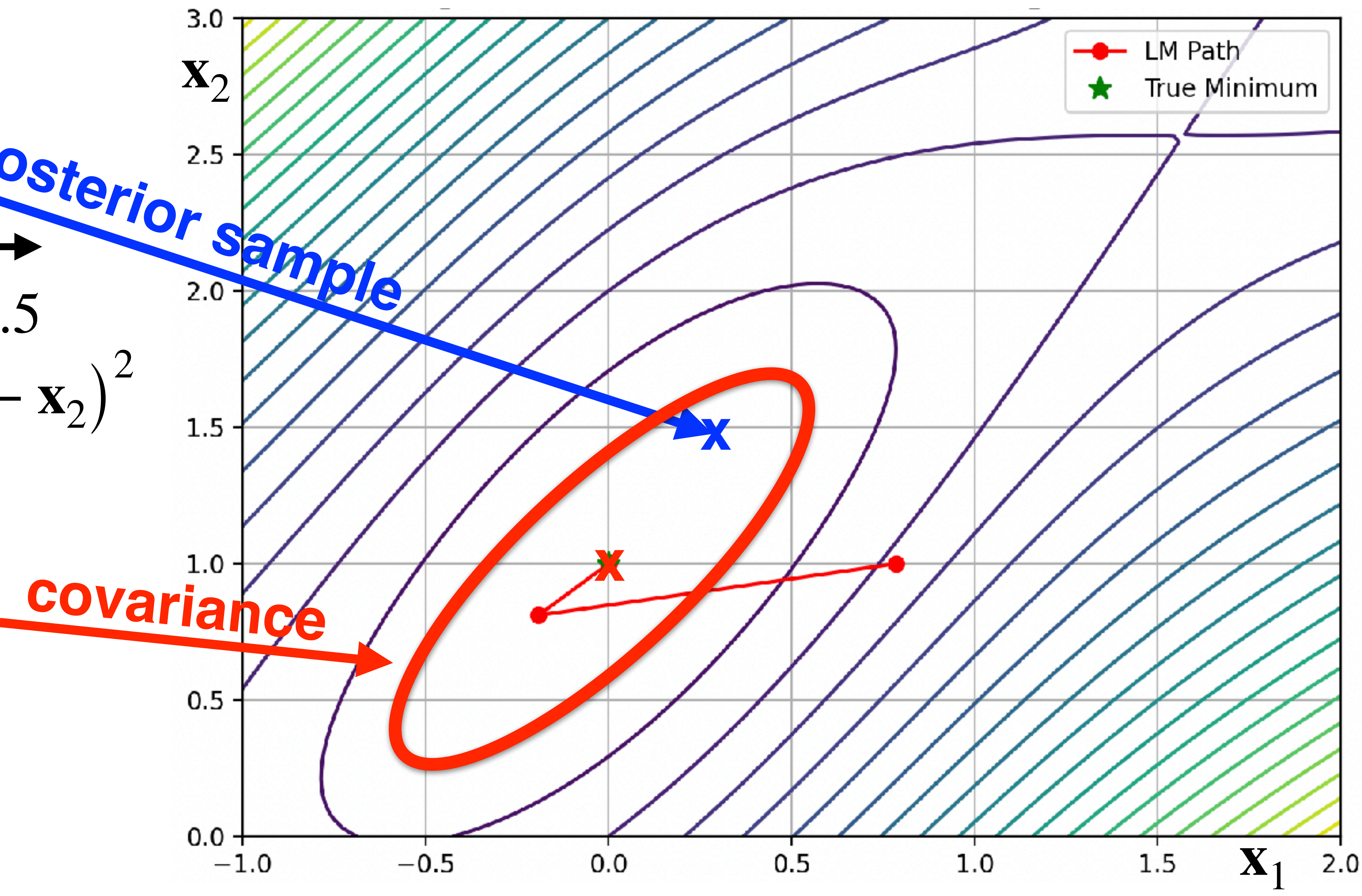

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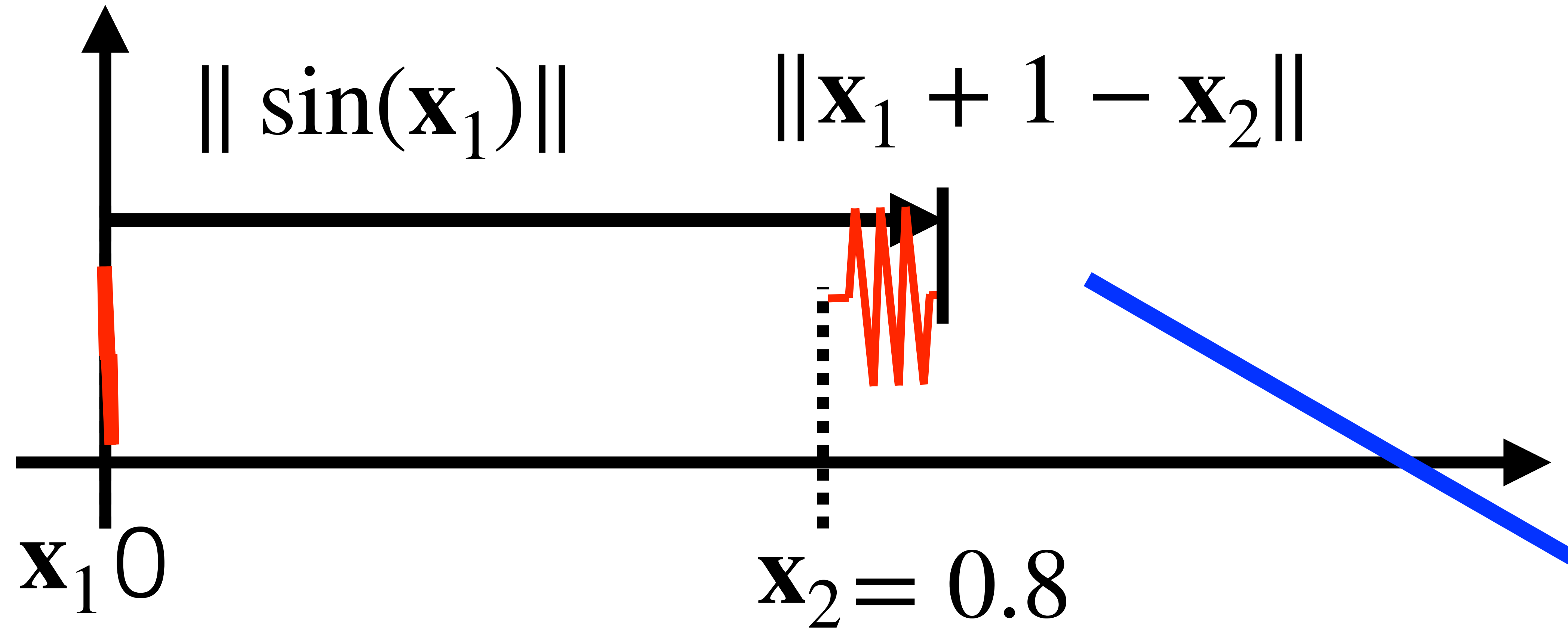

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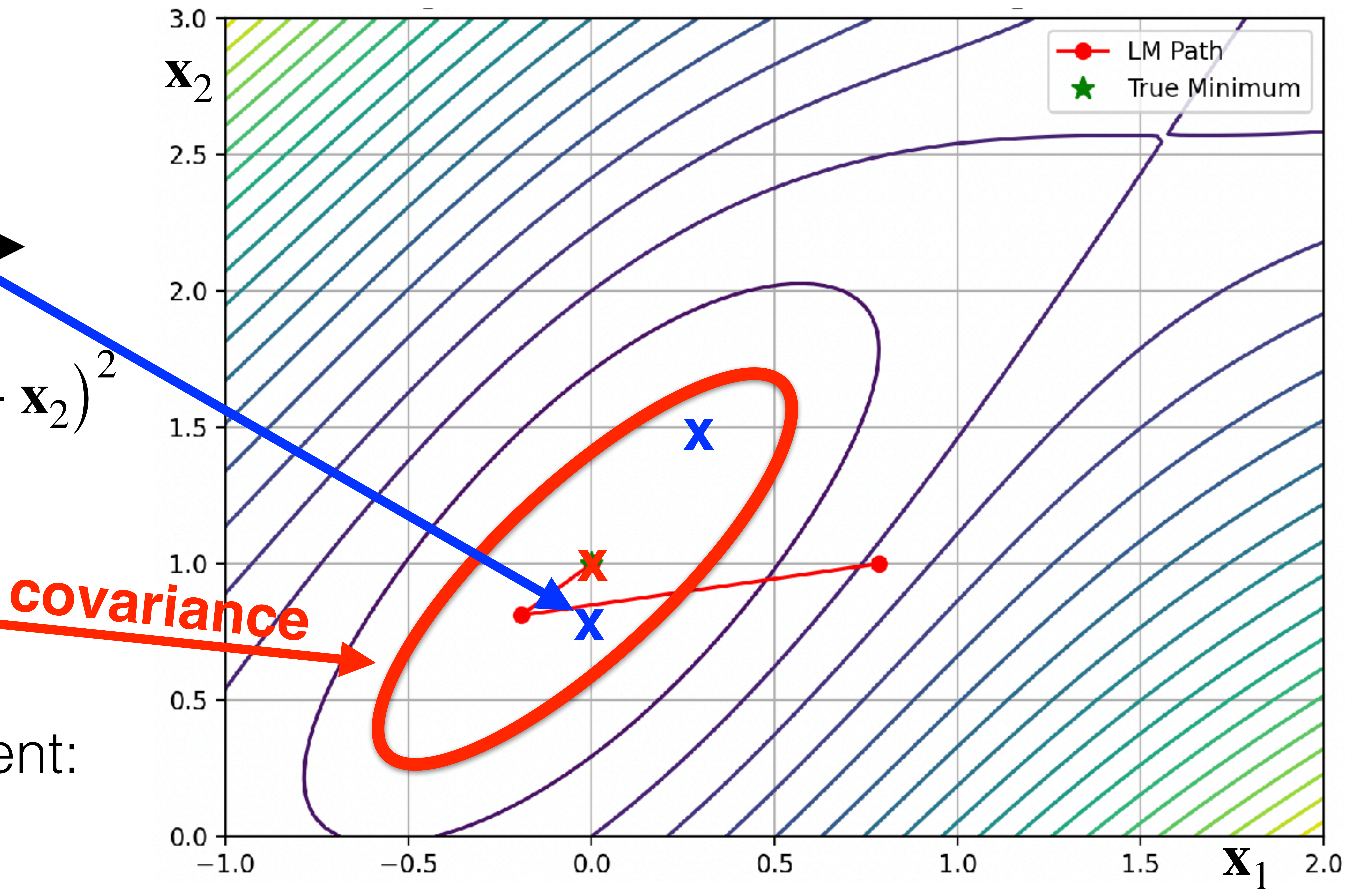
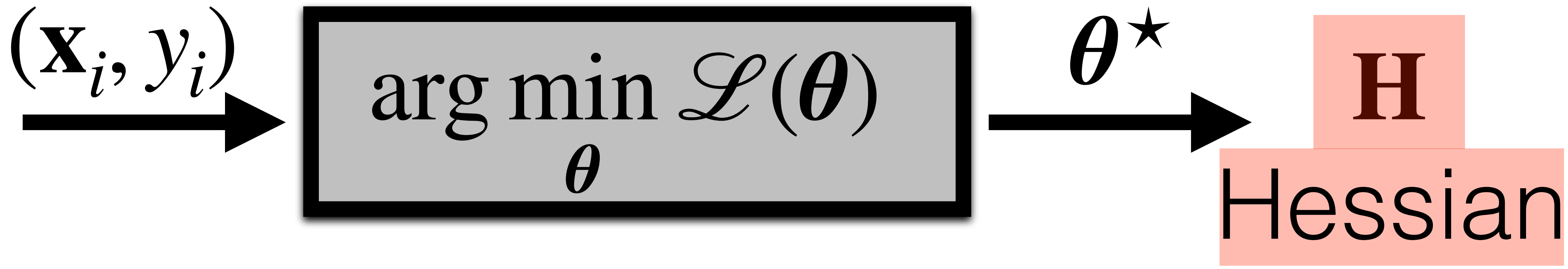


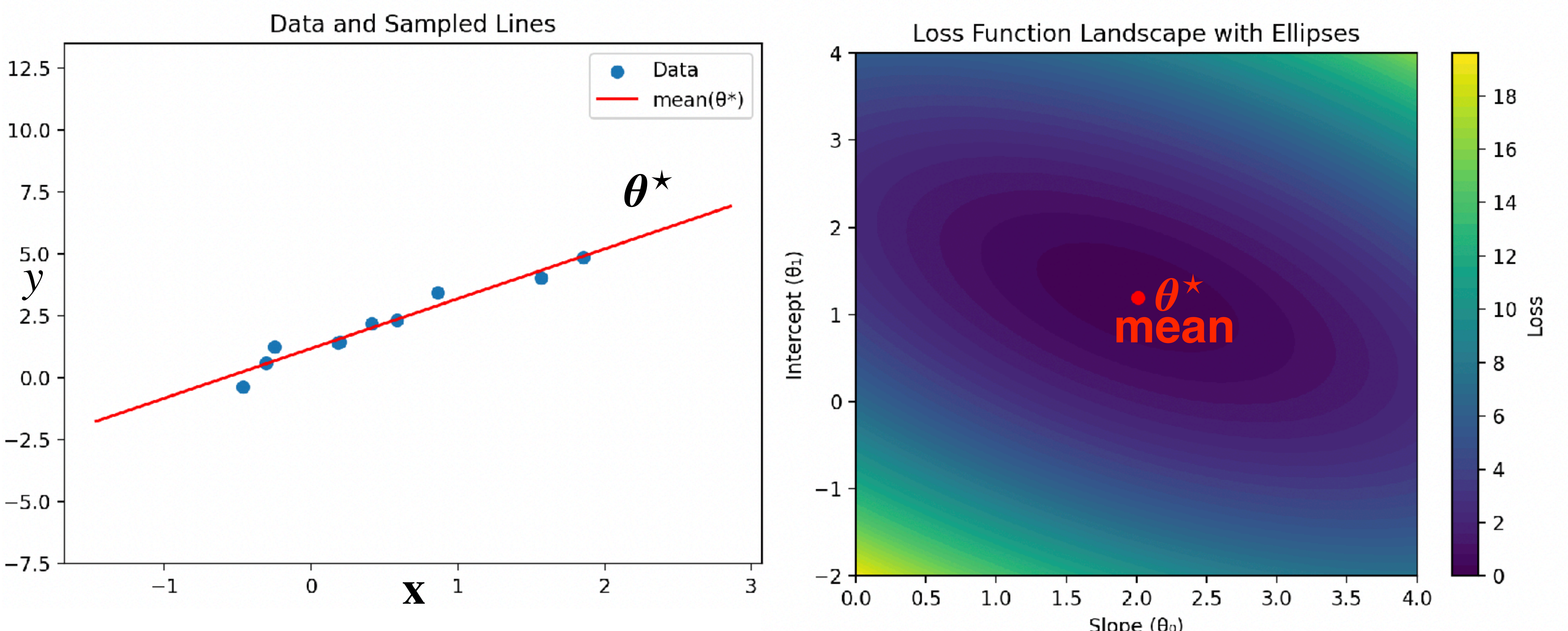
Approximation is typically sufficient:

$$\mathbf{H} \approx 2\mathbf{J}^\top\mathbf{J} = = \begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix}$$

The same issue appears in machine learning approaches

line fitting





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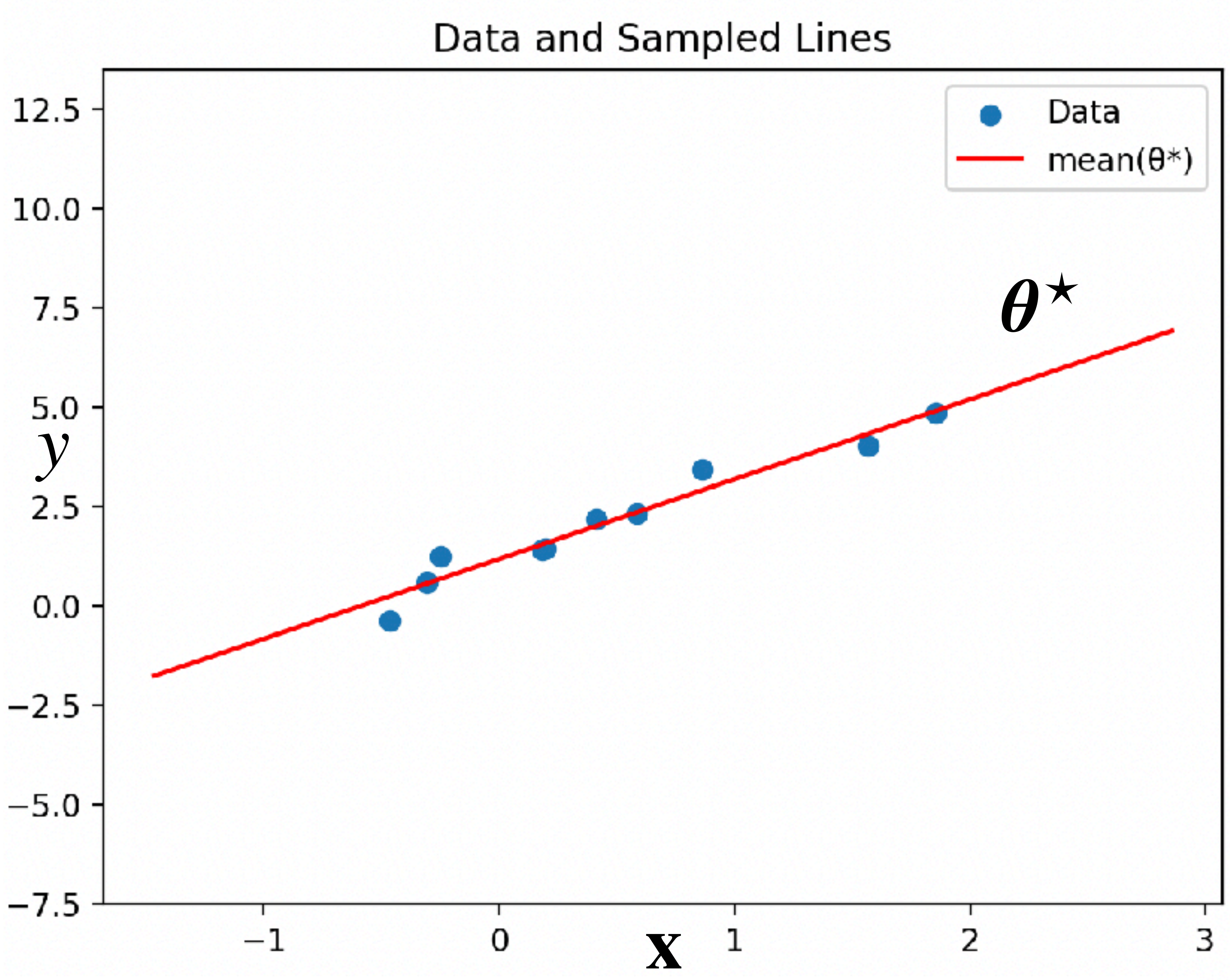
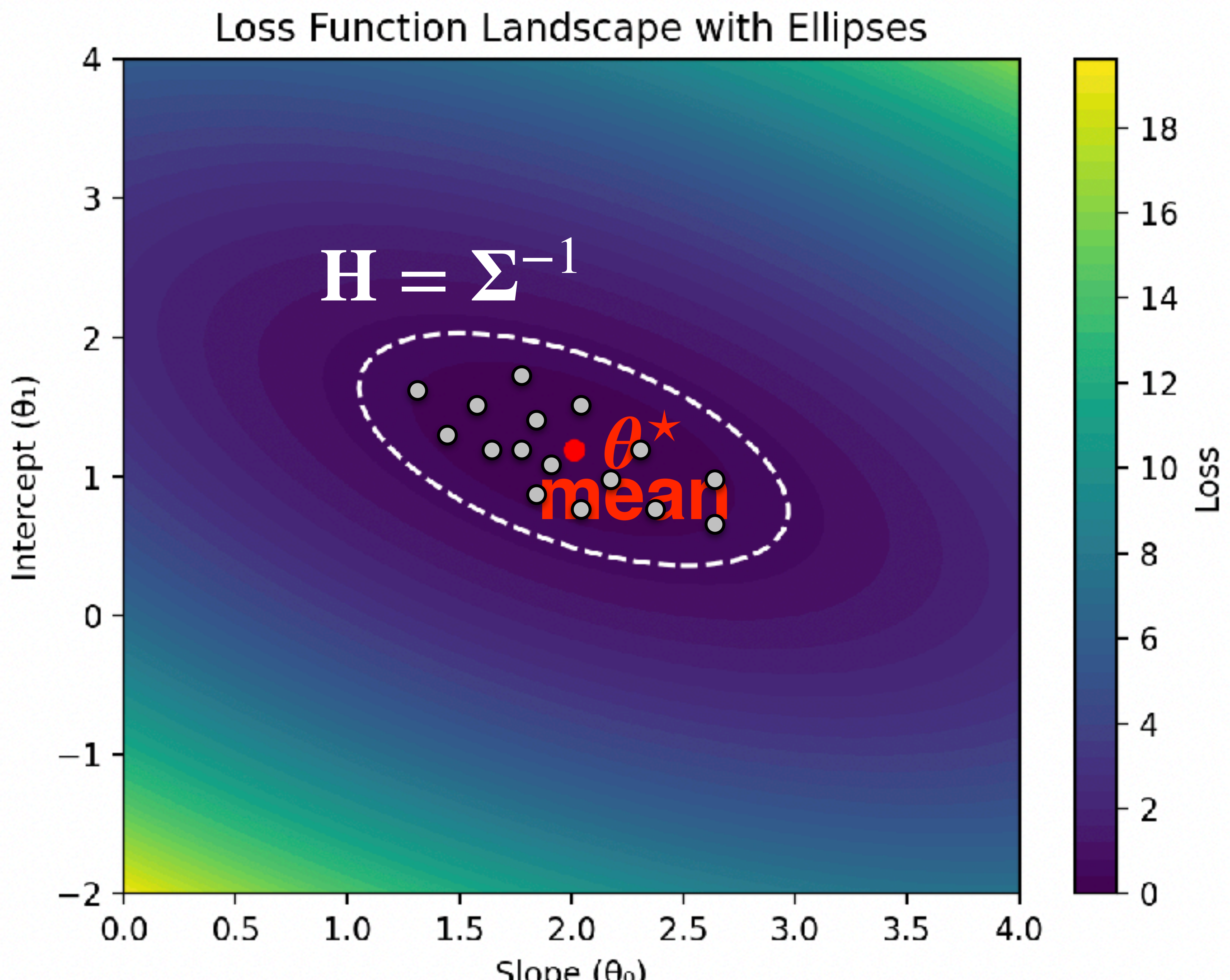
line fitting

$(\mathbf{x}_i, y_i) \rightarrow \arg\min_{\boldsymbol{\theta}} \mathscr{L}(\boldsymbol{\theta}) \rightarrow \boldsymbol{\theta}^\star$ → $\mathbf{H}$ Hessian





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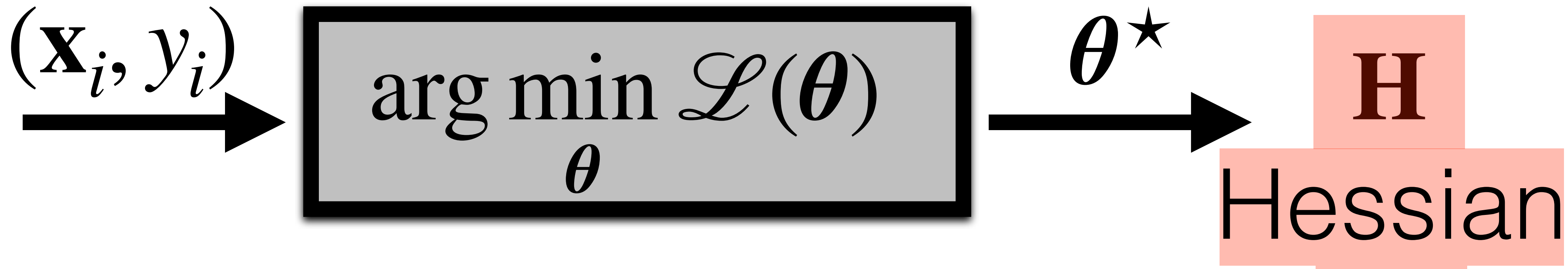

$$\mathscr{L}(\boldsymbol{\theta}) = -\log p\big(\boldsymbol{\theta} \,|\, (\mathbf{x}_i, y_i)\big) \approx \mathscr{N}(\boldsymbol{\theta};\ \boldsymbol{\theta}^\star, \boldsymbol{\Sigma}) = c + 0.5\,(\boldsymbol{\theta} - \boldsymbol{\theta}^\star)^\top \boldsymbol{\Sigma}^{-1} (\boldsymbol{\theta} - \boldsymbol{\theta}^\star)$$