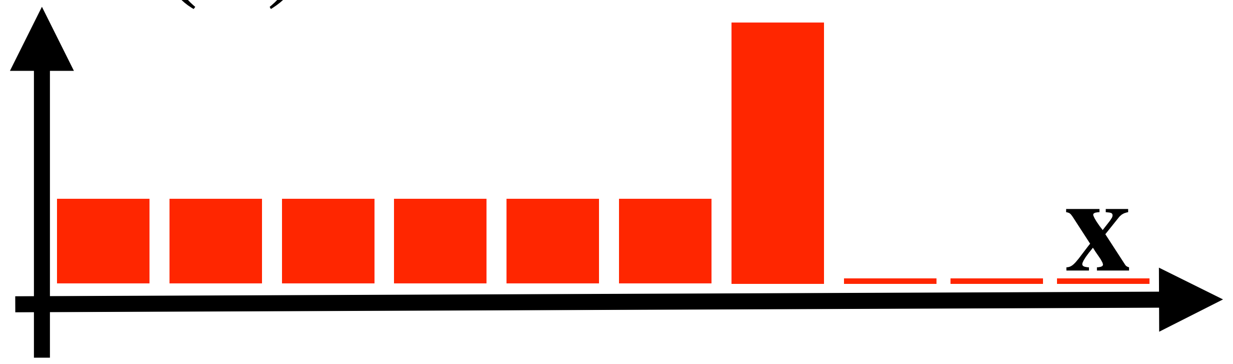
Particle filter

Karel Zimmermann

	Drawbacks	Advantages
Bayes filter	○ course of dimensionality ○ spatial discretization	○ represents arbitrary prob. distribution
Kalman filter	○ represent only gaussians ○ suffers from linearization	○ nicely scales with higher dimensions
Partical filter		○ trade-off between high-dim. scaling and representation power

Bayes filter

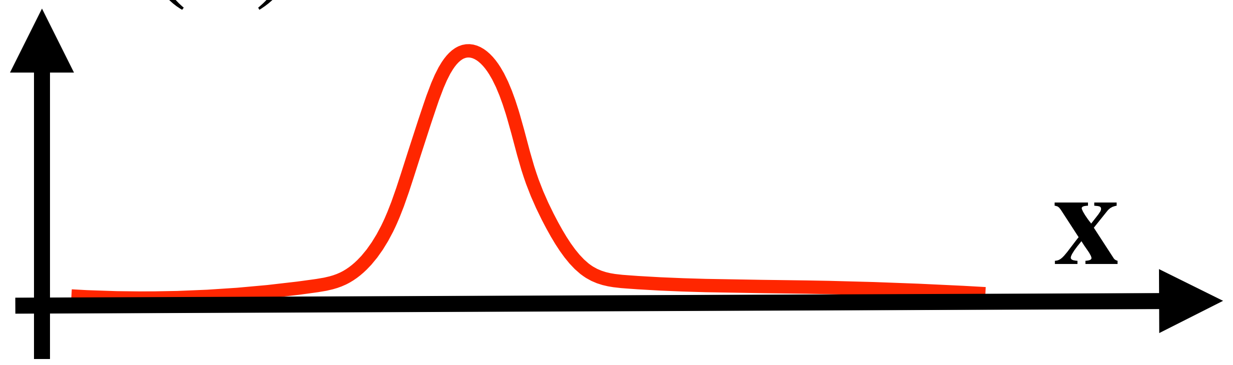
bel(**x**) discrete



$$\text{bel}(\mathbf{x}) = [p_1, p_2, \ldots, p_m]$$

Kalman filter

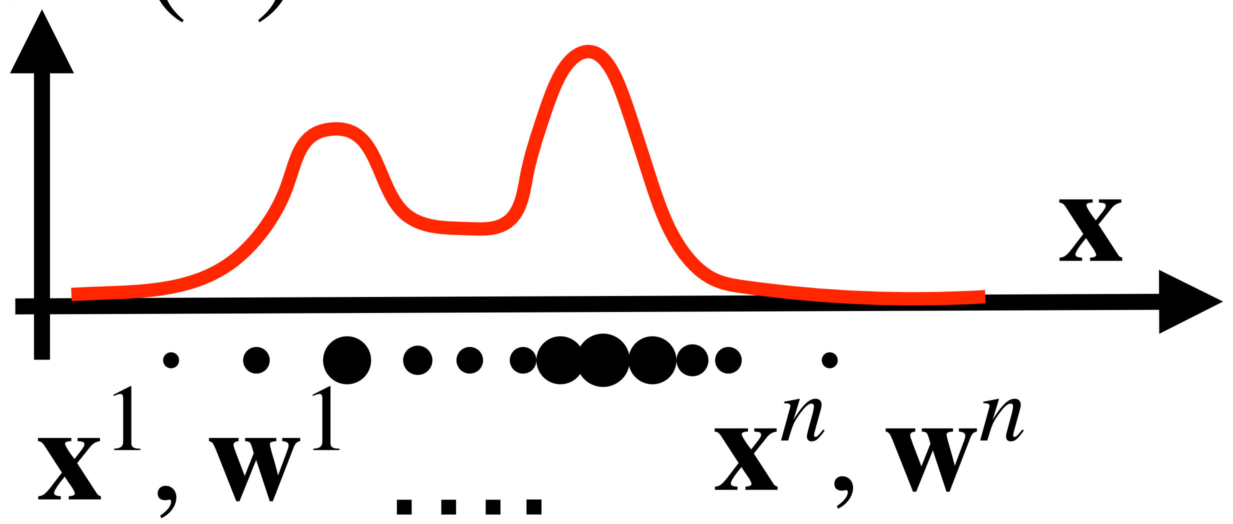
bel(**x**) continuous



$$\text{bel}(\mathbf{x}) = \mathcal{N}(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{\exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)}{\sqrt{(2\pi)^n \det(\boldsymbol{\Sigma})}}$$

Partical filter

bel(**x**) continuous



$$\text{bel}(\mathbf{x}) = \sum_{i=1}^{n} \mathbf{w}^i \cdot \delta_{\mathbf{x}^i}(\mathbf{x})$$

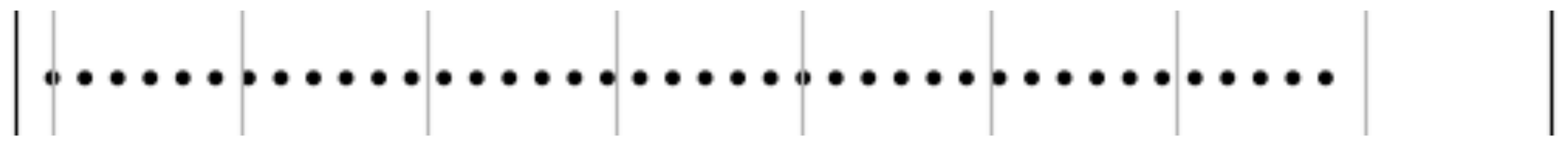
Dirac impulse function / smoothening kernel

1. Initialize particles: $\mathcal{X}_0 = \{\mathbf{x}_0^1, \ldots, \mathbf{x}_0^n\}$

Kidnapped robot problem

Particles = hypothesis about the current state

$\mathbf{x}_0^i$



Particle filter

1. Initialize particles: $\mathcal{X}_0 = \{\mathbf{x}_0^1, \ldots, \mathbf{x}_0^n\}$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_{t-1}^i$

   $\overline{\mathbf{x}}_t^i \sim p(\mathbf{x}_t \mid \mathbf{x}_{t-1}^i, \mathbf{u}_t)$

Particle filter

$p(\mathbf{z}_t = 1 \mid \mathbf{x}_t)$

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3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\overline{\mathbf{x}}_t^i$

   $$\mathbf{w}_t^i = p(\mathbf{z}_t \mid \overline{\mathbf{x}}_t^i)$$

$\mathbf{x}_0^i$

$\overline{\mathbf{x}}_1^i$

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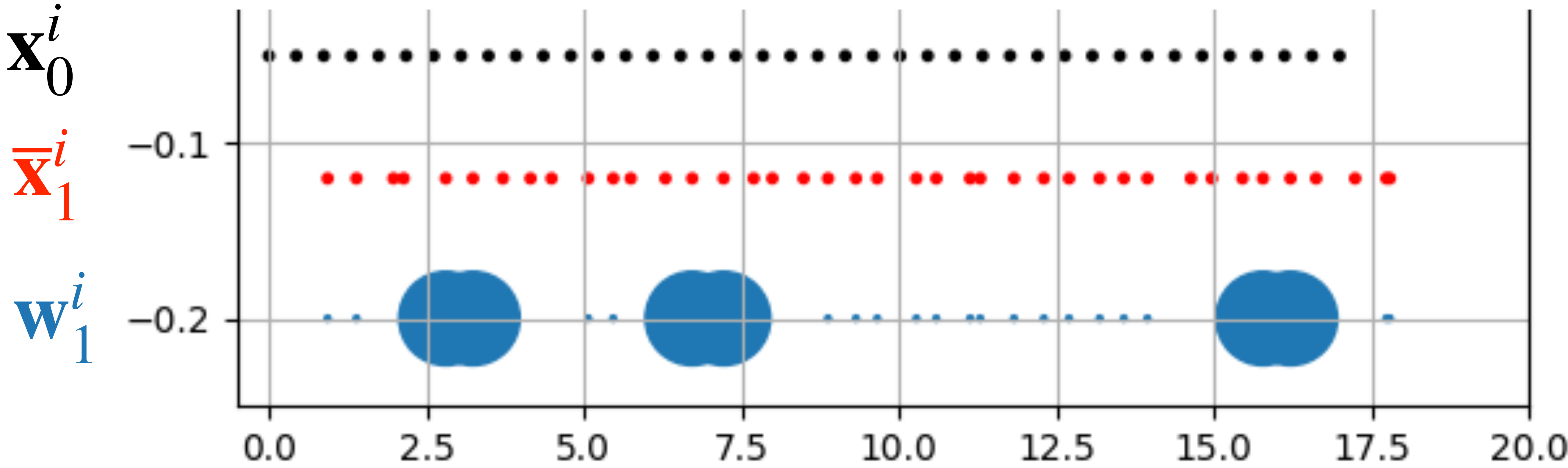
$p(\mathbf{z}_t = 1 \,|\, \mathbf{x}_t)$



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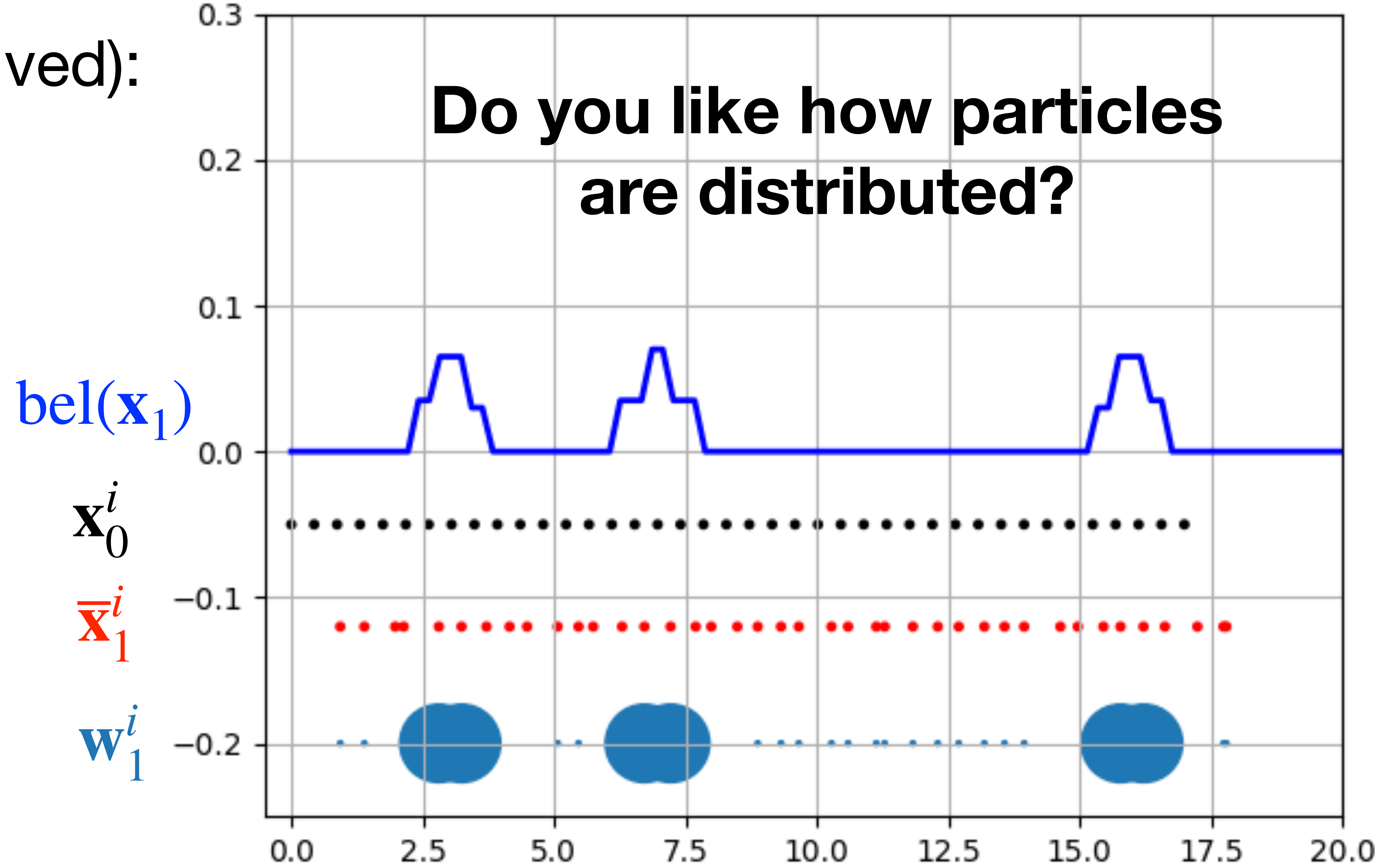

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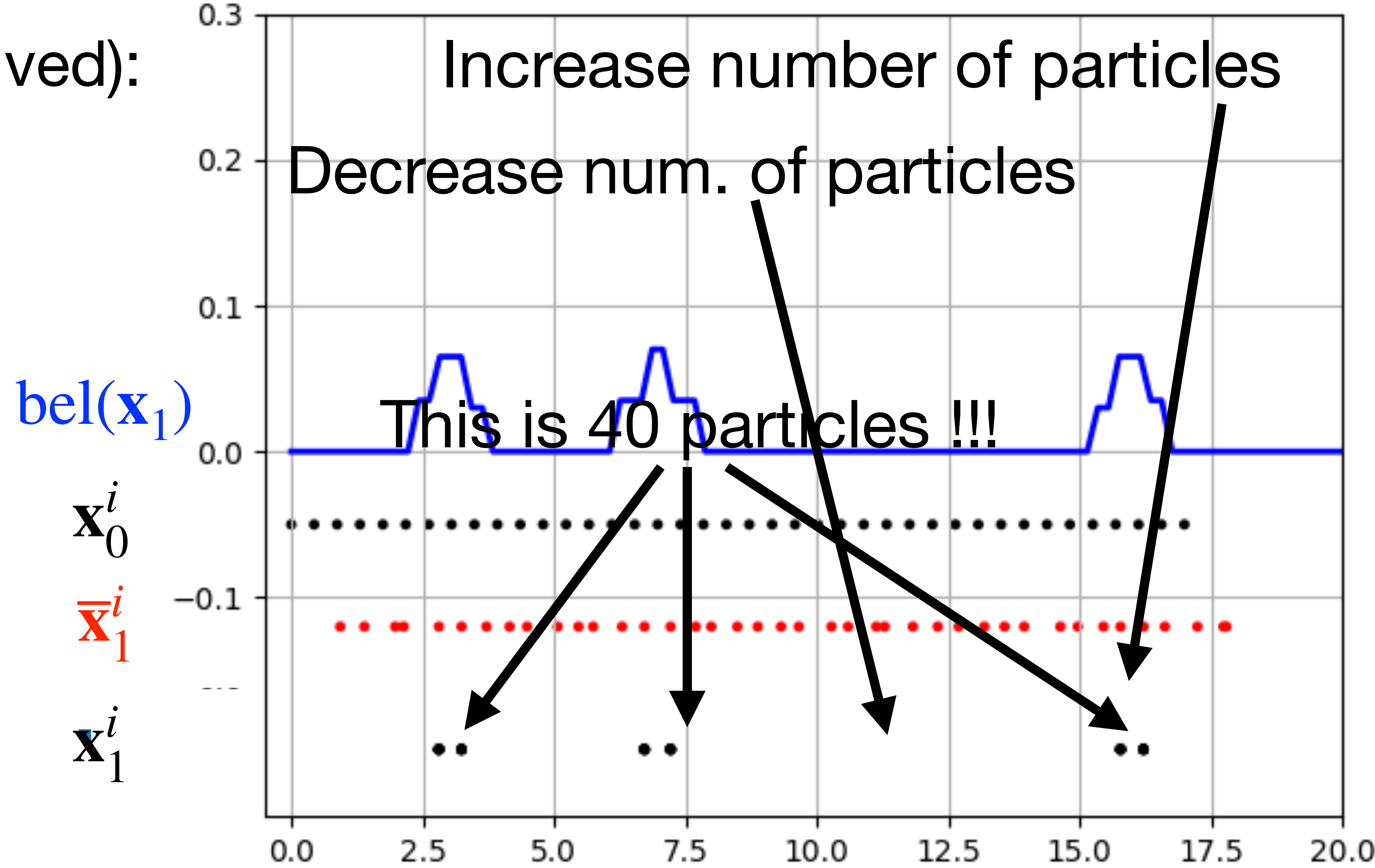
$p(\mathbf{z}_t = 1 \mid \mathbf{x}_t)$



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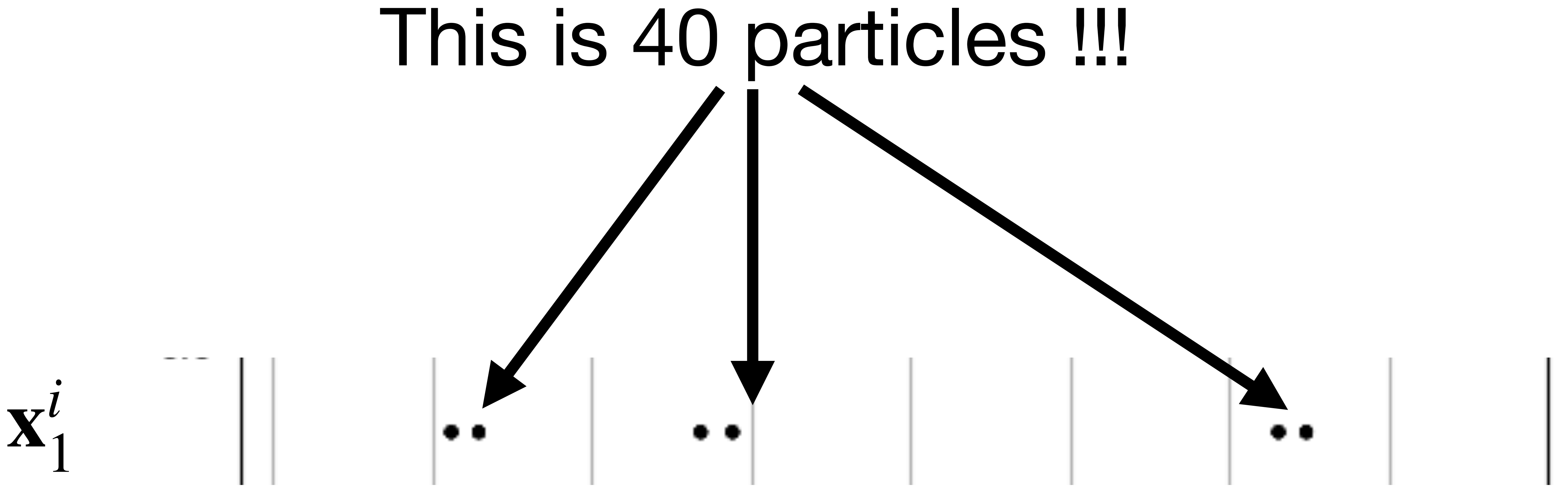
4. Resample
   Draw $\mathcal{X}_t = \{\mathbf{x}_t^1, \ldots, \mathbf{x}_t^n\} \propto \mathbf{w}_t^i$
5. Repeat from 2:
   $t = t + 1$



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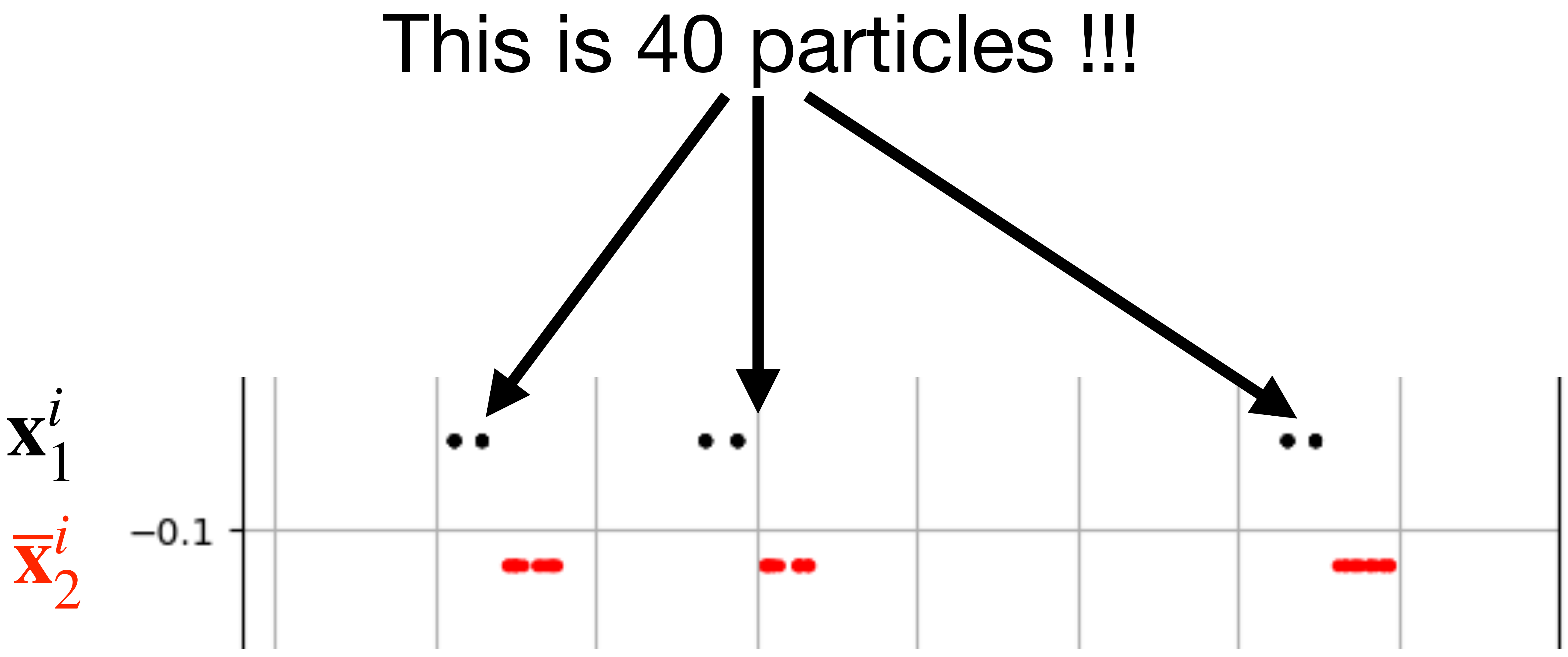

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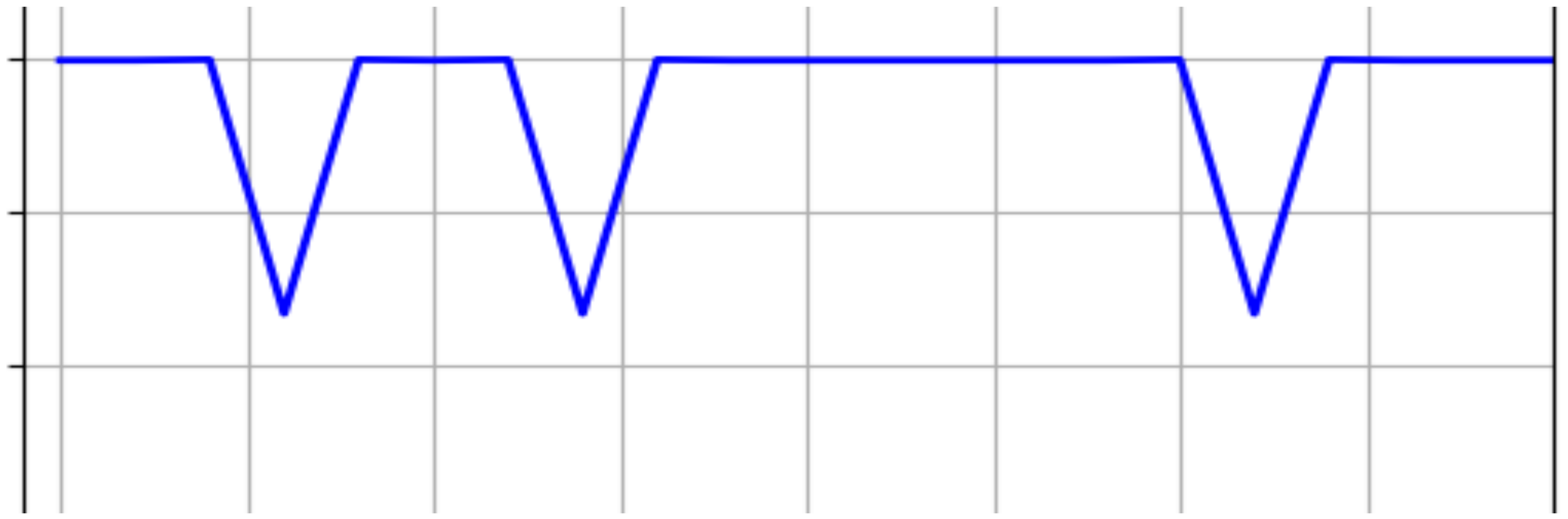
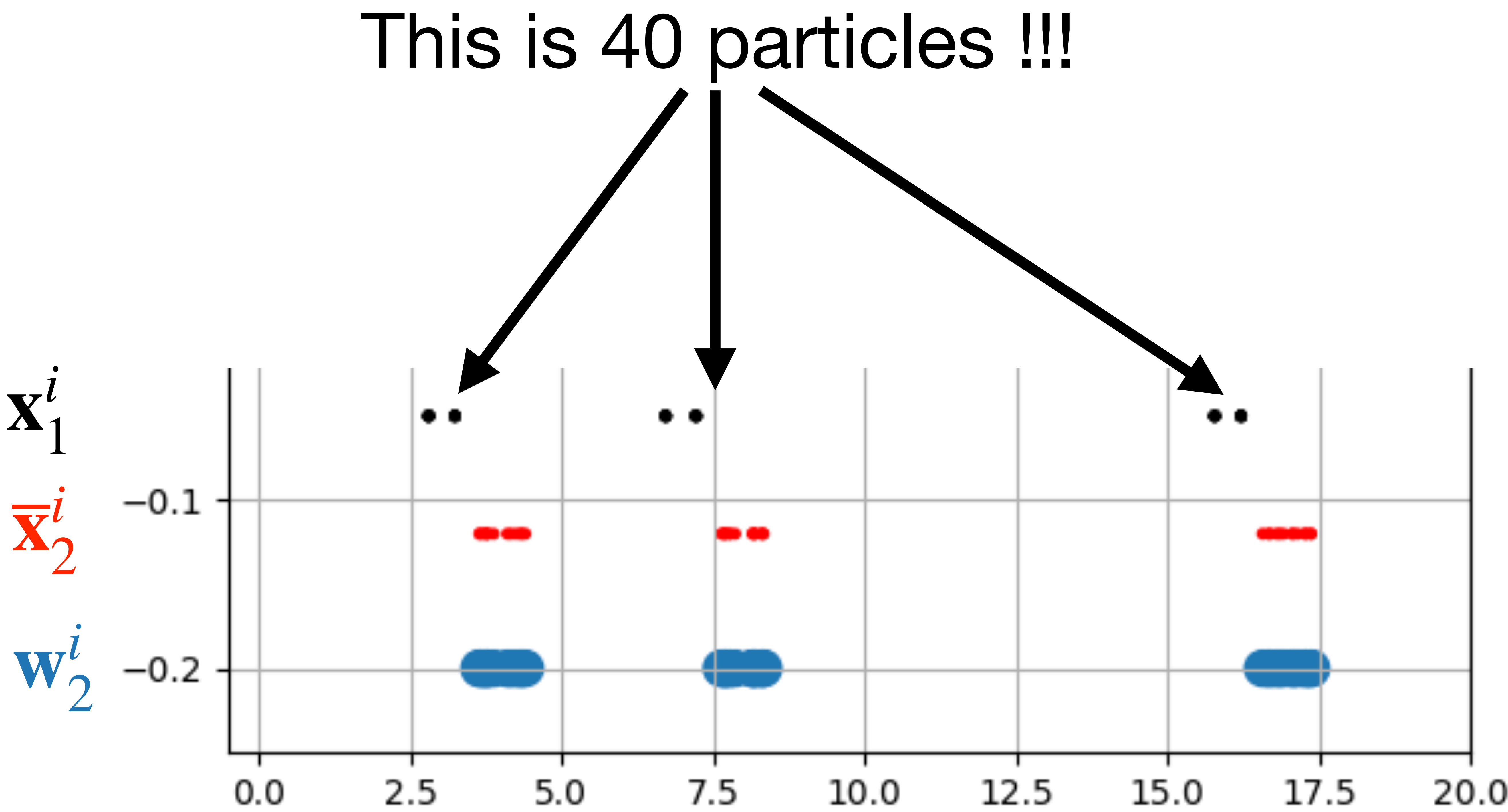
$p(\mathbf{z}_t = 0 \mid \mathbf{x}_t)$





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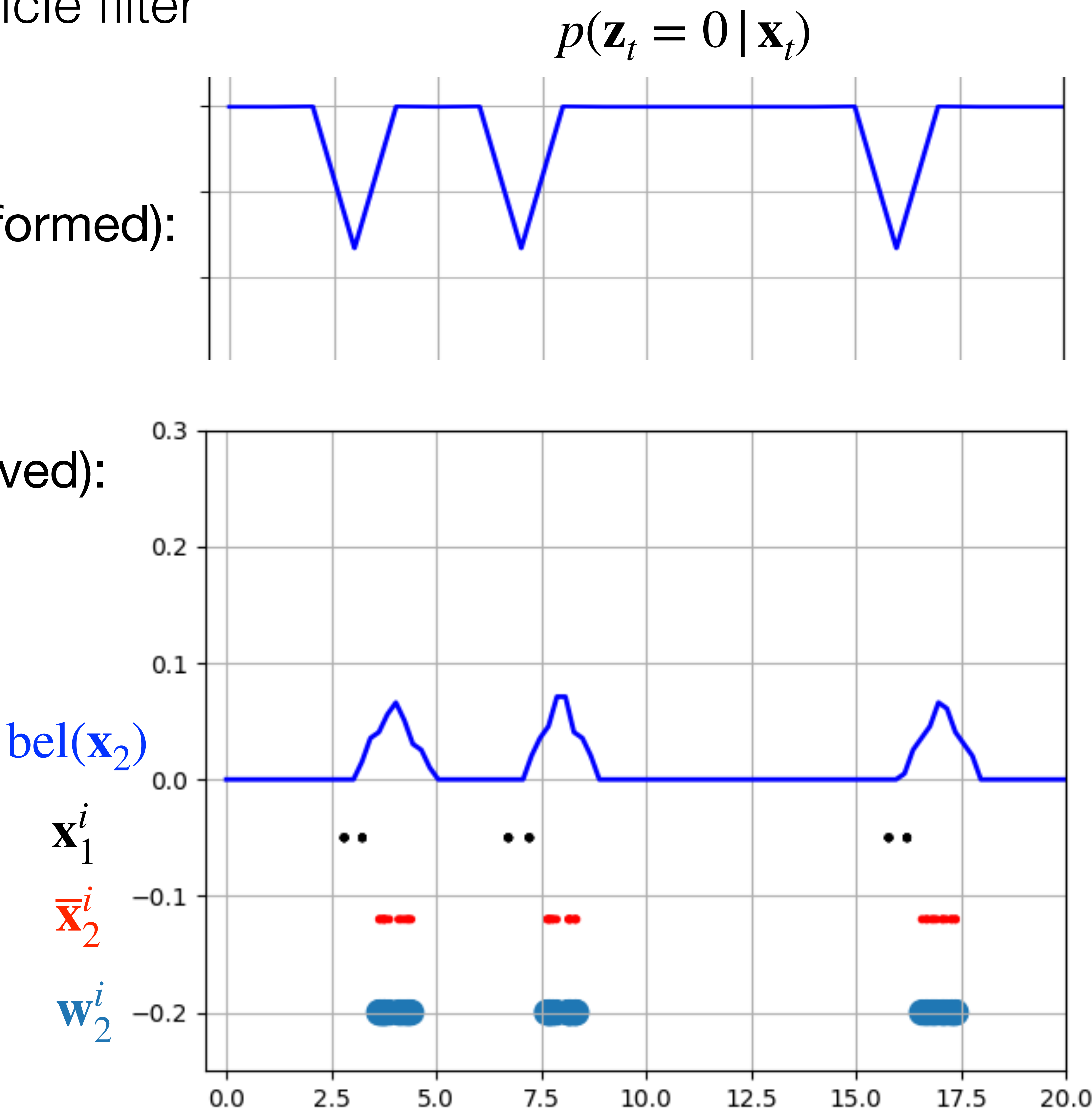

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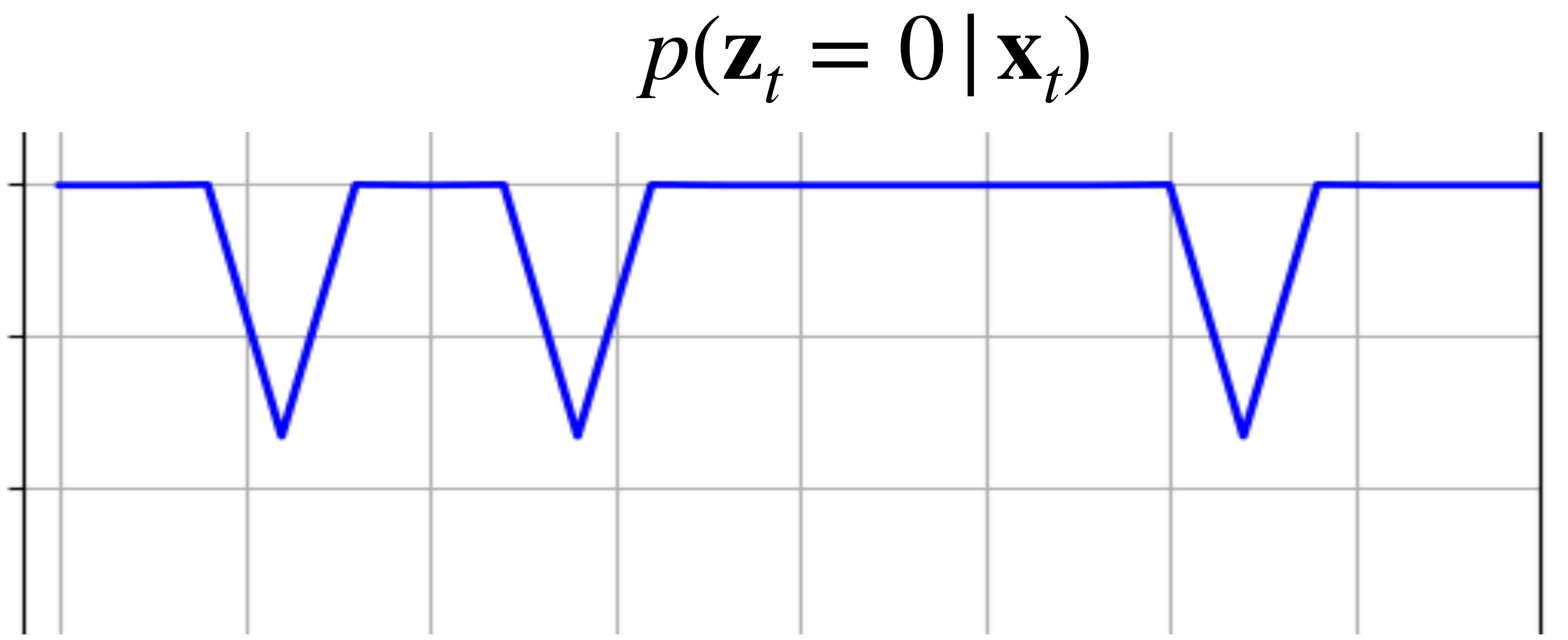

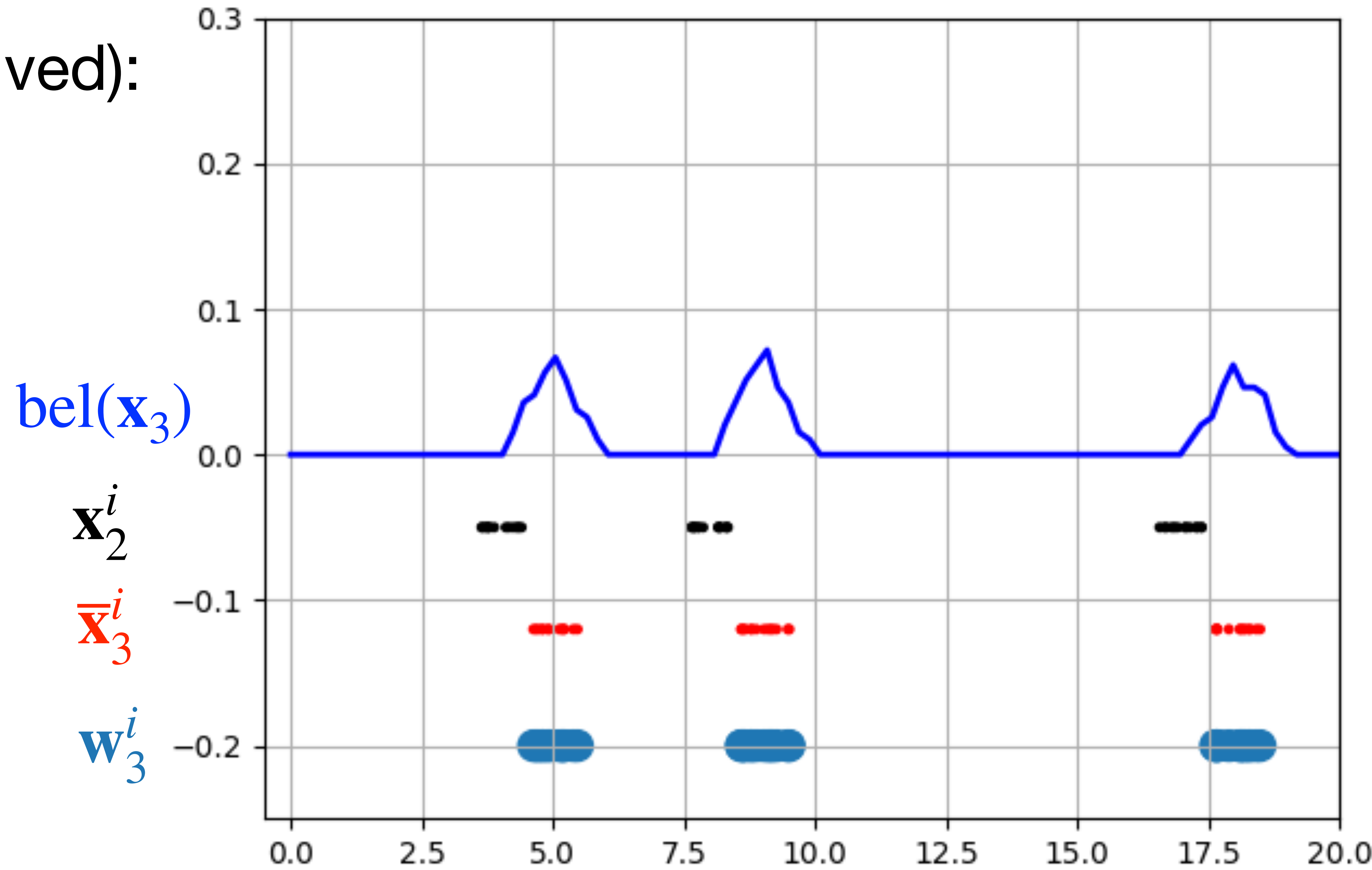

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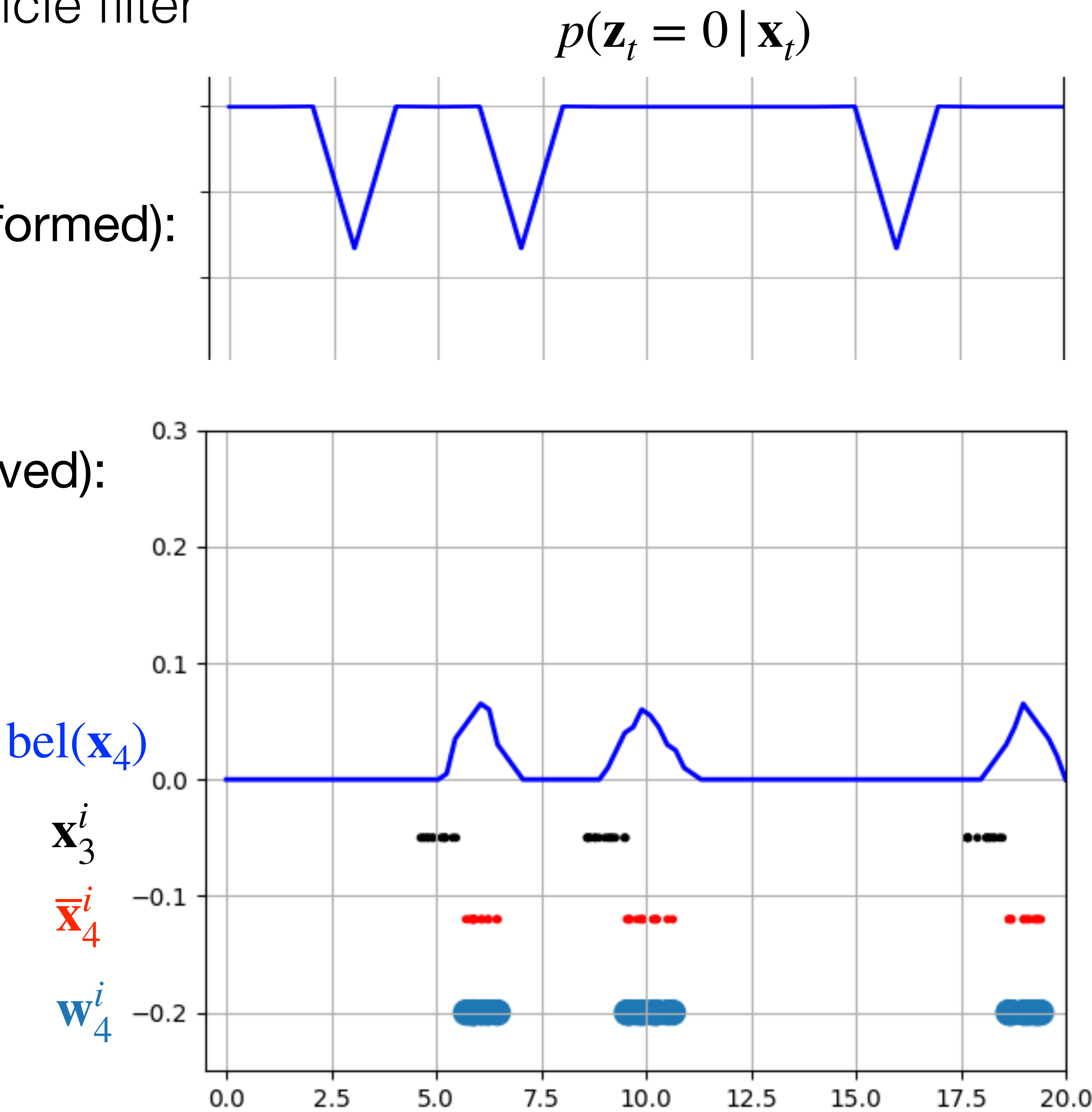

Particle filter

$p(\mathbf{z}_t = 1 \mid \mathbf{x}_t)$

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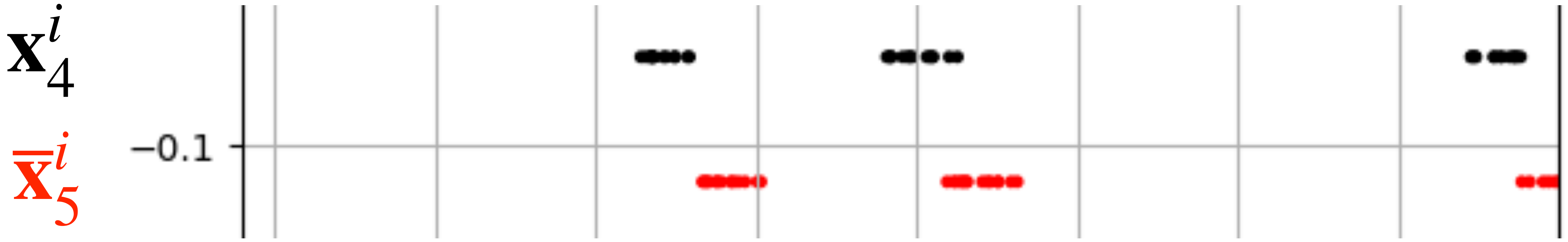

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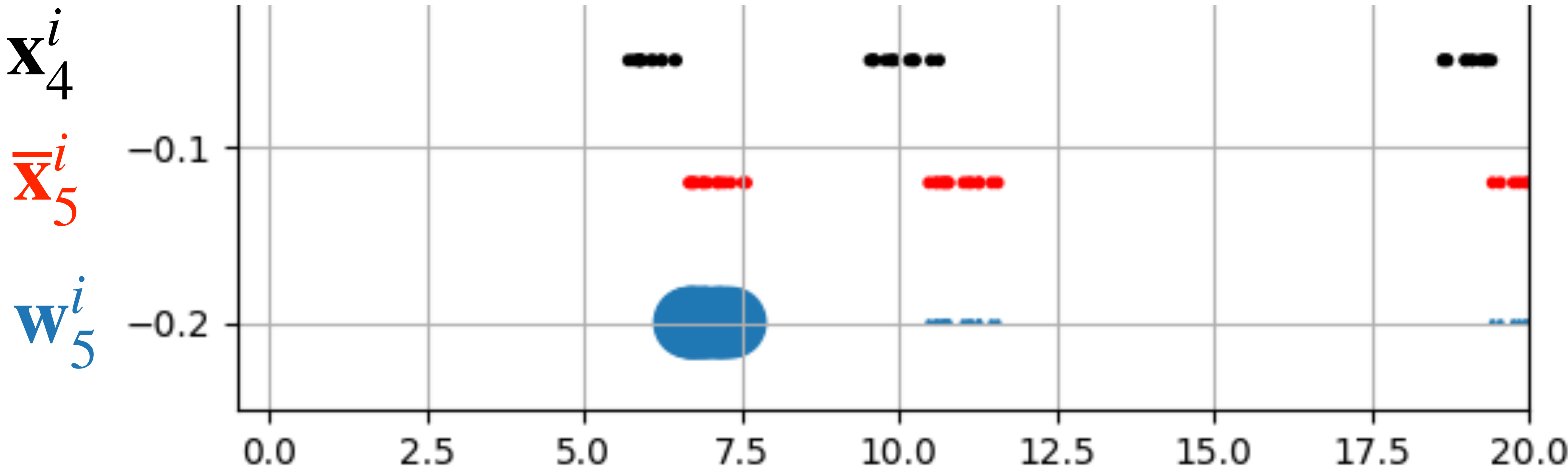

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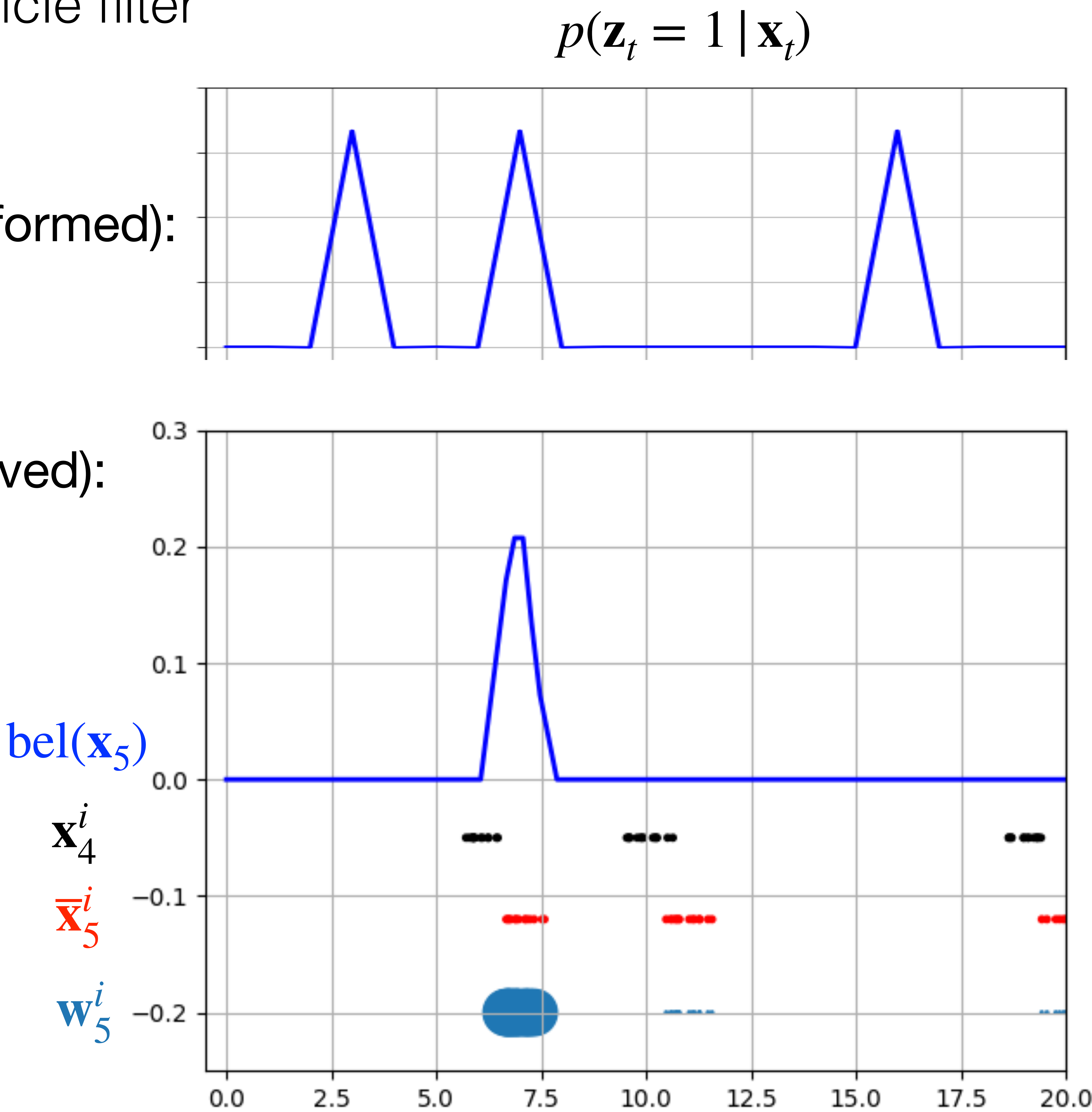

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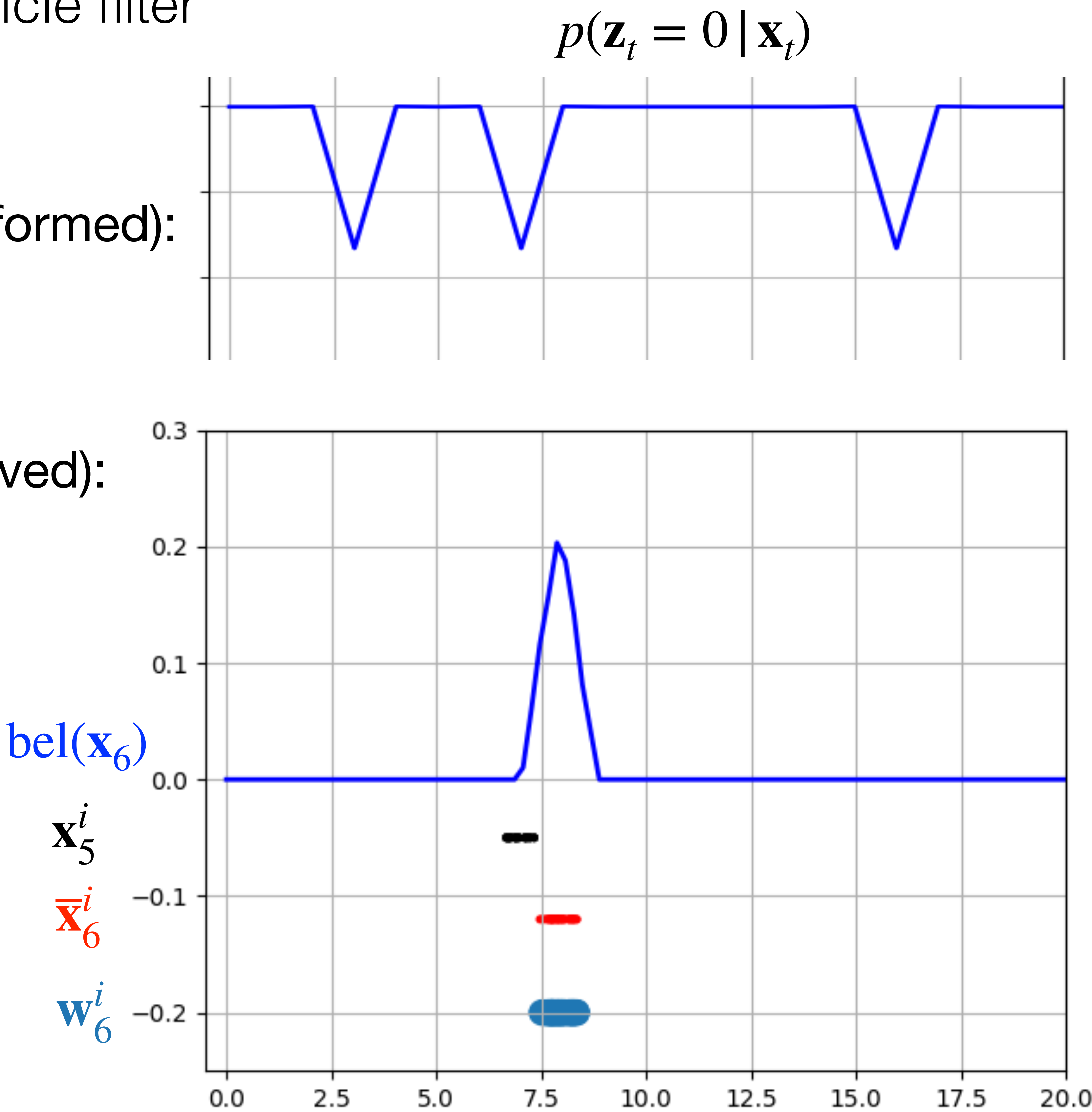

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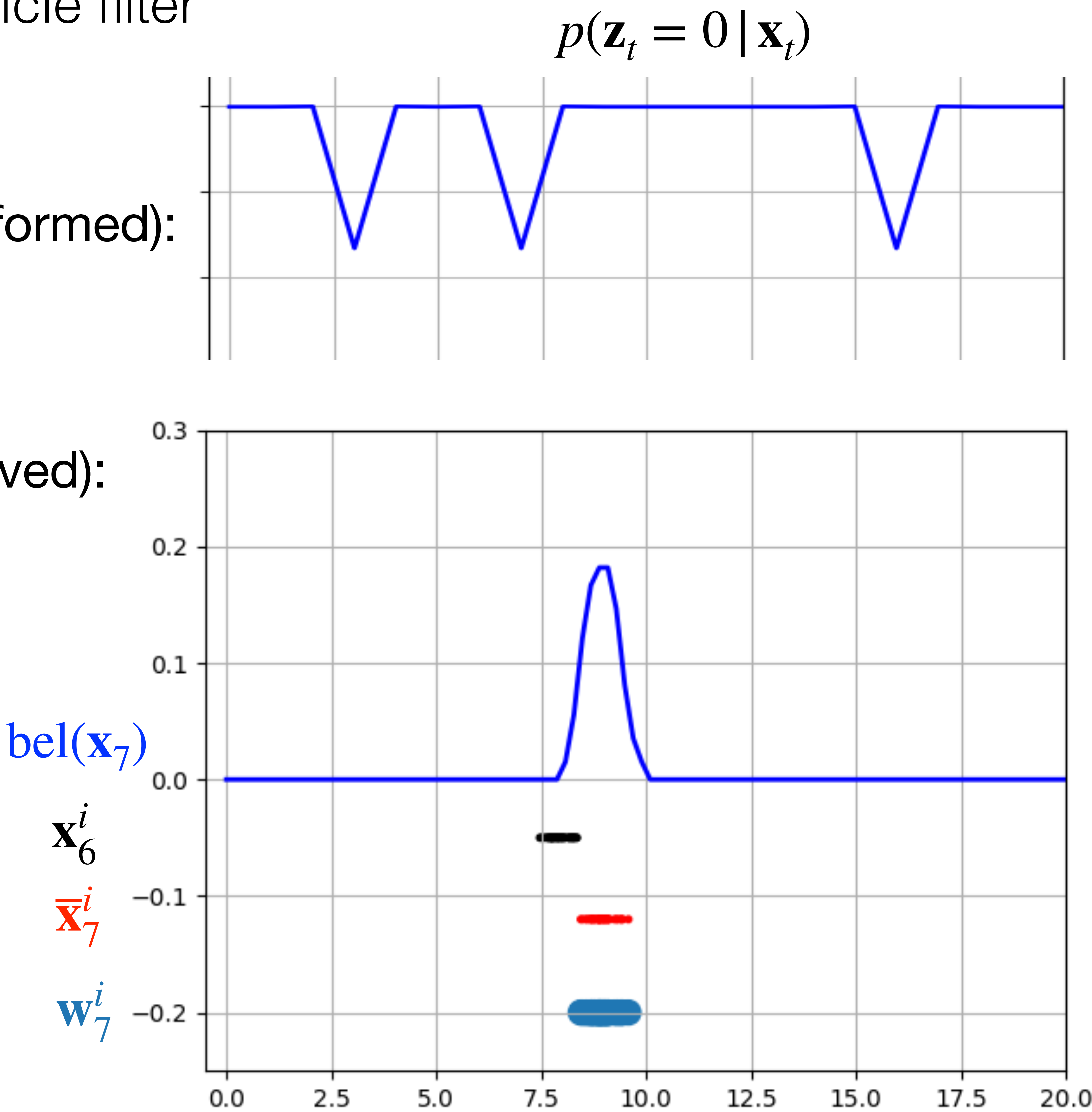

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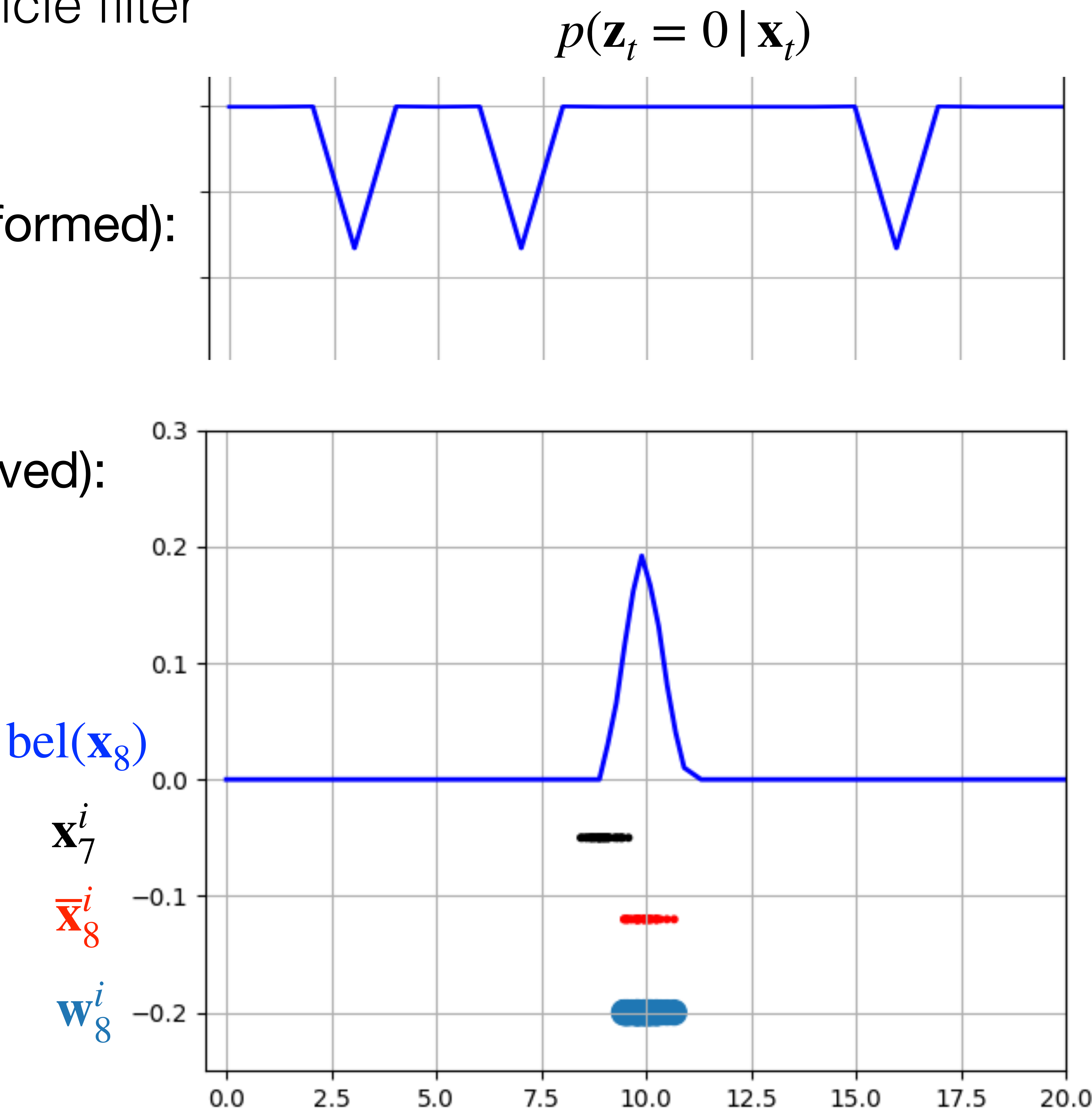


Running the same example multiple times

Running the same example multiple times

Running the same example multiple times

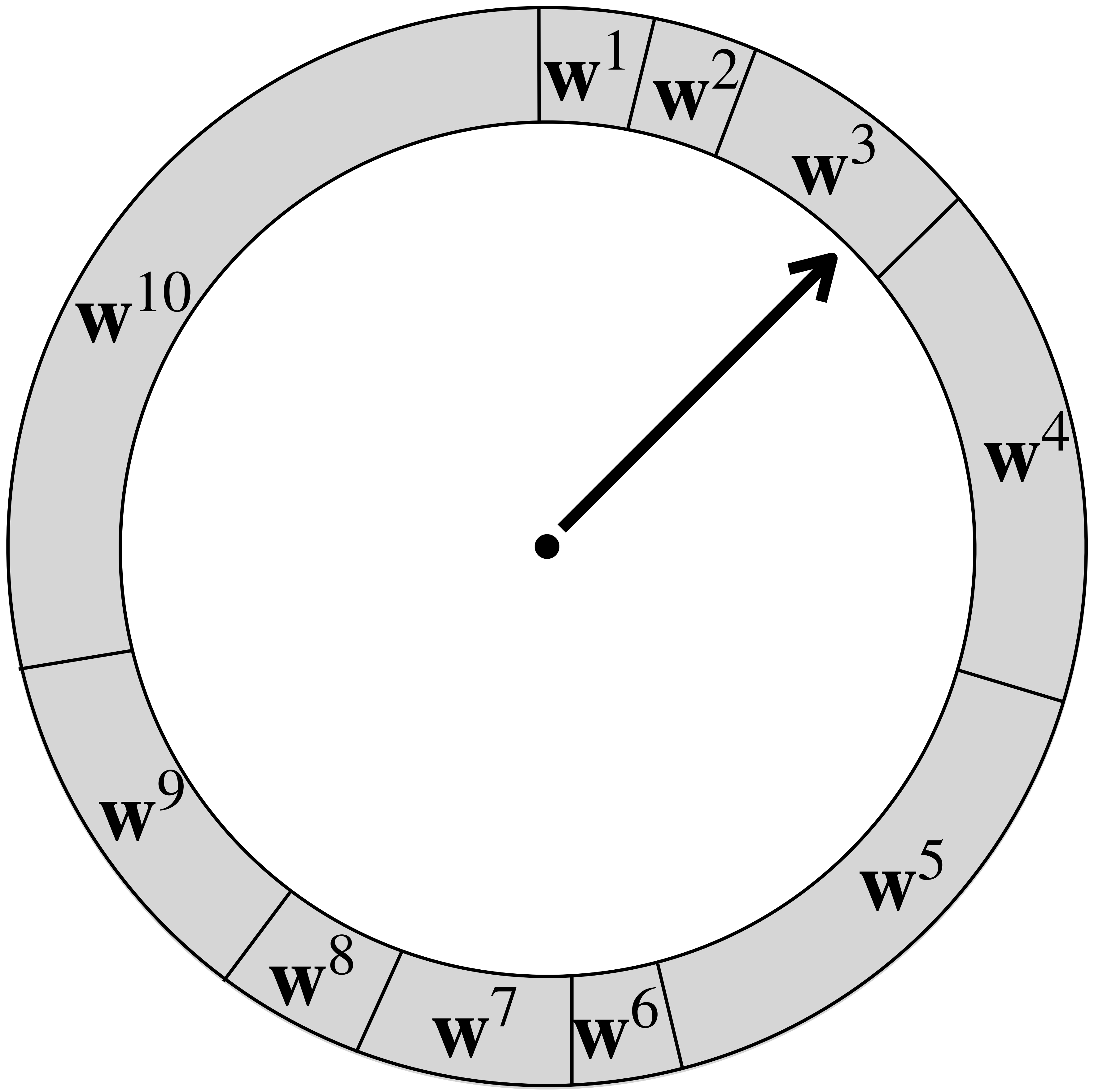
4. Resample

Draw $\mathscr{X}_t = \{\mathbf{x}_t^1, \ldots, \mathbf{x}_t^n\} \propto \mathbf{w}_t^i$

Monte Carlo resampling inspired by roulette wheel

4. Resample

Draw $\mathcal{X}_t = \{\mathbf{x}_t^1, \ldots, \mathbf{x}_t^n\} \propto \mathbf{w}_t^i$



Roulette wheel

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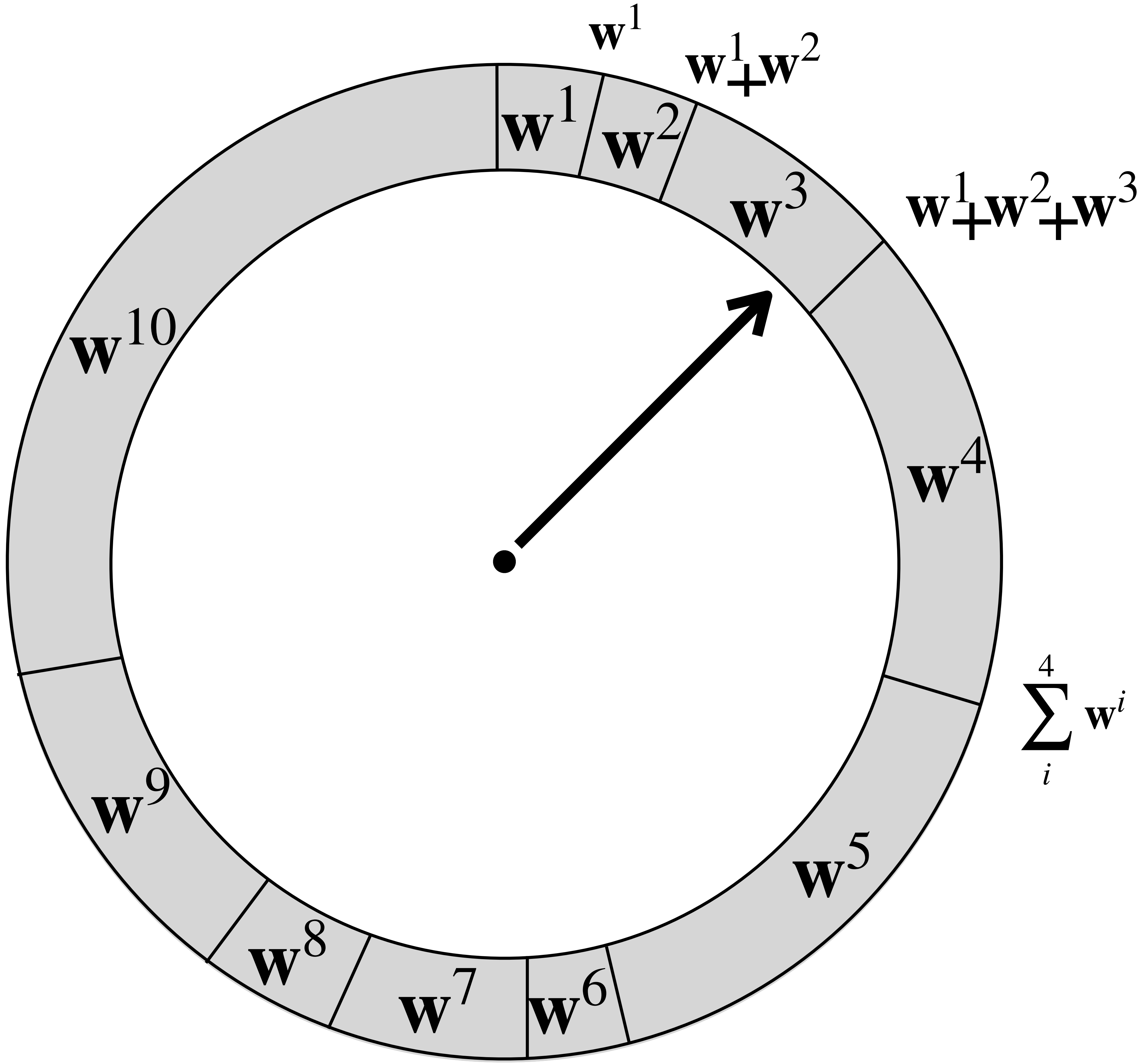
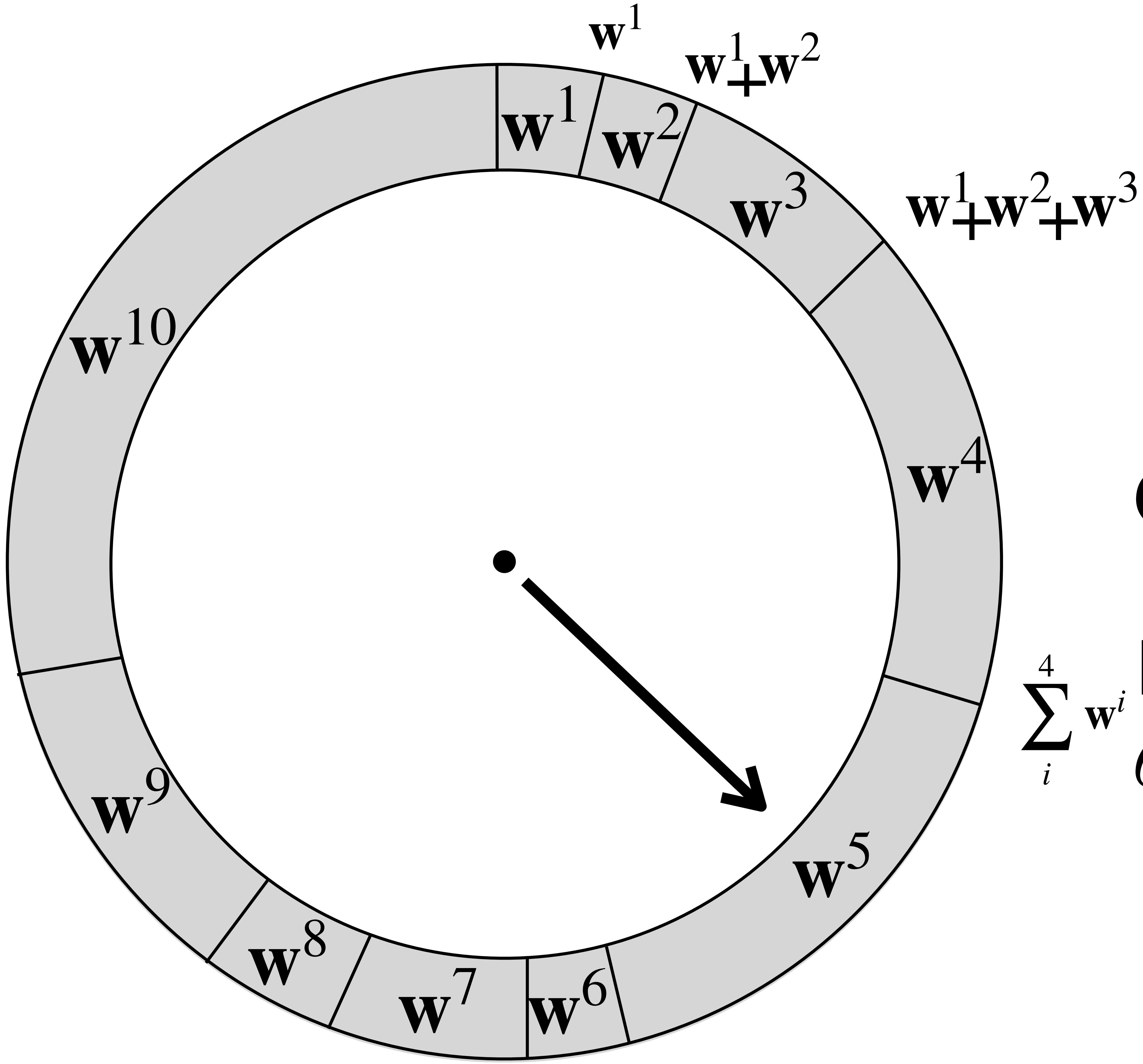


Roulette wheel

Estimate cell borders as cumsum

4. Resample

Draw $\mathcal{X}_t = \{\mathbf{x}_t^1, \ldots, \mathbf{x}_t^n\} \propto \mathbf{w}_t^i$



Generate random number

Find corresponding slot by binary search
$\mathcal{O}\left(\log(N)\right)$

Repeat is N times

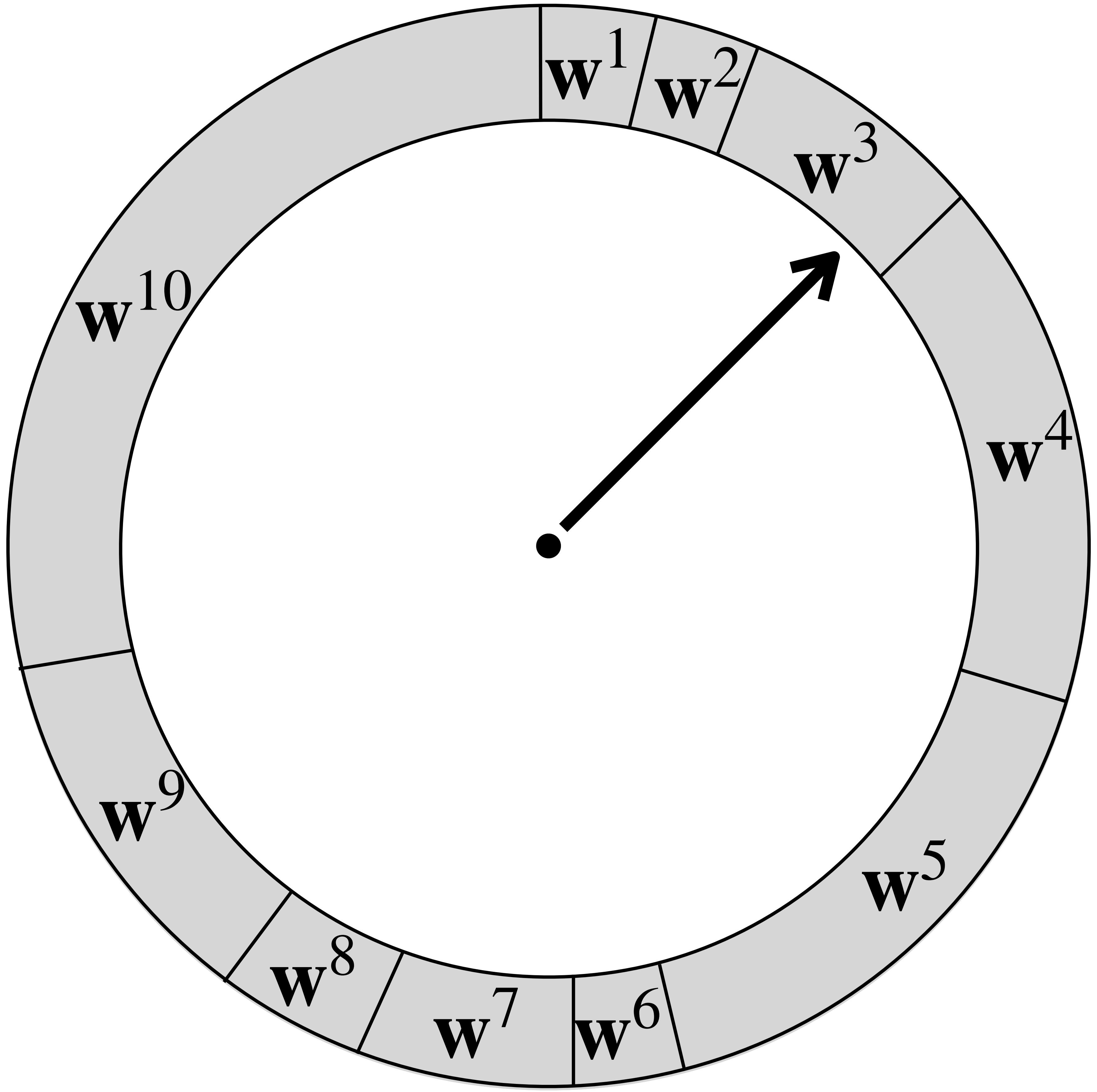
Roulette wheel

- N particles $\mathcal{O}\left(N \log(N)\right)$
- easy to understand

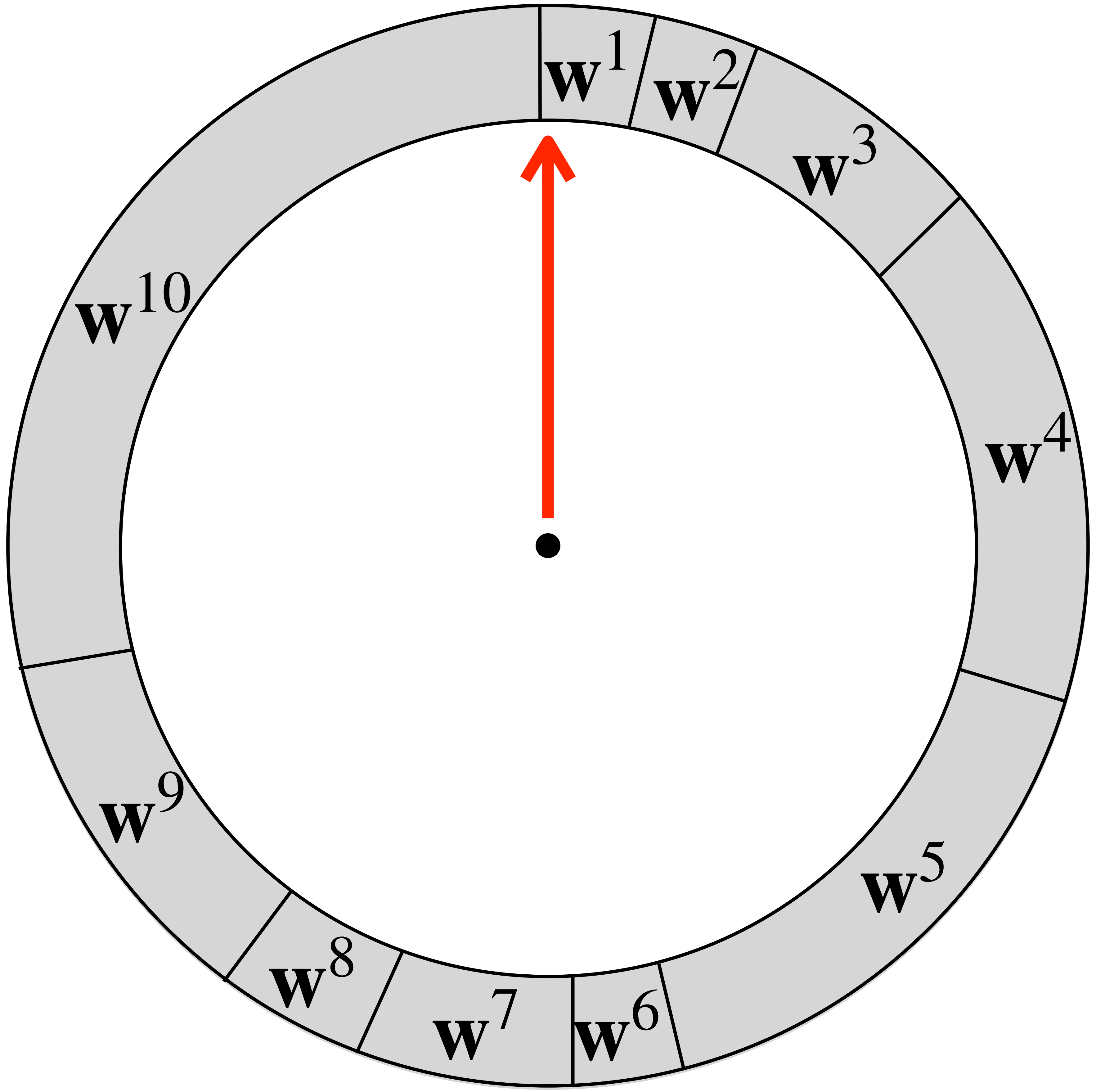
4. Resample

Draw $\mathcal{X}_t = \{\mathbf{x}_t^1, \ldots, \mathbf{x}_t^n\} \propto \mathbf{w}_t^i$

Generate only one small random number



Roulette wheel
- N particles $\mathcal{O}\big(N \log(N)\big)$
- easy to understand

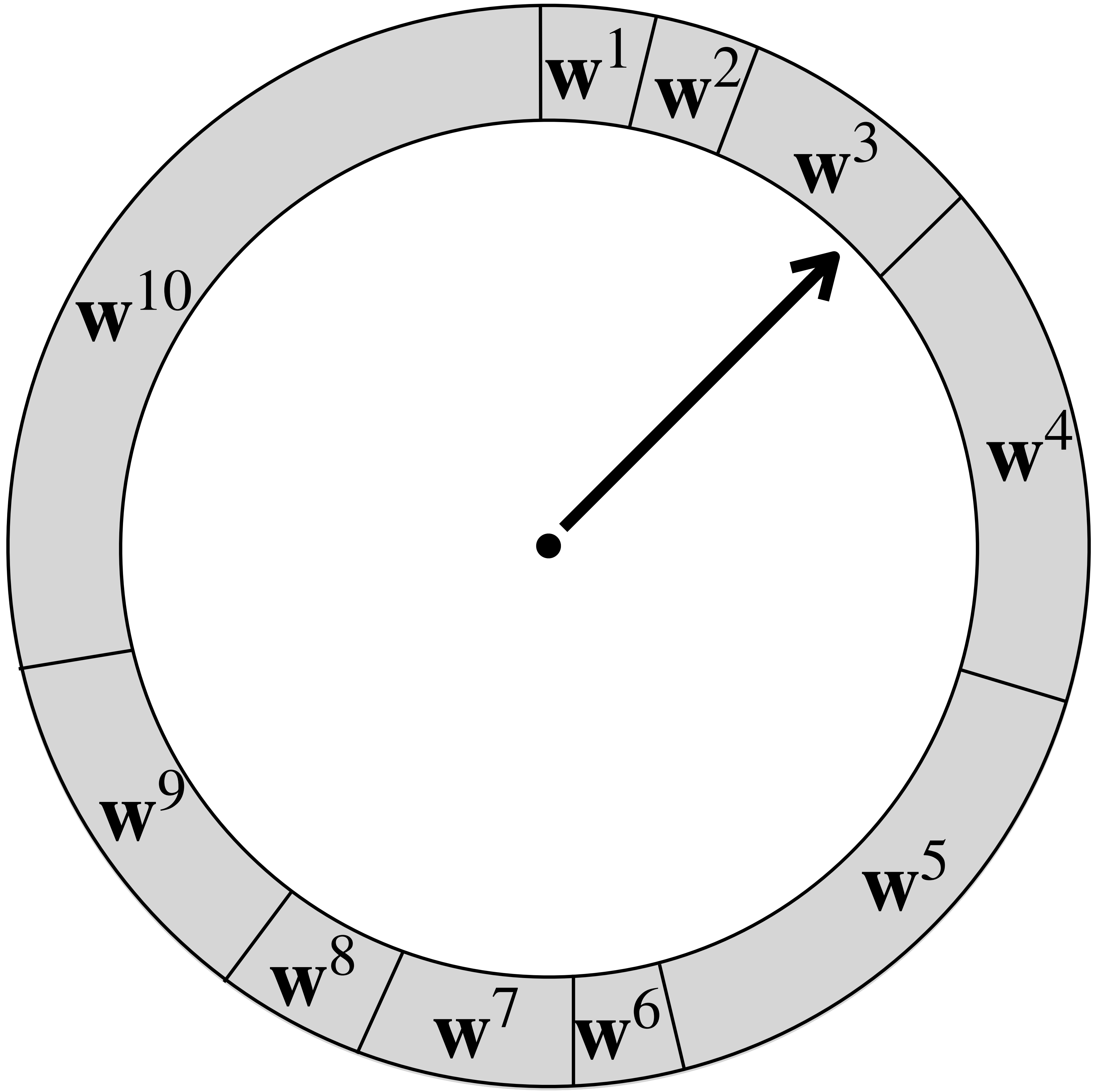


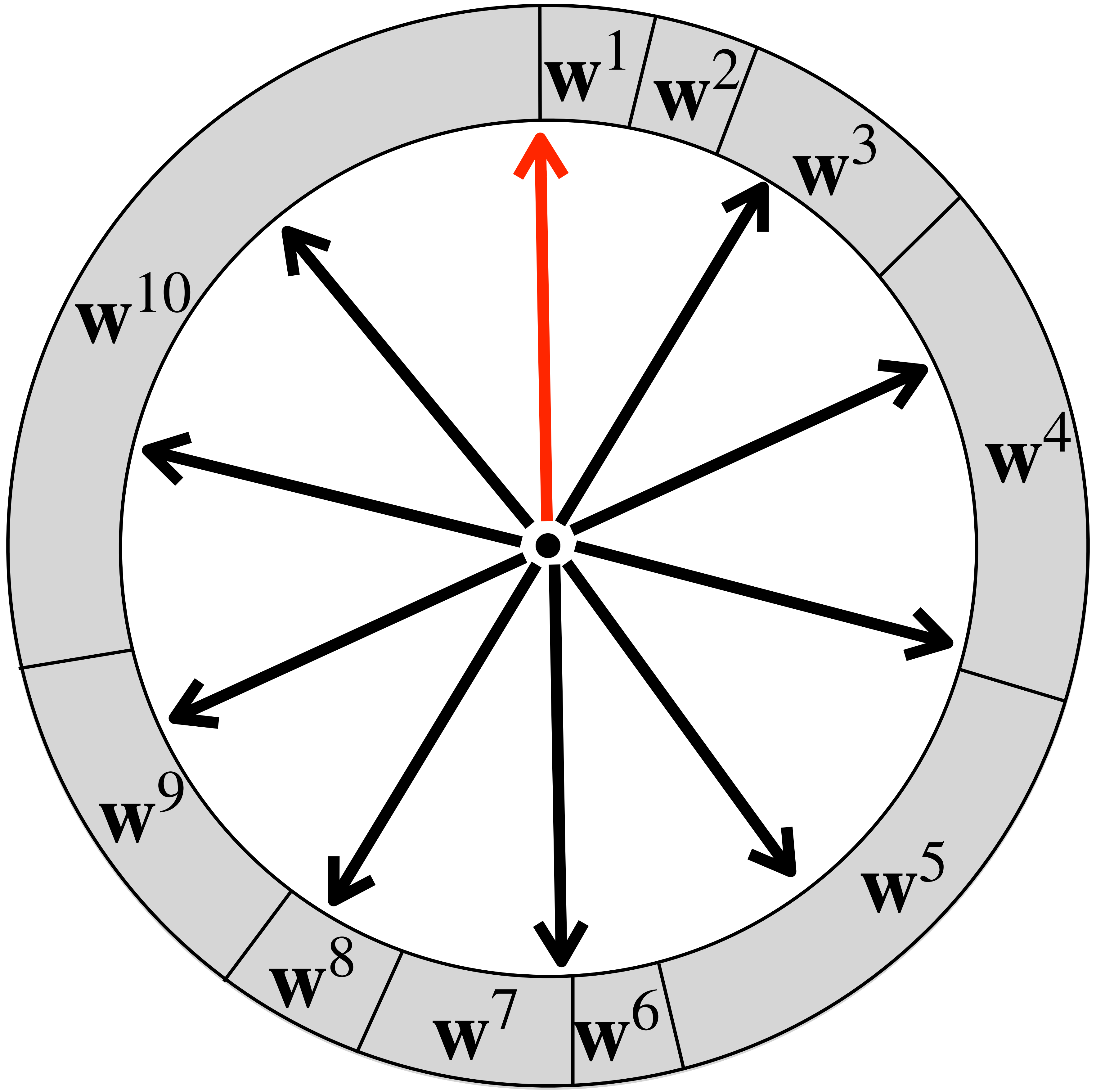
Stochastic universal resampling

4. Resample

Draw $\mathcal{X}_t = \{\mathbf{x}_t^1, \ldots, \mathbf{x}_t^n\} \propto \mathbf{w}_t^i$

Other numbers are equally distributed





Roulette wheel

- N particles $\mathcal{O}\left(N \log(N)\right)$
- easy to understand

Stochastic universal resampling

4. Resample

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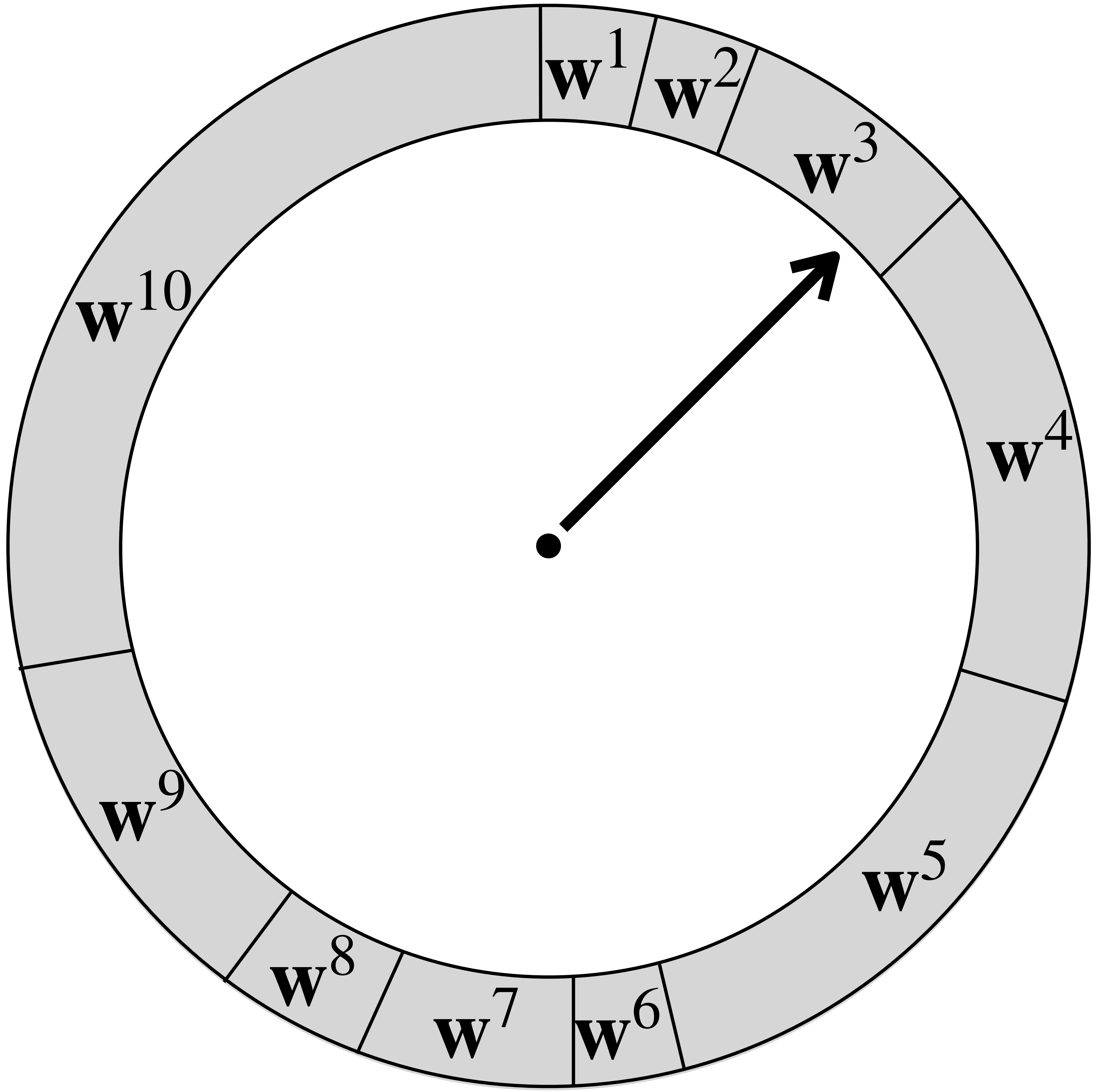
Generate 1 number => Generating N samples
Estimate bins starting at the random number



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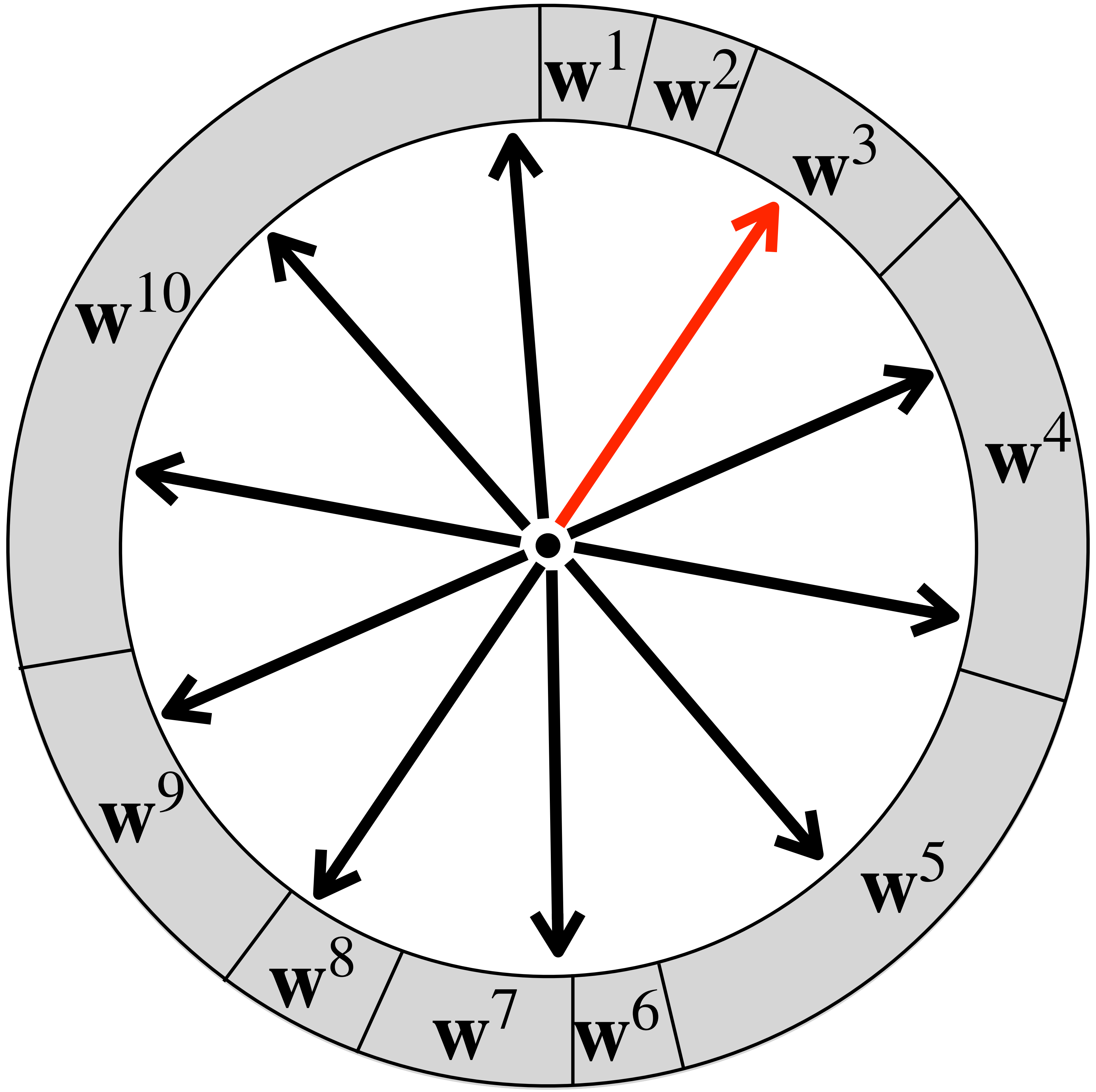
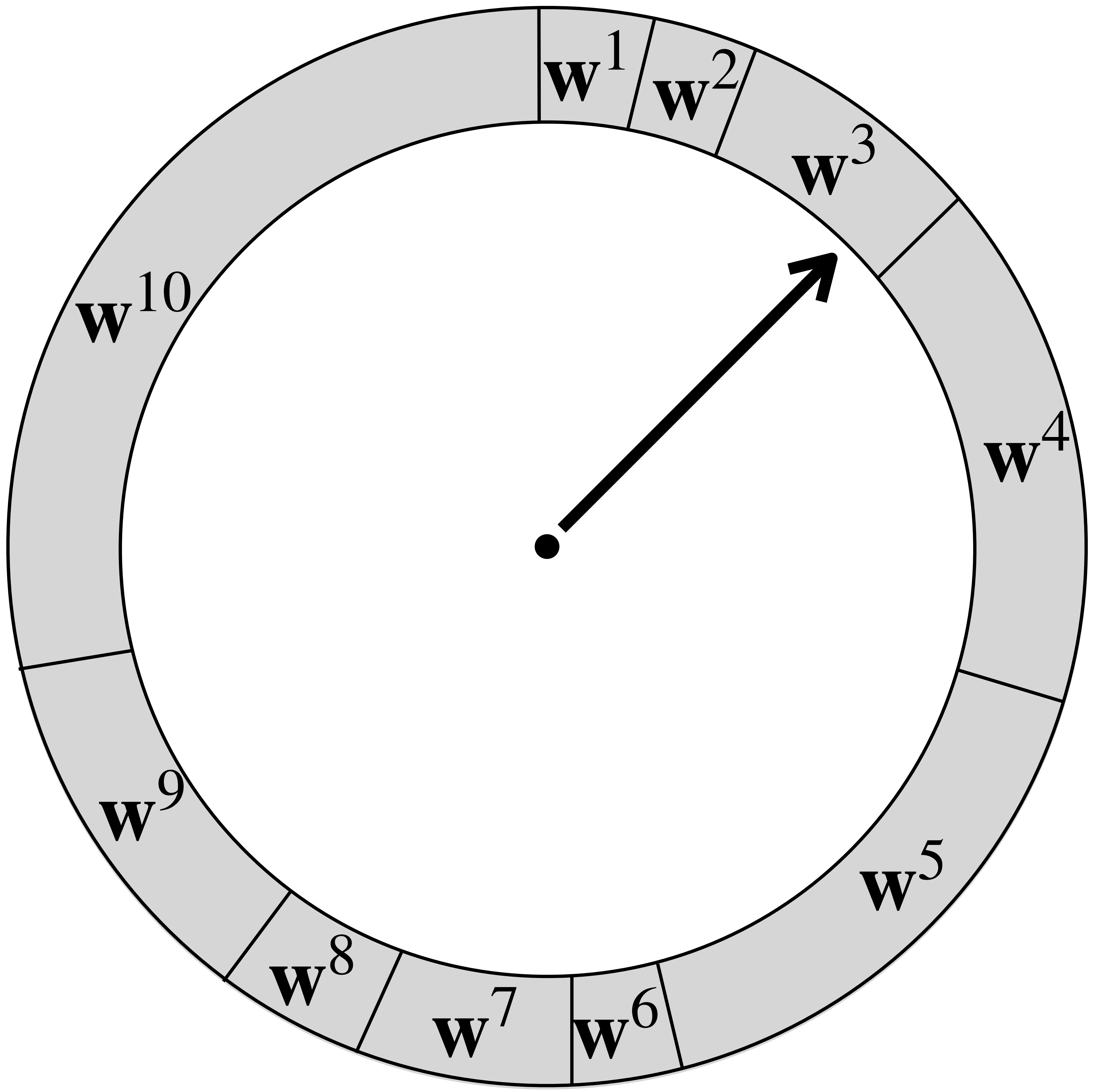


Stochastic universal resampling

4. Resample

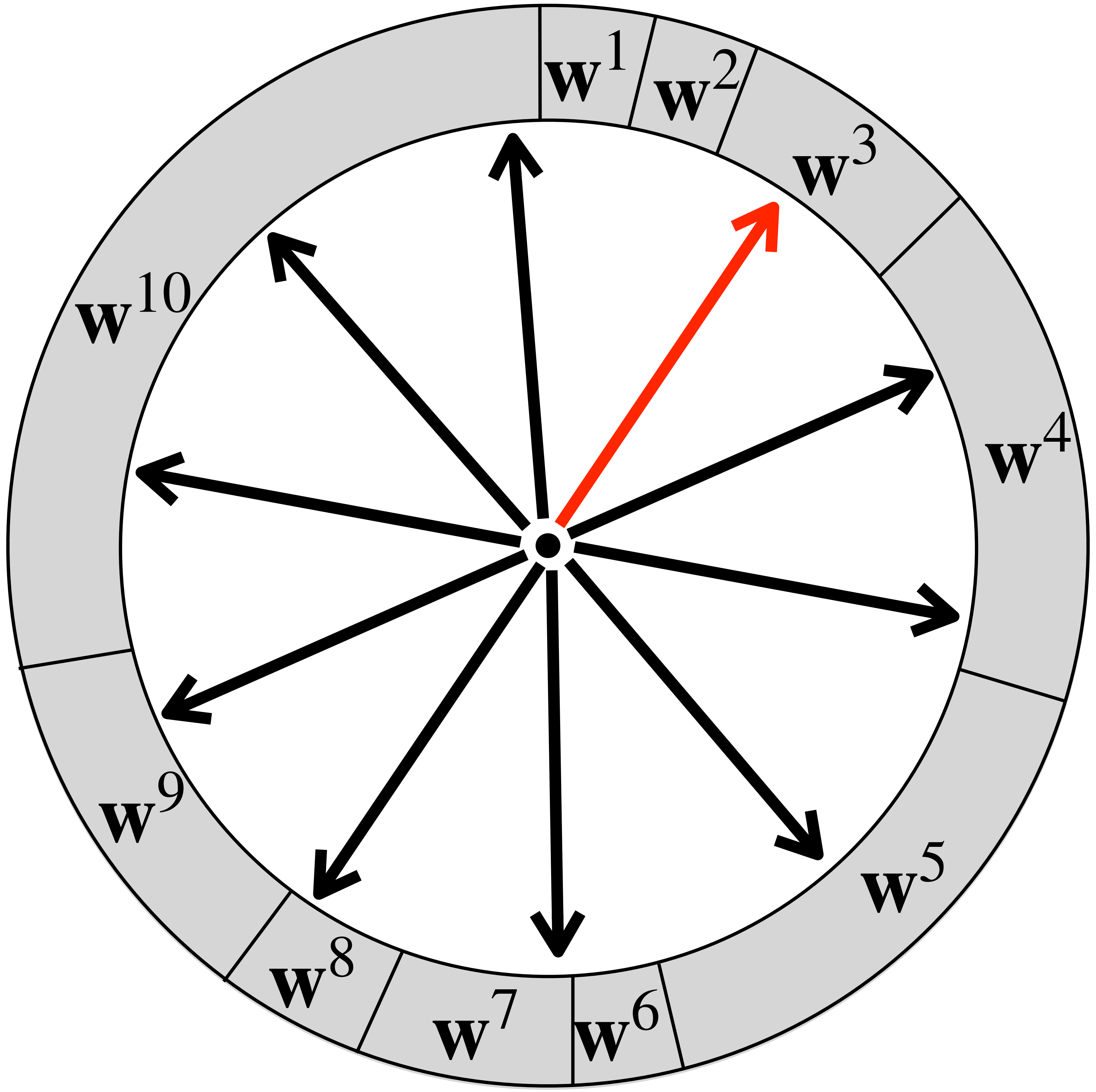
Draw $\mathcal{X}_t = \{\mathbf{x}_t^1, \ldots, \mathbf{x}_t^n\} \propto \mathbf{w}_t^i$

Go through the wheel and update the slot if arrow is higher than cumsum value



Roulette wheel

- N particles $\mathcal{O}(N\log(N))$
- easy to understand

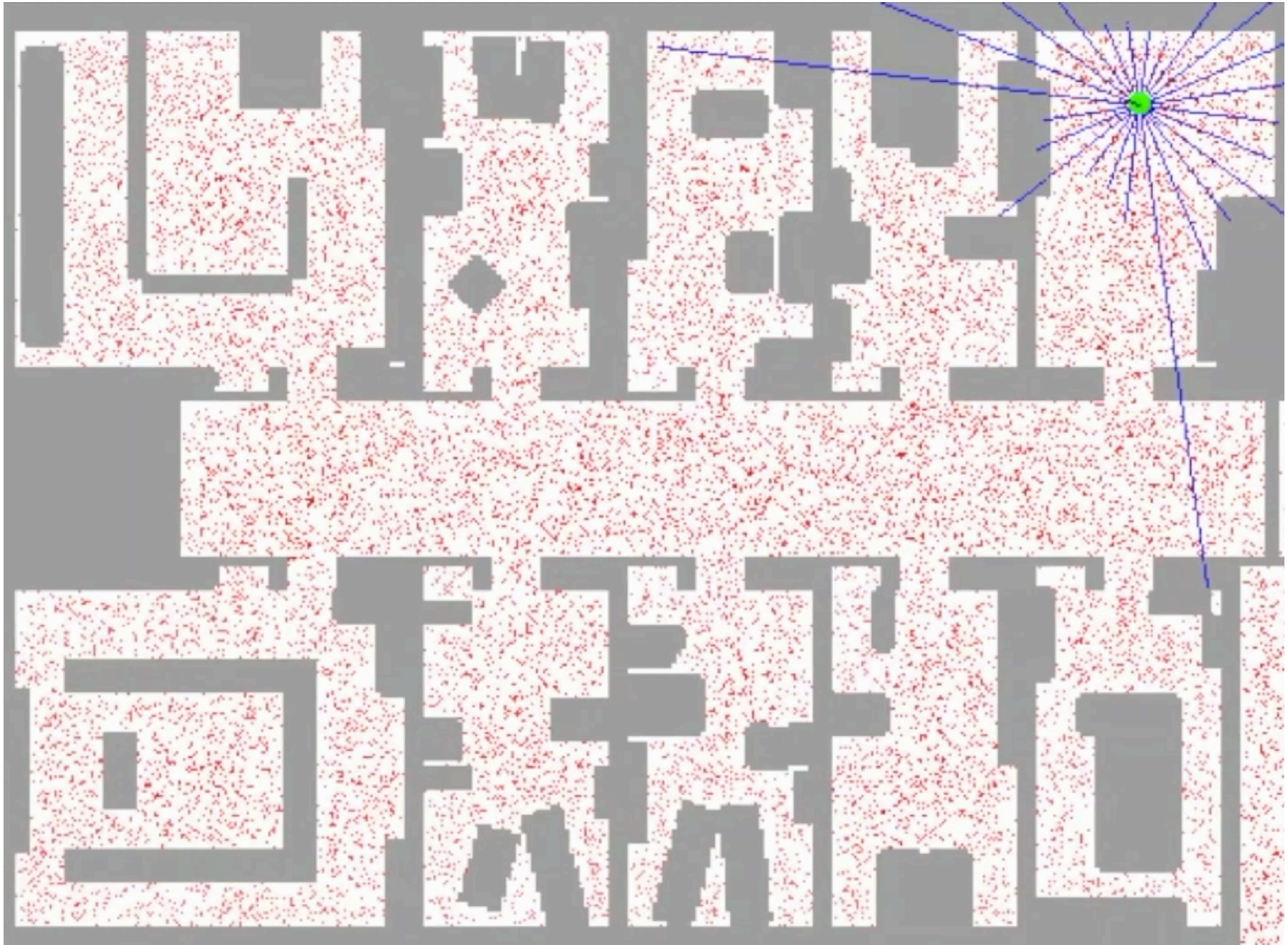


Stochastic universal resampling

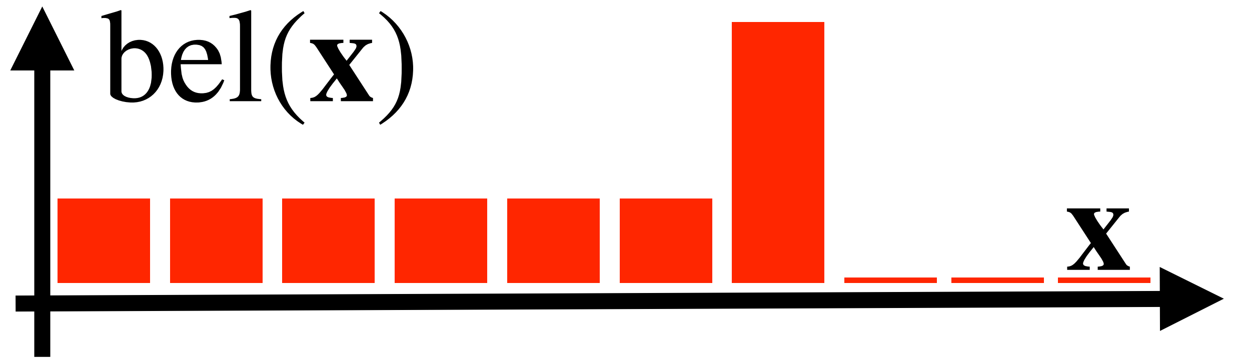
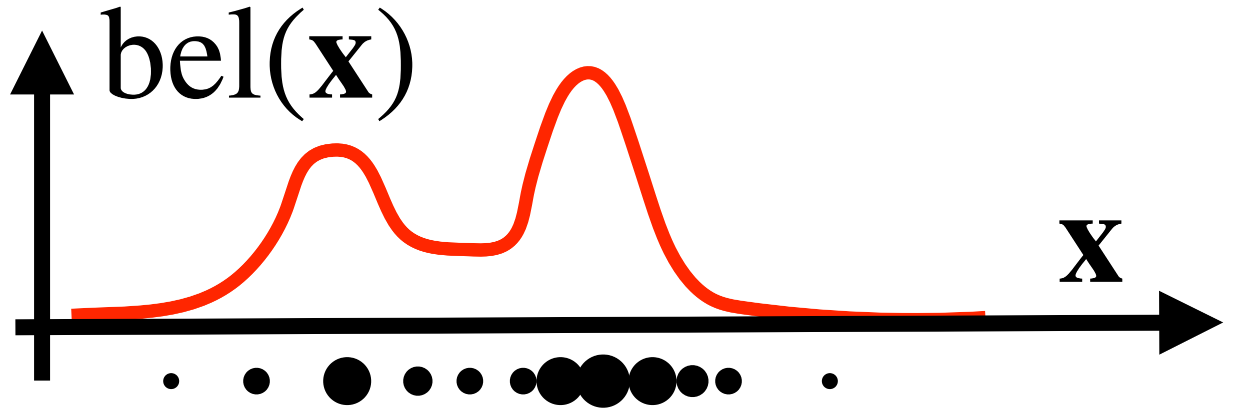
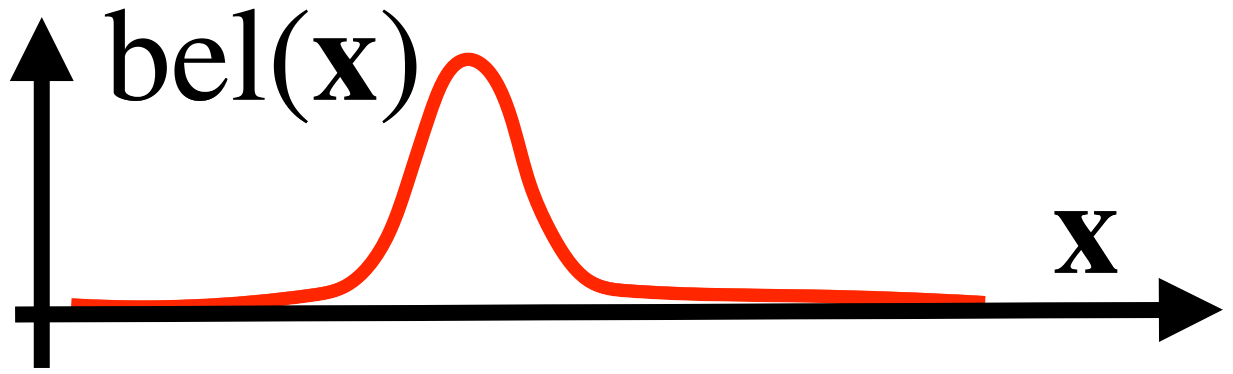
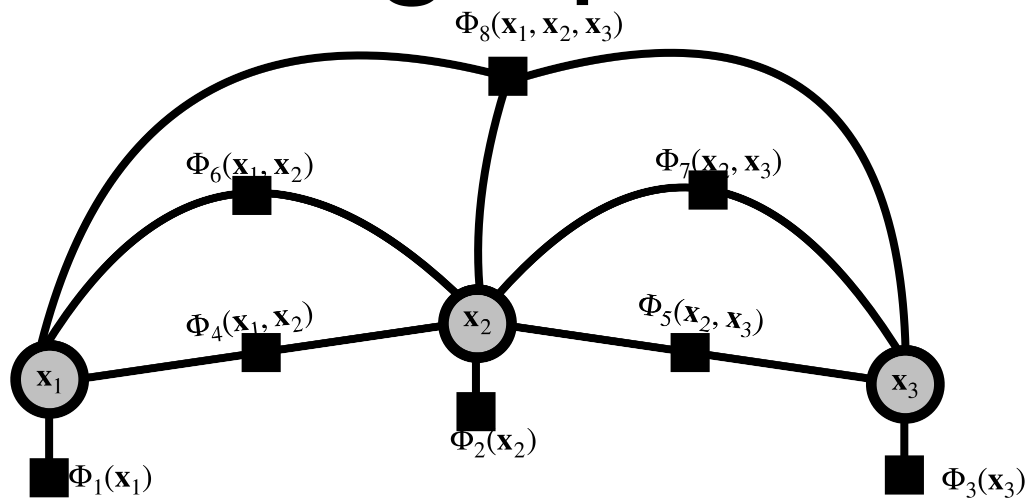
- N particles $\mathcal{O}(N)$
- lower variance

Particle filter example [Dieter Fox]

Particle filter example [Dieter Fox]



Summary

	Drawbacks	Advantages
Bayes filter 	○ course of dimensionality ○ spatial discretization	○ represents arbitrary prob. distribution
Particle filter 	○ course of dimensionality ○ partical quantization	○ represents arbitrary prob. distribution
Kalman filter 	○ represent only gaussians ○ suffers from linearization	○ nicely scales with higher dimensions
Factorgraph 	○ typically represents gaussian factors ○ grows to infinity	○ does not suffer from linearizations ○ allows for arbitrary conditional independences