Perspective Camera

3D Computer Vision – Lab Session Task

(CTU FEE subjects B4M33TDV, BE4M33TDV, XP33VID)

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CENTER FOR MACHINE PERCEPTION

Wire-frame Model

Task: **Develop a simulation of perspective camera projection of a wire-frame model.**

Let the 3D object be given. It is composed from two planar diagrams, that are connected. Coordinates of vertices of both diagrams $\mathbf{X}_1$ and $\mathbf{X}_2$ are:

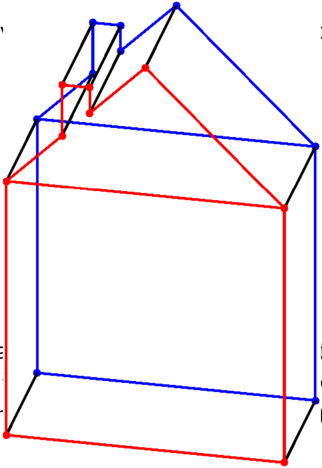
```
X1=[[-0.5,  0.5, 0.5, -0.5, -0.5, -0.3, -0.3, -0.2, -0.2,  0,  0.5 ],
    [-0.5, -0.5, 0.5,  0.5, -0.5, -0.7, -0.9, -0.9, -0.8, -1, -0.5 ],
    [ 4,    4,   4,    4,    4,    4,    4,    4,    4,    4,  4   ]]

X2=[[-0.5,  0.5, 0.5, -0.5, -0.5, -0.3, -0.3, -0.2, -0.2,  0,   0.5],
    [-0.5, -0.5, 0.5,  0.5, -0.5, -0.7, -0.9, -0.9, -0.8, -1,  -0.5],
    [ 4.5,  4.5, 4.5,  4.5,  4.5,  4.5,  4.5,  4.5,  4.5,  4.5, 4.5]]
```

Wire-frame model contains edges, that connects vertices in $\mathbf{X}_1$ and $\mathbf{X}_2$ in given order, and additionally it contains edges connecting vertices between $\mathbf{X}_1$ and $\mathbf{X}_2$, such that $\mathbf{X}_1(:,i)$ is connected to the vertex $\mathbf{X}_2(:,i)$, $\forall i$.

Task: **Develop a simulation of perspective camera projection of a wire-frame model.**

Let the 3D object be gi planar diagrams, that are connected. Coordinates $\mathbf{X}_1$ and $\mathbf{X}_2$ are:



```
X1=[[-0.5,  0.5, 0.5,                                  , -0.2, -0.2,   0,   0.5 ],
    [-0.5, -0.5, 0.5,                                  , -0.9, -0.8, -1,  -0.5 ],
    [ 4,     4,   4,                                      4,    4,    4,   4   ]]

X2=[[-0.5,  0.5, 0.5,                                  , -0.2, -0.2,   0,    0.5],
    [-0.5, -0.5, 0.5,                                  , -0.9, -0.8, -1,   -0.5],
    [ 4.5,  4.5, 4.5,                                  ,  4.5,  4.5,  4.5, 4.5]]
```

Wire-frame model conta tices in $\mathbf{X}_1$ and $\mathbf{X}_2$ in given order, and additionally i vertices between $\mathbf{X}_1$ and $\mathbf{X}_2$, such that $\mathbf{X}_1(:, i)$ is cor), $\forall i$.

Perspective Camera Projection

A 2D projection of a 3D point $\mathbf{X}_w$ in the world coordinate system onto an image plane is given by the formula

$$\mathbf{K}\mathbf{R}\begin{bmatrix}\mathbf{I}_3 & -\mathbf{C}_w\end{bmatrix}\underline{\mathbf{X}_w}$$

- $\mathbf{C}_w$ is the camera position in the world coordinate system
- $\mathbf{R}$ is the camera rotation matrix
- $\mathbf{K}$ is the camera calibration matrix

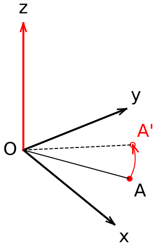
In this assignment, use the following calibration matrix:

$$\mathbf{K} = \begin{pmatrix} 1000 & 0 & 500 \\ 0 & 1000 & 500 \\ 0 & 0 & 1 \end{pmatrix}$$

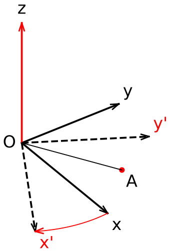
Your task is to construct the extrinsics $\mathbf{R}, \mathbf{C}_w$ for several given situations, perform the projection and draw the result in the image plane.

Positive rotation direction convention: curl right hand rule (thumb = rotation axis, fingers = direction)
this is for rotating a point coordinates:

$$\mathbf{A}' = \mathbf{R}\mathbf{A}$$

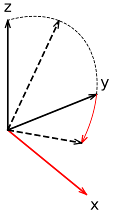


Base is rotated in the opposite direction:
vector $\overrightarrow{\mathbf{OA}'}$ in (x, y, z) base has the same coordinates as $\overrightarrow{\mathbf{OA}}$ in (x', y', z) base.

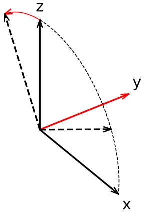


3D Elementary Rotations of Camera Base

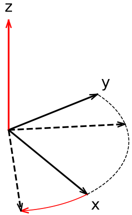
$$\mathbf{R}_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$



$$\mathbf{R}_y(\alpha) = \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}$$



$$\mathbf{R}_z(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

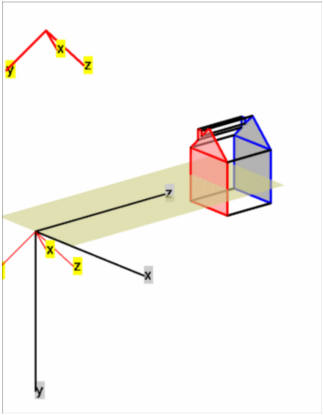


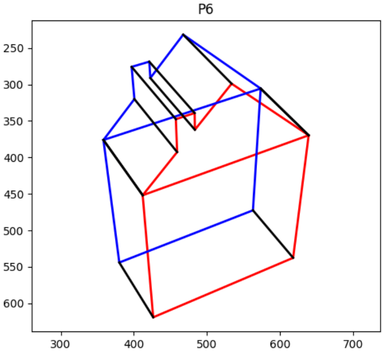
Construct following camera matrices (keep the image u-axis parallel to the scene x-axis):

- $\mathbf{P}_1$: Camera in the origin looking in the direction of z-axis
- $\mathbf{P}_2$: Camera located at $\begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^\top$ looking in the direction of z-axis
- $\mathbf{P}_3$: Camera located at $\begin{bmatrix} 0 & 0.5 & 0 \end{bmatrix}^\top$ looking in the direction of z-axis
- $\mathbf{P}_4$: Camera located at $\begin{bmatrix} 0 & -3 & 0.5 \end{bmatrix}^\top$, with optical axis rotated by 0.5 rad around x-axis towards y-axis (positive)
- $\mathbf{P}_5$: Camera located at $\begin{bmatrix} 0 & -5 & 4.2 \end{bmatrix}^\top$ looking in the direction of y-axis
- $\mathbf{P}_6$: Camera located at $\begin{bmatrix} -1.5 & -3 & 1.5 \end{bmatrix}^\top$, with optical axis rotated by 0.5 rad around y-axis towards x-axis (i.e., -0.5 rad) followed by a rotation by 0.8 rad around x-axis towards y-axis (positive)

Use the cameras $\mathbf{P}_1$ to $\mathbf{P}_6$ for projection of given wire-frame model into an image. The edges inside $\mathbf{X}_1$ should be drawn red, the edges inside $\mathbf{X}_2$ should be drawn blue and the rest should be drawn in black.

Camera located at $\begin{bmatrix}-1.5 & -3 & 1.5\end{bmatrix}^\top$, with optical axis rotated by 0.5 rad around y-axis towards x-axis (i.e., -0.5 rad) followed by a rotation by 0.8 rad around x-axis towards y-axis (positive)





Hint: Projection Visualization in Python

Let $\begin{bmatrix} u_1 & v_1 \end{bmatrix}^\top$ and $\begin{bmatrix} u_2 & v_2 \end{bmatrix}^\top$ be the projections of $\mathbf{X}_1$ and $\mathbf{X}_2$ (respectively) on the image plane by the inferred matrices. The visualization can be done as follows:

```
plt.plot( u1, v1, 'r-', linewidth=2 )
plt.plot( u2, v2, 'b-', linewidth=2 )
plt.plot( [u1, u2], [v1, v2 ], 'k-', linewidth=2 )
plt.gca().invert_yaxis()
plt.axis( 'equal' ) # this kind of plots should be isotropic
```