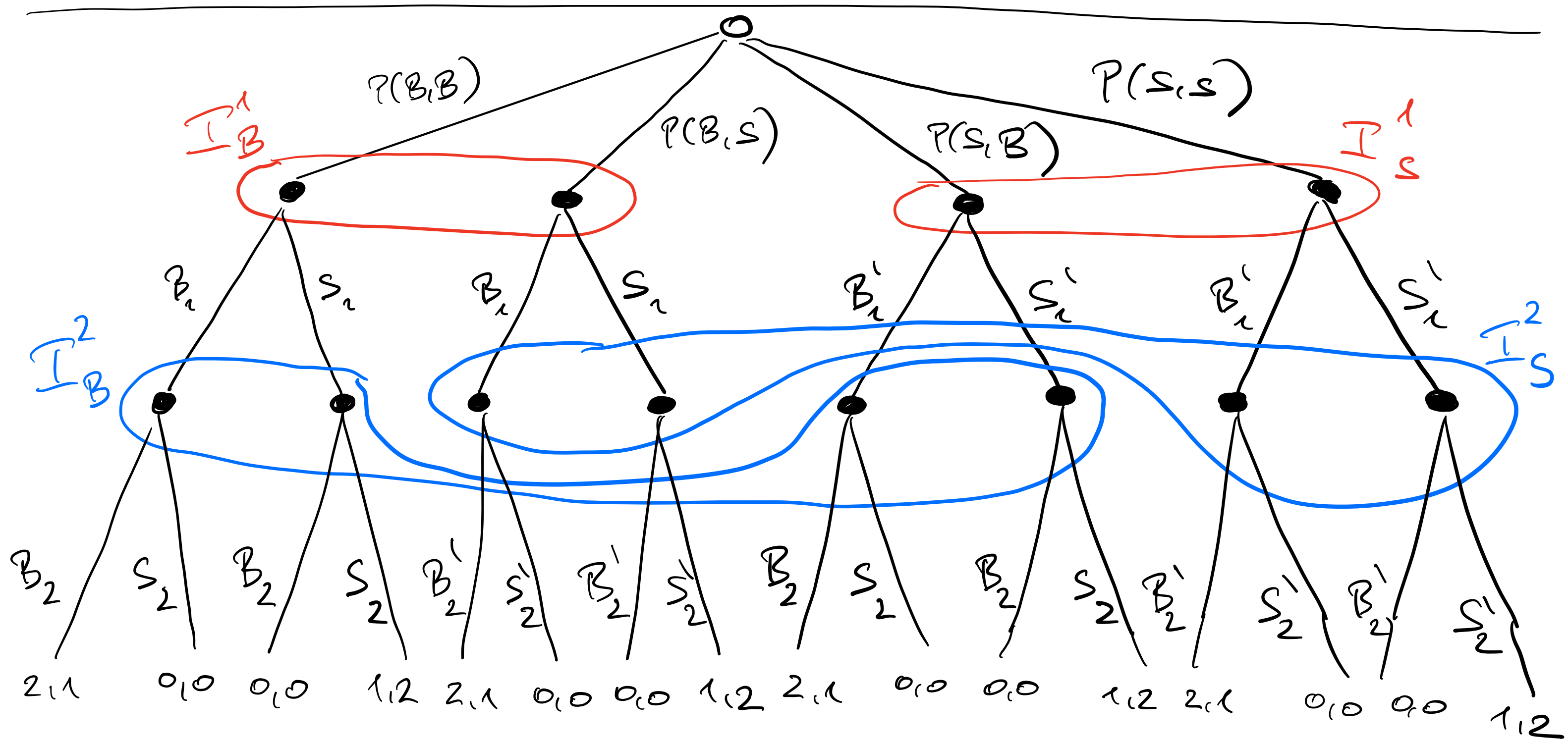
Metagame for the correlated equilibrium



- Expected utility of player 1 for following advice B is $P(B_2|B)\cdot 2 + P(S_2|B)\cdot 0$
- Expected utility of player 1 for not following advice B is $P(B_2|B)\cdot 0 + P(S_2|B)\cdot 1$

CE, equivalently (proof)

Idea: Write $P(s_{-i}|s_i) = \dfrac{P(s_{-i}, s_i)}{P(s_i)}$ and note that the denominator doesn't depend on s_{-i}. Finally, note that if $P(s_i)=0$, then necessarily $P(s_{-i}, s_i)=0$, so the inequality 2. from Proposition is trivially true.

Bach or Stravinski

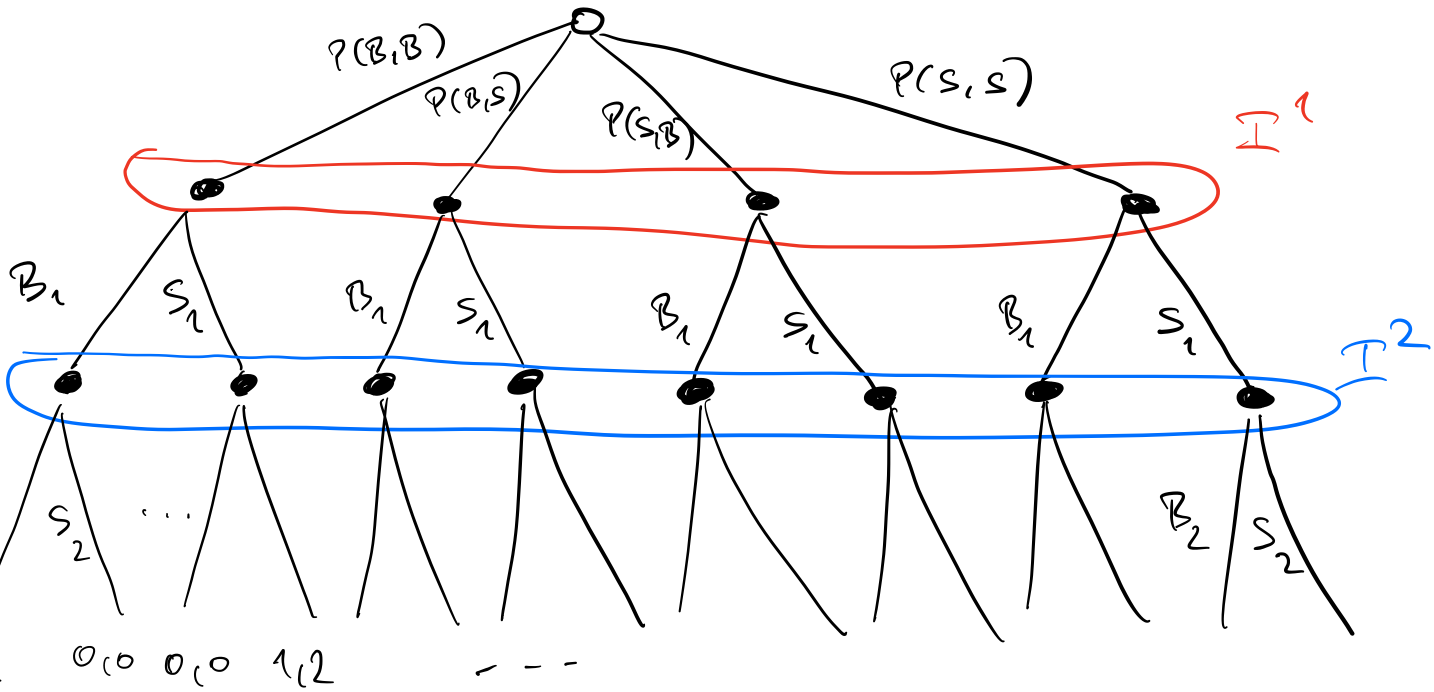
NE induce CE:

	B	S	
B	1	0	1
S	0	0	0
	1	0	

0	0	0
0	1	1
0	1	

2/9	4/9	6/9
1/9	2/9	3/9
3/9	6/9	

Metagame for the coarse CE



CCE for Matching pennies

Utility / probability

1 / α	-1 / β
-1 / γ	1 / δ

(1) $\alpha - \beta + \gamma - \delta \leq \alpha - \beta - \gamma + \delta$

(2) $-\alpha + \beta - \gamma + \delta \leq \alpha - \beta - \gamma + \delta$

(3) $-\alpha + \gamma - \beta + \delta \leq -\alpha + \beta + \gamma - \delta$

(4) $\alpha - \gamma + \beta - \delta \leq -\alpha + \beta + \gamma - \delta$

$(1) \Rightarrow \gamma \le \delta$

$(2) \Rightarrow \beta \le \alpha$

$(3) \Rightarrow \delta \le \beta$

$(4) \Rightarrow \alpha \le \gamma$

$\Rightarrow \gamma \le \delta \le \beta \le \alpha \le \gamma$

$\Downarrow$

all $\le$ are $=$

$\Downarrow$

$\alpha = \beta = \gamma = \delta = 1/4$

CCE for Prisoner's dilemma

$-1,-1$ (α)	$-4,0$ (β)
$0,-4$ (γ)	$-3,-3$ (δ)

$(1)\quad -\alpha - 4\beta - \gamma - 4\delta \le -\alpha - 4\beta - 3\delta$

$(2)\quad -3\beta \quad -3\delta \le -\alpha - 4\beta - 3\delta$

$(3)\quad -\alpha - 4\gamma - \beta - 4\delta \le -\alpha - 4\gamma - 3\delta$

$(4)\quad -3\gamma \quad -3\delta \le -\alpha - 4\gamma - 3\delta$

$(1) \Rightarrow -\gamma \le \delta$

$(2) \Rightarrow \beta \le -\alpha \Rightarrow \alpha = \beta = 0$

$(3) \Rightarrow -\beta \le \delta$

$(4) \Rightarrow \gamma \le -\alpha \Rightarrow \gamma = 0$

$\Downarrow$

$\delta = 1$

Weak SE may not exist

	c	d
a	2,1	4,0
b	1,0	3,1

$p_1 := p_1(a)$

Best response of Follower to a mixed strategy of leader:

$p_1 \in [0, \frac{1}{2}) \Rightarrow U_2(c, p_1) = p_1$, $U_2(d, p_1) = 1 - p_1$

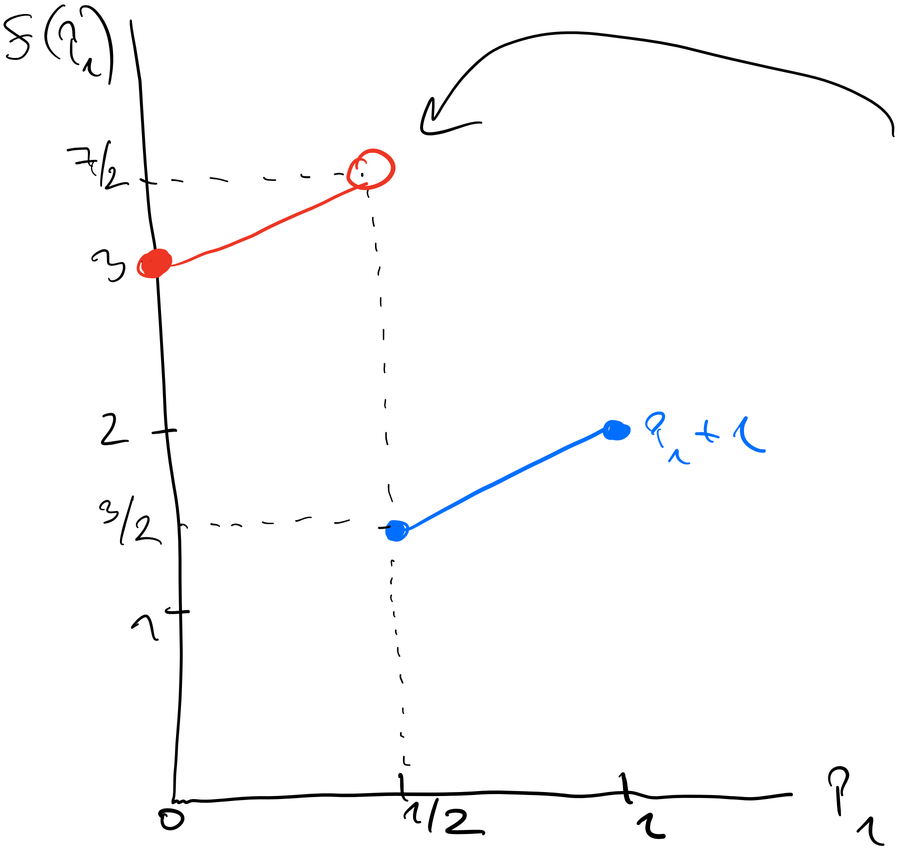
$p_1 = \frac{1}{2} \Rightarrow U_2(c, p_1) = U_2(d, p_1) = \frac{1}{2}$

$p_1 \in (\frac{1}{2}, 1] \Rightarrow U_2(c, p_1) = p_1$, $U_2(d, p_2) = 1 - p_1$

Function $S(p_1) := \min_{s_2 \in BR_2(p_1)} U_1(p_1, s_2)$

$U_1(p_1, c) = 2p_1 + 1 - p_1 = p_1 + 1$

$U_1(p_1, d) = 4p_1 + 3(1 - p_1) = p_1 + 3$



Function S has no maximum only supremum!

SE in 2-player zero-sum games

Let $t_2 \in BR_2(p_1)$:

$$U_2(p_1, t_2) = \max_{s_2 \in S_2} U_2(p_1, s_2)$$

$$\Longleftrightarrow$$

$$\boxed{U_1(p_1, t_2)} = \min_{s_2 \in S_2} U_1(p_1, s_2)$$

Weak SE: leader solves the problem

$$\max_{p_1 \in \Delta_1} \min_{t_2 \in BR_2(p_1)} \boxed{U_1(p_1, t_2)} =$$

$$= \max_{p_1 \in \Delta_1} \min_{t_2 \in BR(p_1)} \min_{s_2 \in S_2} U_1(p_1, s_2) =$$

$$= \max_{p_1 \in \Delta_1} \min_{s_2 \in S_2} U_1(p_1, s_2)$$

Strong SE: analogously, leader solves

$$\max_{p_1 \in \Delta_1} \max_{t_2 \in BR_2(p_1)} \boxed{U_1(p_1, t_2)} =$$

$$= \max_{p_1 \in \Delta_1} \max_{t_2 \in BR_2(p_1)} \min_{s_2 \in S_2} U_1(p_1, s_2)$$

$$= \max_{p_1 \in \Delta_1} \min_{s_2 \in S_2} U_1(p_1, s_2)$$

Bach or Stravinski, revisited

	c	d
a	2,1	0,0
b	0,0	1,2

Let the row player be the leader and $p_1 \in \Delta_1$ be the commitment.

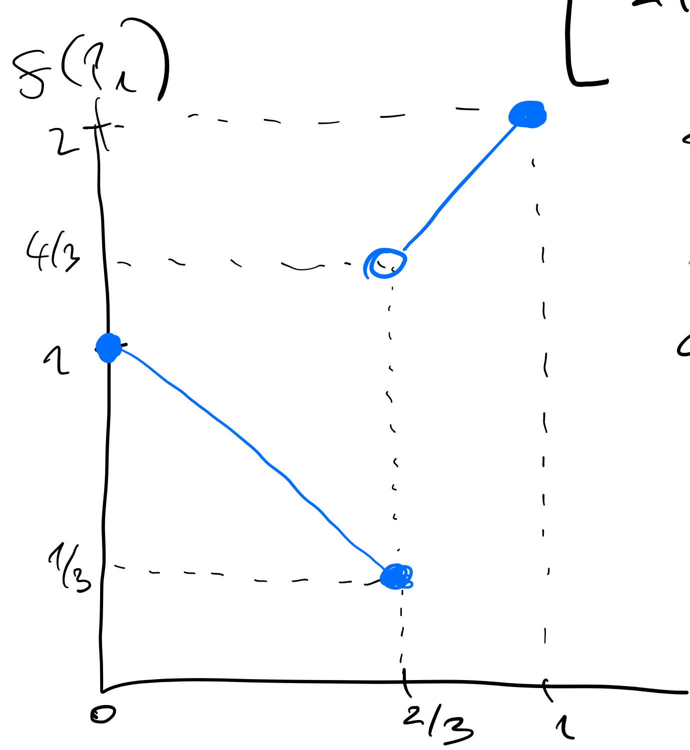
Then $U_2(p_1, c) = p_1$, where $p_1 := p_1(a)$, and $U_2(p_1, d) = 2 - 2p_1$. This means that

$$BR_2(p_1) = \begin{cases} d, & 0 \le p_1 < 2/3 \\ \{c, d\}, & p_1 = 2/3 \\ c, & 2/3 < p_1 \le 1 \end{cases}$$

We analyze function

$$f(p_1) := \min_{s_2 \in BR_2(p_1)} U_1(p_1, s_2)$$

$$= \begin{cases} 1 - p_1 & 0 \le p_1 \le 2/3 \\ 2p_1 & 2/3 < p_1 \le 1 \end{cases}$$



This implies that strong SE = weak SE, and $p_1^* = 1$, $s_2^* = c$ so this is a pure NE.

Note that the payoff of the leader, 2, is greater or equal than π_1 any CE payoff.

We will compute coarse CE of this game.

2,1 $\quad \alpha$	0,0 $\quad \beta$
0,0 $\quad \gamma$	1,2 $\quad \delta$

(1) $2\alpha + 2\gamma \leq 2\alpha + \delta$

(2) $\beta + \delta \leq 2\alpha + \delta$

(3) $\alpha + \beta \leq \alpha + 2\delta$

(4) $2\gamma + 2\delta \leq \alpha + 2\delta$

$$\Longleftrightarrow$$

$$2\gamma \leq \delta$$
$$\beta \leq 2\alpha$$
$$\beta \leq 2\delta$$
$$2\gamma \leq \alpha$$

The last inequality system defines CE, so in this case

CE = coarse CE.