DEPARTMENT OF COMPUTER GRAPHICS AND INTERACTION

WINDOWING

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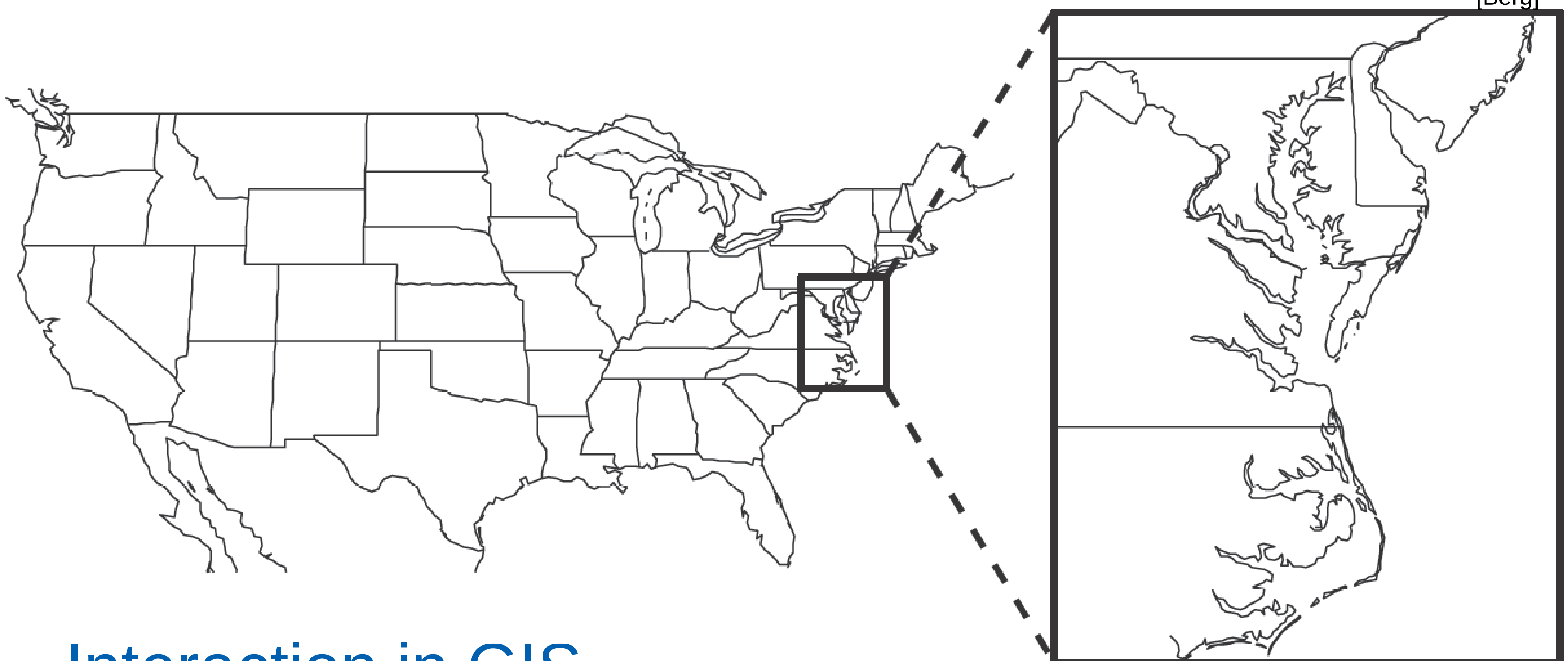
https://cw.felk.cvut.cz/doku.php/courses/a4m39vg/start

Based on [Berg], [Mount]

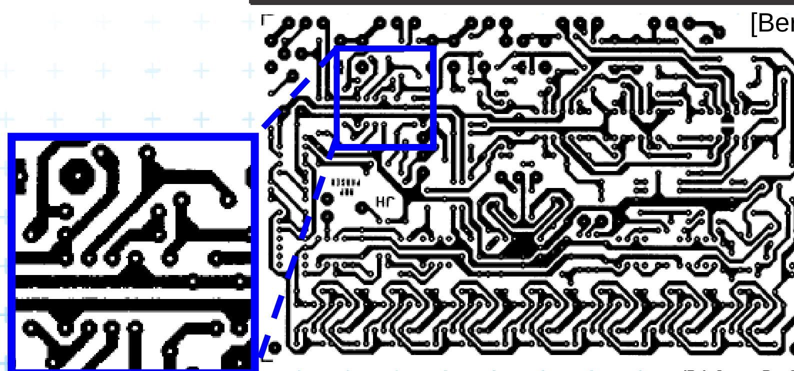
Version from 30.11.2022

Windowing queries - examples

[Berg]



[Berg]



[Vakken]

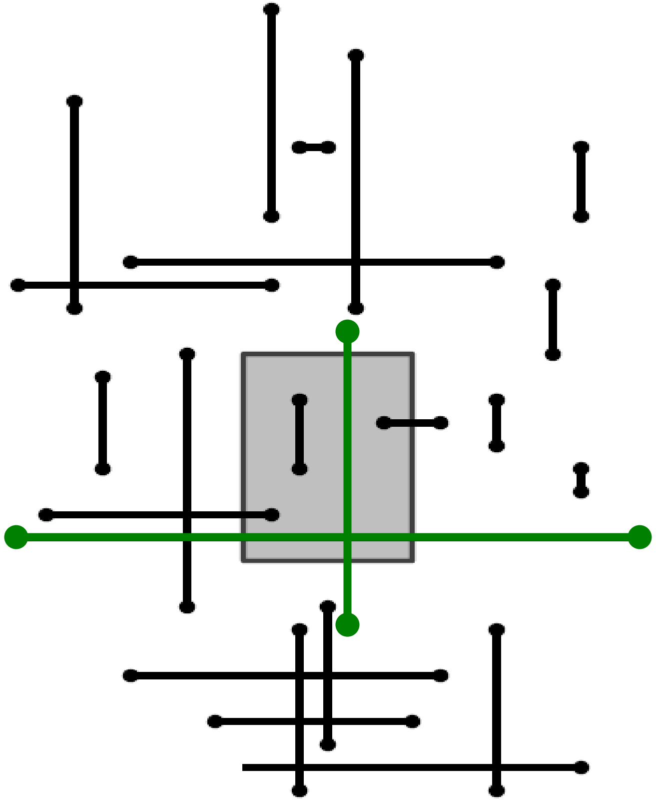
- Interaction in GIS
  - Select subset by outlining
  - Zoom in and re-center
- Circuit board inspection,…

Windowing versus range queries

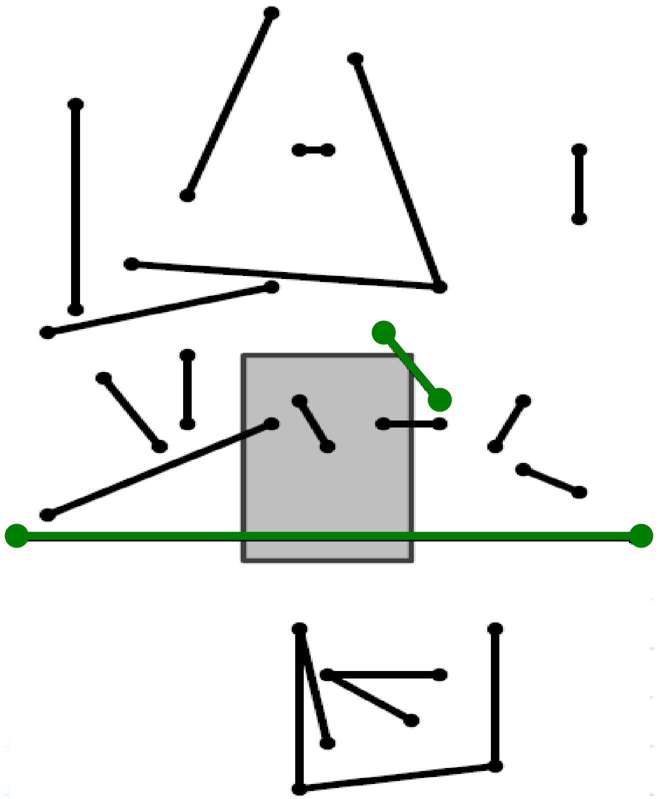
- Range queries (see range trees in Lecture 03)
  - Points
  - Often in higher dimensions
- Windowing queries
  - Line segments, curves, …
  - Usually in low dimension (2D, 3D)

- The goal for both:
  Preprocess the data into a data structure
  - so that the objects intersected by the query rectangle can be reported efficiently

Windowing queries on line segments



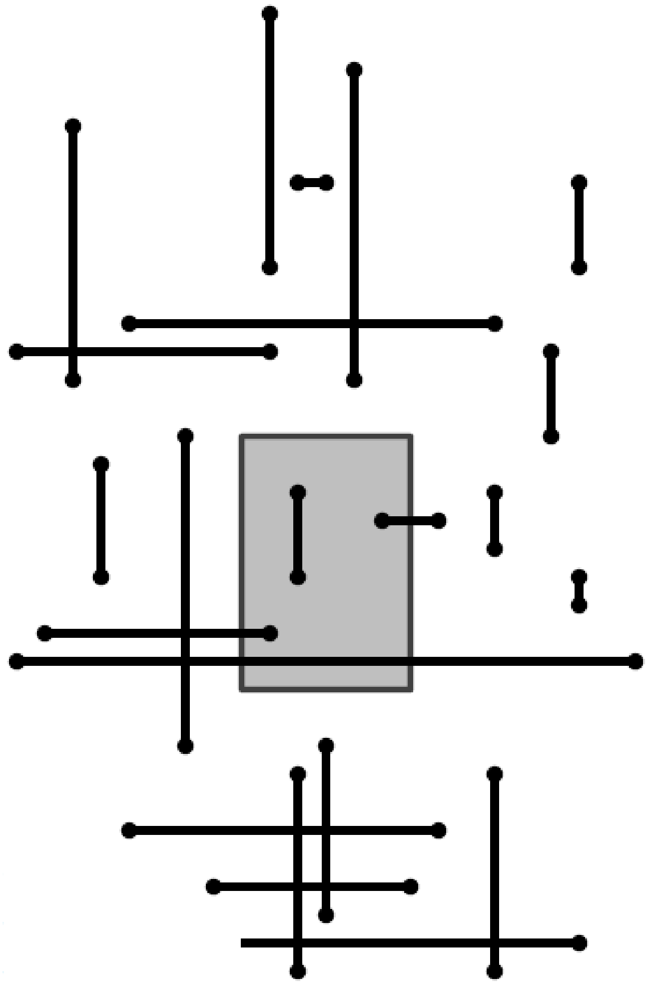
1. Axis parallel line segments



2. Arbitrary line segments (non-crossing)

[Vakken]

1. Windowing of axis parallel line segments

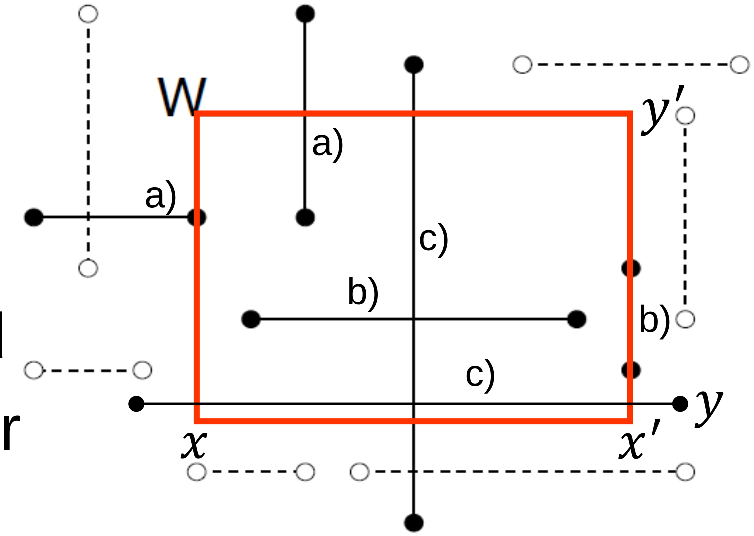


[Vakken]

1. Windowing of axis parallel line segments

Window query

- Given
  - a set of orthogonal line segments *S* (preprocessed),
  - and orthogonal query rectangle $W = [x : x'] \times [y : y']$
- Count or report all the line segments of *S* that intersect *W*
- Such segments have
  a) one endpoint in
  b) two end points in – included
  c) no end point in – cross over

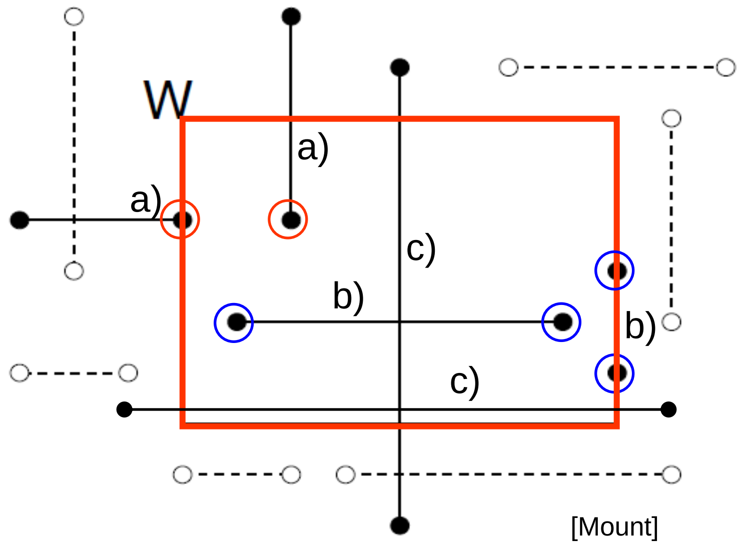


[Mount]

DCGI

Line segments with 1 or 2 points inside

a) one point inside
- Use a 2D range tree (lesson 3)
- $O(n \log n)$ storage
- $O(\log^2 n + k)$ query time or
- $O(\log n + k)$ with fractional cascading



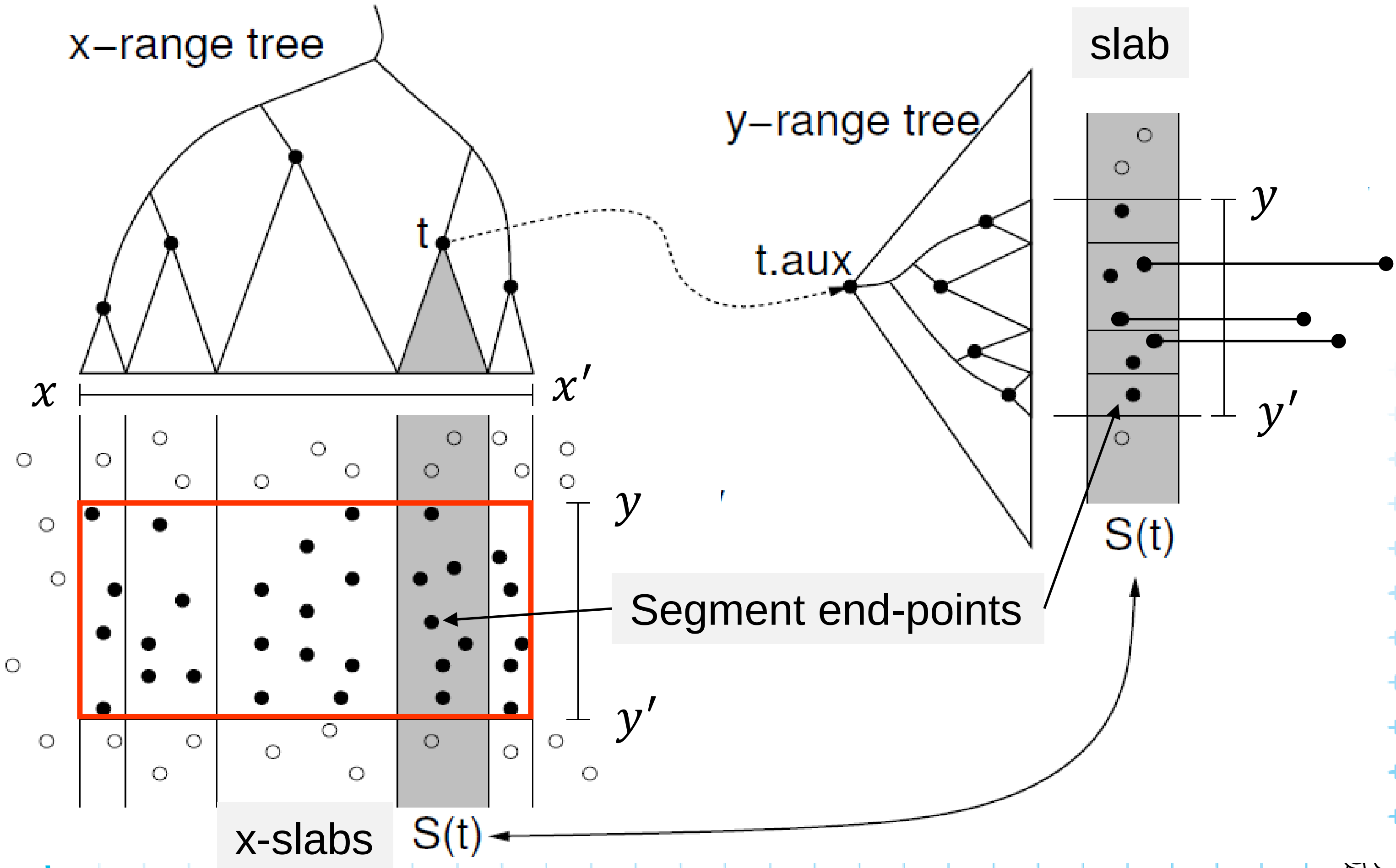
b) two points inside – as a) one point inside
- Avoid reporting twice:
  - Mark segment when reported (clear after the query) and skip marked segments or
  - when end point found, check the other end-point and report only one of them (the leftmost or the bottom).

DCGI

2D range tree (without fractional cascading-more in Lecture 3)

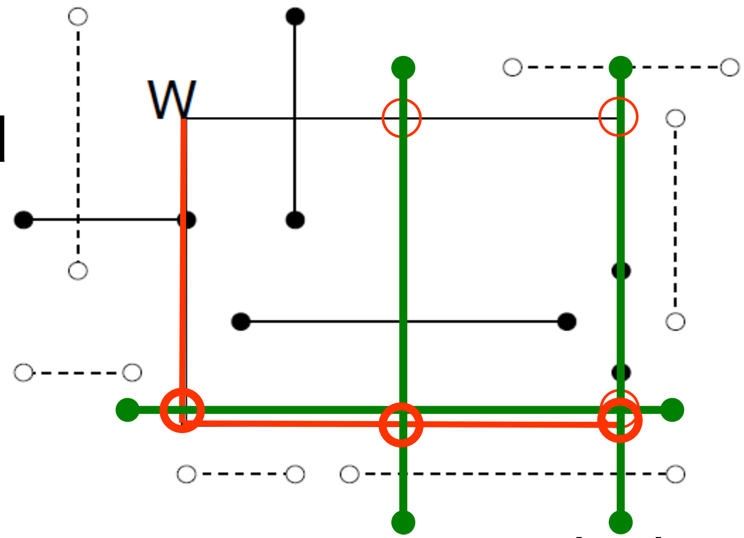
Search space: points
Query: Orthogonal intervals $[x : x'] \times [y : y']$

a), b)

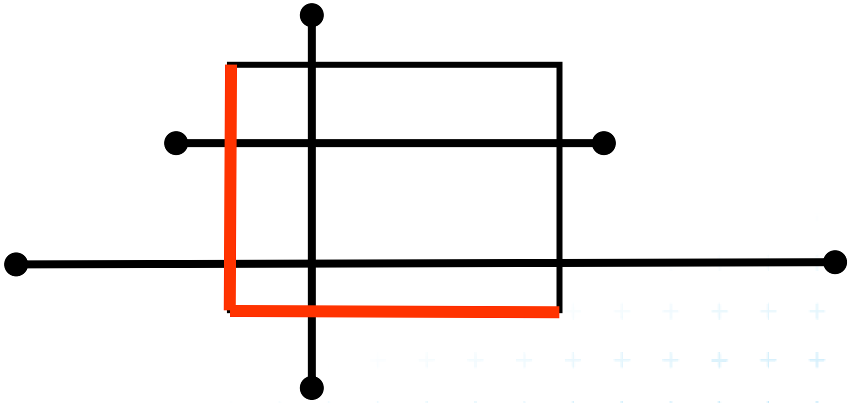


Line segments that cross over the window

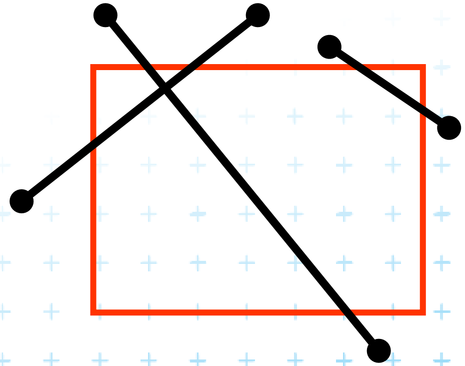
c) No points inside
- Such segments not detected using end-point range tree
- Cross the boundary twice



For axis parallel segments



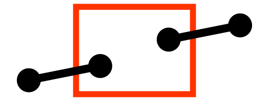
Check left and bottom boundary



For non-parallel segments

Check all 4 boundaries

Windowing problem summary



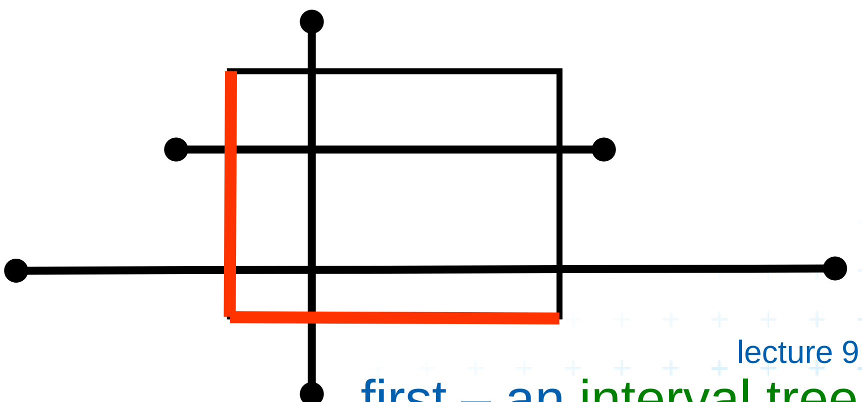
Cases a) and b)

– Segment end-point in the query rectangle (window)

– Solved by 2D range trees (see lecture 3, $O(n \log n)$ time & memory)

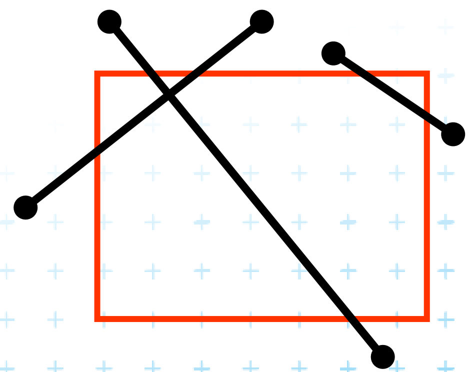
- We will discuss only case c)

  – Segment crosses the window



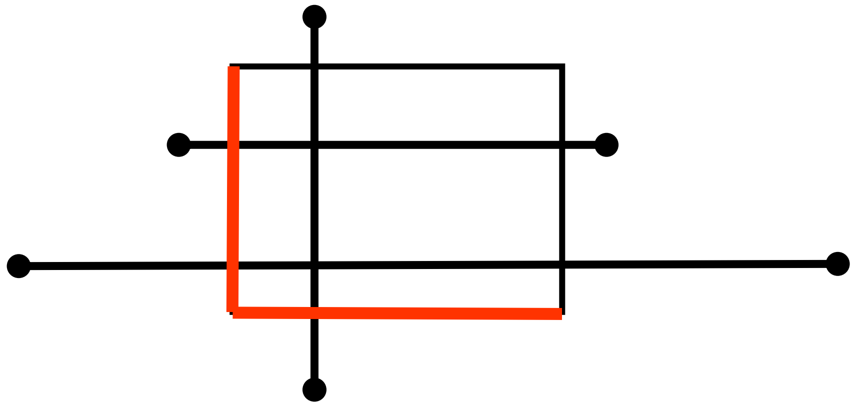
lecture 9

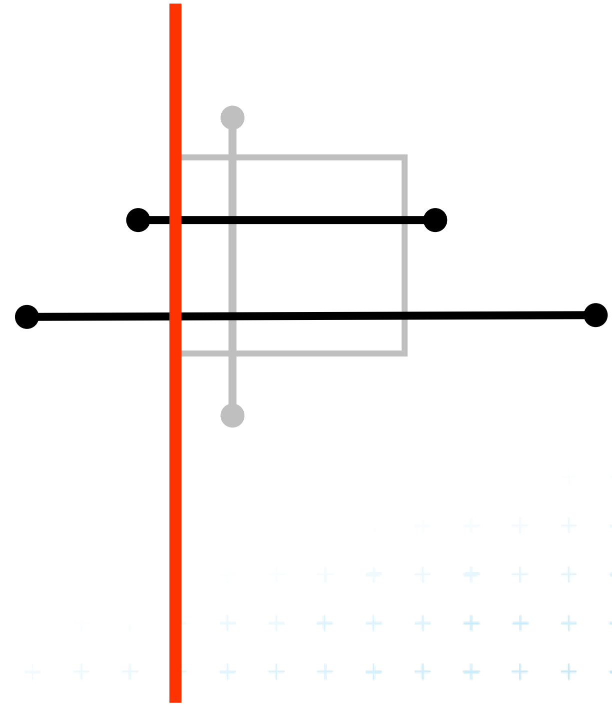
first – an interval tree (three variants)



later – a segment tree

case c) principle



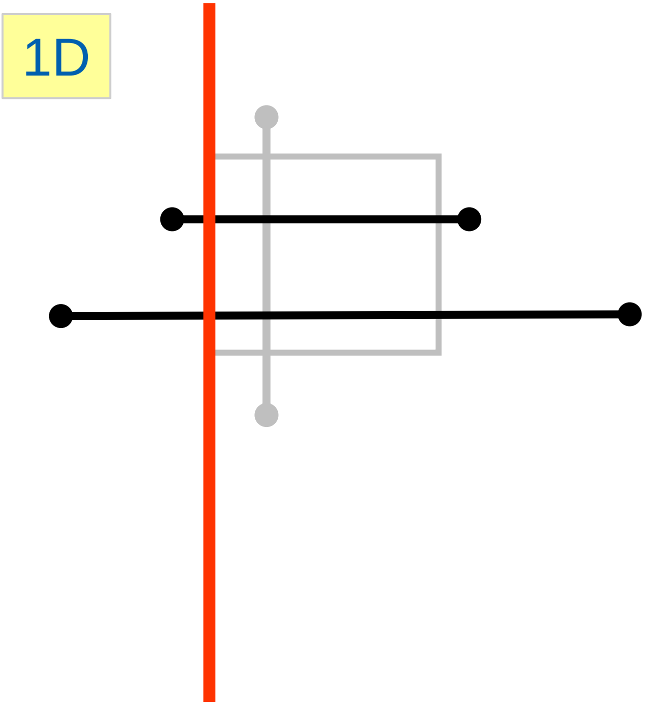


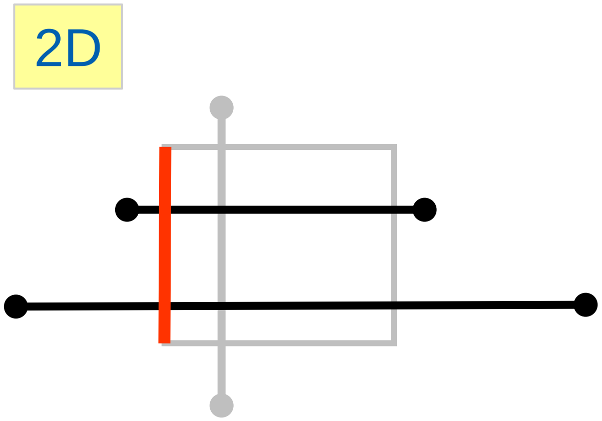
Segments cross the window

solved as

Line crosses the segments
(horizontal + vertical)

Talk Outline



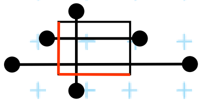


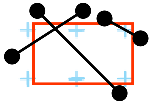
Line x line segments

interval tree

For heat-up

Line segment x line segments

2 variants of interval tree 

1 variant of segment tree 

Data structures for case c)

- Interval tree (1D IT)
  - stores 1D intervals (end-points in sorted lists)
  - computes intersections with query interval
  - see intersection of axis angle rectangles – there is y-overlap used, here is x-overlap
- We must extend Interval tree to 2D
  - variants differ in storage of interval end-points M_L, M_R
    - 2D range trees
    - priority search trees
- Segment tree
  - splits the plane to slabs in x in elementary intervals

Talk overview

1. Windowing of axis parallel line segments in 2D
   – 3 variants of *interval tree – IT in x-direction*
   – Differ in storage of segment end points M_L and M_R

1D
i. Line stabbing (standard *IT* with *sorted lists*) lecture 9 - intersections

2D
ii. Line segment stabbing (*IT* with *range trees*)
iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position

2D
– *segment tree + BST*

DCGI

i. Segment intersected by vertical line

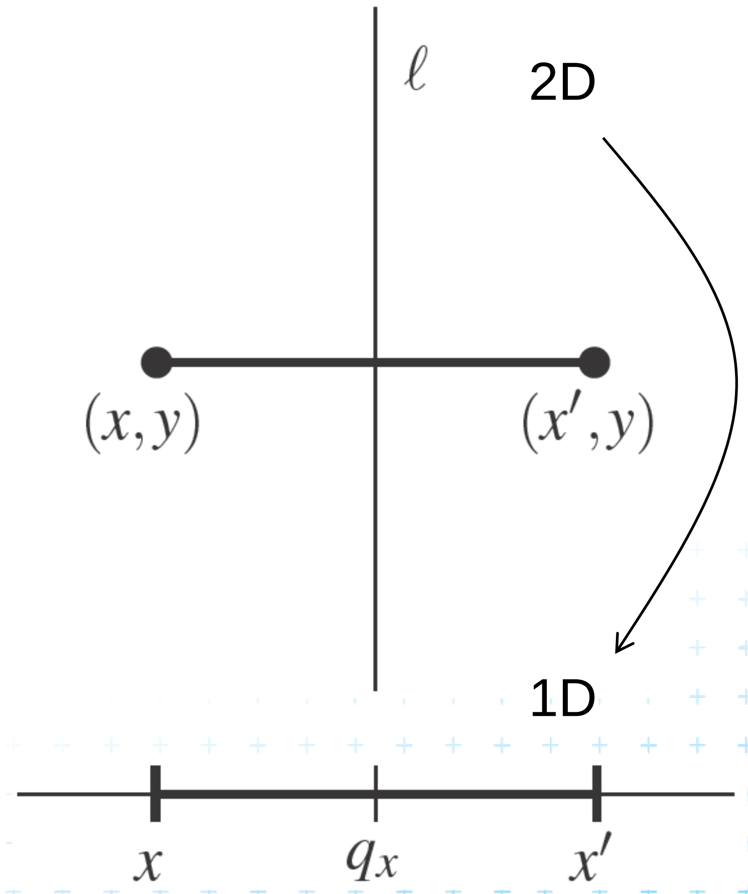
- Query line $\ell := (x = q_x)$

  Report the segments stabbed by a vertical line

  = 1 dimensional problem

  (ignore y coordinate)

$\Rightarrow$ Report the interval $[x : x']$ containing query point q_x

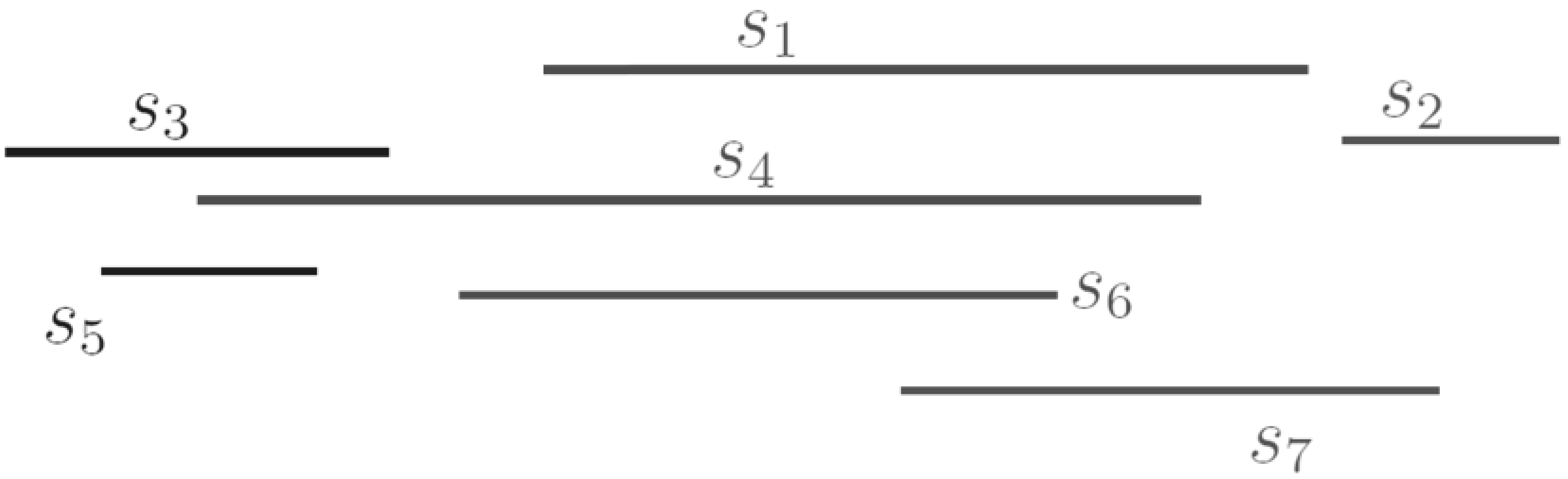


DS: Interval tree with sorted lists

[Mount]

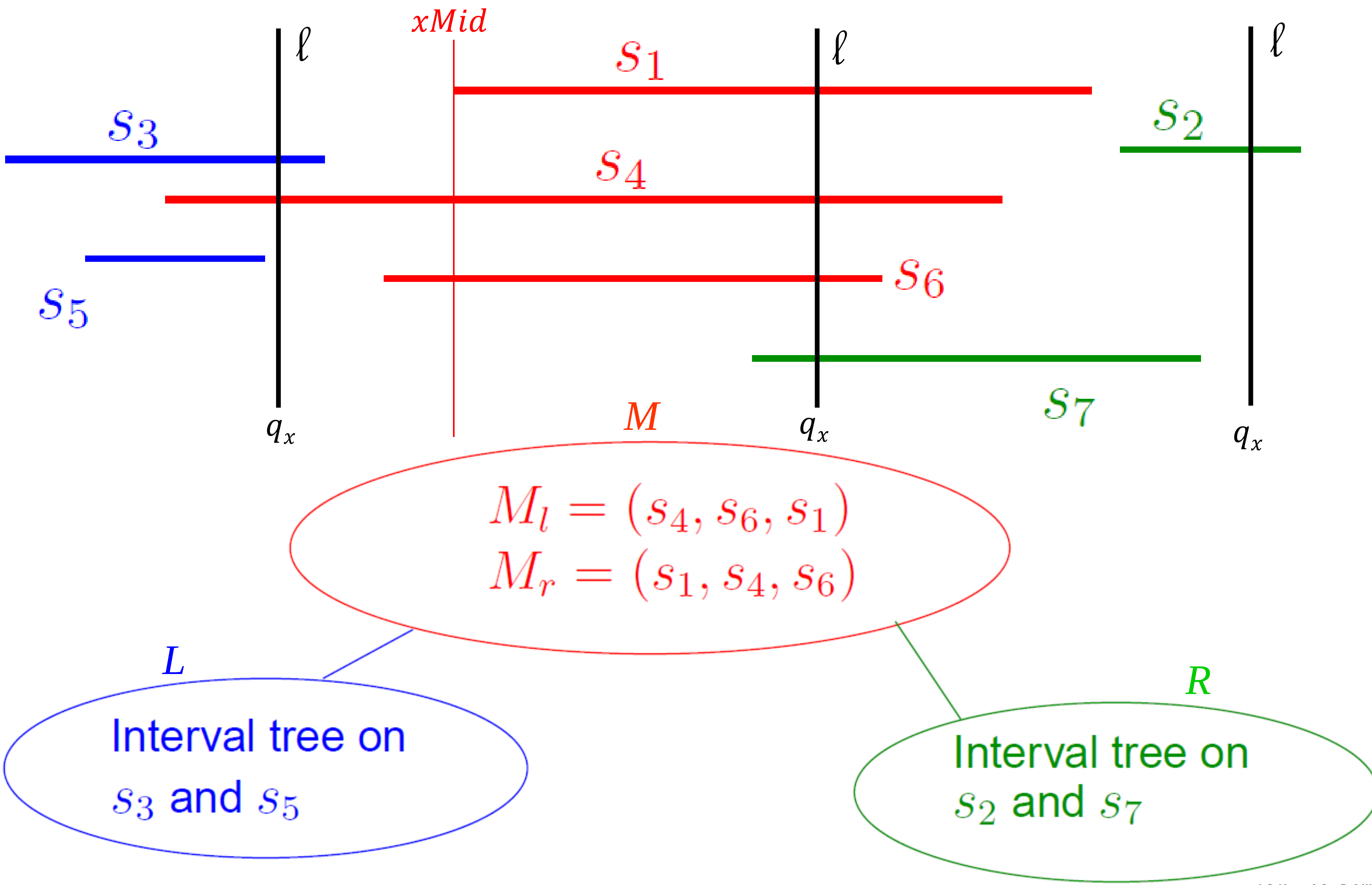
Interval tree principle

(see lecture 9 - intersections)



Interval tree principle

(see lecture 9 - intersections)

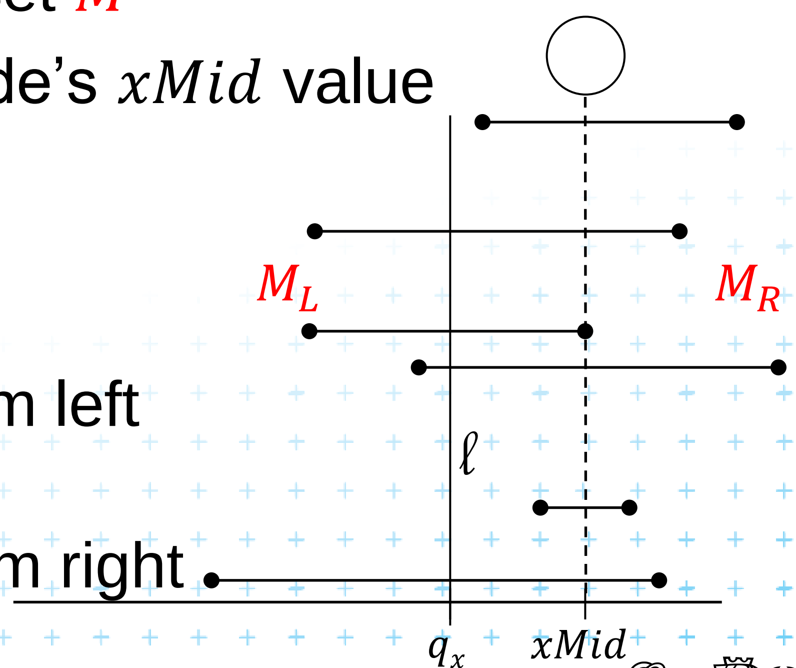


[Vigneron]

i. Segment intersected by vertical line

Principle

- Store input segments in static interval tree
- In each interval tree node
  - Check the segments in the set M
  - These segments contain node's $xMid$ value
    - M_L are left end-points
    - M_R are right end-points
  - q_x is the query value
  - If ($q_x < xMid$) Sweep M_L from left
    p ∈ M_L: if $p_x \leq q_x \Rightarrow$ intersection
  - If ($q_x > xMid$) Sweep M_R from right
    p ∈ M_R: if $p_x \geq q_x \Rightarrow$ intersection



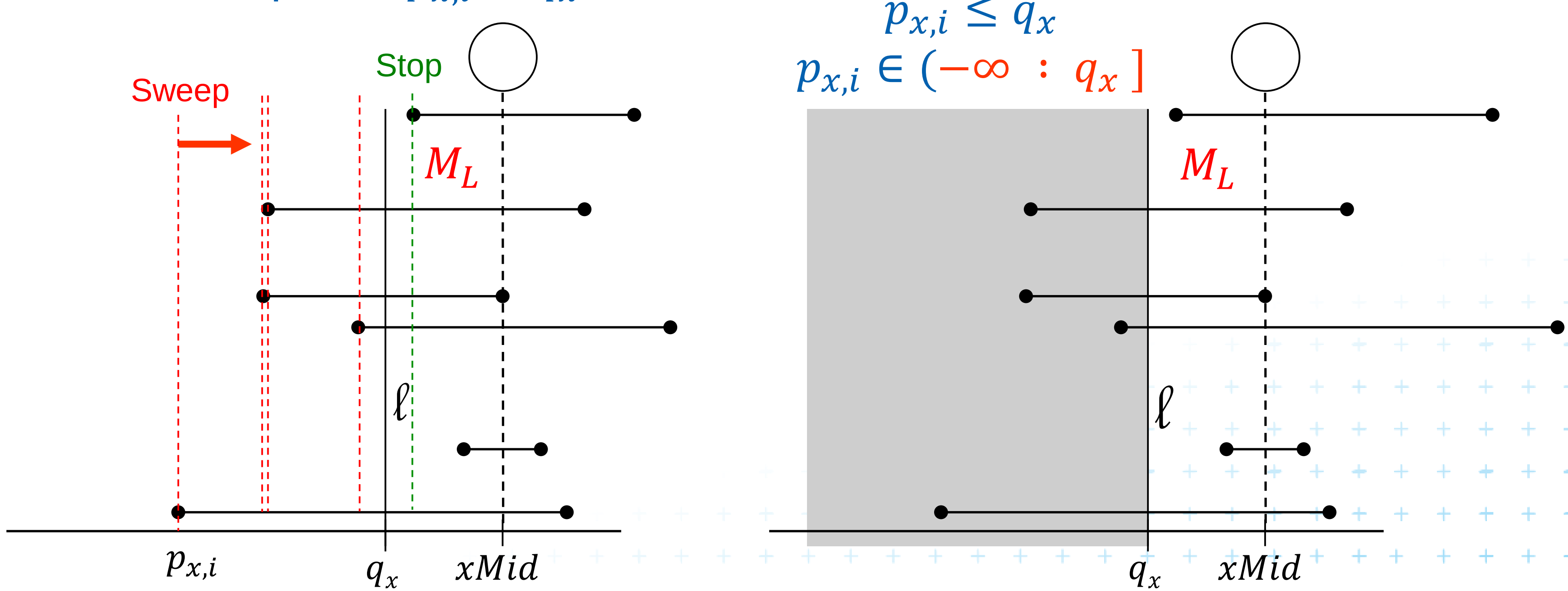
Inspired by [Berg]

Segment intersection (left from $xMid$)

All line segments from M pass through $xMid$

$\Rightarrow q_x$ must be between $p_{x,i}$ and $xMid$ to intersect the line segment i

$\Rightarrow$ left endpoints $p_{x,i} \leq q_x \Rightarrow$ intersection

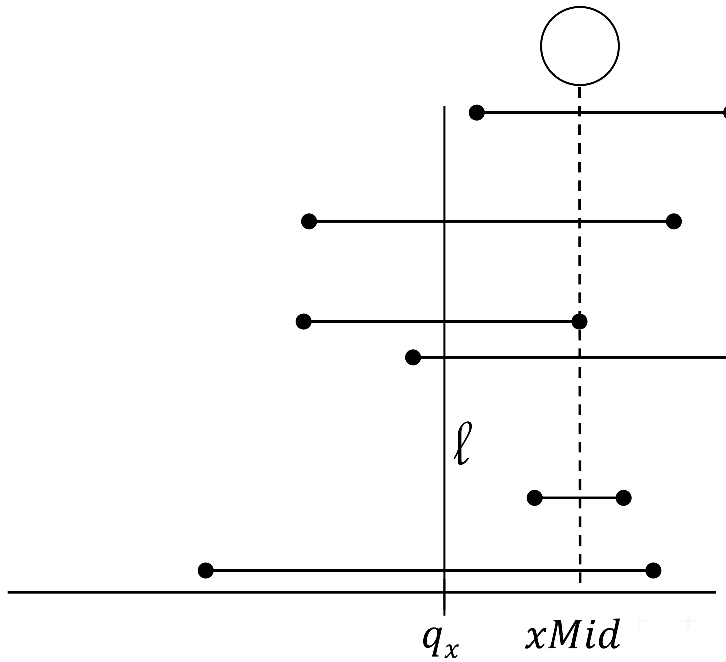


Intersection with line ℓ means Intersection with half space q

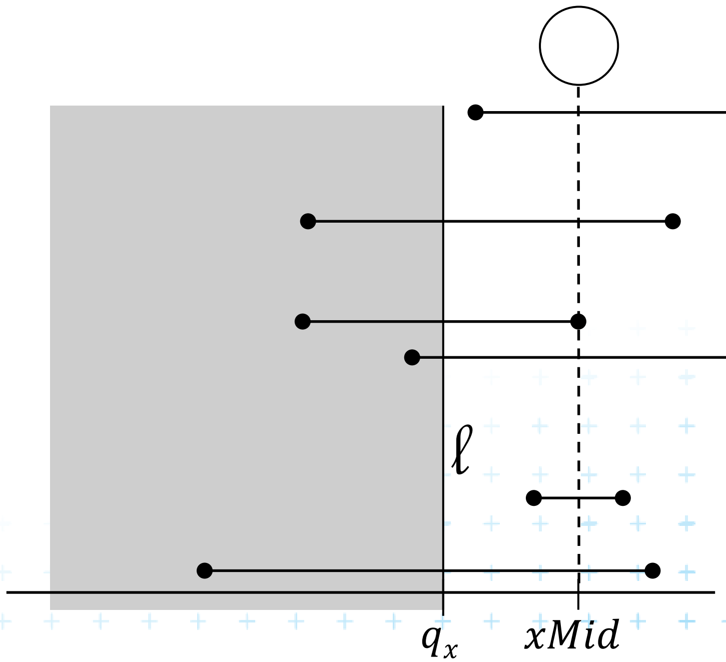
$\ell := q_x \times [-\infty : \infty]$ $\quad$ $q := (-\infty : q_x] \times [-\infty : \infty]$

Principle once more

Instead of intersecting edges by line



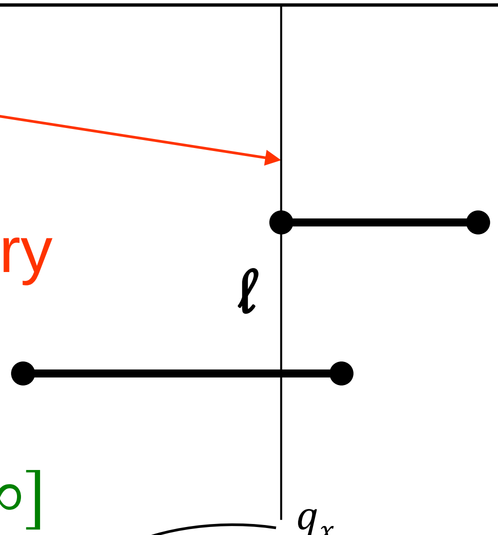
search points in half-space

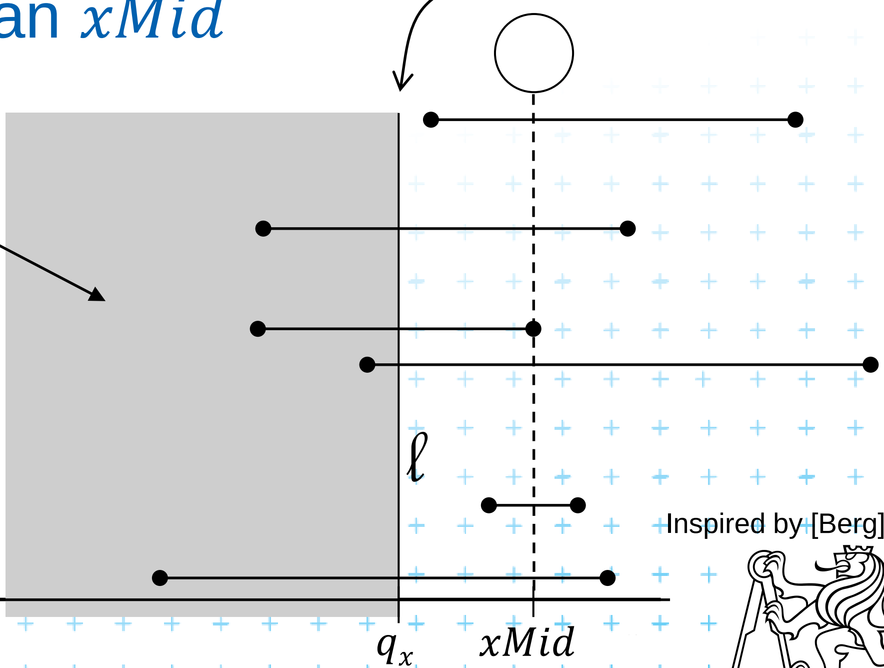


i. Segment intersected by vertical line

De facto a 1D problem

- Query line $\ell := q_x \times [-\infty : \infty]$
- Horizontal segment of M stabs the query line ℓ left of $xMid$ **iff** its (segment's) left endpoint lies in half-space
  $$q := (-\infty : q_x] \times [-\infty : \infty]$$
- In IT node with stored median $xMid$ report all segments from M
  - M_L: whose left point lies in $(-\infty : q_x]$ if ℓ lies left from xMid
  - M_R: whose right point lies in $[q_x : +\infty)$ if ℓ lies right from xMid

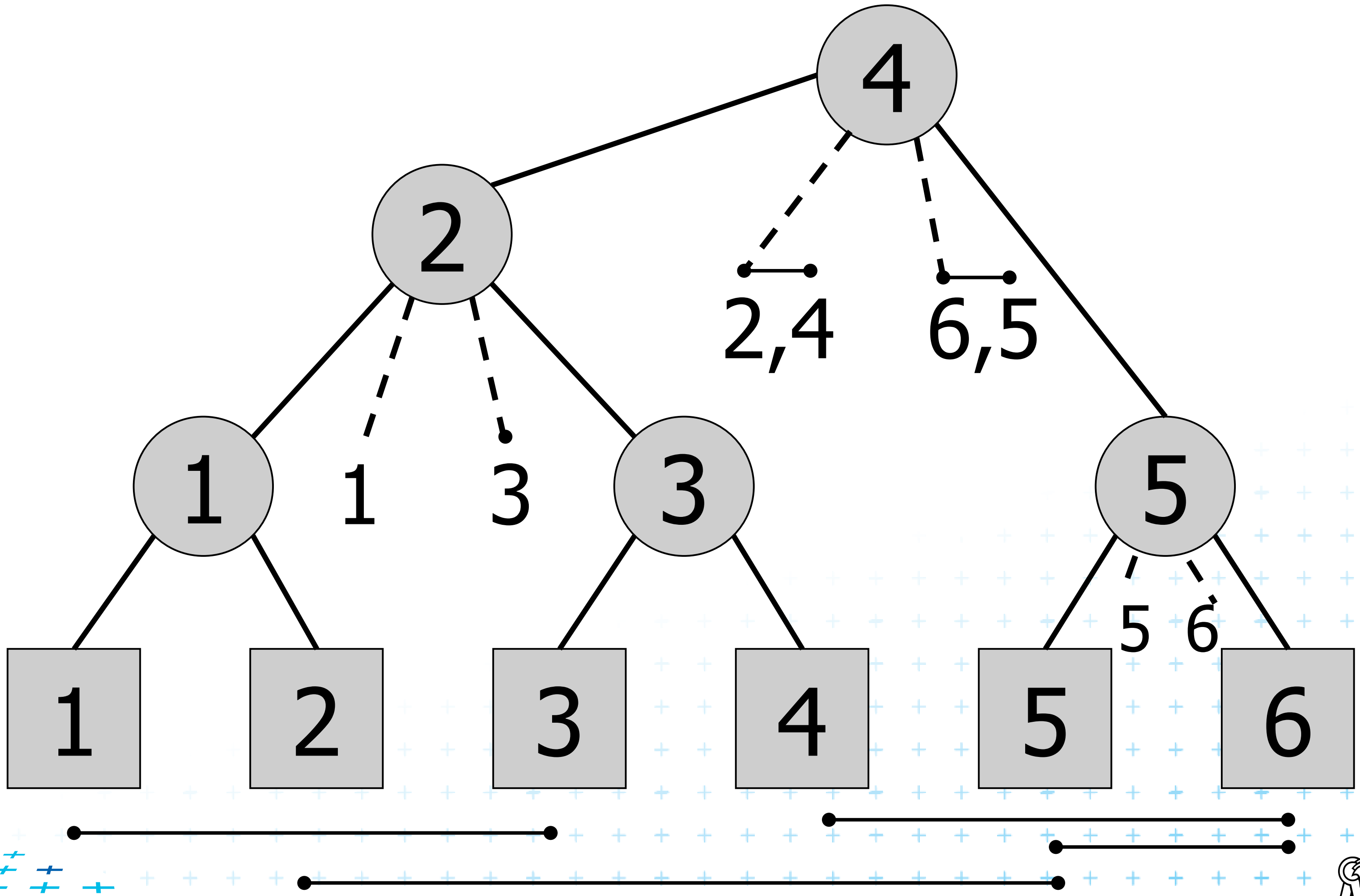




Inspired by [Berg]

Static interval tree [Edelsbrunner80]

Tree over sorted segment end-points



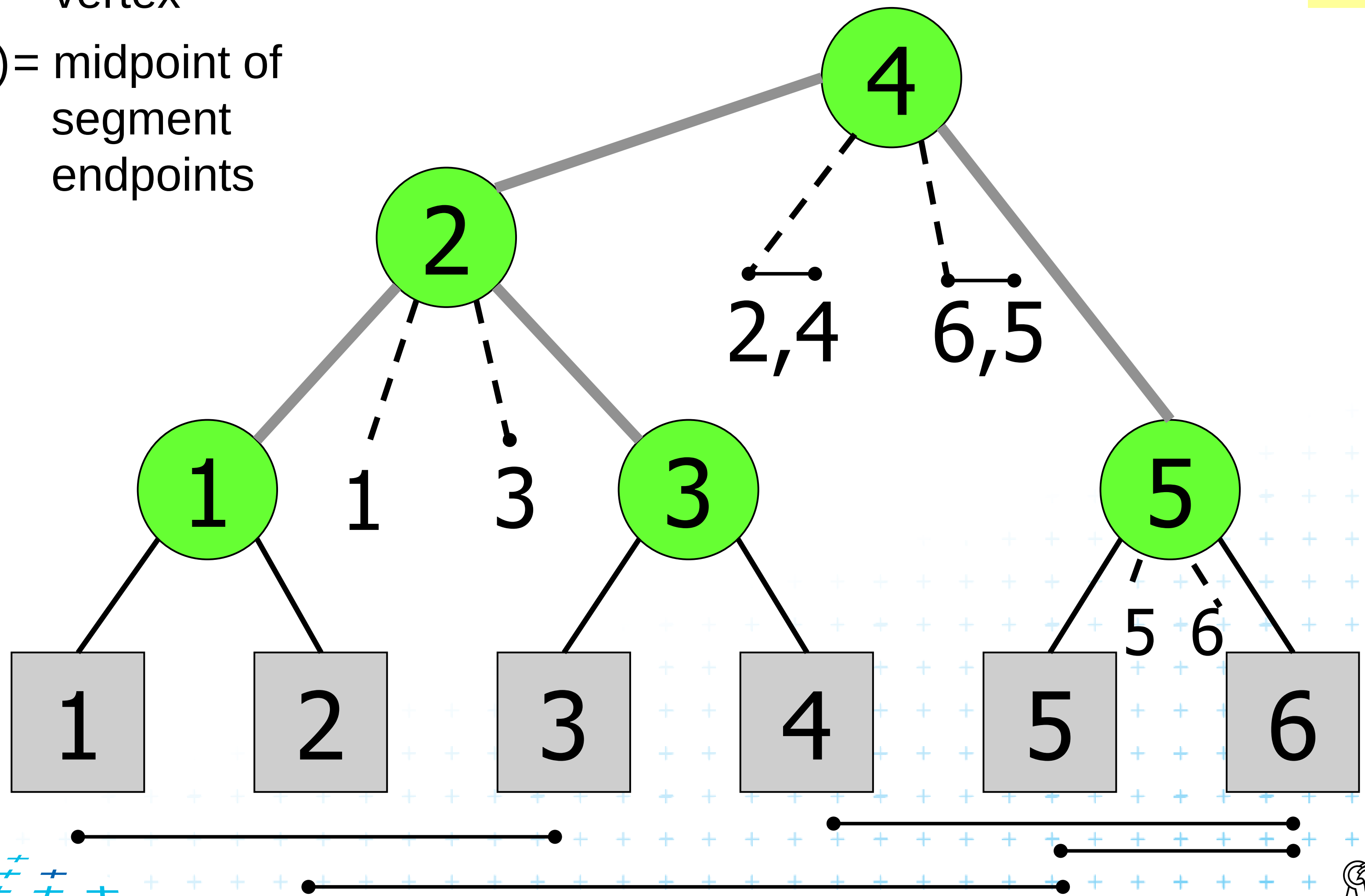
[Kukral]

Primary structure – static tree for endpoints

Static

v = vertex

d(v)= midpoint of segment endpoints



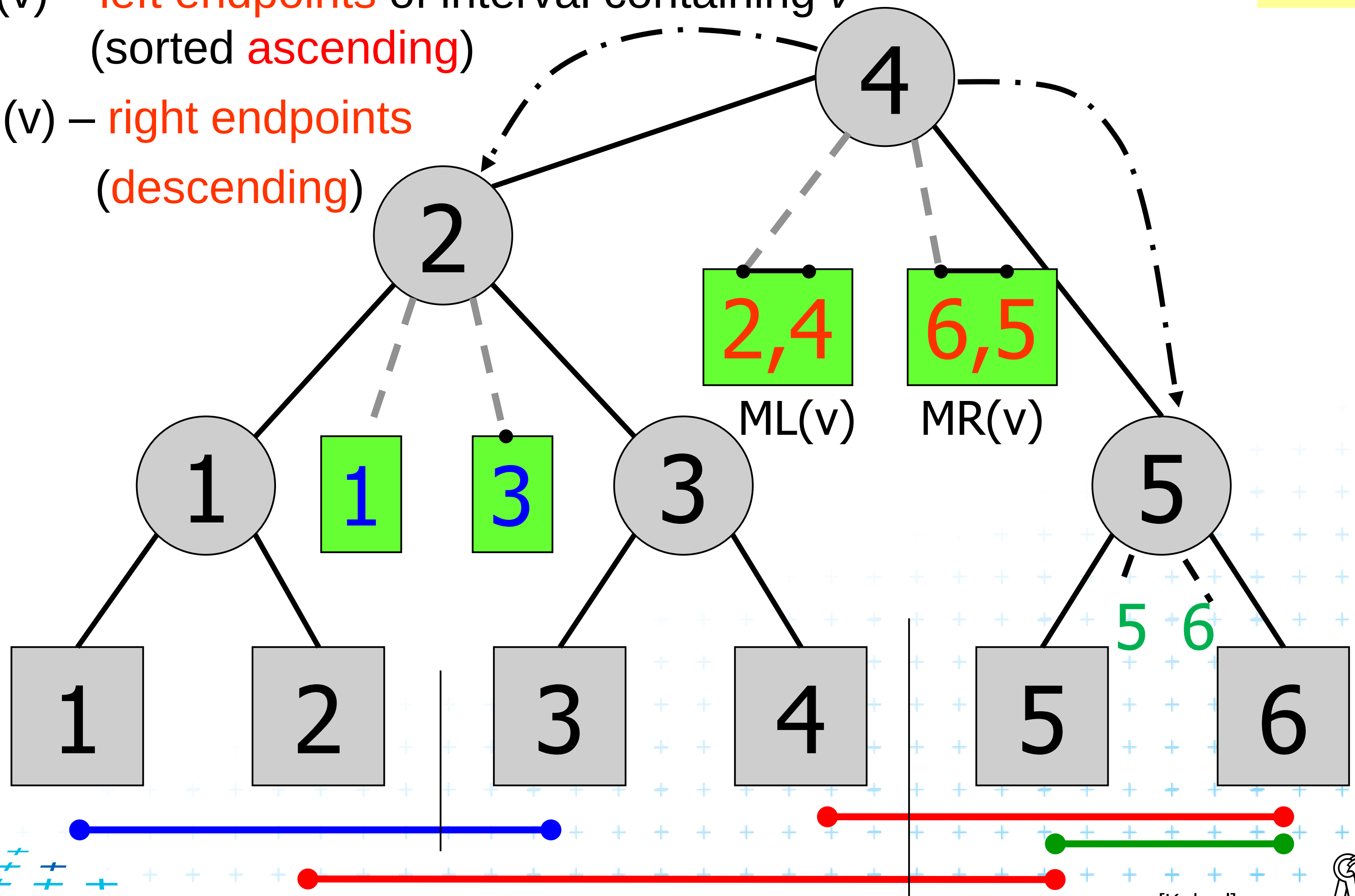
[Kukral]

Secondary lists of incident interval end-pts.

Dynamic

ML(v) – left endpoints of interval containing *v*
(sorted ascending)

MR(v) – right endpoints
(descending)



[Kukral]

Interval tree construction

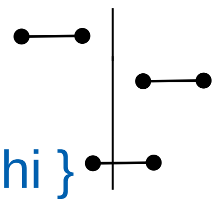
Merged procedures from in lecture 09
- PrimaryTree(*S*) on slide 33
- InsertInterval (*b, e, T*) on slide 35

ConstructIntervalTree(*S*) **// Intervals all active – no active lists**

Input: Set S of intervals on the real line – on *x-axis*

Output: The root of an interval tree for *S*

```
1.  if (|S| == 0) return null                              // no more intervals
2.  else
3.      xMed = median endpoint of intervals in S           // median endpoint
4.      L = { [xlo, xhi] in S | xhi < xMed }               // left of median
5.      R = { [xlo, xhi] in S | xlo > xMed }               // right of median
6.      M = { [xlo, xhi] in S | xlo <= xMed <= xhi }       // contains median
7.      ML = sort M in increasing order of xlo             // sort M
8.      MR = sort M in decreasing order of xhi
9.      t = new IntTreeNode(xMed, ML, MR)                  // this node
10.     t.left  = ConstructIntervalTree(L)                 // left subtree
11.     t.right = ConstructIntervalTree(R)                 // right subtree
12.     return t
```



steps 4.,5.,6. done in one step if presorted [Mount]

Line stabbing query for an interval tree

Less effective variant of QueryInterval (b, e, T)
on slide 34 in lecture 09
with merged parts: fork and search right

Stab(t, qx)
Input: IntTreeNode t, Scalar qx
Output: prints the intersected intervals

```
if (t == null) return                           // no leaf: fell out of the tree
if (qx < t.xMed)                                // left of median?
    for (i = 0; i < t.ML.length; i++)           // traverse M_L left end-points
        if (t.ML[i].lo ≤ qx) print (t.ML[i])    // ..report if in range
        else break                              // ..else done
    Stab (t.left, qx)                           // recurse on left subtre
else  // (qx ≥ t.xMed)                          // right of or equal to median
    for (i = 0; i < t.MR.length; i++) {         // traverse M_R right end-points
        if (t.MR[i].hi ≥ qx) print (t.MR[i])    // ..report if in range
        else break                              // ..else done
    Stab (t.right, qx)                          // recurse on right subtree
```

Note: Small inefficiency for qx == t.xMed – recurse on right

[Mount]

Complexity of line stabbing via interval tree

with *sorted lists*

- Construction - $O(n \log n)$ time
  - Each step divides at maximum into two halves or less (minus elements of M) => tree of height $h = O(\log n)$
  - If presorted endpoints in three lists L,R, and M then median in O(1) and copy to new L,R,M in $O(n)$
- Vertical line stabbing query - $O(k + \log n)$ time
  - One node processed in $O(1 + k')$, k' reported intervals
  - v visited nodes in $O(v + k)$, k total reported intervals
  - $v = h =$ tree height = $O(\log n)$ $k = \Sigma k'$
- Storage - $O(n)$
  - Tree has $O(n)$ nodes, each segment stored twice (two endpoints)

Talk overview

1. Windowing of axis parallel line segments in 2D
   - 3 variants of *interval tree – IT in x-direction*
   - Differ in storage of segment end points M_L and M_R

1D i. Line stabbing (standard *IT* with *sorted lists*) lecture 9 - intersections

2D ii. Line segment stabbing (*IT* with *range trees*)

iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position

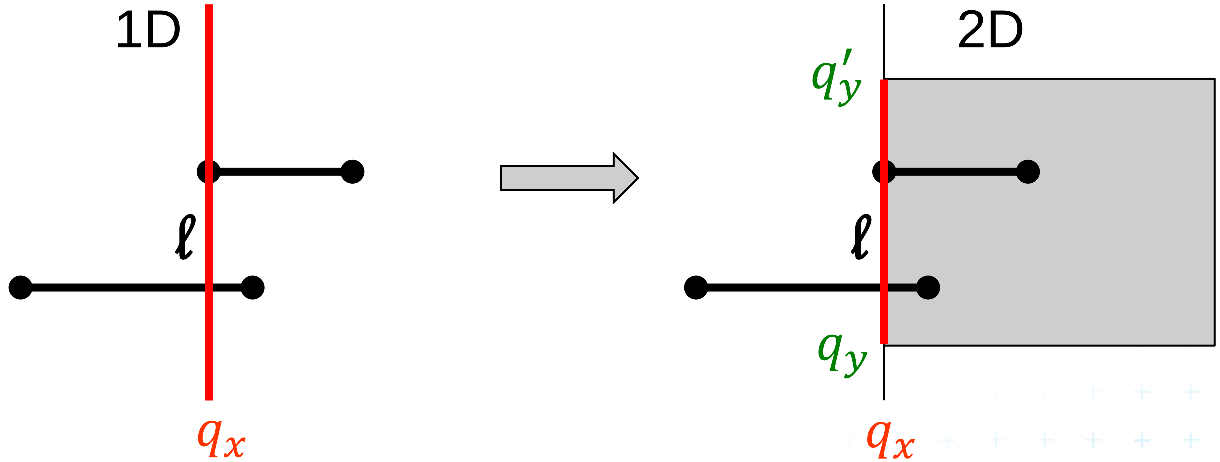
2D – *segment tree + BST*

Line segment stabbing (*IT* with *range trees*)

Enhance 1D interval trees to 2D

change lines

to segments



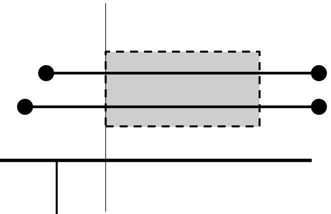
$q_x \times [-\infty : \infty]$ (no y-test)

$q_x \times [q_y : q'_y]$ (additional y-test)

Sorted lists

Range trees

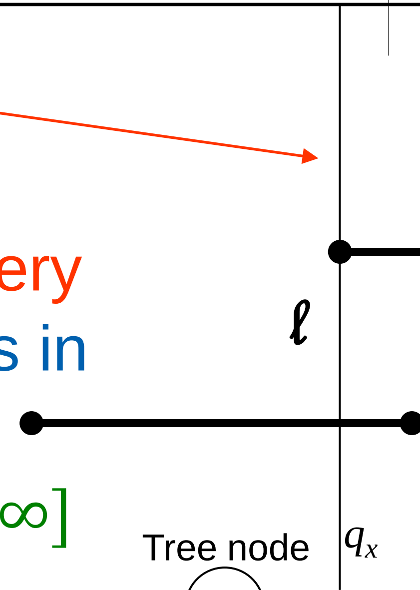
i. Segments × vertical line



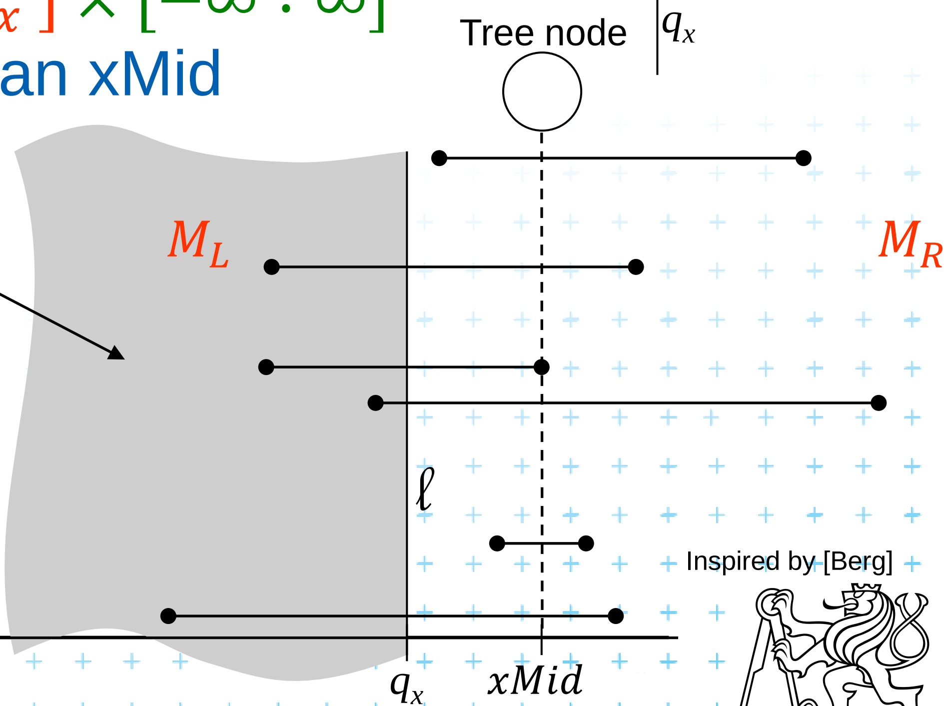
De facto a 1D problem

- Query line $\ell := q_x \times [-\infty : \infty]$
- Horizontal segment of M_L stabs the query line ℓ left of $xMid$ **iff** its left endpoint lies in half-space

$$q := (-\infty : q_x] \times [-\infty : \infty]$$



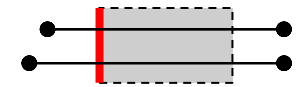
- In IT node with stored median xMid report all segments from M
  - M_L: whose left point lies in $(-\infty : q_x]$ if ℓ lies left from xMid
  - M_R: whose right point lies in $[q_x : +\infty)$ if ℓ lies right from xMid



Inspired by [Berg]

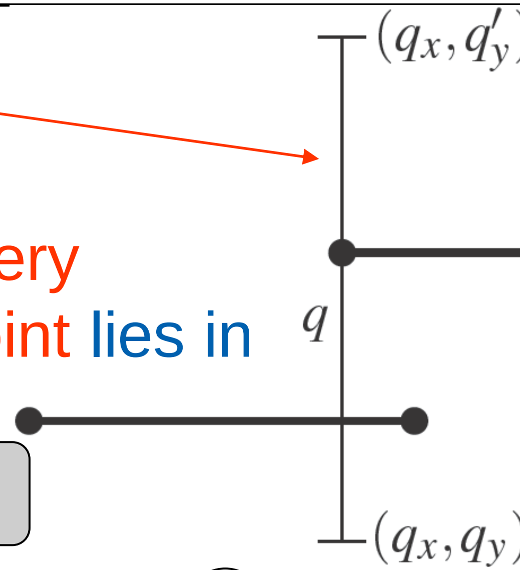
DCGI

ii. Segments × vertical line segment



A 2D problem

- Query segment $q := q_x \times [q_y : q'_y]$

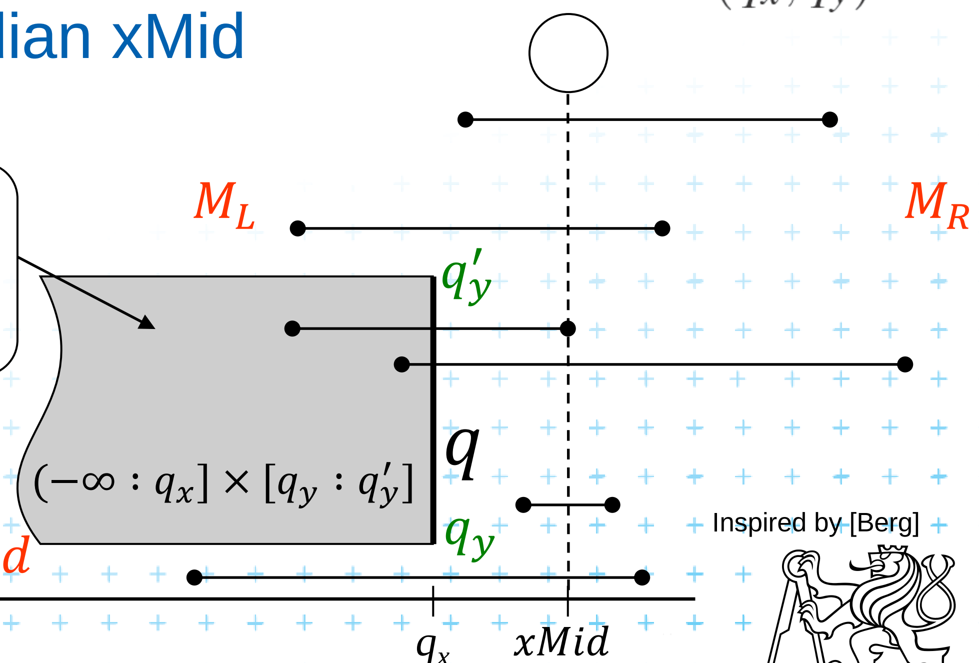


- Horizontal segment of M_L stabs the query segment q left of $xMid$ **iff** its left endpoint lies in semi-infinite rectangular region

New test

$$q := (-\infty : q_x] \times [q_y : q'_y]$$

- In IT node with stored median xMid report all segments
  - M_L: whose left points lie in $(-\infty : q_x] \times [q_y : q'_y]$ where q_x lies left from $xMid$
  - M_R: whose right point lies in $[q_x : +\infty) \times [q_y : q'_y]$ where q_x lies right from $xMid$

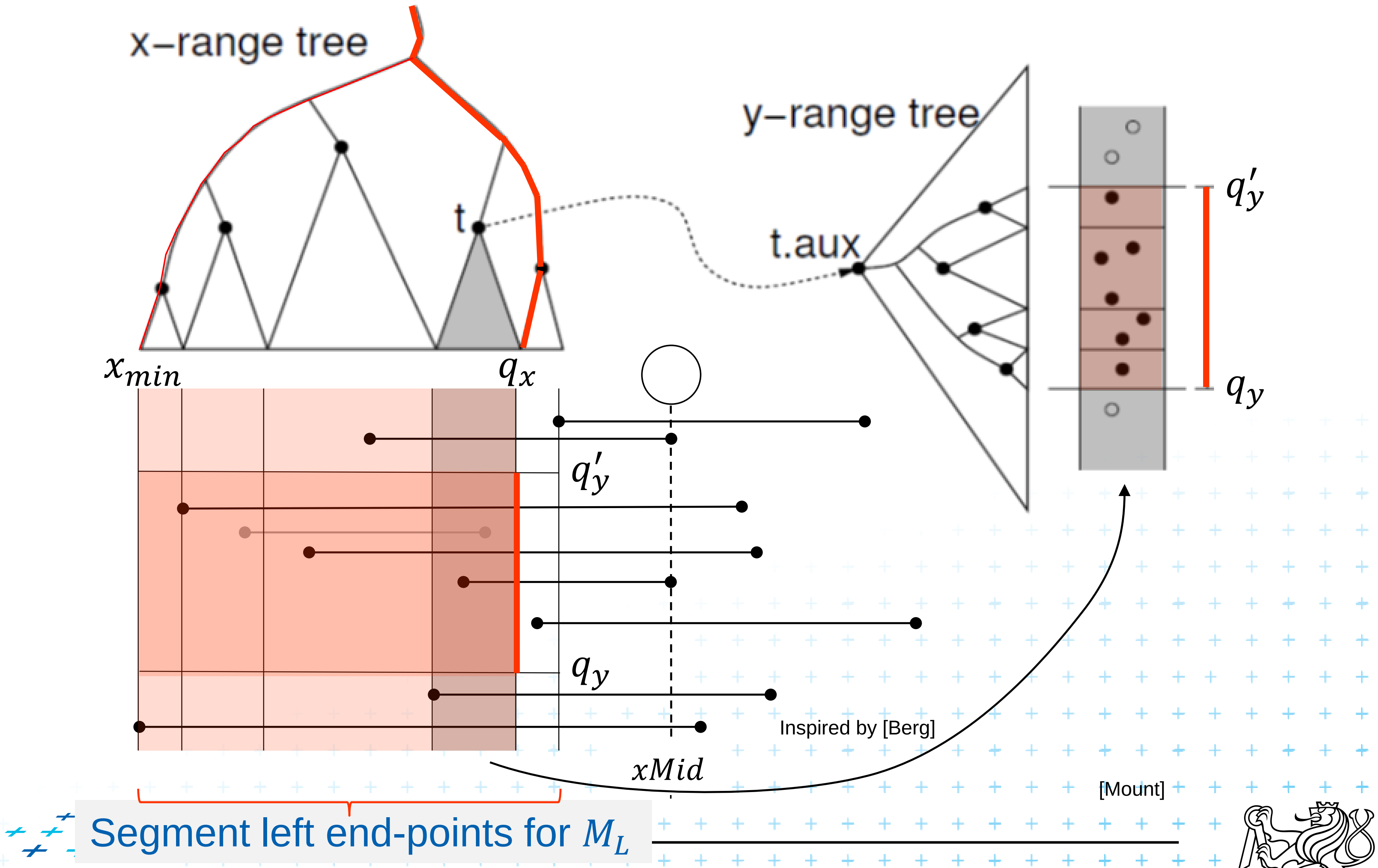


Inspired by [Berg]

Data structure for endpoints

- Storage of M_L and M_R
  - 1D Sorted lists is not enough for line segments
  - We need to test in y too
  - Use 2D range trees
    (one for M_L and one for M_R in each node)
- Instead $O(n)$ sequential search in M_L and M_R perform $O(\log n)$ search
  in range tree with fractional cascading

2D range tree (without fractional cascading-more in Lecture 3)



Complexity of range tree line segment stabbing

- Construction - $O(n \log n)$ time
  - Each step divides at maximum into two halves L,R or less (minus elements of M) => int. tree height $O(\log n)$
  - If the range trees are efficiently build in $O(n)$ after points sorted
- Vertical line segment stab. q. - $O(k + \log^2 n)$ time
  - One node processed in $O(\log n + k')$, k' reported segm. (interval tree; 2D range tree search with Fractional Cascading)
  - v-visited nodes in $O(v \log n + k)$, k total reported segm. (interval tree)
  - v = interval tree height = $O(\log n)$ $\quad \mathrm{k} = \sum k'$
  - $O(k + \log^2 n)$ time - range tree with fractional cascading
  - $O(k + \log^3 n)$ time - range tree without fractional casc.
- Storage - $O(n \log n)$

  Can be done better?
  - Dominated by the range trees

DCGI

Talk overview

1. Windowing of axis parallel line segments in 2D
   - 3 variants of *interval tree – IT in x-direction*
   - Differ in storage of segment end points M_L and M_R

1D i. Line stabbing (standard *IT* with *sorted lists*) lecture 9 - intersections

2D ii. Line segment stabbing (*IT* with *range trees*)

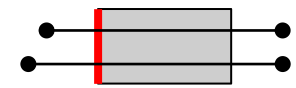
iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position

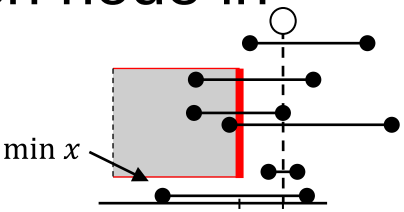
2D – *segment tree + BST*

iii. Priority search trees

[McCreight85]



- Another variant for case c) on slide 9
  - Exploit the fact that query rectangle in each node in interval tree is unbounded (in x direction)



- Priority search trees
  - as secondary data structure for both left and right endpoints (M_L and M_R) of segments in nodes of interval tree – one for ML, one for MR
  - Improve the storage to $O(n)$ for horizontal segment intersection with left window edge (2D range tree has $O(n \log n)$)
- For cases a) and b) - $O(n \log n)$ storage remains
  - we need range trees for windowing segment endpoints

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Rectangular range queries variants

- Let $P = \{p_1, p_2, \ldots, p_n\}$ is set of points in plane
- Goal: rectangular range queries of the form
  $(-\infty : q_x] \times [q_y : q'_y]$ – unbounded (in x direction)
- In 1D: search for nodes v with $v_x \in (-\infty : q_x]$
  - range tree $O(\log n + k)$ time (search the end, report left)
  - ordered list $O(1 + k)$ time 1 is for possibly fail test of the first
    (start in the leftmost, stop on v with $v_x > q_x$)
  - use heap $O(1 + k)$ time !
    (traverse all children, stop when $v_x > q_x$)
- In 2D – use heap for points with $x \in (-\infty : q_x]$
  + integrate information about y-coordinate
  = Priority search tree

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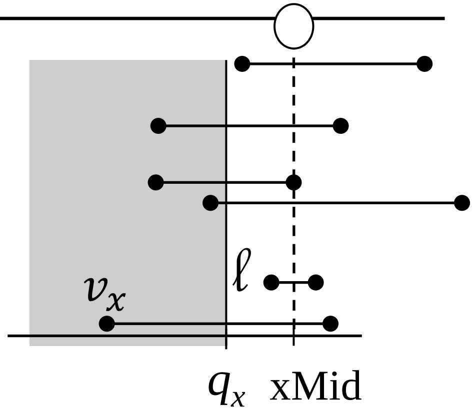
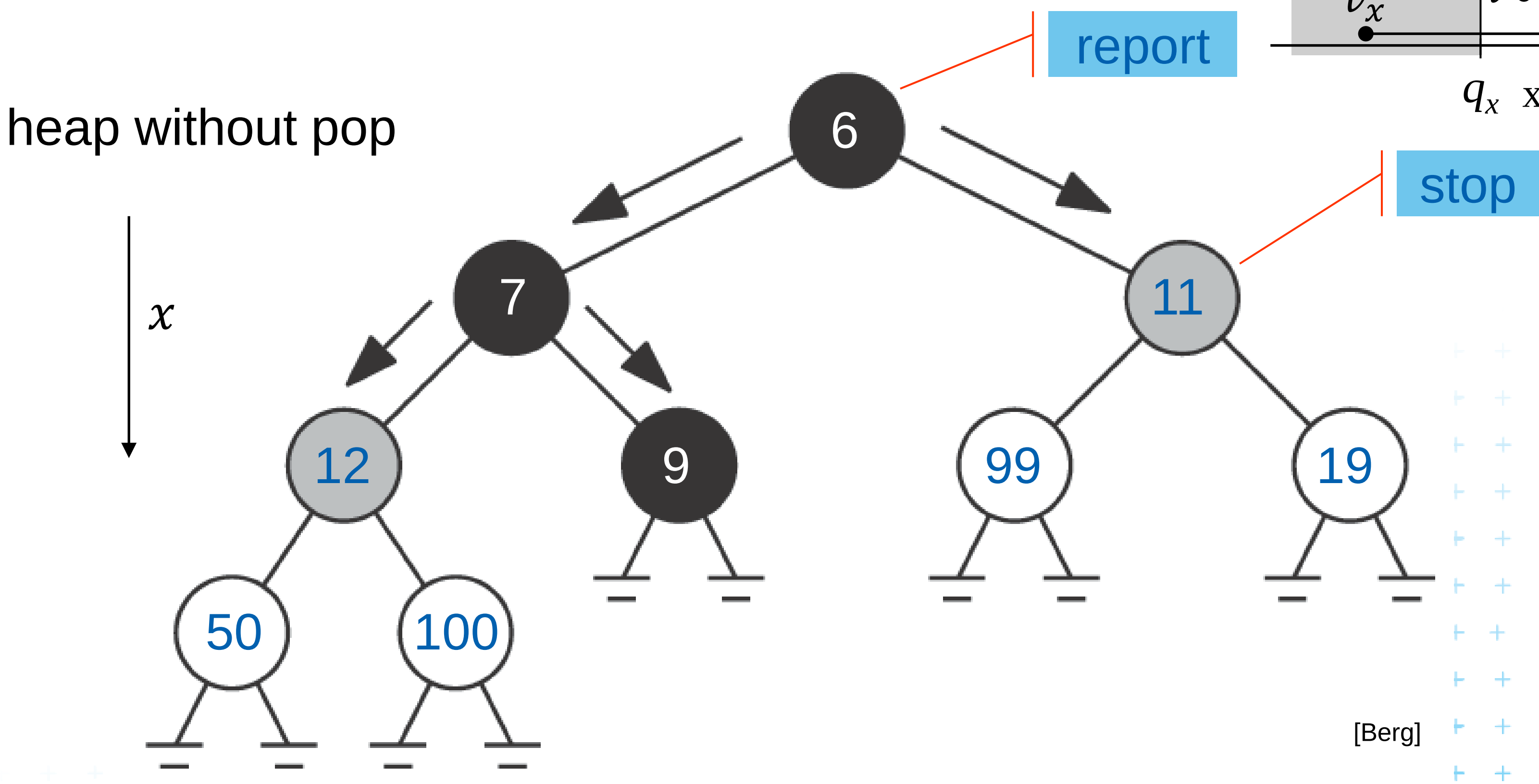
Heap for 1D unbounded range queries

- Traverse all children, stop if $v_x > q_x$
- Example: Query $(-\infty : 10]$, $q_x = 10$

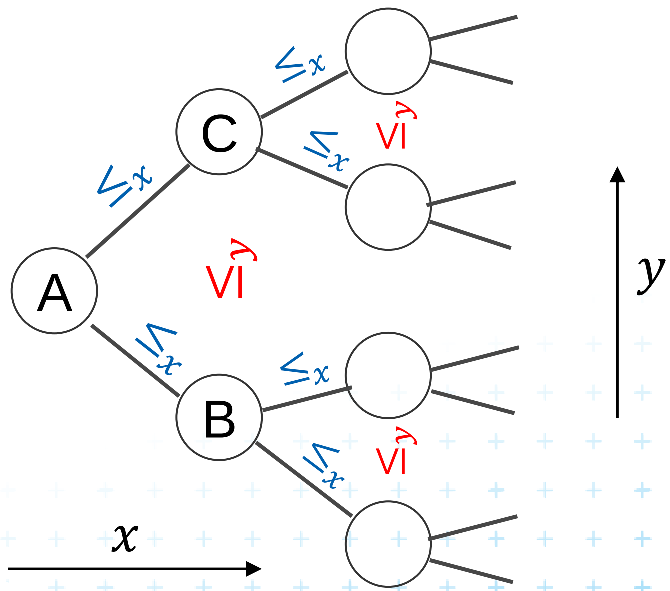




[Berg]

Principle of priority search tree

- Heap $\leq_x$
  - relation between parent and its child nodes only
  - no relation between the child nodes themselves
- Priority search tree
  - relate the child nodes according to y $\leq_y$



x Heap

A $\leq_x$ B

A $\leq_x$ C

y BVS

B $\leq_y$ A $\leq_y$ C $\Rightarrow$ B $\leq_y$ C

DCGI

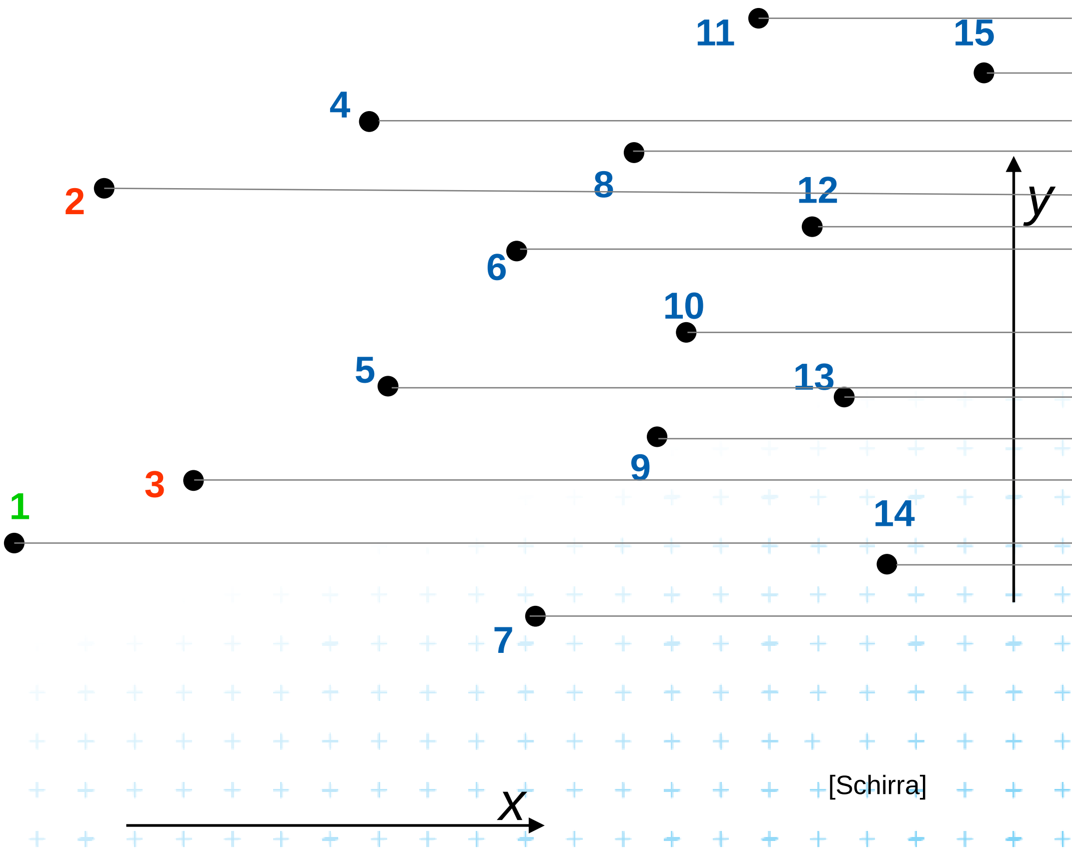
Priority search tree (PST)

= Heap in 2D that can incorporate info about both x, y

- BST on y-coordinate (horizontal slabs) ~ 1D range tree
- Heap on x-coordinate (minimum *x* from slab along *x*)

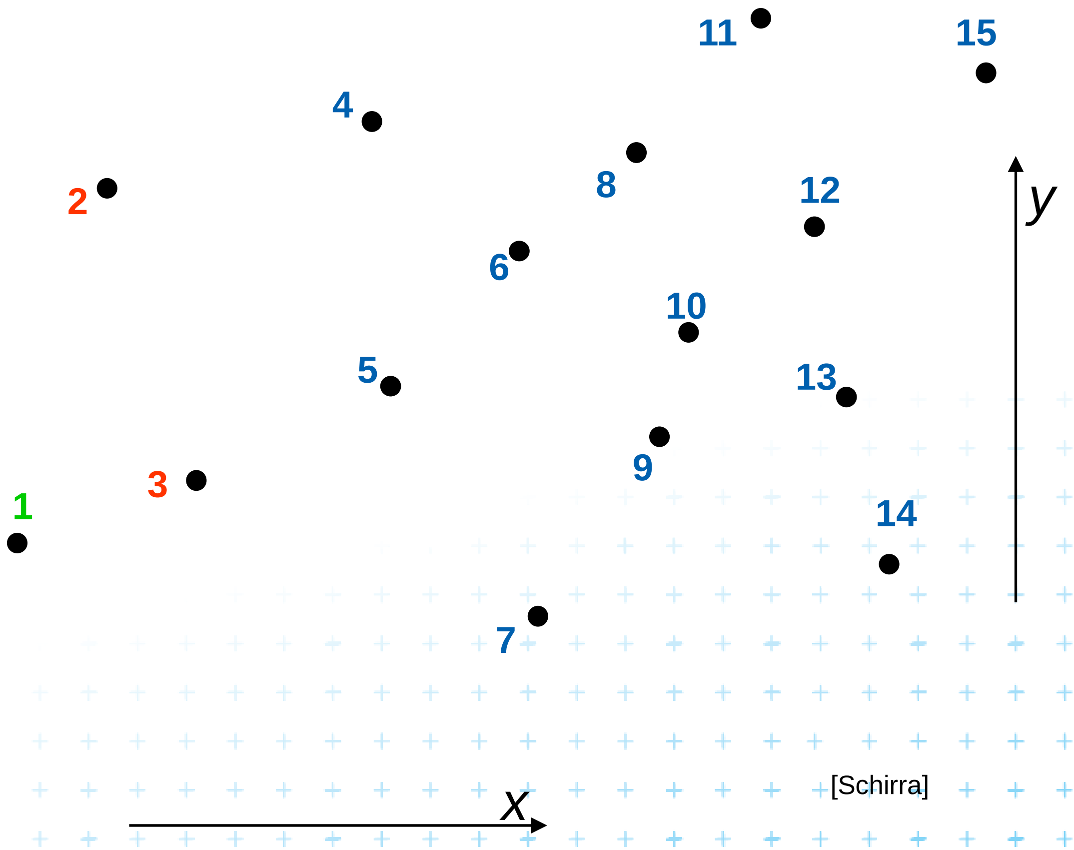
- If P is empty, PST is empty leaf
- else
  - p_{min} = point with smallest x-coordinate in P – a heap root
  - y_{med} = y-coord. median of points $P \setminus \{p_{min}\}$ – BST root
  - $P_{below} \coloneqq \{ p \in P \setminus \{p_{min}\} : p_y \leq y_{med} \}$
  - $P_{above} \coloneqq \{ p \in P \setminus \{p_{min}\} : p_y > y_{med} \}$
- Point p_{min} and scalar y_{med} are stored in the PST root
- The left subtree is PST of P_{below}
- The right subtree is PST of P_{above}

Priority search tree construction example



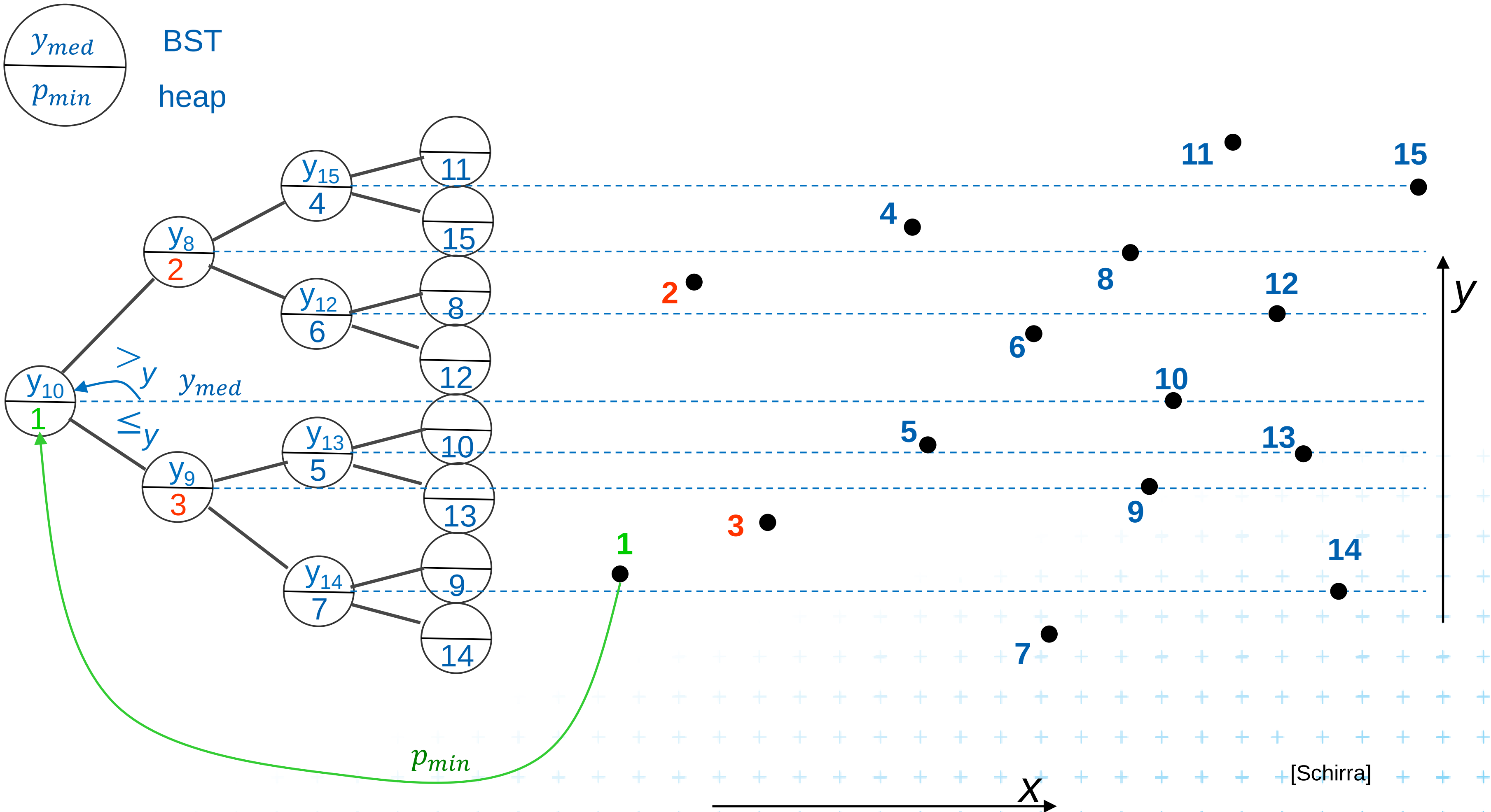
[Schirra]

Priority search tree construction example



[Schirra]

Priority search tree construction example



Priority search tree construction

PrioritySearchTree(P)

Input: set P of points in plane

Output: priority search tree T

1. if $P = \emptyset$ then PST is an empty leaf
2. else
3. p_{min} = point with smallest x-coordinate in P // heap on x root
4. y_{med} = y-coord. median of points $P \setminus \{p_{min}\}$ // BST on y root
5. Split points $P \setminus \{p_{min}\}$ into two subsets – according to y_{med}
6. $P_{below} := \{ p \in P \setminus \{p_{min}\} : p_y \leq y_{med} \}$
7. $P_{above} := \{ p \in P \setminus \{p_{min}\} : p_y > y_{med} \}$
8. *T* = newTreeNode()
9. *T.p* = p_{min} // point $[\,x, y\,]$
10. *T.y* = y_{med} // scalar
11. *T.left* = PrioritySearchTree(P_{below})
12. *T.rigft* = PrioritySearchTree(P_{above})

… Notation on the next slide:

… $p(v)$, v = tree node

… $y(v)$

… $l(v)$

… $r(v)$

13. $O(n \log n)$, but $O(n)$ if presorted on y-coordinate and bottom up

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Query Priority Search Tree

QueryPrioritySearchTree(T**,** $(-\infty : q_x] \times [q_y : q'_y]$**)**

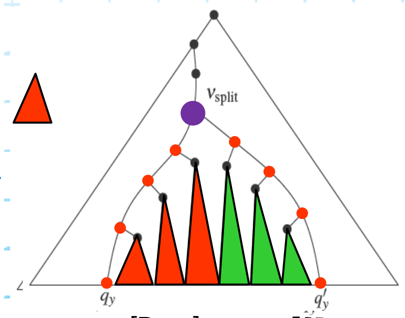
Input: A priority search tree and a range, unbounded to the left

Output: All points lying in the range

1. Search with q_y and q'_y in T // BST on y-coordinate – select y range
   Let v_{split} be the node where the two search paths split (split node)

2. for each node v on the search path of q_y or q'_y // points along the paths
3. if $p(v) \in (-\infty : q_x] \times [q_y : q'_y]$ then Report $p(v)$ // starting in tree root

4. for each node v on the path of q_y in the left subtree of v_{split} // inner trees
5. if the search path goes left at v
6. ReportInSubtree($r(v)$, q_x) // report right subtree
7. for each node v on the path of q'_y in right subtree of v_{split}
8. if the search path goes right at v
9. ReportInSubtree($l(v)$, q_x) // rep. left subtree



[Berg]

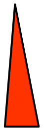
Reporting of subtrees between the y-paths

ReportInSubtree(v, q_x)

Input: The root v of a subtree of a priority search tree and a value q_x.

Output: All points p in the subtree with x-coordinate at most q_x.

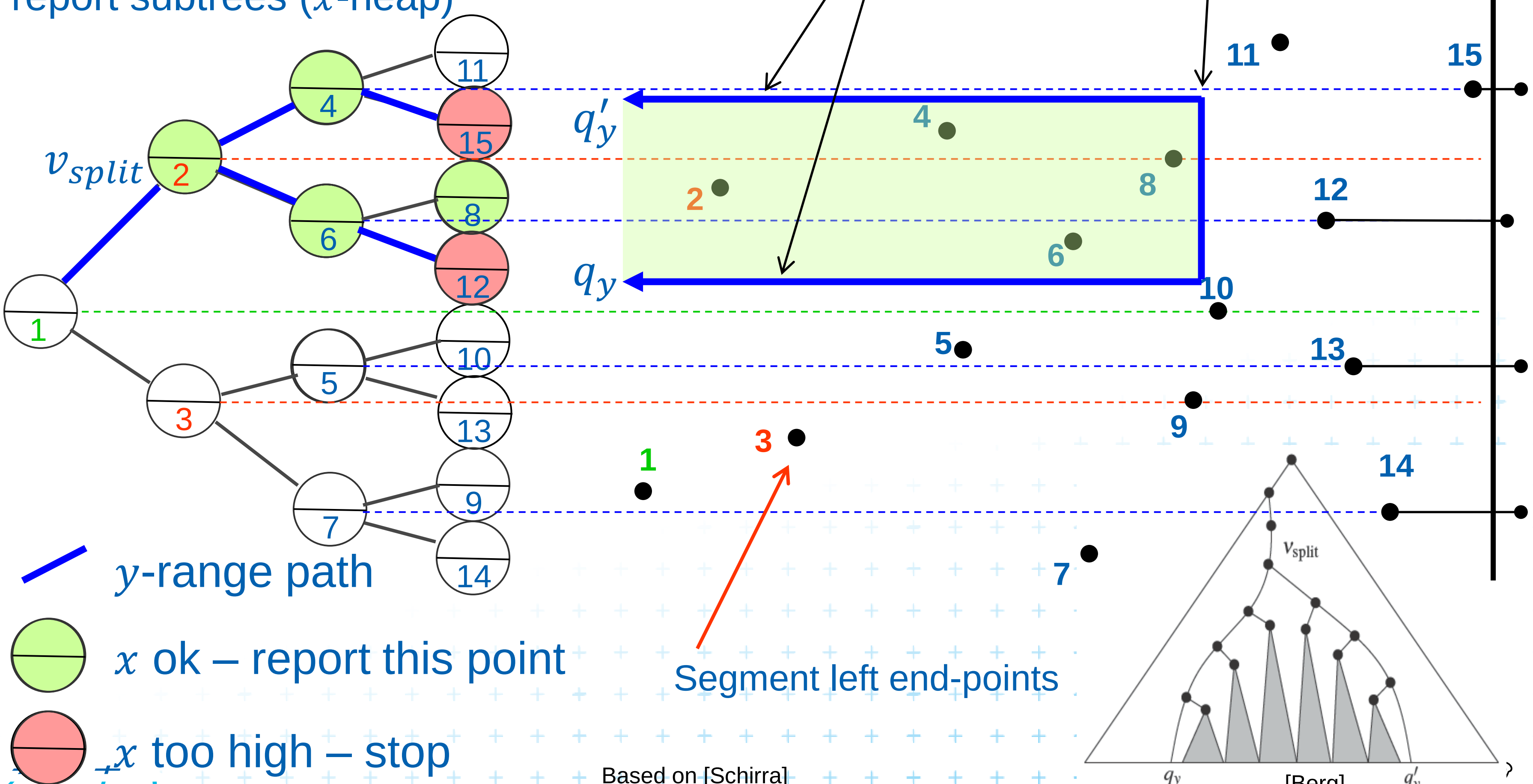
```
if x(p(v)) ≤ q_x          // x ∈ (−∞ : q_x]   -- heap condition
    Report point p(v).
if v is not a leaf
    ReportInSubtree( l(v), q_x )
    ReportInSubtree( r(v), q_x )
```



Search according to x in the heap

Priority search tree query $(-\infty : q_x] \times [q_y : q'_y]$

1. select y range (y-BVS~ 1D range tree)
2. report points on paths (x-heap)
3. report subtrees (x-heap)



Based on [Schirra]

Priority search tree complexity

For set of n points in the plane

- Build $O(n \log n)$
- Storage $O(n)$
- Query $O(k + \log n)$
  - points in query range $(-\infty : q_x] \times [q_y : q'_y]$
  - k is number of reported points

- Use Priority search tree as associated data structure for interval trees for storage of set M (one for M_L, one for M_R)

Talk overview

1. Windowing of axis parallel line segments in 2D
   – 3 variants of *interval tree – IT in x-direction*
   – Differ in storage of segment end points M_L and M_R

1D i. Line stabbing (standard *IT* with *sorted lists*) lecture 9 - intersections

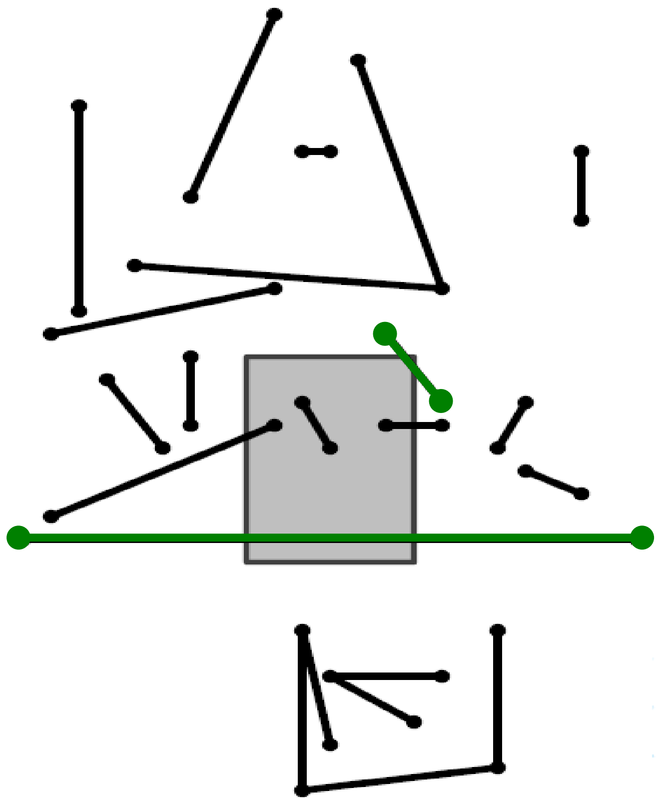
2D ii. Line segment stabbing (*IT* with *range trees*)

iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position

2D – *segment tree + BST*

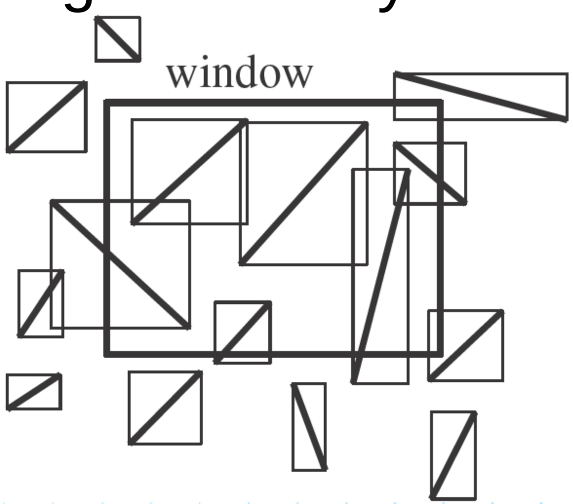
2. Windowing of line segments in general position

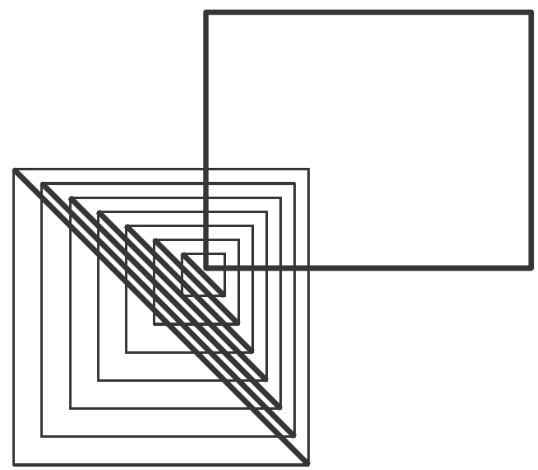


[Vakken]

Windowing of arbitrary oriented line segments

- Two cases of intersection
  - a,b) Endpoint inside the query window => range tree
  - c) Segment intersects side of query window => ???
- Intersection with BBOX (segment bounding box)?
  - – Intersection with 4n sides of the segment BBOX?
  - – But segments may not intersect the window –> query y





NOT

Talk overview

1. Windowing of axis parallel line segments in 2D
   (variants of *interval tree - IT*)

1D
   i. Line stabbing (*IT* with *sorted lists*)

2D
   ii. Line segment stabbing (*IT* with *range trees*)
   iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position

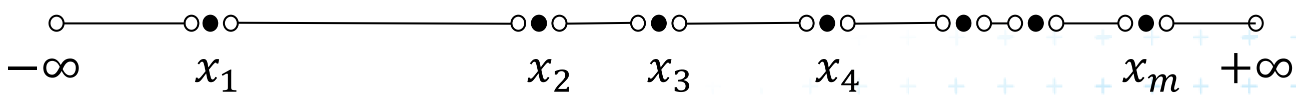
2D
   – *segment tree*

Note: segment = interval
it consists of elementary intervals

Segment tree

[Bentley, 1977]

- Exploits locus approach
  - Partition parameter space into regions of same answer
  - Localization of such region = knowing the answer
- For given set S of n intervals (segments) on real line
  - Finds m elementary intervals (induced by interval end-points)
  - Partitions 1D parameter space into these elementary intervals



$$(-\infty : x_1), [x_1 : x_1], (x_1 : x_2), [x_2 : x_2], \ldots ,$$
$$(x_{m-1} : x_m), [x_m : x_m], (x_m : +\infty)$$

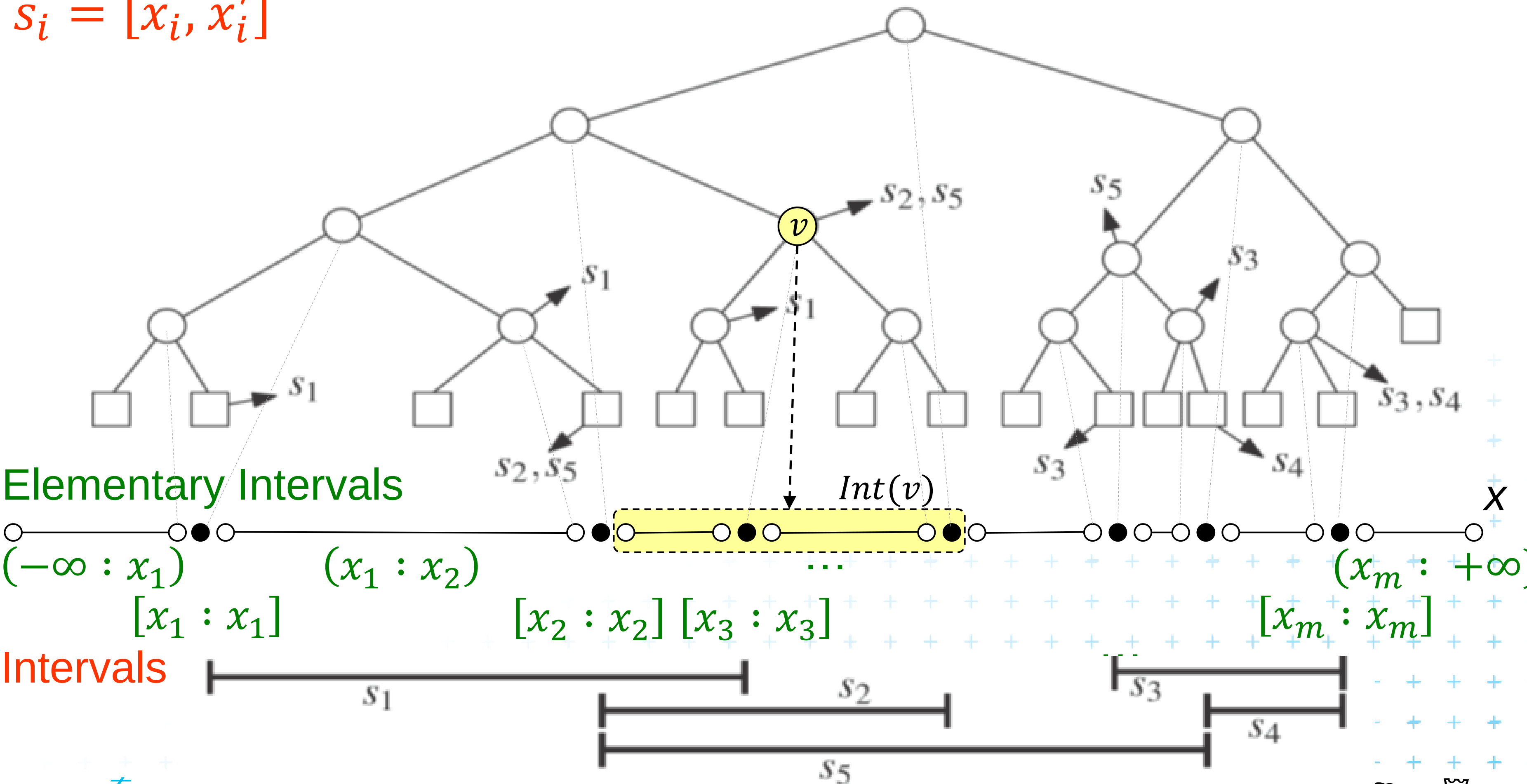
  - Stores line segments s_i with the elementary intervals
  - Reports the segments s_i containing query point q_x.

Plain is partitioned into vertical slabs

Segment tree example

Segments $S = \{s_1, s_2, \dots, s_n\}$

$s_i = [x_i, x'_i]$

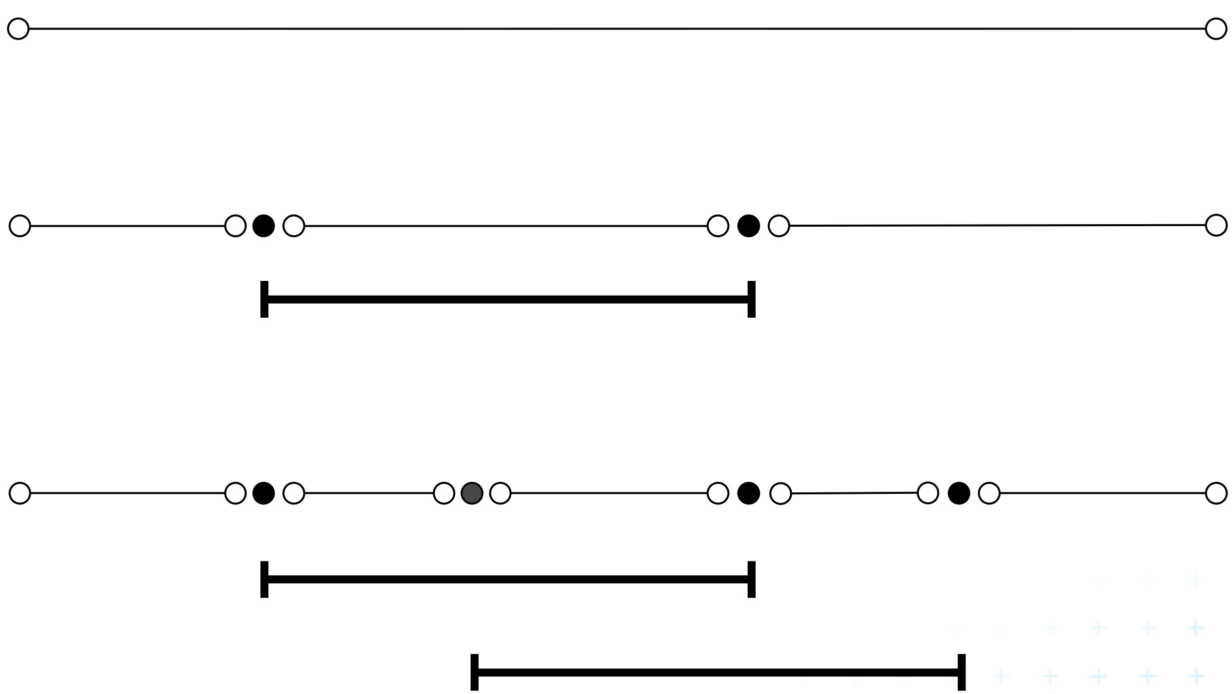


Number of elementary intervals for n segments

$n = 0$ $\# = 1$

$n = 1$ $\# = 4 + 1$

$n = 2$ $\# = 4 * 2 + 1$



n Each end-point adds two elementary intervals

Each segment four…

$\# = 4n + 1$

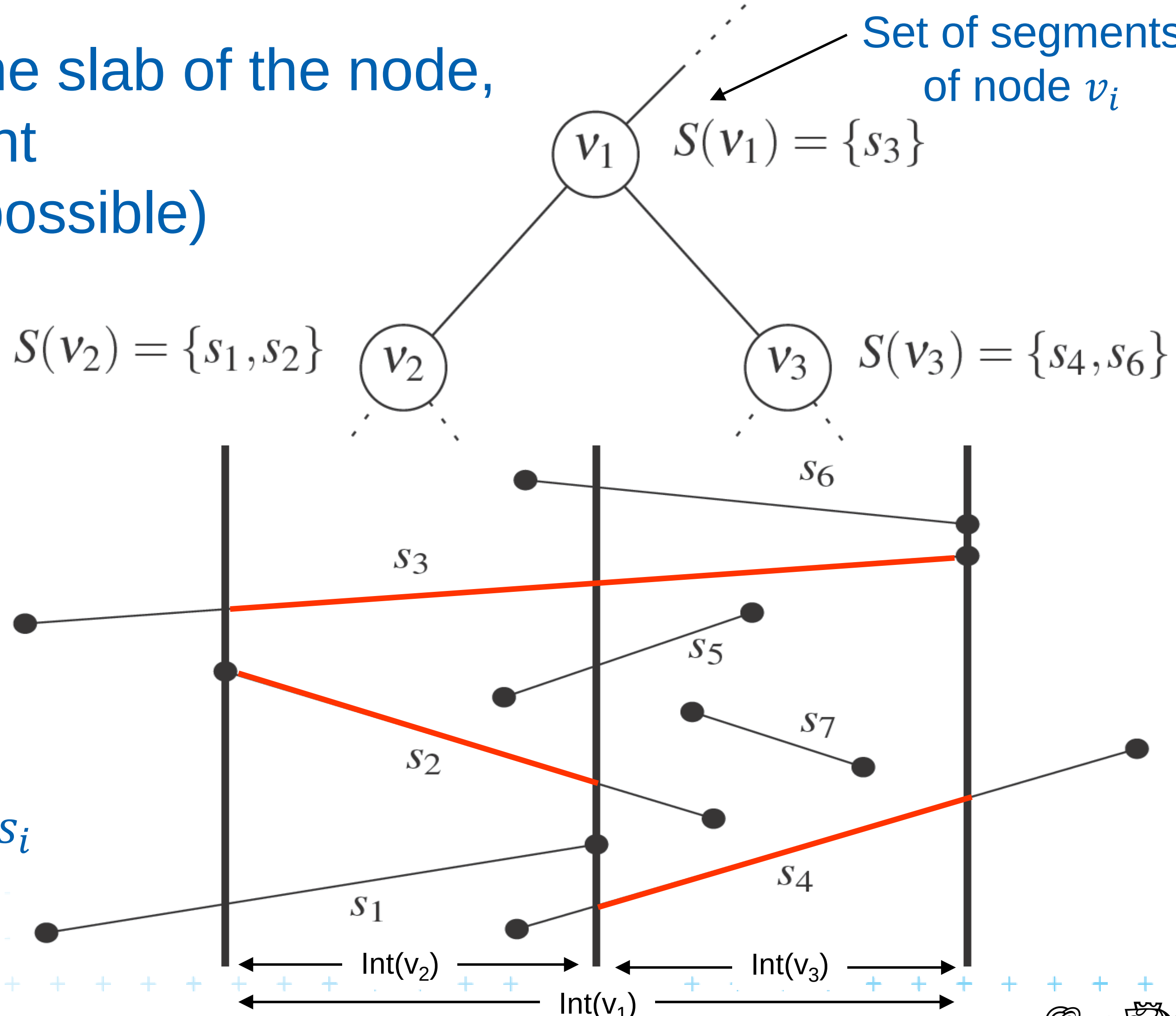
Segment tree definition

Segment tree

- Skeleton is a balanced binary tree T
- Leaves ~ elementary intervals
- Internal nodes v
  ~ union of elementary intervals of its children
  - Store: 1. interval $Int(v)$ = union of elementary intervals of its children
    segments s_i
    2. canonical set $S(v)$ of segments $[x_i : x_i']\in S$
  - Holds $Int(v) \subseteq [x_i : x_i']$ and $Int(\text{parent}(v))] \nsubseteq [x_i : x_i']$ (node interval is not larger than the segment)
  - Segments $[x_i : x_i']$ are stored as high as possible, such that $Int(v)$ is completely contained in the segment

Segments span the slab

Segments span the slab of the node, but not of its parent (stored as up as possible)



$Int(v_j) \subseteq s_i$

and

$Int(\text{parent}(v)) \nsubseteq s_i$

[Berg]

Query segment tree – stabbing query (1D)

QuerySegmentTree(v, q_x)
Input: The root of a (subtree of a) segment tree and a query point q_x
Output: All intervals (=segments) in the tree containing q_x.

1. Report all the intervals s_i in $S(v)$. // covered by the current node
2. **if** v is not a leaf // root covers "all"$(-\infty, +\infty)$
3. if $q_x \in$ Int($l(v)$) // go left
4. QuerySegmentTree($l(v)$, q_x)
5. else // or go right
6. QuerySegmentTree($r(v)$, q_x)

Query time $O(\log n + k)$, where k is the number of reported intervals
$O(1 + k_v)$ for one node
Height $O(\log n)$

Segment tree construction

ConstructSegmentTree(S)
Input: Set of intervals (segments) S
Output: segment tree

1. Sort endpoints of segments in S, get elementary intervals … $O(n \log n)$
2. Construct a binary search tree T on elementary intervals … $O(n)$
   (bottom up) and determine the interval $Int(v)$ it represents
3. Compute the canonical subsets for the nodes
   (lists of their segments s_i):
4. v = root(T)
5. for all segments $s_i = [x_i : x_i'] \in S$
6. InsertSegmentTree(v, $[x_i : x_i']$)

Segment tree construction – interval insertion

InsertSegmentTree(v, $[x : x']$)

Input: The root of (a sub-tree of) a segment tree and an interval.

Output: The interval will be stored in the sub-tree.

```
if Int(v) ⊆ [x : x']                          // Int(v) contains s_i = [x : x']
    store s_i = [x : x'] at v
else if Int( l(v) ) ∩ [x : x'] ≠ ∅            // part of s_i to the left
        InsertSegmentTree( l(v), [x : x'] )
    if Int( r(v) ) ∩ [x : x'] ≠ ∅             // part of s_i to the right
        InsertSegmentTree( r(v), [x : x'] )
```

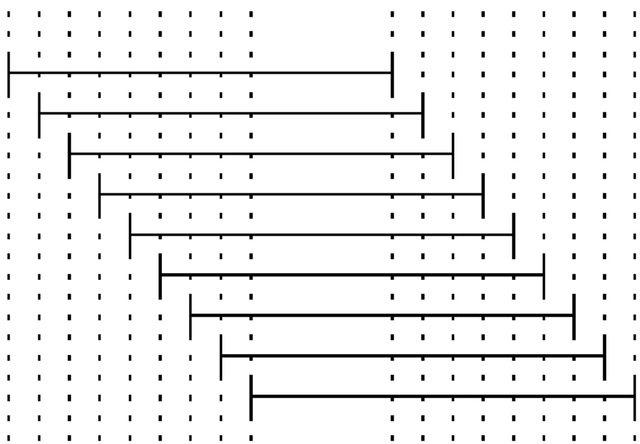
One interval is stored at most twice in one level =>

- Single interval insert $O(\log n)$, insert n intervals $O(\cancel{2}n \log n)$
- Construction total $O(n \log n)$

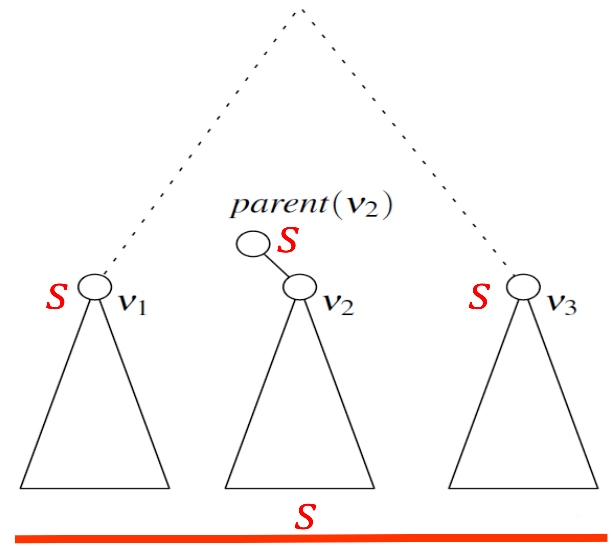
Storage $O(n \log n)$

- Tree height $O(\log n)$, name stored max 2x in one level
- Storage total $O(n \log n)$ – see next slide

Space complexity - notes



[Berg]



[Berg]

Worst case – $O(n^2)$ segments in leafs

But

Store segments as high, as possible

Segment max 2 times in one level $\Leftarrow$

- s covered by v_1 and v_3
- $\Rightarrow v_2$ covered, $Int(v_2) \in s$
- As v_2 lies between v_1 and v_3
- $\Rightarrow Int(parent(v_2)) \in s \Rightarrow$ segment s will not be stored in v_2

$\max 4n + 1$ elementary intervals (leaves)

$\Rightarrow O(n)$ space for the tree

$\Rightarrow O(n \log n)$ space for interval names

Segment tree complexity

A segment tree for set S of n intervals in the plane,

- Build $O(n \log n)$
- Storage $O(n \log n)$
- Query $O(k + \log n)$
  - Report all intervals that contain a query point
  - k is number of reported intervals

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Segment tree versus Interval tree

- Segment tree
  - $O(n \log n)$ storage versus $O(n)$ of Interval tree
  - But returns exactly the intersected segments s_i, interval tree must search the lists M_L and/or M_R
- Good for
  1. extensions (allows different structuring of intervals)
  2. stabbing counting queries
     – store number of intersected intervals in nodes
     – $O(n)$ storage and $O(\log n)$ query time = optimal
  3. higher dimensions – multilevel segment trees
     (Interval and priority search trees do not exist in ^dims)

Talk overview

1. Windowing of axis parallel line segments in 2D (variants of *interval tree - IT*)

1D
i. Line stabbing (standard *IT* with *sorted lists*)

2D
ii. Line segment stabbing (*IT* with *range trees*)
iii. Line segment stabbing (*IT* with *priority search trees*)

2. Windowing of line segments in general position

2D
– *segment tree*
– the windowing algorithm

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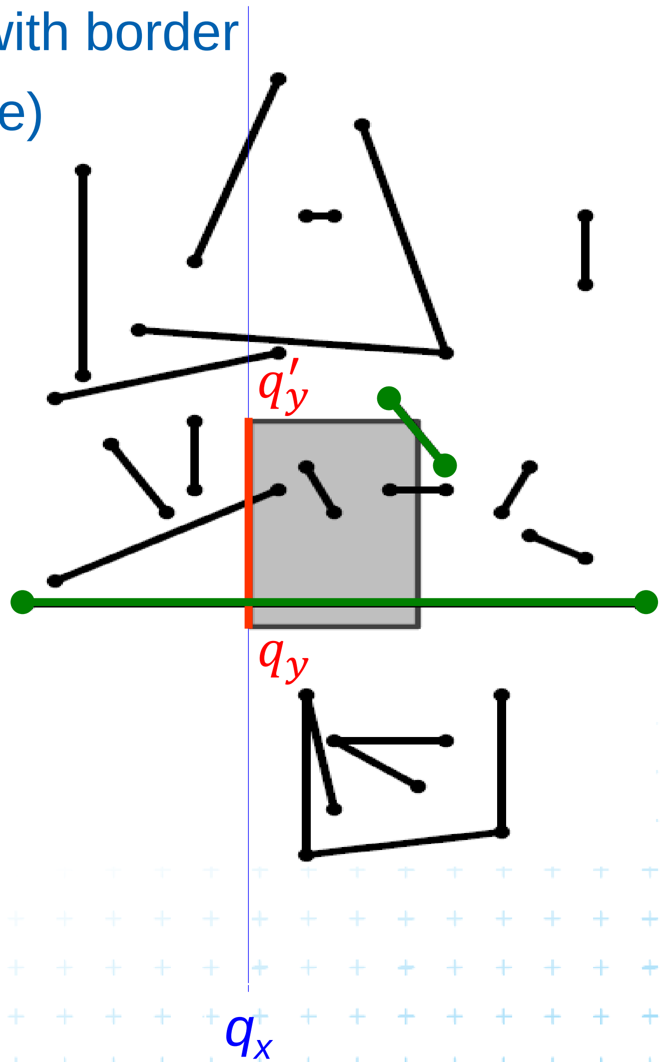
2. Windowing of line segments in general position

Test intersection with border

Done 4x (rectangle)

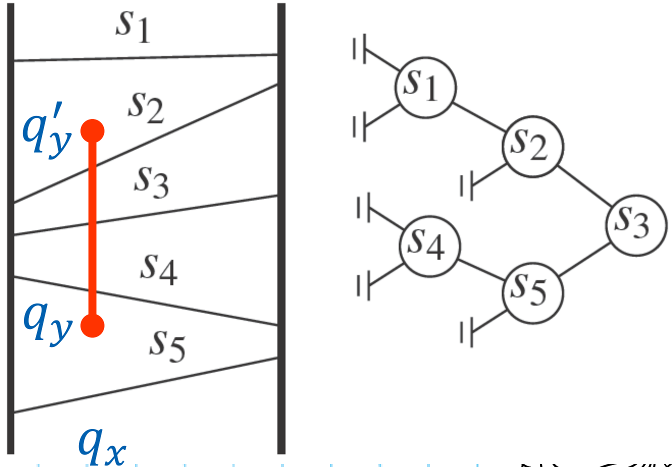


[Vakken]

Windowing of arbitrary oriented line segments

- Let S be a set of arbitrarily oriented line segments in the plane.
- Report the segments intersecting a vertical query segment $q := q_x \times [q_y : q'_y]$ – window border
- Segment tree T on x intervals of segments in S
  - node v of T corresponds to vertical slab $Int(v) \times (-\infty : \infty)$
  - segments span the slab of the node, but not of its parent
  - segments do not intersect

  => segments in the slab (node) can be vertically ordered – BST

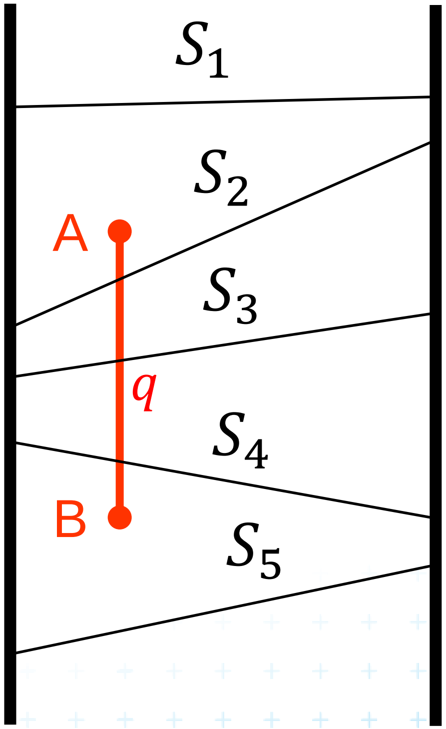


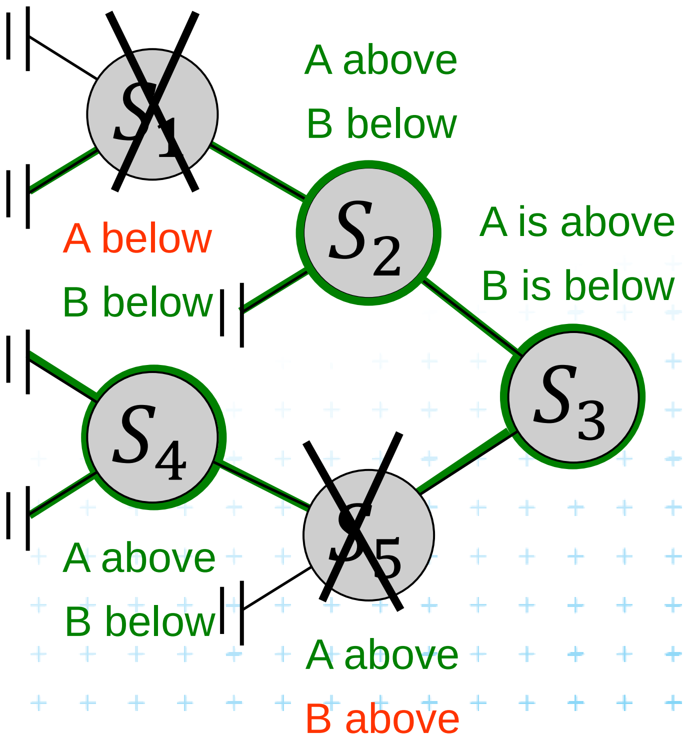
[Berg]

Segments between vertical segment endpoints

Segment s is intersected by vert.query segment q iff

- The lower endpoint (B) of q is below s and
- The upper endpoint (A) of q is above s





[Berg]

Segments between vertical segment endpoints

- Segments (in the slab) do not mutually intersect
  => segments can be vertically ordered and stored in BST

  – Each node v of the x segment tree (vertical slab) has an associated y-BST
  – BST $T(v)$ of node v stores the canonical subset $S(v)$ according to the vertical order
  – Intersected segments can be found by searching $T(v)$ in $O(k_v + \log n)$, k_v is the number of intersected segments

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Windowing of arbitrary oriented line segments complexity

Structure associated to node (BST) uses storage linear in the size of $S(v)$

- Build $O(n \log n)$
- Storage $O(n \log n)$
- Query $O(k + \log^2 n)$ … $O(\log n)$ segm tree $+O(\log n)$ BST
  - Report all segments that contain a query point
  - k is number of reported segments

Windowing of line segments in 2D – conclusions

Construction: all interval tree variants $O(n \log n)$

		Search	Memory
	1. Axis parallel		
1D	i. Line (*sorted lists*)	$O(k + \log n)$	$O(n)$
2D	ii. Segment (*range trees*)	$O(k + \log^2 n)$	$O(n \log n)$
	iii. Segment (*priority s. tr.*)	$O(k + \log n)$	$O(n)$
	2. In general position		
2D	– *segment tree + BST*	$O(k + \log^2 n)$	$O(n \log n)$

DCGI

References

[Berg] Mark de Berg, Otfried Cheong, Marc van Kreveld, Mark Overmars: Computational Geometry: Algorithms and Applications, Springer-Verlag, 3rd rev. ed. 2008. 386 pages, 370 fig. ISBN: 978-3-540-77973-5, Chapters 3 and 9, http://www.cs.uu.nl/geobook/

[Mount] Mount, D.: *Computational Geometry Lecture Notes for Fall 2016*, University of Maryland, Lecture 33. http://www.cs.umd.edu/class/fall2016/cmsc754/Lects/cmsc754-fall16-lects.pdf

[Rourke] Joseph O´Rourke: Computational Geometry in C, Cambridge University Press, 1993, ISBN 0-521- 44592-2 http://maven.smith.edu/~orourke/books/compgeom.html

[Vigneron] Segment trees and interval trees, presentation, INRA, France, http://w3.jouy.inra.fr/unites/miaj/public/vigneron/cs4235/slides.html

[Schirra] Stefan Schirra. Geometrische Datenstrukturen. Sommersemester 2009 http://wwwisg.cs.uni-magdeburg.de/ag/lehre/SS2009/GDS/slides/S10.pdf