

Electromagnetic Field Theory (BAB17EMP)

Useful Mathematical Identities

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1 Operations on Radius Vectors

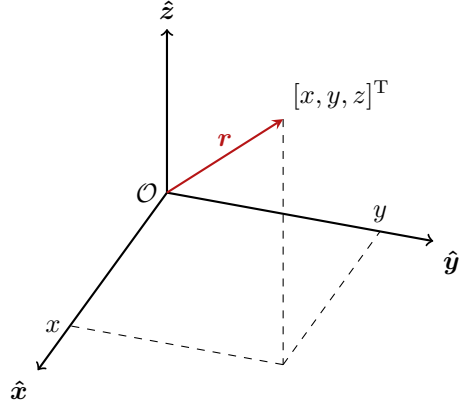


Figure 1: Radius vector \mathbf{r} .

Observation vector:

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}} \quad (1)$$

Vector magnitude:

$$r = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2} \quad (2)$$

Unit vector:

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} \quad (3)$$

Gradient applied to r :

$$\nabla r = \frac{x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}}{\sqrt{x^2 + y^2 + z^2}} = \hat{\mathbf{r}} \quad (4)$$

Gradient applied to $1/r$:

$$\nabla \frac{1}{r} = -\frac{x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}}{\left(\sqrt{x^2 + y^2 + z^2}\right)^3} = -\frac{\mathbf{r}}{r^3} \quad (5)$$

Divergence applied to \mathbf{r} :

$$\nabla \cdot \mathbf{r} = 3 \quad (6)$$

Curl applied to \mathbf{r} :

$$\nabla \times \mathbf{r} = 0 \quad (7)$$

1.1 Identities Involving Separation Vector

Source vector:

$$\mathbf{r}' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = x'\hat{\mathbf{x}} + y'\hat{\mathbf{y}} + z'\hat{\mathbf{z}} \quad (8)$$

Separation vector \mathbf{R}

$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = \begin{bmatrix} x - x' \\ y - y' \\ z - z' \end{bmatrix} = (x - x') \hat{\mathbf{x}} + (y - y') \hat{\mathbf{y}} + (z - z') \hat{\mathbf{z}} \quad (9)$$

Magnitude of Separation vector \mathbf{R}

$$R = |\mathbf{r} - \mathbf{r}'| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \quad (10)$$

$$R = |\mathbf{r} - \mathbf{r}'| = \sqrt{r^2 + (r')^2 - 2rr' \cos(\alpha)} \quad (11)$$

$$\nabla R = \frac{(x - x') \hat{\mathbf{x}} + (y - y') \hat{\mathbf{y}} + (z - z') \hat{\mathbf{z}}}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} = \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} \quad (12)$$

$$\nabla \left(\frac{1}{R} \right) = - \frac{(x - x') \hat{\mathbf{x}} + (y - y') \hat{\mathbf{y}} + (z - z') \hat{\mathbf{z}}}{\left(\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \right)^3} = - \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \quad (13)$$

$$\nabla \cdot \mathbf{R} = 3 \quad (14)$$

$$\nabla \times \mathbf{R} = 0 \quad (15)$$

2 Trigonometric Identities

$$\sin^2 \xi + \cos^2 \xi = 1 \quad (16)$$

$$\sin^2 \xi = \frac{1 - \cos 2\xi}{2} \quad (17)$$

$$\cos^2 \xi = \frac{1 + \cos 2\xi}{2} \quad (18)$$

$$e^{j\xi} = \cos \xi + j \sin \xi \quad (19)$$

$$\cos 2\xi = \cos^2 \xi - \sin^2 \xi \quad (20)$$

$$\sin 2\xi = 2 \sin \xi \cos \xi \quad (21)$$

$$\sin (\xi \pm \zeta) = \sin \xi \cos \zeta \pm \cos \xi \sin \zeta \quad (22)$$

$$\cos (\xi \pm \zeta) = \cos \xi \cos \zeta \mp \sin \xi \sin \zeta \quad (23)$$

3 Coordinate System Transformations

3.1 Point Transformations

Cylindrical to Cartesian:

$$x = \rho \cos \phi \quad (24)$$

$$y = \rho \sin \phi \quad (25)$$

$$z = z \quad (26)$$

Cartesian to cylindrical:

$$\rho = \sqrt{x^2 + y^2} \quad (27)$$

$$\phi = \arctan \frac{y}{x} \quad (28)$$

$$z = z \quad (29)$$

Spherical to Cartesian:

$$x = r \cos \phi \sin \theta \quad (30)$$

$$y = r \sin \phi \sin \theta \quad (31)$$

$$z = r \cos \theta \quad (32)$$

Cartesian to Spherical:

$$r = \sqrt{x^2 + y^2 + z^2} \quad (33)$$

$$\theta = \arctan \frac{\sqrt{x^2 + y^2}}{z} \quad (34)$$

$$\phi = \arctan \frac{y}{x} \quad (35)$$

3.2 Vector Transformations

Cylindrical to Cartesian:

$$\hat{\mathbf{x}} = \hat{\rho} \cos \phi - \hat{\phi} \sin \phi \quad (36)$$

$$\hat{\mathbf{y}} = \hat{\rho} \sin \phi + \hat{\phi} \cos \phi \quad (37)$$

$$\hat{\mathbf{z}} = \hat{z} \quad (38)$$

Cartesian to Cylindrical:

$$\hat{\rho} = \hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi = \hat{\mathbf{x}} \frac{x}{\sqrt{x^2 + y^2}} + \hat{\mathbf{y}} \frac{y}{\sqrt{x^2 + y^2}} \quad (39)$$

$$\hat{\phi} = -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi = -\hat{\mathbf{x}} \frac{y}{\sqrt{x^2 + y^2}} + \hat{\mathbf{y}} \frac{x}{\sqrt{x^2 + y^2}} \quad (40)$$

$$\hat{\mathbf{z}} = \hat{z} \quad (41)$$

Spherical to Cartesian:

$$\hat{\mathbf{x}} = \hat{\mathbf{r}} \sin \theta \cos \phi + \hat{\boldsymbol{\theta}} \cos \theta \cos \phi - \hat{\boldsymbol{\phi}} \sin \phi \quad (42)$$

$$\hat{\mathbf{y}} = \hat{\mathbf{r}} \sin \theta \sin \phi + \hat{\boldsymbol{\theta}} \cos \theta \sin \phi + \hat{\boldsymbol{\phi}} \cos \phi \quad (43)$$

$$\hat{\mathbf{z}} = \hat{\mathbf{r}} \cos \theta - \hat{\boldsymbol{\theta}} \sin \theta \quad (44)$$

Cartesian to Spherical:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}} \sin \theta \cos \phi + \hat{\mathbf{y}} \sin \theta \sin \phi + \hat{\mathbf{z}} \cos \phi \quad (45)$$

$$\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}} \cos \theta \cos \phi + \hat{\mathbf{y}} \cos \theta \sin \phi - \hat{\mathbf{z}} \sin \theta \quad (46)$$

$$\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi \quad (47)$$

3.3 Conversion Between Coordinate Systems (Example)

The relation

$$\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \sin \theta \hat{\boldsymbol{\phi}} \quad (48)$$

holds since the vector

$$\hat{\boldsymbol{\phi}} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}} \quad (49)$$

can be compared with the vector multiplication and the vector multiplication

$$\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 0 & 0 & 1 \\ \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \end{vmatrix} = -\sin \theta \sin \phi \hat{\mathbf{x}} + \sin \theta \cos \phi \hat{\mathbf{y}} = \sin \theta \hat{\boldsymbol{\phi}}, \quad (50)$$

where we used

$$\hat{\mathbf{r}} = \sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}}. \quad (51)$$

4 Differential Operators

4.1 Rectangular Coordinate System

$$\mathbf{F} = F_x \hat{\mathbf{x}} + F_y \hat{\mathbf{y}} + F_z \hat{\mathbf{z}} \quad (52)$$

$$\nabla f = \frac{\partial f}{\partial x} \hat{\mathbf{x}} + \frac{\partial f}{\partial y} \hat{\mathbf{y}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}} \quad (53)$$

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \quad (54)$$

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \hat{\mathbf{x}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \hat{\mathbf{y}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \hat{\mathbf{z}} \quad (55)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \quad (56)$$

$$\nabla^2 \mathbf{F} = \nabla^2 F_x \hat{\mathbf{x}} + \nabla^2 F_y \hat{\mathbf{y}} + \nabla^2 F_z \hat{\mathbf{z}} \quad (57)$$

4.2 Polar Coordinate System

$$\mathbf{F} = F_\rho \hat{\boldsymbol{\rho}} + F_\phi \hat{\boldsymbol{\phi}} \quad (58)$$

$$\nabla f = \frac{\partial f}{\partial \rho} \hat{\boldsymbol{\rho}} + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\boldsymbol{\phi}} \quad (59)$$

$$\nabla^2 f = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} \quad (60)$$

4.3 Cylindrical Coordinate System

$$\mathbf{F} = F_\rho \hat{\boldsymbol{\rho}} + F_\phi \hat{\boldsymbol{\phi}} + F_z \hat{\mathbf{z}} \quad (61)$$

$$\nabla f = \frac{\partial f}{\partial \rho} \hat{\boldsymbol{\rho}} + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}} \quad (62)$$

$$\nabla \cdot \mathbf{F} = \frac{1}{\rho} \frac{\partial (\rho F_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \quad (63)$$

$$\nabla \times \mathbf{F} = \left(\frac{1}{\rho} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z} \right) \hat{\boldsymbol{\rho}} + \left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial \rho} \right) \hat{\boldsymbol{\phi}} + \left(\frac{1}{\rho} \frac{\partial (\rho F_\phi)}{\partial \rho} - \frac{1}{\rho} \frac{\partial F_\rho}{\partial \phi} \right) \hat{\mathbf{z}} \quad (64)$$

$$\nabla^2 f = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} \quad (65)$$

4.4 Spherical Coordinate System

$$\mathbf{F} = F_r \hat{\mathbf{r}} + F_\theta \hat{\boldsymbol{\theta}} + F_\phi \hat{\boldsymbol{\phi}} \quad (66)$$

$$\nabla f = \frac{\partial f}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\boldsymbol{\phi}} \quad (67)$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r^2} \frac{\partial (r^2 F_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (F_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial F_\phi}{\partial \phi} \quad (68)$$

$$\nabla \times \mathbf{F} = \frac{1}{r \sin \theta} \left(\frac{\partial (F_\phi \sin \theta)}{\partial \theta} - \frac{\partial F_\theta}{\partial \phi} \right) \hat{\mathbf{r}} + \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial F_r}{\partial \phi} - \frac{\partial (r F_\phi)}{\partial r} \right) \hat{\boldsymbol{\theta}} + \frac{1}{r} \left(\frac{\partial (r F_\theta)}{\partial r} - \frac{\partial F_r}{\partial \theta} \right) \hat{\boldsymbol{\phi}} \quad (69)$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2} \quad (70)$$

5 Vector Identities

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 \quad (71)$$

$$|\mathbf{u}| = \sqrt{\mathbf{u} \cdot \mathbf{u}} \quad (72)$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} u_1 & v_1 & \hat{\mathbf{e}}_1 \\ u_2 & v_2 & \hat{\mathbf{e}}_2 \\ u_3 & v_3 & \hat{\mathbf{e}}_3 \end{vmatrix} = (u_2 v_3 - u_3 v_2) \hat{\mathbf{e}}_1 + (u_3 v_1 - u_1 v_3) \hat{\mathbf{e}}_2 + (u_1 v_2 - u_2 v_1) \hat{\mathbf{e}}_3 \quad (73)$$

$$\frac{\partial \mathbf{F}}{\partial q} = \frac{\partial F_x}{\partial q} \hat{\mathbf{x}} + \frac{\partial F_y}{\partial q} \hat{\mathbf{y}} + \frac{\partial F_z}{\partial q} \hat{\mathbf{z}} \quad (74)$$

$$\mathbf{A}^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}, \quad (75)$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w}) \mathbf{v} - (\mathbf{u} \cdot \mathbf{v}) \mathbf{w} \quad (76)$$

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1} \mathbf{A}^{-1} \quad (77)$$

$$(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T \quad (78)$$

$$\frac{\partial \mathbf{A}}{\partial x} = \begin{bmatrix} \frac{\partial a_{11}}{\partial x} & \cdots & \frac{\partial a_{1n}}{\partial x} \\ \vdots & \ddots & \vdots \\ \frac{\partial a_{n1}}{\partial x} & \cdots & \frac{\partial a_{nn}}{\partial x} \end{bmatrix} \quad (79)$$

6 Differential Identities

$$\nabla \times \nabla f = 0 \quad (80)$$

$$\nabla \cdot \nabla \times \mathbf{F} = 0 \quad (81)$$

$$\nabla \times \nabla \times \mathbf{F} = \nabla \nabla \cdot \mathbf{F} - \nabla^2 \mathbf{F} \quad (82)$$

$$\nabla \cdot (f\mathbf{F}) = f(\mathbf{r}) \nabla \cdot \mathbf{F} + \mathbf{F}(\mathbf{r}) \cdot \nabla f \quad (83)$$

$$\nabla \times (f\mathbf{F}) = f\nabla \times \mathbf{F} + \nabla f \times \mathbf{F} \quad (84)$$

$$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = (\nabla \times \mathbf{F}) \cdot \mathbf{G} - (\nabla \times \mathbf{G}) \cdot \mathbf{F} \quad (85)$$

7 Integration Identities

Substitution of cylindrical coordinates:

$$\iiint_V f(x, y, z) \, dx \, dy \, dz = \iiint_V f(\rho, \phi, z) \rho \, d\rho \, d\phi \, dz \quad (86)$$

Substitution of spherical coordinates:

$$\iiint_V f(x, y, z) \, dx \, dy \, dz = \iiint_V f(r, \theta, \phi) r^2 \sin \theta \, dr \, d\theta \, d\phi \quad (87)$$

The line integral of the scalar field:

$$\int_{\ell} f(\mathbf{r}) \, dl = \int_a^b f(\mathbf{r}(u)) \cdot \left| \frac{d\mathbf{r}(u)}{du} \right| \, du \quad (88)$$

The line integral of the vector field:

$$\int_{\ell} \mathbf{F}(\mathbf{r}) \cdot d\mathbf{l} = \int_{\ell} \mathbf{F}(\mathbf{r}) \cdot \boldsymbol{\tau} \, dl = \int_a^b \mathbf{F}(\mathbf{r}(u)) \cdot \frac{d\mathbf{r}(u)}{du} \, du \quad (89)$$

example

$$\int_{\ell: u\hat{\mathbf{x}}+u\hat{\mathbf{y}}, u \in [0,1]} (u^2\hat{\mathbf{x}} + u^2\hat{\mathbf{y}}) \cdot d\mathbf{l} = \int_0^1 (u^2\hat{\mathbf{x}} + u^2\hat{\mathbf{y}}) \cdot \frac{d(u\hat{\mathbf{x}} + u\hat{\mathbf{y}})}{du} \, du = \int_0^1 2u^2 \, du = \frac{2}{3} \quad (90)$$

The surface integral of the scalar field:

$$\iint_S f(\mathbf{r}) \, dS = \iint_S f(\mathbf{r}(u, s)) \left| \frac{\partial \mathbf{r}(u, s)}{\partial u} \times \frac{\partial \mathbf{r}(u, s)}{\partial s} \right| \, du \, ds \quad (91)$$

The surface integral of the vector field:

$$\iint_S \mathbf{F}(\mathbf{r}) \cdot d\mathbf{S} = \iint_S \mathbf{F}(\mathbf{r}) \cdot \mathbf{n} \, dS = \iint_S \mathbf{F}(\mathbf{r}(u, s)) \cdot \left(\frac{\partial \mathbf{r}(u, s)}{\partial u} \times \frac{\partial \mathbf{r}(u, s)}{\partial s} \right) \, du \, ds \quad (92)$$

example $S : r(\rho, \phi) = \hat{\mathbf{x}}\rho \cos \phi + \hat{\mathbf{y}}\rho \sin \phi, 0 \leq \rho \leq 1, 0 \leq \phi \leq 2\pi$

$$\iint_S \hat{\mathbf{z}} \cdot d\mathbf{S} = \int_0^1 \int_0^{2\pi} \hat{\mathbf{z}} \cdot [(\hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi) \times (-\hat{\mathbf{x}}\rho \sin \phi + \hat{\mathbf{y}}\rho \cos \phi)] \, d\phi \, d\rho = \int_0^1 \int_0^{2\pi} \rho \, d\phi \, d\rho = \pi \quad (93)$$

Curl theorem:

$$\oint_{\partial S} \mathbf{F}(\mathbf{r}) \cdot d\mathbf{l} = \iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S} \quad (94)$$

Divergence theorem:

$$\oint_{\partial V} \mathbf{F}(\mathbf{r}) \cdot d\mathbf{S} = \iiint_V \nabla \cdot \mathbf{F} \, dV \quad (95)$$

Other useful identities:

$$\oint_{l'} d\mathbf{l}' = 0 \quad (96)$$

$$\oint_{l'} (\hat{\mathbf{r}} \cdot \mathbf{r}') \, d\mathbf{l} = -\hat{\mathbf{r}} \times \iint_{S'} d\mathbf{S}' \quad (97)$$

8 Fourier Transform

$$\mathcal{F}\{\mathbf{F}(\mathbf{r}, t)\} = \hat{\mathbf{F}}(\mathbf{r}, \omega) \quad (98)$$

$$\mathcal{F}\left\{\frac{\partial \mathbf{F}(\mathbf{r}, t)}{\partial t}\right\} = j\omega \hat{\mathbf{F}}(\mathbf{r}, \omega) \quad (99)$$

$$\mathcal{F}\{f(\omega) * \mathbf{F}(\mathbf{r}, t)\} = \hat{f}(\omega) \hat{\mathbf{F}}(\mathbf{r}, \omega) \quad (100)$$

9 Useful Functions

9.1 Sinc Function

$$\text{Sinc}(x) = \frac{\sin x}{x} = \int_{-1}^1 e^{-jkx} \, dk \quad (101)$$

9.2 Dirac Delta Function

$$\int_{-\infty}^{\infty} \delta(x) dx = 1 \quad (102)$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(y) \delta(x) \delta(z) dx dy dz = 1 \quad (103)$$

$$\int_{-\infty}^{\infty} \int_0^{2\pi} \int_0^{\infty} \frac{1}{\rho} \delta(\rho) \delta(\phi) \delta(z) d\rho d\phi dz = 1 \quad (104)$$

$$\int_0^{\pi} \int_0^{2\pi} \int_0^{\infty} \frac{1}{r^2 \sin \theta} \delta(\rho) \delta(\phi) \delta(z) dr d\phi d\theta = 1 \quad (105)$$

$$\int_{V'} f(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') dV' = \begin{cases} f(\mathbf{r}'), & \mathbf{r}' \in V' \\ 0, & \text{otherwise} \end{cases} \quad (106)$$

9.3 Heaviside Step Function

$$H(x) = \int_{-\infty}^x \delta(x') dx' = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad (107)$$

$$\delta(x) = \frac{\partial H(x)}{\partial x} \quad (108)$$

9.4 Rectangular Function

$$\text{rect}\left(\frac{t-X}{Y}\right) = H\left(t-X + \frac{Y}{2}\right) - H\left(t-X - \frac{Y}{2}\right) \quad (109)$$

10 Series Expansions

10.1 Taylor Series

$$f(a) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n = f(a) + \frac{f'(a)}{1!} (x-a) + \frac{f''(a)}{2!} (x-a)^2 + \dots \quad (110)$$

10.2 The Generalized Binomial Theorem

$$(x + y)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^{r-k} y^k \quad (111)$$

for $|x| > |y|$ real numbers and any complex number r .

10.3 Fourier Series

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{jk_0 x} \quad (112)$$