Electromagnetic Field Theory (BAB17EMP)
Formulas

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Contents

Constant	**Symbol**	**Value**	**Units**
Speed of light in vacuum	c	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Planck's constant	h	6.626×10^{-34}	$\mathrm{J\,s}$
Gravitational constant	G	6.674×10^{-11}	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$
electronvolt	eV	1.602×10^{-19}	J
Elementary charge	e	1.602×10^{-19}	C
Electron mass	m_e	9.109×10^{-31}	kg
Proton mass	m_p	1.673×10^{-27}	kg
Permittivity of free space	ϵ_0	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	1.257×10^{-6}	$\mathrm{H\,m^{-1}}$
Avogadro's number	N_A	6.022×10^{23}	$\mathrm{mol^{-1}}$
Boltzmann constant	k_B	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$

Table 1: Physical constants.

Quantity	**Symbol**	**Unit**	**SI Units**
Charge	q	Coulomb (C)	$\mathrm{A\,s}$
Potential	φ, V	Volt (V)	$\mathrm{kg\,m^2\,A^{-1}\,s^{-3}}$
Electric field intensity	$\boldsymbol{E}$	Volt per meter (V/m)	$\mathrm{kg\,m\,A^{-1}\,s^{-3}}$
Electric displacement field	$\boldsymbol{D}$	Coulomb per meter (C/m)	$\mathrm{m^{-1}\,A\,s}$
Capacitance	C	Farad (F)	$\mathrm{A^2\,s^4\,kg^{-1}\,m^{-2}}$
Electric flux	Φ_E	Volt meter (V/m)	$\mathrm{kg\,m\,A^{-1}\,s^{-3}}$
Polarization	$\boldsymbol{P}$	Coulomb per square meter (C/m^2)	$\mathrm{m^{-2}\,A\,s}$
Current	I	Ampere (A)	A
Resistance	R	Ohm (Ω)	$\mathrm{kg\,m^2\,A^{-2}\,s^{-3}}$
Magnetic field intensity	$\boldsymbol{H}$	Ampere per meter (A/m)	$\mathrm{A\,m^{-1}}$
Magnetic field	$\boldsymbol{B}$	Tesla (T)	$\mathrm{kg\,A^{-1}\,s^{-2}}$
Inductance	L	Henry (H)	$\mathrm{kg\,m^2\,A^{-2}\,s^{-2}}$
Magnetic flux	Φ_B	Weber (Wb)	$\mathrm{kg\,m^2\,A^{-1}\,s^{-2}}$
Magnetization	$\boldsymbol{M}$	Ampere per meter (A/m)	$\mathrm{A\,m^{-1}}$
Electromagnetic force	$\boldsymbol{F}$	Netwon (N)	$\mathrm{kg\,m\,s^{-2}}$
Energy	U	Joule (J)	$\mathrm{kg\,m^2\,s^{-2}}$
Energy density	u	Joule per cubic meter (J/m^3)	$\mathrm{kg\,m^{-1}\,s^{-2}}$
Power	P	Watt (W)	$\mathrm{kg\,m^2\,s^{-3}}$

Table 2: Several fundamental electromagnetic quantities and their units.

1 Physical Constants

2 Electromagnetic Quantities and Units

3 Fundamental Relations

3.1 General Electromagnetism

Linear, surface, volumetric charge density

$$Q = \int_{\ell'} \tau(\boldsymbol{r}') \, \mathrm{d}\ell' \tag{1}$$

$$Q = \iint_{S'} \sigma(\boldsymbol{r}') \, \mathrm{d}S' \tag{2}$$

$$Q = \iiint_{V'} \rho(\boldsymbol{r}') \, \mathrm{d}V' \tag{3}$$

Electric current

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} \tag{4}$$

$$I = \iint_{S} \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S} \tag{5}$$

Maxwell's equations

differential form

$$\nabla \cdot \boldsymbol{D} = \rho_{\mathrm{free}} \tag{6}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{7}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \tag{8}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{9}$$

integral form

$$\oiint_{\partial V} \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{S} = Q_{\text{free}} \tag{10}$$

$$\oiint_{\partial V} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0 \tag{11}$$

$$\oint_{\partial S} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I + \iint_{S} \frac{\partial \boldsymbol{D}}{\partial t} \cdot \mathrm{d}\boldsymbol{S} \tag{12}$$

$$\oint_{\partial S} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\iint_{S} \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{S} \tag{13}$$

Material relations

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} \tag{14}$$

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M}\right) \tag{15}$$

linear media

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \epsilon_0 \chi_e \boldsymbol{E} = \varepsilon \boldsymbol{E} \tag{16}$$

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \chi_m \boldsymbol{M}\right) = \mu \boldsymbol{H} \tag{17}$$

free space

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \tag{18}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{19}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c_0^2} \frac{\partial \boldsymbol{E}}{\partial t} \tag{20}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{21}$$

3.2 Electrostatics

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \tag{22}$$

$$\nabla \times \boldsymbol{E} = 0 \tag{23}$$

$$\boldsymbol{E} = -\nabla \varphi \tag{24}$$

$$\varphi\left(\boldsymbol{r}\right) = \int \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} + K \tag{25}$$

$$\nabla \cdot \nabla \varphi = \nabla^2 \varphi = -\frac{\rho}{\epsilon_0} \tag{26}$$

$$\nabla \times \nabla \varphi = 0 \tag{27}$$

$$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2 \left(\boldsymbol{r}_1 - \boldsymbol{r}_2\right)}{\left|\boldsymbol{r}_1 - \boldsymbol{r}_2\right|^3} \tag{28}$$

$$\boldsymbol{E}\left(\boldsymbol{r}\right) = \frac{1}{4\pi\epsilon_0} \sum_n \frac{Q_n \left(\boldsymbol{r} - \boldsymbol{r}'_n\right)}{\left|\boldsymbol{r} - \boldsymbol{r}'_n\right|^3} \tag{29}$$

$$\boldsymbol{E}\left(\boldsymbol{r}\right) = \frac{1}{4\pi\epsilon_0} \iiint_{V'} \frac{\rho\left(\boldsymbol{r}'\right) \left(\boldsymbol{r} - \boldsymbol{r}'\right)}{\left|\boldsymbol{r} - \boldsymbol{r}'\right|^3} \, \mathrm{d}V' \tag{30}$$

3.3 Magnetostatics

$$\nabla \cdot \boldsymbol{B} = 0 \tag{31}$$

$$\nabla \times \boldsymbol{B} = \mu \boldsymbol{J} \tag{32}$$

Biot-Savart law

$$\boldsymbol{B}\left(\boldsymbol{r}\right) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\boldsymbol{J}\left(\boldsymbol{r}'\right) \times \left(\boldsymbol{r} - \boldsymbol{r}'\right)}{\left|\boldsymbol{r} - \boldsymbol{r}'\right|^3} \, \mathrm{d}^3\left(\boldsymbol{r}\right) \tag{33}$$

magnetic vector potential

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{34}$$

$$\boldsymbol{A}\left(\boldsymbol{r}\right) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\boldsymbol{J}\left(\boldsymbol{r}'\right)}{\left|\boldsymbol{r} - \boldsymbol{r}'\right|} \, \mathrm{d}^3\left(\boldsymbol{r}\right) \tag{35}$$

3.4 Electrodynamics

Lorentz Force Law

$$\boldsymbol{F} = Q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right) \tag{36}$$

Continuity equation

$$\nabla \cdot \boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0 \tag{37}$$

Poynting vector

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H} \tag{38}$$

Poynting theorem

$$\frac{\partial u}{\partial t} + \nabla \cdot \boldsymbol{S} = -\boldsymbol{J} \cdot \boldsymbol{E} \tag{39}$$

Wave equation

$$\nabla^2 \boldsymbol{E} - \mu_0 \epsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0 \tag{40}$$

$$\nabla^2 \boldsymbol{B} - \mu_0 \epsilon_0 \frac{\partial^2 \boldsymbol{B}}{\partial t^2} = 0 \tag{41}$$

Boundary conditions

$$\hat{\boldsymbol{n}} \times (\boldsymbol{E}_2 - \boldsymbol{E}_1) = 0 \tag{42}$$

$$\hat{\boldsymbol{n}} \cdot (\boldsymbol{D}_2 - \boldsymbol{D}_1) = \sigma \tag{43}$$

$$\hat{\boldsymbol{n}} \times (\boldsymbol{H}_2 - \boldsymbol{H}_1) = \boldsymbol{K} \tag{44}$$

$$\hat{\boldsymbol{n}} \cdot (\boldsymbol{B}_2 - \boldsymbol{B}_1) = 0 \tag{45}$$

4 Materials

Source:

4.1 Dielectrics

Material	**Relative permittivity** ε_{r}	E_{max} (MV/m)
Air (standard conditions)	$1.00058986 \pm 0.00000050$	3.0
Alumina	10	13.4
Benzene	2.3	163
Distilled Water (20°C)	80.1	65–70
Glass	4–10	9.8–13.8
Mica	4.5–8	118
Neoprene rubber	6.7	15.7–26.7
Paper	3.7	16
Polyethylene	2.2	18.9–21.7
Polystyrene	2.56	19.7
PTFE (Teflon, Extruded)	2.1	19.7
PTFE (Teflon, Insulating Film)	2.1	60–173
Quartz	3.3	8
Silicone oil	2.5	10–15

Table 3: Dielectric constant and dielectric strength of various common materials.

4.2 Conductors

4.3 Magnetic Materials

Material	**Resistivity** ρ_0 ($\Omega\,\mathrm{m}$)	α ($1/C^\circ$)
Metals		
Iron	9.71×10^{-8}	0.0065
Aluminum	2.65×10^{-8}	0.0043
Copper	1.67×10^{-8}	0.0039
Silver	1.59×10^{-8}	0.0041
Gold	2.35×10^{-8}	0.004
Nickel	6.84×10^{-8}	0.0069
Mercury	95.8×10^{-8}	0.0009
Tungsten	5.51×10^{-8}	0.0045
Alloys		
Nichrome	100.0×10^{-8}	0.0004
Constantan	49.0×10^{-8}	0.000008
Manganin	48.2×10^{-8}	0.000002
Semiconductors		
Germanium	0.46	-0.048
Silicon	4300	-0.075
Graphite	1.4×10^{-5}	-0.0005
Insulators		
Sulfur	2×10^{15}	
Quartz (SiO_2)	1×10^{14}	
Wood	10^{8}–10^{11}	
Glass	10^{10}–10^{14}	
Lucite	$> 10^{13}$	
Mica	10^{11}–10^{15}	
Diamond	10^{13}	

Table 4: Resistivity values ($T_0 = 20\mathrm{C}^\circ$) and temperature coefficient of various common materials.

Material	**Relative permeability μ_r**	**Magnetic Type**
Bismuth	0.999983	diamagnetic
Copper	0.9999906	diamagnetic
Silver	0.9999936	diamagnetic
Lead	0.9999831	diamagnetic
Water	0.999991	diamagnetic
Air	1.00000036	paramagnetic
Aluminum	1.000021	paramagnetic
Platinum	1.00026	paramagnetic
Nickel	100	ferromagnetic
Cobalt	250	ferromagnetic
Iron	5000	ferromagnetic
Mild Steel (0.2 C)	600	ferromagnetic
Permalloy (78.5 Ni)	5000	ferromagnetic
Supermalloy	100000	ferromagnetic
Mu-metal (78 Ni, 5 Cu, 2 Cr)	100000	ferromagnetic

Table 5: Relative magnetic permeability of various common materials.