Electromagnetic Field Theory

Week 13

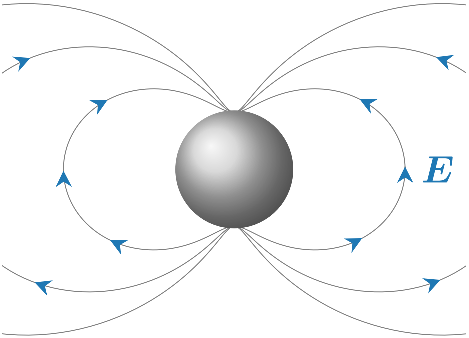
Miloslav Čapek

Department of Electromagnetic Field
Czech Technical University in Prague
Czech Republic
em@fel.cvut.cz

December 18, 2024
Winter semester 2024/25

Outline

1. Faraday's Law of Induction
2. Inductance
3. Magnetic Field Energy
4. Displacement Current
5. Maxwell's Equations



Faraday's Law of Induction

Motional electromotive (emf) force

$$U_{\mathrm{emf}} = \oint_l \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}.$$

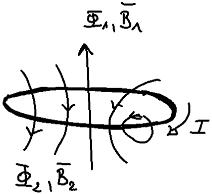
Faraday's law of induction

$$\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\iint \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{l}.$$

Faraday's law of induction in differential form

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}.$$

Lenz's Law



Ideal Transformer

Current

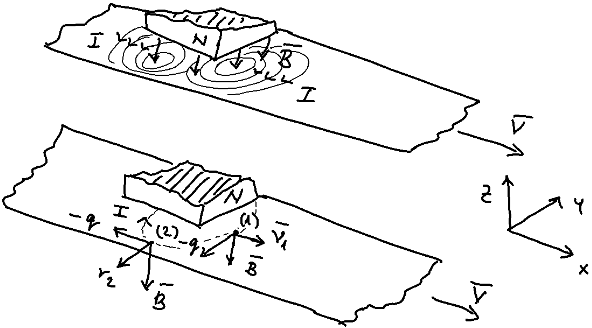
$$\frac{i_1}{i_2} = \frac{N_2}{N_1}$$

and voltage

$$\frac{u_1}{u_2} = \frac{N_1}{N_2}$$

transformation of ideal transformer.

Eddy Currents



Static and Dynamic Definition

Static definition

$$L = \frac{\Phi_{\mathrm{t}}}{I} \quad [\mathrm{H}].$$

Dynamic definition

$$u(t) = L\frac{\mathrm{d}i}{\mathrm{d}t}.$$

Mutual Inductance

Mutual inductance M_{21}

$$\Phi_2 = M_{21} I_1.$$

Neumann Formula

$$\Phi_2 = \frac{\mu_0}{4\pi} I_1 \oint_{l_1} \oint_{l_2} \frac{\mathrm{d}\boldsymbol{l}_1 \cdot \mathrm{d}\boldsymbol{l}_2}{R} = M_{21} L_1.$$

Energy in Magnetic Field

Using currents and potential

$$W = \frac{1}{2} \iiint\limits_V \boldsymbol{A} \cdot \boldsymbol{J} \, \mathrm{d}V, \quad [\mathrm{J}].$$

Using magnetic field

$$W = \frac{1}{2} \iiint\limits_V \boldsymbol{H} \cdot \boldsymbol{B} \, \mathrm{d}V, \quad [\mathrm{J}].$$

Need For Displacement Current

Maxwell's equations before Maxwell

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0},$$
$$\nabla \cdot \boldsymbol{B} = 0,$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t},$$
$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}.$$

Displacement Current

Complete Ampère's law:

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

with

$$\boldsymbol{J}_{\mathrm{d}} = \frac{\partial \boldsymbol{D}}{\partial t}$$

being the displacement current.

Energy in Magnetic Field

Complete Maxwell's equations in differential and integral form:

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c_0^2} \frac{\partial \boldsymbol{E}}{\partial t}$$

$$\iint\limits_{S'} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S} = \frac{1}{\varepsilon_0} \iiint\limits_{V'} \rho \, \mathrm{d}V'$$

$$\oiint\limits_{S'} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0$$

$$\oint\limits_{l'} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l}' = -\frac{\mathrm{d}}{\mathrm{d}t} \iint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S}$$

$$\oint\limits_{l'} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l}' = \mu_0 \iint\limits_{S'} \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S} + \frac{1}{c_0^2} \frac{\mathrm{d}}{\mathrm{d}t} \iint\limits_{S'} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S}$$

Questions?

BAB17EMP
em@fel.cvut.cz

December 18, 2024
Winter semester 2024/25

This document has been created as a part of BAB17EMP course.