Electromagnetic Field Theory

Week 12

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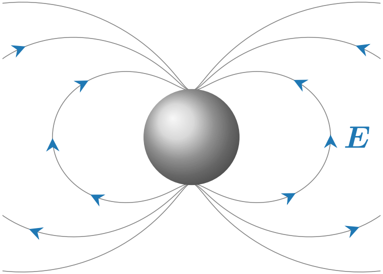
Outline

1. Magnetic Materials



Magnetic Materials

$$\boldsymbol{M} = \chi_{\mathrm{m}} \boldsymbol{H}$$

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \chi_{\mathrm{m}} \boldsymbol{H} = \mu_0 \left(1 + \chi_{\mathrm{m}}\right) \boldsymbol{H} = \mu_0 \mu_{\mathrm{r}} = \mu \boldsymbol{H}$$

Classification	Magnetic Moments	$\boldsymbol{B}$ Values	Comments
Diamagnetic	$\lvert \boldsymbol{m}_{\mathrm{o}} + \boldsymbol{m}_{\mathrm{s}} \rvert = 0$	$B_{\mathrm{int}} < B_0$	Nonmagnetic matrix
Paramagnetic	$\lvert \boldsymbol{m}_{\mathrm{o}} + \boldsymbol{m}_{\mathrm{s}} \rvert \to 0$	$B_{\mathrm{int}} > B_0$	Low magnetic response
Ferromagnetic	$\lvert \boldsymbol{m}_{\mathrm{s}} \rvert \gg \lvert \boldsymbol{m}_{\mathrm{o}} \rvert$	$B_{\mathrm{int}} \gg B_0$	Domains
Antiferromagnetic	$\lvert \boldsymbol{m}_{\mathrm{s}} \rvert \gg \lvert \boldsymbol{m}_{\mathrm{o}} \rvert$	$B_{\mathrm{int}} = B_0$	Adjacent moments oppose
Ferrimagnetic	$\lvert \boldsymbol{m}_{\mathrm{s}} \rvert \gg \boldsymbol{m}_{\mathrm{o}} \rvert$	$B_{\mathrm{int}} > B_0$	Unequal adjacent moments oppose
Superparamagnetic	$\lvert \boldsymbol{m}_{\mathrm{s}} \rvert \gg \lvert \boldsymbol{m}_0 \rvert$	$B_{\mathrm{int}} > B_0$	Nonmagnetic matrix

Classification of magnetic materials (the table is adopted from Hyat & Buck), $\boldsymbol{m}_{\mathrm{o}}$ stands for an orbital moment, m_{s} for a spin moment, B_{int} stands for the magnitude of magnetic field inside the material, and B_0 stands for the magnitude of external magnetic field.

Diamagnetic Materials

- $\chi_\mathrm{m} \approx -10^{-5}$, $\mu_\mathrm{r} < 1$.
- Weakly repelled by a magnetic field.
- No permanent magnetic moment in the absence of a magnetic field.

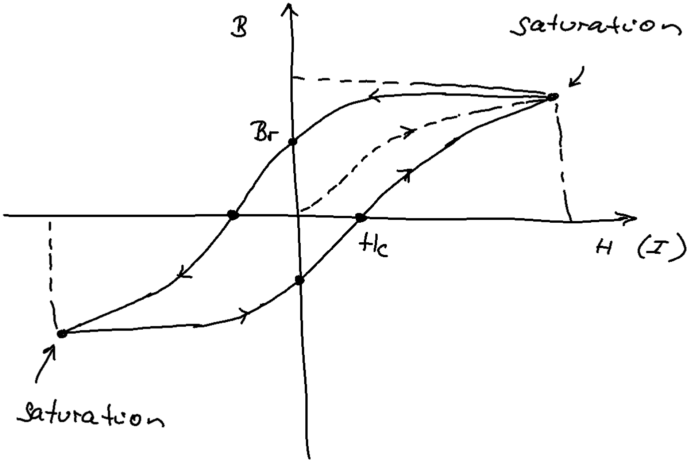
Paramagnetic Materials

- $\chi_\mathrm{m} > 0$, $\mu_\mathrm{r} > 1$.
- Weakly attracted to a magnetic field.
- Magnetic moments align with the external field but only in its presence; the alignment is random when the field is removed.

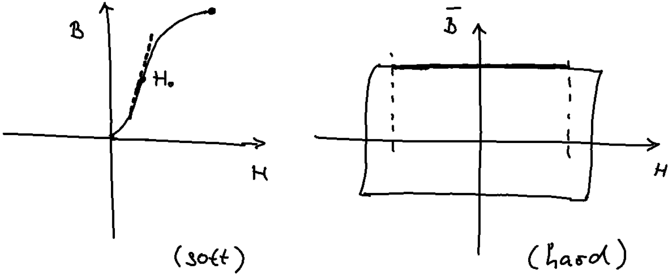
Ferromagnetic Materials

- $\chi_\mathrm{m} \approx 10^3$-$10^5$
- Strongly attracted to a magnetic field.
- Highly nonlinear materials.
- Exhibit hysteresis in their magnetization curve.

Hysteresis

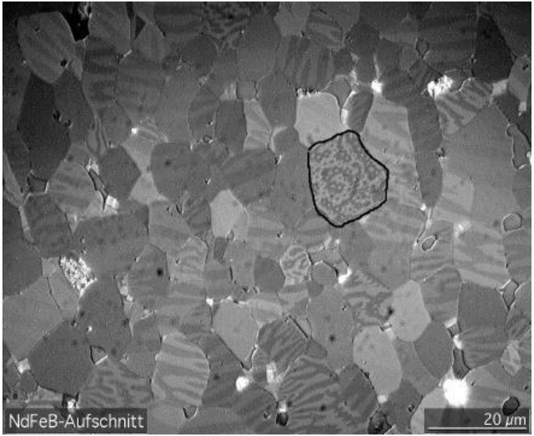


Soft and Hard Materials



Soft materials are usually used around a point where the permeability can be linearized. Hard materials (permanent magnets) are used with magnetic field intensity so they are never de-magnetized.

Soft and Hard Materials



Microcrystalline grains within a piece of $Nd_2Fe_{14}B$ (neodymium magnet). The domains are the light and dark stripes visible within each grain. The outlined grain has an almost vertical magnetocrystalline axis, so the domains are seen end-on. Credits: Wikipedia.

Boundary Conditions in Magnetic Field

$$\mu_1 H_{1,\text{norm}} = \mu_2 H_{2,\text{norm}}$$

$$\frac{B_{1,\tan}}{\mu_1} = \frac{B_{2,\tan}}{\mu_2}$$

$$H_{1,\tan} - H_{2,\tan} = \frac{NI}{\mathrm{d}l} = nI$$

Magnetic Circuits

Electric circuit	**Magnetic circuit**
electromotive voltage U	magnetomotive voltage NI
electric current I	magnetic flux Φ
resistance R	reluctance R_{m}
conductance G	permeance P_{m}
conductivity σ	permeability μ

Analogies between electric and magnetic circuits.

Hopkinson's law

$$U_{\mathrm{m}} = \Phi R_{\mathrm{m}}$$

Motional Elecromotive (emf) Force

$$U_{\mathrm{emf}} = \oint_{l} \boldsymbol{F}_{\mathrm{m}} \cdot \mathrm{d}\boldsymbol{l}$$

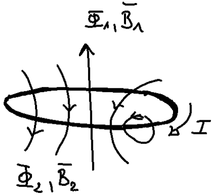
Faraday's Law of Induction

$$U_{\mathrm{emf}} = \oint\limits_{l} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\iint \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{l}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

Lenz's Law



Questions?

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