Electromagnetic Field Theory

Week 11

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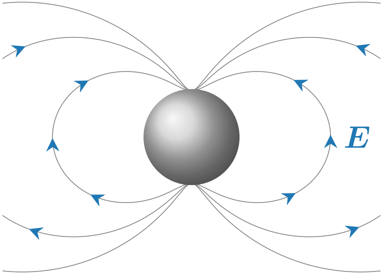
December 7, 2024
Winter semester 2024/25

Outline

1. Magnetic Field
2. Magnetization
3. Ampère's Law for Magnetic Field Intensity $\boldsymbol{H}$
4. Magnetic Susceptibility



Magnetic Flux

$$\Phi = \iint\limits_S \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} \quad [\mathrm{Wb}]$$

$$\Phi = \oint\limits_l \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}$$

Comparison of Electrostatics and Magnetostatics

Electrostatic field	Magnetostatic field
$\nabla \cdot \boldsymbol{E} = \frac{1}{\varepsilon_0}\rho$	$\nabla \cdot \boldsymbol{B} = 0$
$\nabla \times \boldsymbol{E} = \boldsymbol{0}$	$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$
BC $\boldsymbol{E} \to \boldsymbol{0}$ far from sources	BC $\boldsymbol{B} \to \boldsymbol{0}$ far from sources
$\boldsymbol{E}(\boldsymbol{r}_2) = \frac{1}{4\pi\varepsilon_0} \int\limits_{V_1} \frac{\rho(\boldsymbol{r}_1)(\boldsymbol{r}_2 - \boldsymbol{r}_1)}{\lvert\boldsymbol{r}_2 - \boldsymbol{r}_1\rvert^3}\,\mathrm{d}V_1$	$\boldsymbol{B}(\boldsymbol{r}_2) = \frac{\mu_0}{4\pi} \int\limits_{V_1} \frac{\boldsymbol{J}(\boldsymbol{r}_1) \times (\boldsymbol{r}_2 - \boldsymbol{r}_1)}{\lvert\boldsymbol{r}_2 - \boldsymbol{r}_1\rvert^3}\,\mathrm{d}V_1$
$\boldsymbol{F}_\mathrm{e} = q\boldsymbol{E}$	$\boldsymbol{F}_\mathrm{m} = q\left(\boldsymbol{v} \times \boldsymbol{B}\right)$

Magnetic Dipole Moment

Multipole expansion for

$$\frac{1}{R} = \frac{1}{r} + \frac{\cos(\alpha) r'}{r^2} + \left(-\frac{1}{2} + \frac{3}{2}\cos^2(\alpha)(r')^2\right)\frac{1}{r^3} + \mathcal{O}((r')^3).$$

Magnetic dipole moment

$$\boldsymbol{A}_\mathrm{d}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{\boldsymbol{m}\times\hat{\boldsymbol{r}}}{r^2}, \quad [\mathrm{Wb/m}]$$

with

$$\boldsymbol{m} = I \iint_{S'} \mathrm{d}\boldsymbol{S}', \quad [\mathrm{A\,m^2}].$$

The magnetic field of small loop of current is

$$\boldsymbol{B}_\mathrm{d} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos(\theta)\hat{\boldsymbol{r}} + \sin(\theta)\hat{\boldsymbol{\theta}}\right).$$

Atomic Model and Material Classification

1. Electron in orbit.
2. Spin of the electron.
3. Nuclear spin.

The magnetic materials are classified as

- diamagnetic,
- paramagnetic,
- ferromagnetic,
- antiferromagnetic,
- ferrimagnetic,
- superparamagnetic.

Magnetization

Magnetization as the average of loop magnetic moments

$$\boldsymbol{M} = \lim_{\Delta V \to 0} \frac{\sum_i \boldsymbol{m}_i}{\Delta V} \quad [\mathrm{A/m}].$$

Ampère's law including bound current I_b

$$\oint_{l'} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l}' = \mu_0 I_\mathrm{tot} = \mu_0 \left(I_0 + I_\mathrm{b}\right).$$

We have

$$\boldsymbol{m}_\mathrm{b} = I_\mathrm{b} \int \mathrm{d}\boldsymbol{S} = I_b \boldsymbol{S},$$

and

$$I_\mathrm{b} = hM.$$

Intensity of Magnetic Field

Ampère's law for free and bound current expressed as magnetization

$$\oint_{l'} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l}' = \mu_0 I_0 + \mu_0 \int_{l'} \boldsymbol{M} \cdot \mathrm{d}\boldsymbol{l}'.$$

Definition of the intensity of magnetic field

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu_0} - \boldsymbol{M}, \quad [\mathrm{A/m}].$$

The integral form of Ampère's law for free current only

$$\oint_{l'} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I_0 = \iint_{S'} \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S},$$

and its differential form

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_0.$$

Magnetic Susceptibility

Linear (linearized) materials

$$\boldsymbol{M} = \chi_{\mathrm{m}} \boldsymbol{H},$$

where χ_{m} is magnetic susceptibility.

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \chi_{\mathrm{m}} \boldsymbol{H} = \mu \boldsymbol{H},$$

Questions?

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This document has been created as a part of BAB17EMP course.