# Electromagnetic Field Theory

Week 9

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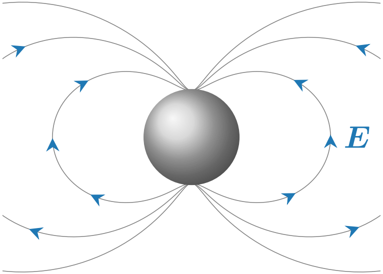
November 27, 2024
Winter semester 2024/25

# Outline

1. Stationary Magnetic Field
2. Biot-Savart Law
3. Ampère's Law
4. Magnetic Flux
5. Vector Potential



# Stationary Magnetic Field

Conditions

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = 0, \quad \frac{\mathrm{d}\boldsymbol{J}}{\mathrm{d}t} = 0.$$

The equivalent terms in the stationary magnetic field are

$$\mathrm{d}Q\,\boldsymbol{v}, \quad I\,\mathrm{d}\boldsymbol{l}, \quad \boldsymbol{J}\,\mathrm{d}V, \quad [\mathrm{A\,m}].$$

# Biot-Savart Law

For linear current

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int\limits_{l'} I(\boldsymbol{r}') \frac{\mathrm{d}\boldsymbol{l}' \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3}, \quad [\mathrm{T}],$$

or for the conductor's volume

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int\limits_{V'} \frac{\boldsymbol{J}(\boldsymbol{r}')\,\mathrm{d}V' \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3}, \quad [\mathrm{T}].$$

where

- $\mu_0$ is vacuum permeability,
- $\boldsymbol{r}$ points to the observation region,
- $\boldsymbol{r}'$ points to the source region,
- T is physical unit Tesla, equivalent to N/(A m).

# Magnetic Field of a Straight Wire

Start from

$$\boldsymbol{B}(s) = \frac{\mu_0}{4\pi} I \int \frac{\mathrm{d}\boldsymbol{l}' \times \hat{\boldsymbol{R}}}{R^2}$$

with $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$, $R = |\boldsymbol{R}|$, $\hat{\boldsymbol{R}} = \boldsymbol{R}/R$ to get

$$\boldsymbol{B}(s) = \frac{\mu_0 I}{2\pi s} \hat{\boldsymbol{\phi}}.$$

Force between two parallel wires:

$$\frac{\boldsymbol{F}_{\mathrm{m}}}{l} = -\frac{\mu_0 I_1 I_2}{2\pi d} \hat{\boldsymbol{x}}.$$

# Ampère's Law

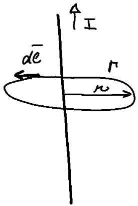
Ampère's law dictates that

$$\oint_l \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \mu_0 I_{\mathrm{tot}},$$

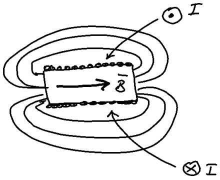
or equivalently

$$\oint_l \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \mu_0 \iint_S \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S}.$$

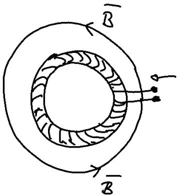
# Coils: Basic Arrangements



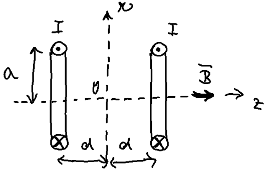
A loop.



A solenoid.



A toroid.



A loop.

# Divergence of Magnetic Field

$$\nabla \cdot \boldsymbol{B}(\boldsymbol{r}) = 0.$$

# Magnetic Flux

The flux of magnetic field through a closed surface

$$\oiint_S \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0.$$

Magnetic flux

$$\Phi = \iint_S \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S}, \quad [\mathrm{Wb}].$$

(Wb is the physical unit Weber, equivalent to V s.)

# Vector Potential

Use

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

where $\boldsymbol{A}$ is the magnetic vector potential, with the choice

$$\nabla \cdot \boldsymbol{A} = 0$$

which gives

$$\nabla^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J},$$

with

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int\limits_{V'} \frac{\boldsymbol{J}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|} \, \mathrm{d}V', \quad [\mathrm{Wb/m^2}].$$

# Questions?

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This document has been created as a part of BAB17EMP course.