Electromagnetic Field Theory

Week 8

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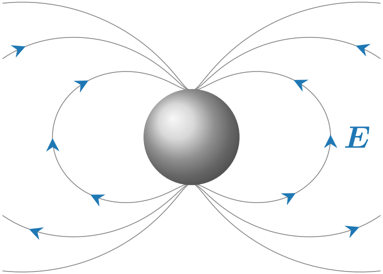
Outline



Magnetostatic field

Magnetostatic field

$$\begin{aligned} \nabla \cdot \boldsymbol{B} &= 0, \\ \nabla \times \boldsymbol{B} &= \mu_0 \boldsymbol{J}. \end{aligned}$$

Magnetic force

$$\boldsymbol{F}_{\mathrm{m}} = q(\boldsymbol{v} \times \boldsymbol{B}), \quad [\mathrm{N}].$$

Lorentz force

$$\boldsymbol{F} = \boldsymbol{F}_{\mathrm{e}} + \boldsymbol{F}_{\mathrm{m}} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right), \quad [\mathrm{N}].$$

Steady current field

$$I = \iint\limits_S N q \boldsymbol{v} \cdot \mathrm{d}\boldsymbol{S} = \iint\limits_S \rho \boldsymbol{v} \cdot \mathrm{d}\boldsymbol{S} = \iint\limits_S \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S}.$$

Different current types:

- conductive currents (in conductors and semiconductors),
- convective currents (e^- or ions in vacuum),
- in electrolyte (ions, *e.g.*, in battery).

Steady current condition:

$$\frac{\partial \boldsymbol{J}}{\partial t} = \mathbf{0}.$$

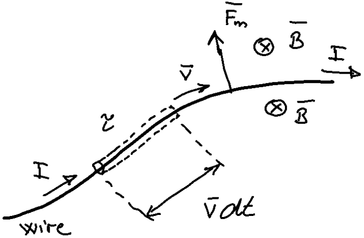
Current Continuity Equation – Steady Currents

$$\oint\!\!\!\oint_S \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S} = 0$$

$$\nabla \cdot \boldsymbol{J} = 0$$

(valid only in for steady currents, *i.e.*, $\mathrm{d}d\rho/\,\mathrm{d}t = 0$)

Magnetic Force Defined by Current



$$\boldsymbol{F}_{\mathrm{m}} = \int_{l} I \left(\mathrm{d}\boldsymbol{l} \times \boldsymbol{B} \right) = I \int_{l} \left(\mathrm{d}\boldsymbol{l} \times \boldsymbol{B} \right)$$

Ohm's Law in Differential Form

$$\boldsymbol{J} = \sigma \boldsymbol{E}, \quad [\mathrm{A/m^2}]$$

with conductivity σ in [S/m]and resistivity

$$\rho = \frac{1}{\sigma}, \quad [\Omega\,\mathrm{m}].$$

Ohm’s Law in Integral Form

$$R = \int_{0}^{l} \frac{\mathrm{d}l}{\sigma S}, \quad [\Omega].$$

$$U = RI, \quad [\mathrm{V}, \Omega, \mathrm{A}],$$

with R being resistance and

$$G = \frac{1}{R}, \quad [\mathrm{S}]$$

being conductance.

Connection of Resistors

Serial connection

$$R = \sum_i R$$

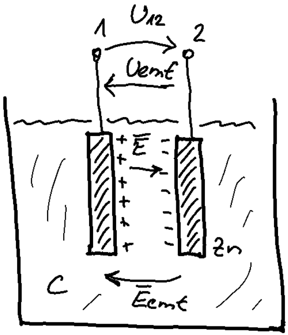
Parallel connection

$$\frac{1}{R} = \sum_i \frac{1}{R_i}$$

Electromotive Force

Many possible mechanisms of emf

- chemical force,
- mechanical pressure,
- temperature gradient,
- light,
- running belt,
- moving charges in the magnetic field.



$$U_{\mathrm{emf}} = \int_2^1 \boldsymbol{E}_{\mathrm{emf}} \cdot \mathrm{d}\boldsymbol{l} = -\int_2^1 \boldsymbol{E}_{\mathrm{c}} \cdot \mathrm{d}\boldsymbol{l} = \int_1^2 \boldsymbol{E}_{\mathrm{c}} \cdot \mathrm{d}\boldsymbol{l} = U_{12}$$

Boundary Conditions for Current Density

Normal Component

$$J_{1,\mathrm{norm}} = J_{2,\mathrm{norm}}$$
$$\sigma_1 E_{1,\mathrm{norm}} - \sigma_2 E_{2,\mathrm{norm}} = 0$$
$$\varepsilon_1 E_{1,\mathrm{norm}} - \varepsilon_2 E_{2,\mathrm{norm}} = \sigma_0$$

Tangential Component

$$\frac{J_{2,\tan}}{\sigma_2} = \frac{J_{1,\tan}}{\sigma_1}$$
$$E_{2,\tan} = E_{1,\tan}$$

Kirchhoff's Circuit Laws

Kirchhoff's Current Law

- The current entering a note escapes it:

$$\oiint_S \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S} = 0$$

$$\sum_i I_i = 0$$

Kirchhoff's Voltage Law

- The closed-loop integral of the electric field is zero everywhere except EMF sources:

$$\oint_l \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = 0$$

$$\sum_i U_i = 0$$

Analogies Between Electrostatic and Steady Current Field

Electrostatics	$\boldsymbol{E}$	φ	U	$\boldsymbol{D}$	ε	Q	C
Steady current	$\boldsymbol{E}$	φ	U	$\boldsymbol{J}$	σ	I	G

What about differences?

Questions?

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