Electromagnetic Field Theory

Week 5

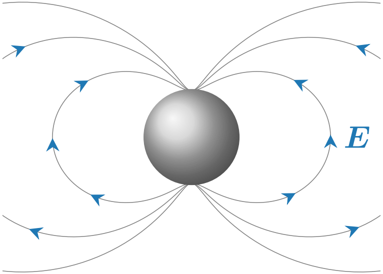
Miloslav Čapek

Department of Electromagnetic Field
Czech Technical University in Prague
Czech Republic
em@fel.cvut.cz

October 22, 2024
Winter semester 2024/25

Outline

1. Poisson's and Laplace's Equation
2. Dielectrics
3. Capacity



Poisson's and Laplace's Equation

Poisson's equation

$$\Delta\varphi = -\frac{\rho}{\epsilon_0}$$

Laplace's equation

$$\Delta\varphi = 0,$$

Dirichlet boundary condition

$$\varphi(\mathit{\Gamma}) = K$$

Neumann boundary condition

$$\frac{\partial\varphi(\mathit{\Gamma})}{\partial\hat{\boldsymbol{n}}} = K$$

Fundamental Solution: Dirac Delta

Dirac delta $\delta(x)$ if defined by its properties:

$$\int_{-\infty}^{+\infty} \delta(x)\,\mathrm{d}x = 1,$$

$$\int_{-\infty}^{+\infty} f(x)\delta(x - x')\,\mathrm{d}x = f(x').$$

Fundamental Solution: Green's Function

Consider linear integro-differential equation

$$\mathcal{L}\boldsymbol{u}(\boldsymbol{r}) = \boldsymbol{v}(\boldsymbol{r}),$$

and attempt to solve

$$\mathcal{L}\boldsymbol{G}(\boldsymbol{r}, \boldsymbol{r}') = \delta(\boldsymbol{r} - \boldsymbol{r}').$$

Then, the solution can be written as

$$\boldsymbol{u}(\boldsymbol{r}) = \int \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{r}')\boldsymbol{v}(\boldsymbol{r}')\,\mathrm{d}V'.$$

Polarization

Surface polarization

$$\boldsymbol{P} = \sigma_{\mathrm{b}}\,\hat{\boldsymbol{p}}, \quad [\mathrm{Cm}^{-2}]$$

Volumetric polarization

$$\rho_{\mathrm{b}} = -\nabla \cdot \boldsymbol{P}$$

Electric Susceptibility

Relation between electric field and polarization:

$$\boldsymbol{P} = \epsilon_0 \boldsymbol{\chi}_{\mathrm{e}} \boldsymbol{E},$$

where $\boldsymbol{\chi}_{\mathrm{e}}$ is susceptibility

$$\boldsymbol{\chi}_{\mathrm{e}} = \begin{bmatrix} \chi_{\mathrm{e},xx} & \chi_{\mathrm{e},xy} & \chi_{\mathrm{e},xz} \\ \chi_{\mathrm{e},yx} & \chi_{\mathrm{e},yy} & \chi_{\mathrm{e},yz} \\ \chi_{\mathrm{e},zx} & \chi_{\mathrm{e},zy} & \chi_{\mathrm{e},zz} \end{bmatrix}, \quad [-]. \tag{1}$$

Material classification:

Linearity × Nonlinearity

Homogeneity × Inhomogeneity

Isotropy × Anisotropy

Electric Displacement, Permittivity

Total charge

$$\rho = \rho_0 + \rho_{\mathrm{b}}.$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_0 + \rho_{\mathrm{b}}}{\epsilon_0}.$$

$$\nabla \cdot \boldsymbol{D} = \rho_0.$$

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

$$\boldsymbol{P} = \epsilon_0 \chi_{\mathrm{e}} \boldsymbol{E}$$

Electric displacement field

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \epsilon_0 \chi_{\mathrm{e}} \boldsymbol{E} = \epsilon_0 \left(1 + \chi_{\mathrm{e}}\right) \boldsymbol{E} = \epsilon_0 \varepsilon_{\mathrm{r}} \boldsymbol{E} = \varepsilon \boldsymbol{E}.$$

Boundary Conditions in Dielectrics

Tangential components:

$$E_{1,\mathrm{tan}} = E_{2,\mathrm{tan}}.$$

Normal components:

$$D_{1,\mathrm{norm}}\,\mathrm{d}S - D_{2,\mathrm{norm}}\,\mathrm{d}S = \sigma_0\,\mathrm{d}S.$$

Condition	General case	Two dielectrics	Dielectric-conductor
Tan. comp.	$E_{1,\mathrm{tan}} = E_{2,\mathrm{tan}}$	$E_{1,\mathrm{tan}} = E_{2,\mathrm{tan}}$	$E_{\mathrm{tan}} = 0$
Norm. comp.	$D_{1,\mathrm{norm}} - D_{2,\mathrm{norm}} = \sigma_0$	$D_{1,\mathrm{norm}} = D_{2,\mathrm{norm}}$	$D_{\mathrm{norm}} = \sigma_0$
Electric pot.		$\varphi_1 = \varphi_2$	

Capacity

Capacity is defined as

$$C = \frac{Q_0}{U}, \quad [\mathrm{F}].$$

Serial connection

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots .$$

Parallel connection

$$C = C_1 + C_2 + \dots .$$

Questions?

BAB17EMP
em@fel.cvut.cz

October 22, 2024
Winter semester 2024/25

This document has been created as a part of BAB17EMP course.