# Electromagnetic Field Theory

## Week 3

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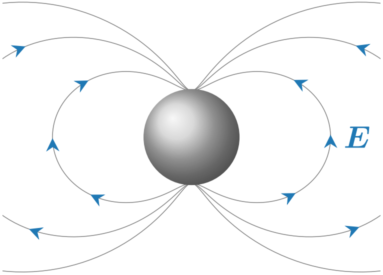
October 9, 2024
Winter semester 2024/25

# Outline

1. Gauss's Law
2. Work in Electrostatics, Electric Potential
3. Elementary Electric Dipole
4. Good Conductors



# Gauss's Law

- integral form

$$\oiint_S \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S} = \frac{1}{\epsilon_0} \int_{V'} \rho(\boldsymbol{r}') \, \mathrm{d}V' = \frac{Q}{\epsilon_0}$$

- differential form

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$

# Work in Electrostatic Field

$$W = -\int_a^b \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{l}, \quad [\mathrm{J}]$$

Electric potential

$$-\int_a^b \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = \varphi(b) - \varphi(a)$$

$$\varphi = -\int \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} + K, \quad [\mathrm{V}]$$

# Electric Field Over Closed Path

- integral form

$$\oint_l \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = 0$$

- differential form

$$\nabla \times \boldsymbol{E} = \mathbf{0}$$

# Relation Between Electric Field and Potential

$$\boldsymbol{E} = -\nabla \varphi$$

voltage

$$U_{\mathrm{AB}} = \varphi(\mathrm{B}) - \varphi(\mathrm{A}), \quad [\mathrm{V}]$$

# Summary

## Coulomb's law

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0} \int\limits_{V'} \rho(\boldsymbol{r}') \frac{(\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \, \mathrm{d}V$$

## Integral form

$$\oiint\limits_S \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S} = \frac{Q}{\epsilon_0}$$

$$\oint\limits_l \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = 0$$

## Differential form

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \boldsymbol{E} = \boldsymbol{0}$$

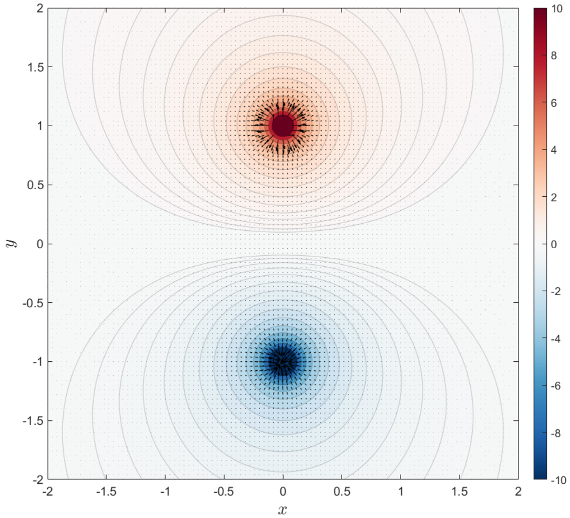
# Elementary Electric Dipole

The potential generated by elementary dipole

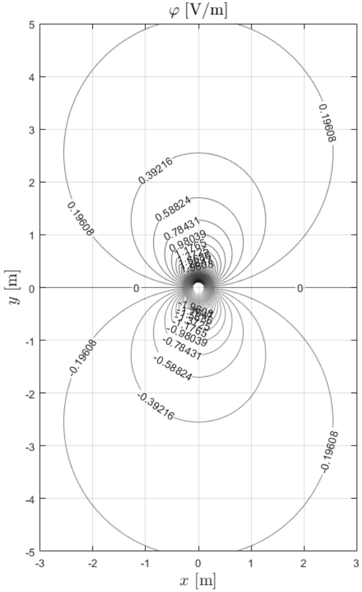
$$\varphi(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\frac{p\cos(\theta)}{R^2} = \frac{1}{4\pi\epsilon_0}\frac{\boldsymbol{p}\cdot\hat{\boldsymbol{r}}}{R^2}, \quad [\mathrm{V}]$$

Electric field of elementary dipole? Solved in the exercises!

# Electric Field of Two Opposite Charges



- Electric potential of the elementary dipole:



# Conductors

Inside the conductor

$$\boldsymbol{E} = \mathbf{0}$$

$$\varphi = K$$

Boundary condition

$$\boldsymbol{E}(\boldsymbol{r} \in \partial V) = \frac{\sigma}{\epsilon_0}\hat{\boldsymbol{n}}$$

# Parallel-Plate Capacitor

Potential from the charged plane

$$\varphi = -\int \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} + K = -\frac{\sigma}{2\epsilon_0}|z| + K.$$

Electric potential from two planes charged with opposite surface charge density

$$\varphi(z) = \varphi_+(z) + \varphi_(z)- = \frac{\sigma}{2\epsilon_0}\left(\left|z+\frac{d}{2}\right| - \left|z-\frac{d}{2}\right|\right)$$

Voltage and capacity

$$U = Q\frac{d}{\epsilon_0 S} \quad \Rightarrow \quad CU = Q.$$

# Questions?

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This document has been created as a part of BAB17EMP course.