Rendering Participating Media

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SIGGRAPH 2009 Course: Scattering

Thursday, August 6th, 12:45 - 16:30, Room 265-266

Fog

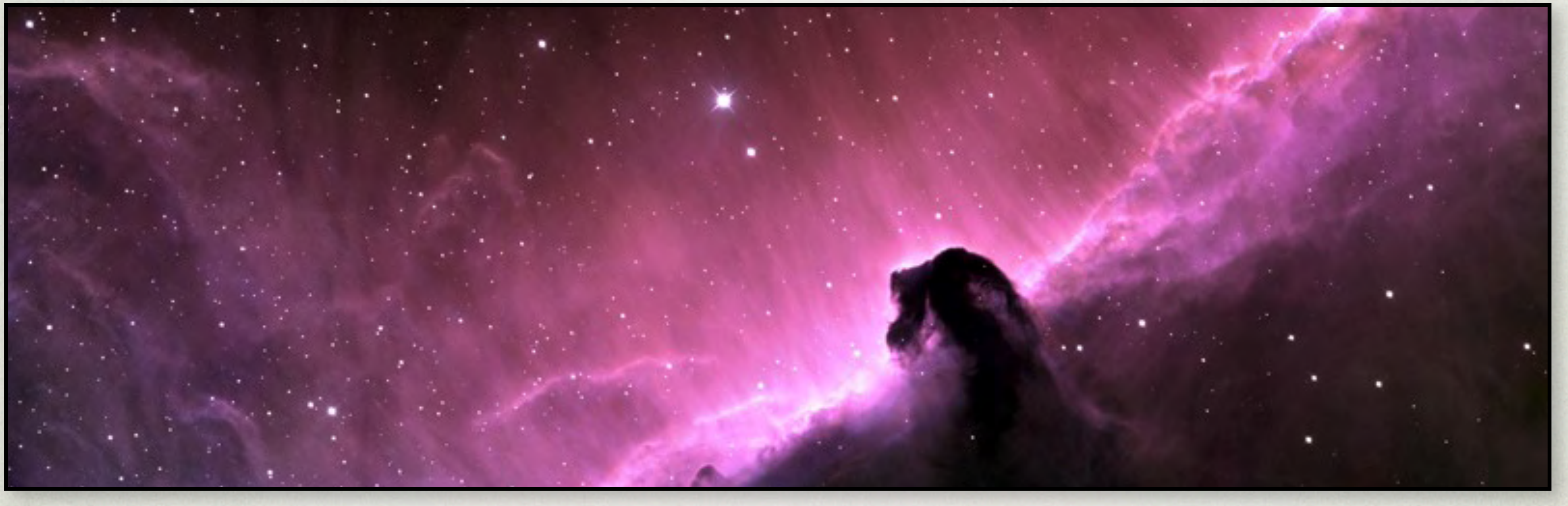
Clouds & Crespucular rays

Aerial (Atmospheric)Perspective

Leonardo da Vinci (1480)

"Thus, if one is to be five times as distant, make it five times bluer."
—*Treatise on Painting*, Leonardo Da Vinci, pp 295, circa 1480.

Nebula



T.A.Rector (NOAO/AURA/NSF) and the Hubble Heritage Team (STScI/AURA/NASA)

Outline

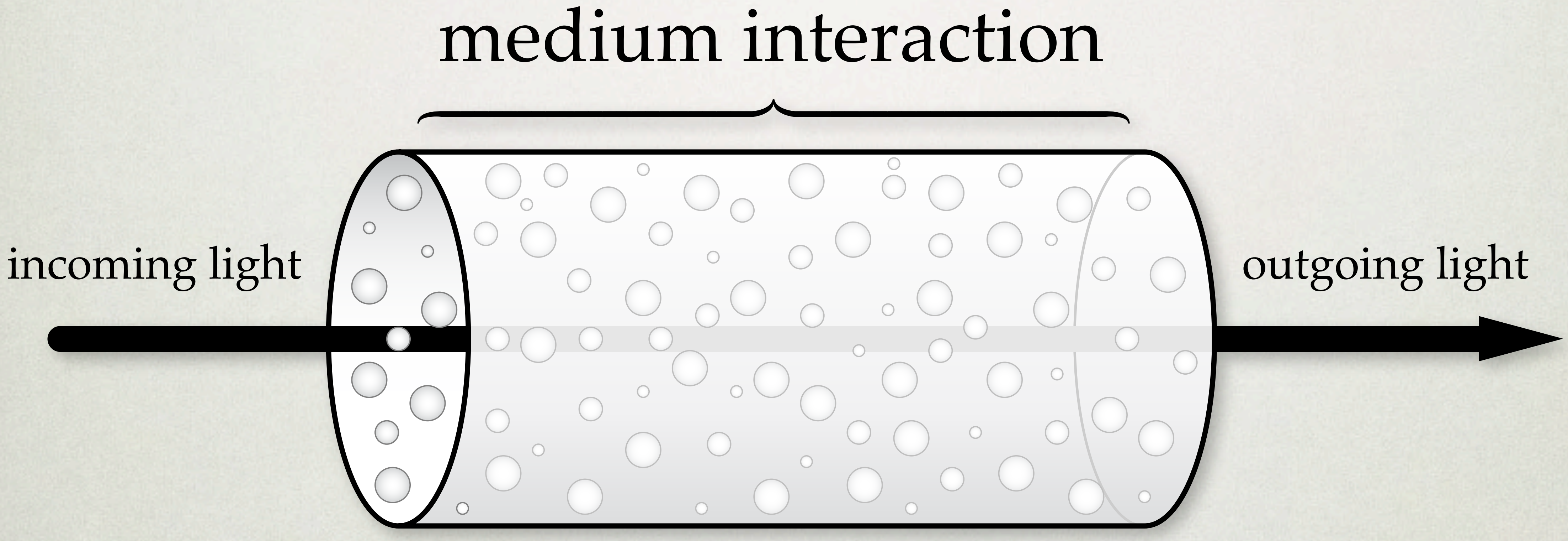
- Theoretical background
- Methods for rendering participating media

Radiance

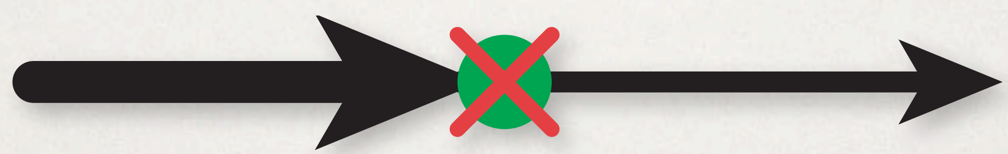
- The main quantity we are interested in for rendering is radiance:

$L(\mathbf{x} \rightarrow \vec{\omega})$, $L(\mathbf{x}, \vec{\omega})$: radiance, or "light"

Participating Media



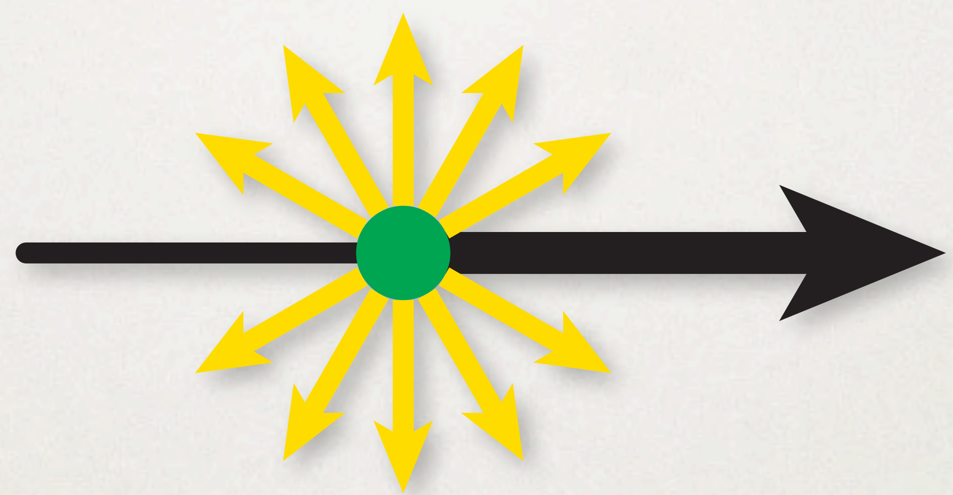
Absorption



$$(\vec{\omega} \cdot \nabla_a)\, L(\mathbf{x} \to \vec{\omega}) = -\sigma_a(\mathbf{x})\, L(\mathbf{x} \to \vec{\omega})$$

$\sigma_a(\mathbf{x})$: absorption coefficient [1/m]

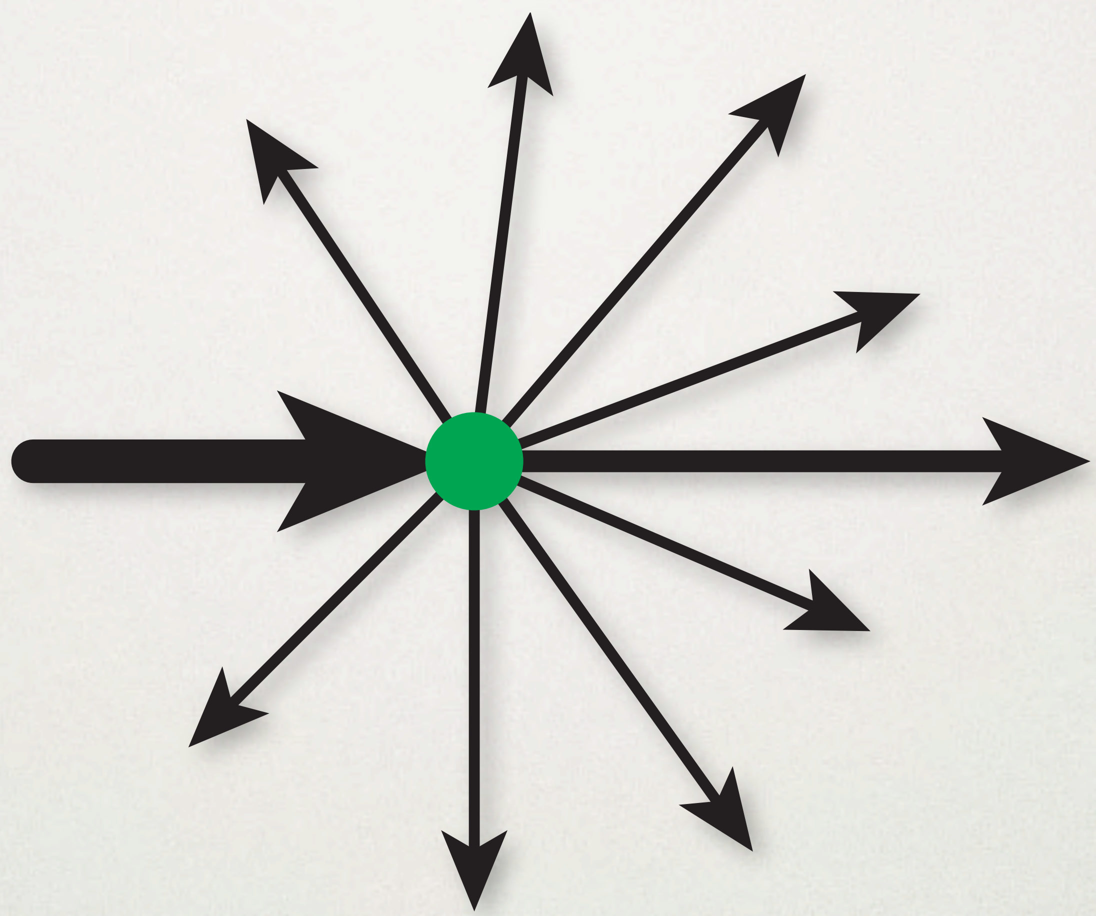
Emission



$$(\vec{\omega} \cdot \nabla_e) L(\mathbf{x} \rightarrow \vec{\omega}) = \sigma_a(\mathbf{x}) L_e(\mathbf{x} \rightarrow \vec{\omega})$$

$\sigma_a(\mathbf{x})$: absorption coefficient [1/m]

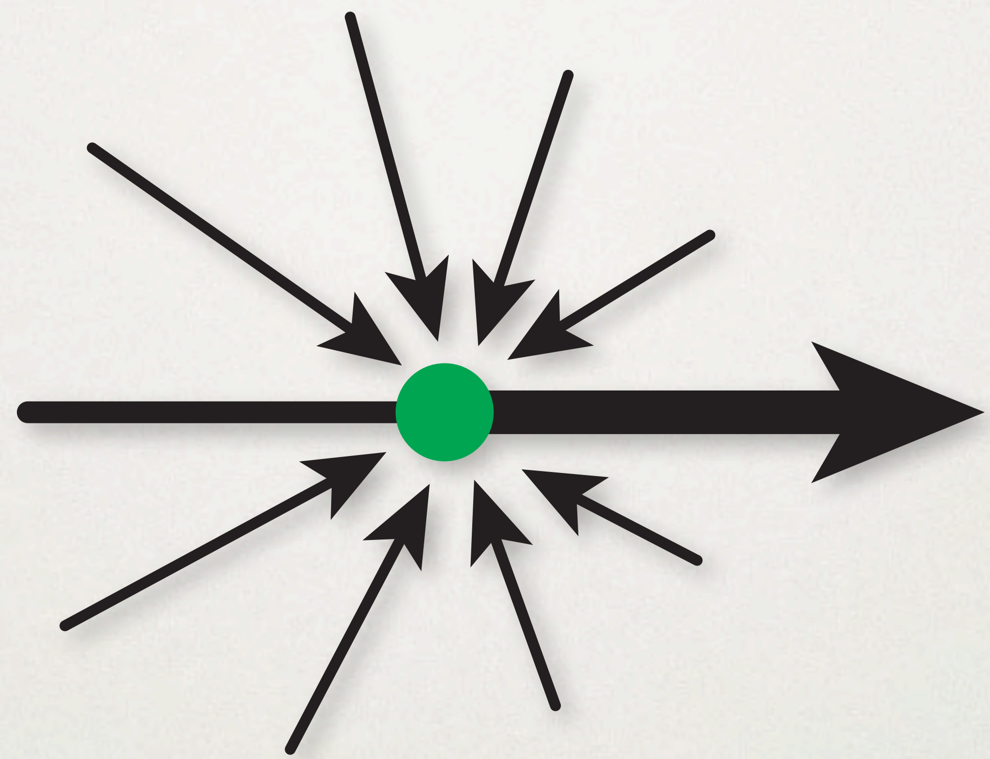
Out-Scattering



$$(\vec{\omega} \cdot \nabla_o) L(\mathbf{x} \to \vec{\omega}) = -\sigma_s(\mathbf{x})\, L(\mathbf{x} \to \vec{\omega})$$

$\sigma_s(\mathbf{x})$: scattering coefficient [1/m]

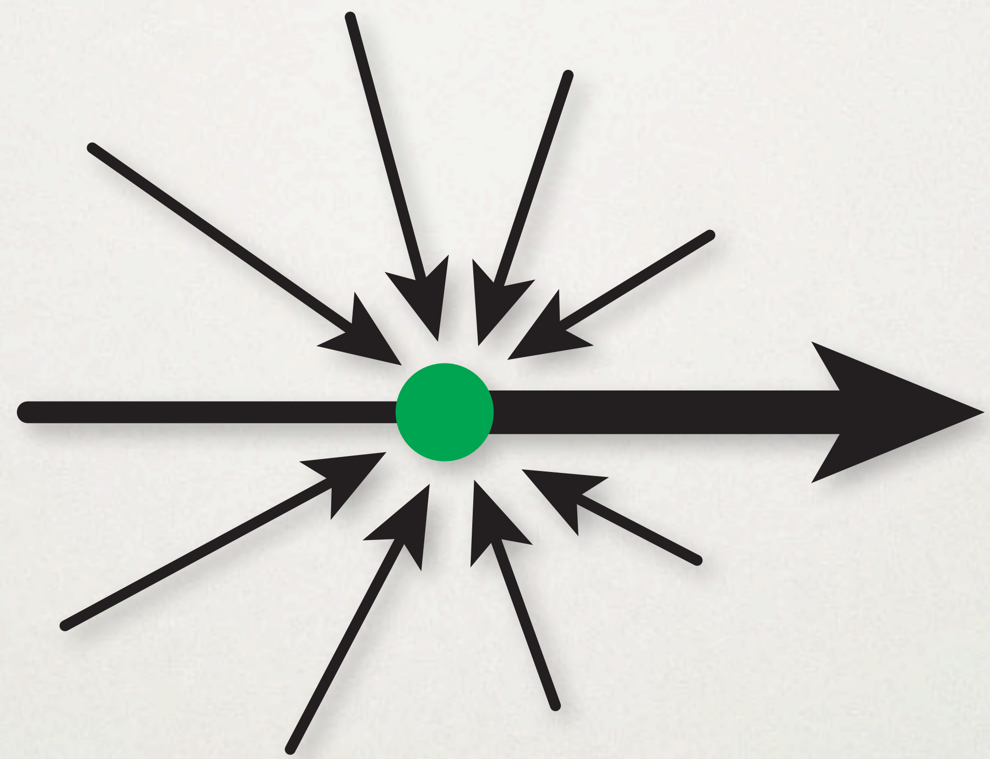
In-Scattering



$$(\vec{\omega} \cdot \nabla_i) L(\mathbf{x} \rightarrow \vec{\omega}) = \sigma_s(\mathbf{x}) \, L_i(\mathbf{x} \rightarrow \vec{\omega})$$

$\sigma_s(\mathbf{x})$: scattering coefficient [1/m]

In-Scattering



$$(\vec{\omega} \cdot \nabla_i) L(\mathbf{x} \rightarrow \vec{\omega}) = \sigma_s(\mathbf{x}) L_i(\mathbf{x} \rightarrow \vec{\omega})$$

$$L_i(\mathbf{x} \rightarrow \vec{\omega}) = \int_{\Omega_{4\pi}} p(\mathbf{x}, \vec{\omega}', \vec{\omega}) L(\mathbf{x} \leftarrow \vec{\omega}') \, d\vec{\omega}'$$

The Phase Function

$$p(\mathbf{x}, \vec{\omega}' \to \vec{\omega})$$

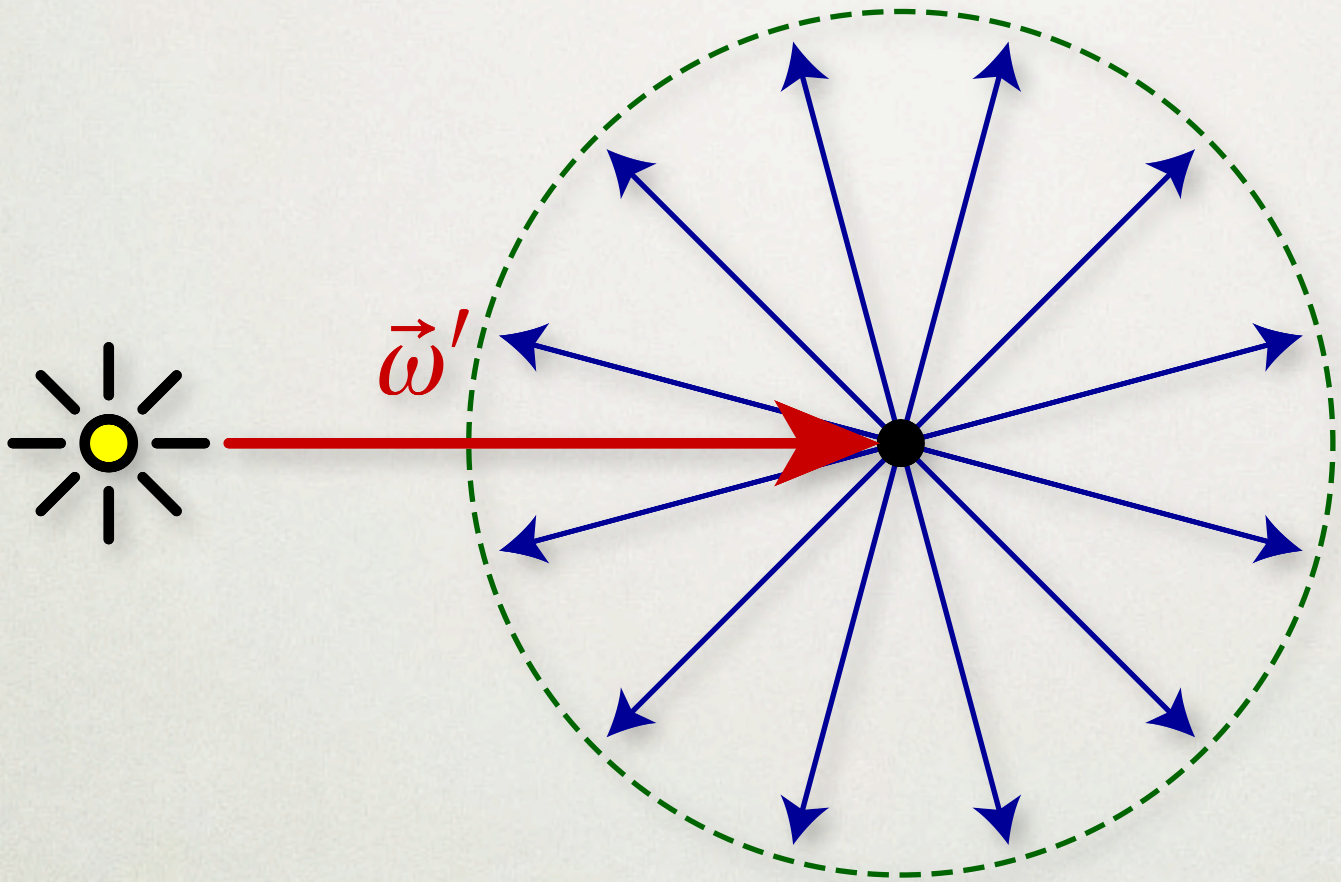
or

$$p(\mathbf{x}, \vec{\omega}', \vec{\omega})$$

- Local, directional distribution of scattering
- Integrates to 1 over all directions:

$$\int_{\Omega_{4\pi}} p(\mathbf{x}, \vec{\omega}', \vec{\omega})\, d\vec{\omega}' = 1$$

The Phase Function



isotropic scattering

$$p(\mathbf{x}, \vec{\omega}', \vec{\omega}) = \frac{1}{4\pi}$$

Anisotropic Scattering

- Anisotropy parameter g (average cosine):

$$g = \int_{\Omega_{4\pi}} p(\mathbf{x}, \vec{\omega}', \vec{\omega}) \cos(\theta)\, d\vec{\omega}'$$

where,

$$\cos(\theta) = \vec{\omega} \cdot \vec{\omega}'$$

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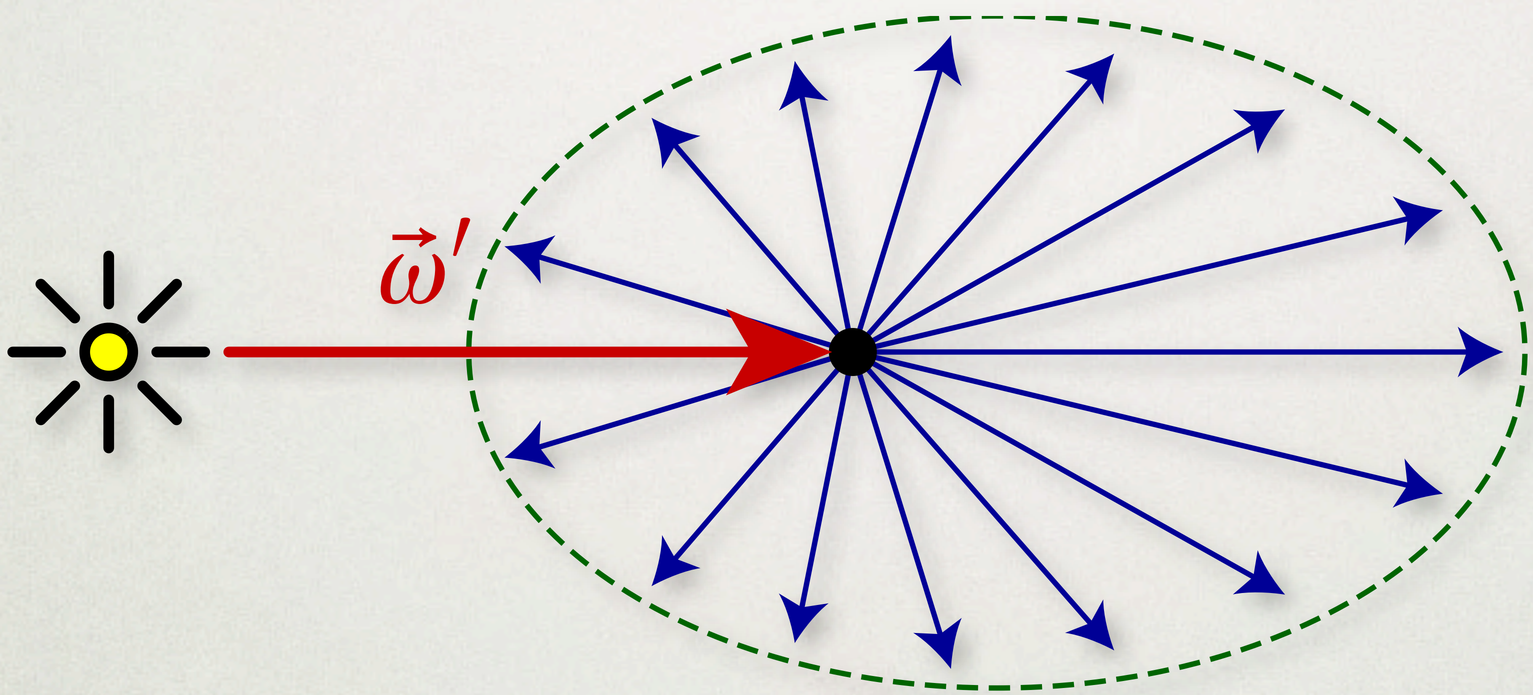
where,

$$\cos(\theta) = \vec{\omega} \cdot \vec{\omega}'$$

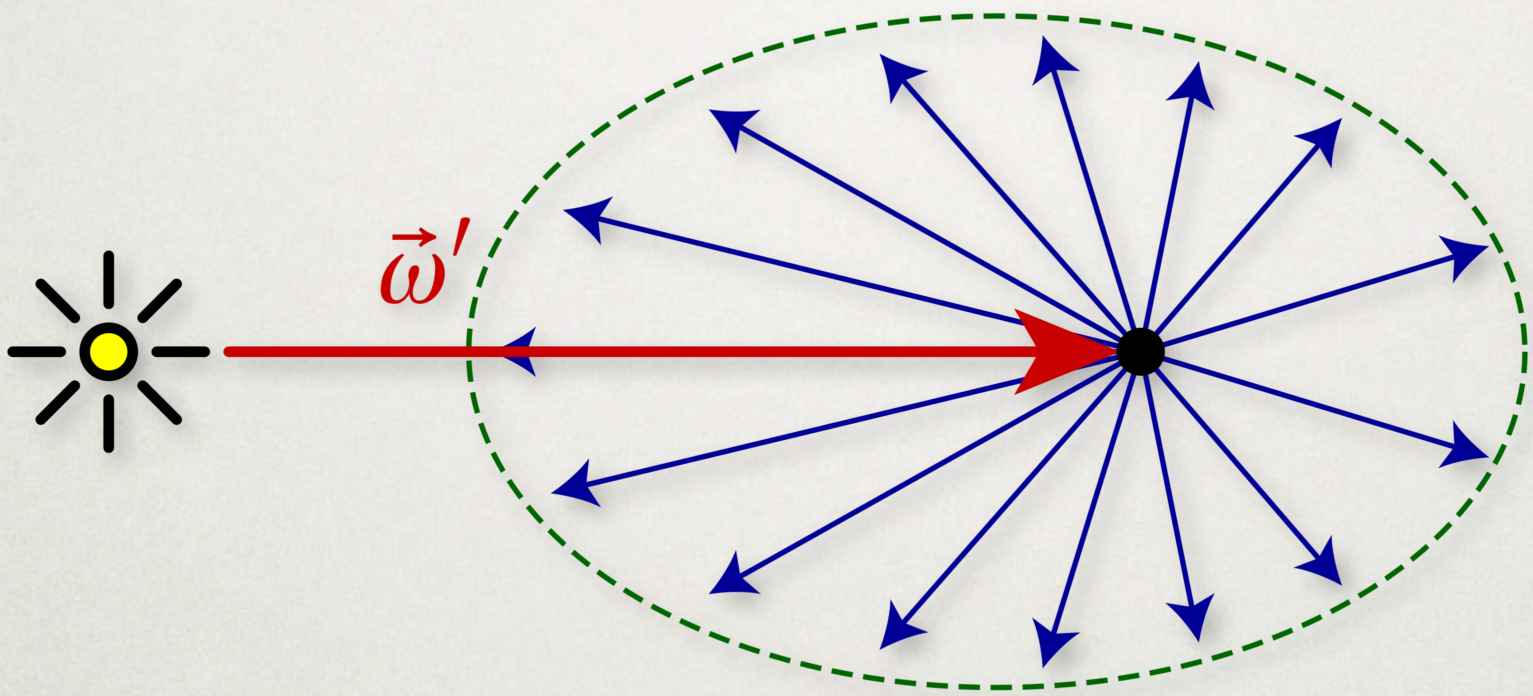
- g=0: isotropic scattering

The Phase Function



g > 0: forward scattering

The Phase Function

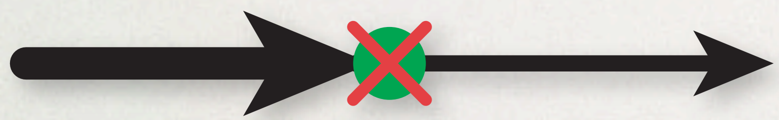


g < 0: backward scattering

The 4 Scattering Events

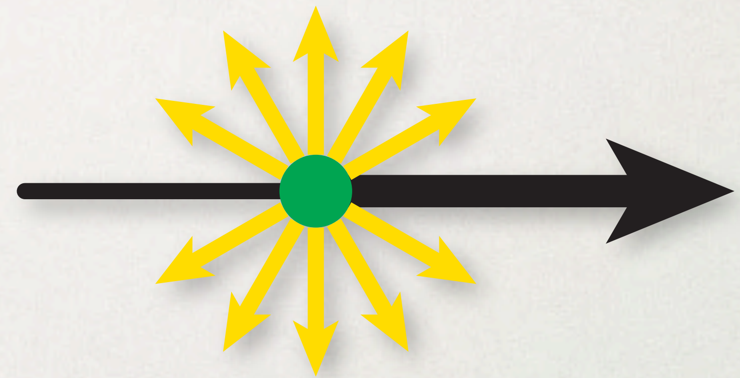
Absorption

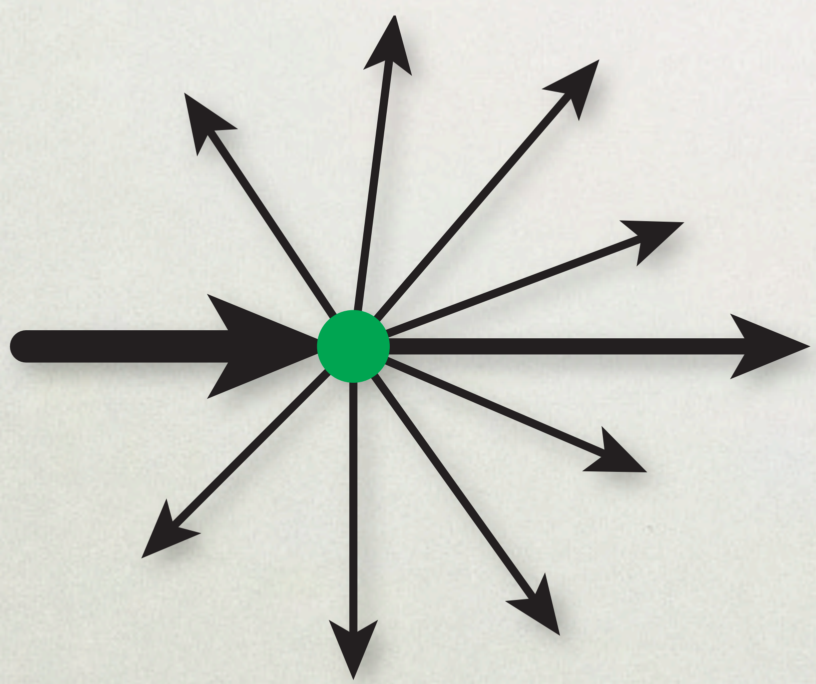
$(\vec{\omega} \cdot \nabla_a) L(\mathbf{x} \to \vec{\omega})$



Emission

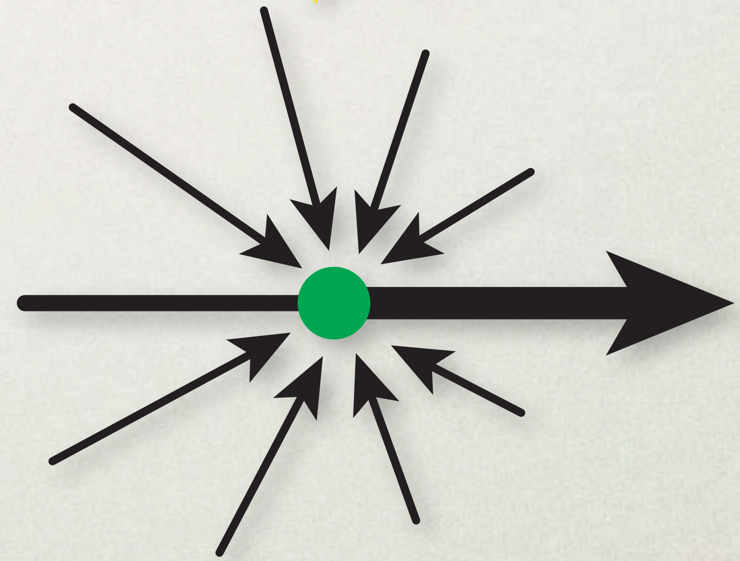
$(\vec{\omega} \cdot \nabla_e) L(\mathbf{x} \to \vec{\omega})$





Out-scattering

$(\vec{\omega} \cdot \nabla_o) L(\mathbf{x} \to \vec{\omega})$



In-scattering

$(\vec{\omega} \cdot \nabla_i) L(\mathbf{x} \to \vec{\omega})$

Radiative Transport Eqn

$$(\vec{\omega} \cdot \nabla) L(\mathbf{x} \rightarrow \vec{\omega}) = -\underbrace{\sigma_a(\mathbf{x}) L(\mathbf{x} \rightarrow \vec{\omega})}_{\text{absorption}}$$

Radiative Transport Eqn

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$$\underbrace{\sigma_a(\mathbf{x})L(\mathbf{x}\to\vec{\omega}) - \sigma_s(\mathbf{x})\,L(\mathbf{x}\to\vec{\omega})}_{\text{extinction}}$$

Radiative Transport Eqn

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$$\underbrace{\qquad\qquad\qquad\qquad\qquad}_{\text{extinction}}$$

$$\underbrace{\sigma_a(\mathbf{x})\,L_e(\mathbf{x}\to\vec{\omega})}_{\text{emission}}$$

Radiative Transport Eqn

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$$\underbrace{\sigma_a(\mathbf{x})\,L_e(\mathbf{x}\rightarrow\vec{\omega})}_{\text{emission}} + \underbrace{\sigma_s(\mathbf{x})\,L_i(\mathbf{x}\rightarrow\vec{\omega})}_{\text{in-scattering}}$$

Radiative Transport Eqn

$$(\vec{\omega} \cdot \nabla) L(\mathbf{x} \rightarrow \vec{\omega}) = -\underbrace{\sigma_t(\mathbf{x}) L(\mathbf{x} \rightarrow \vec{\omega})}_{\text{extinction}} +$$

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$$\sigma_t(\mathbf{x}) = \sigma_a(\mathbf{x}) + \sigma_s(\mathbf{x})$$

extinction coefficient

Volume Rendering Eqn

$$L(\mathbf{x} \leftarrow \vec{\omega}) = \underbrace{T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s)\, L(\mathbf{x}_s \rightarrow -\vec{\omega})}_{\text{reduced surface radiance}} +$$

$$\underbrace{\int_0^s T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t)\, \sigma_a(\mathbf{x})\, L_e(\mathbf{x} \rightarrow -\vec{\omega})\; dt}_{\text{accumulated emitted radiance}} +$$

$$\underbrace{\int_0^s T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t)\, \sigma_s(\mathbf{x}_t)\, L_i(\mathbf{x}_t \rightarrow -\vec{\omega})\; dt}_{\text{accumulated in-scattered radiance}}$$

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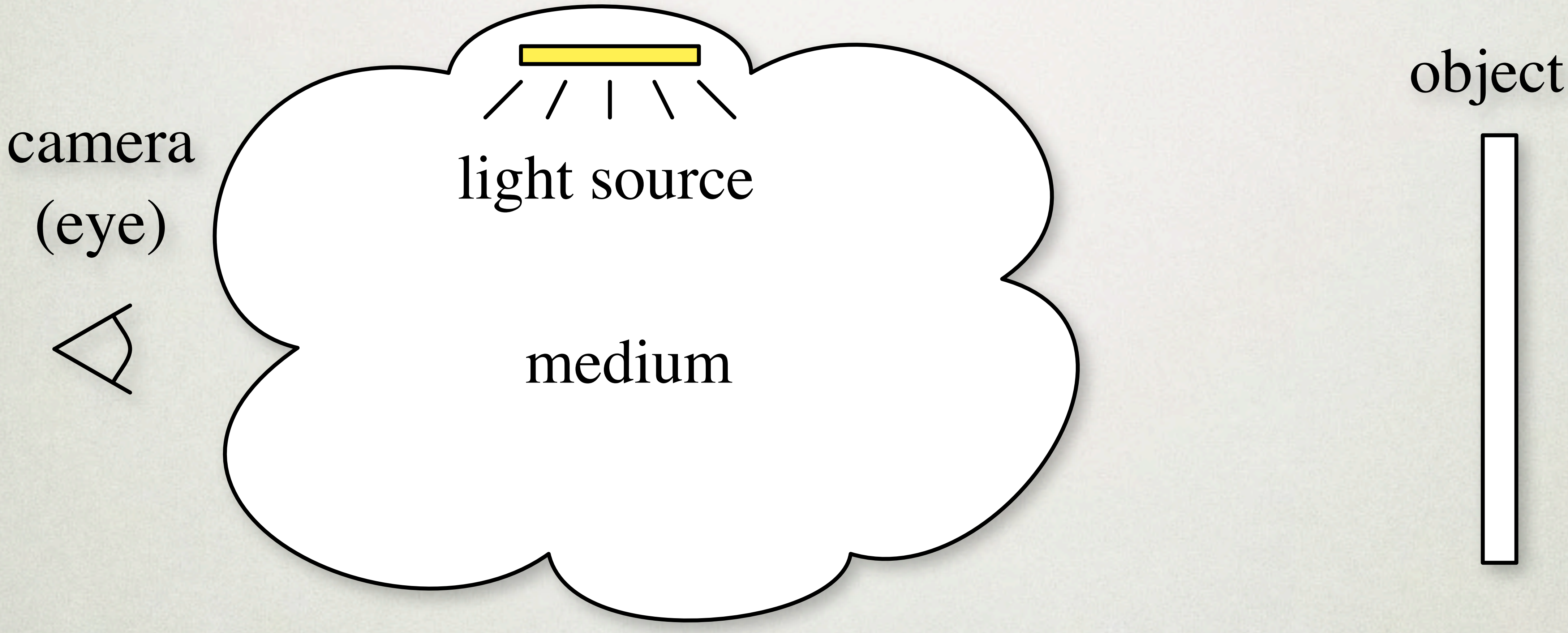
Scene with Medium



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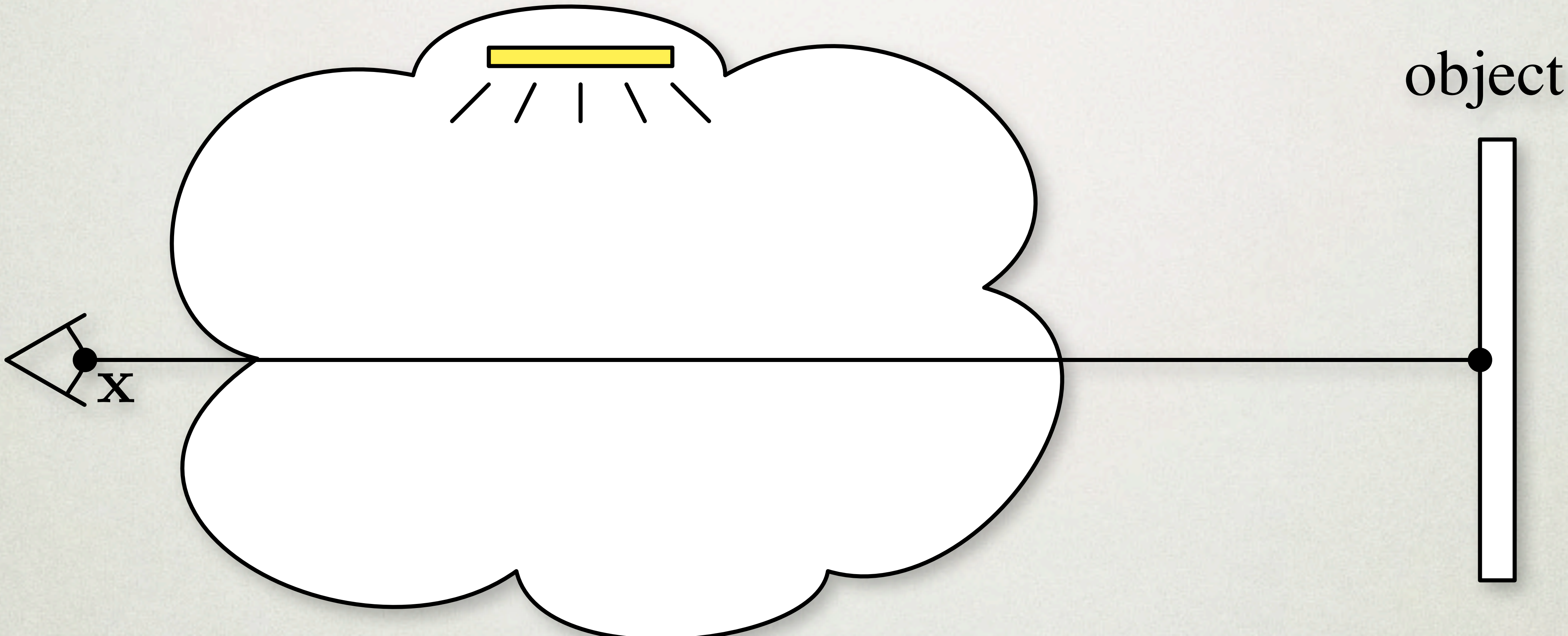

$$L(\mathbf{x}, \vec{\omega}) = \int_0^s T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_i(\mathbf{x}_t, \vec{\omega})\ dt + T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s)L(\mathbf{x}_s, \vec{\omega})$$

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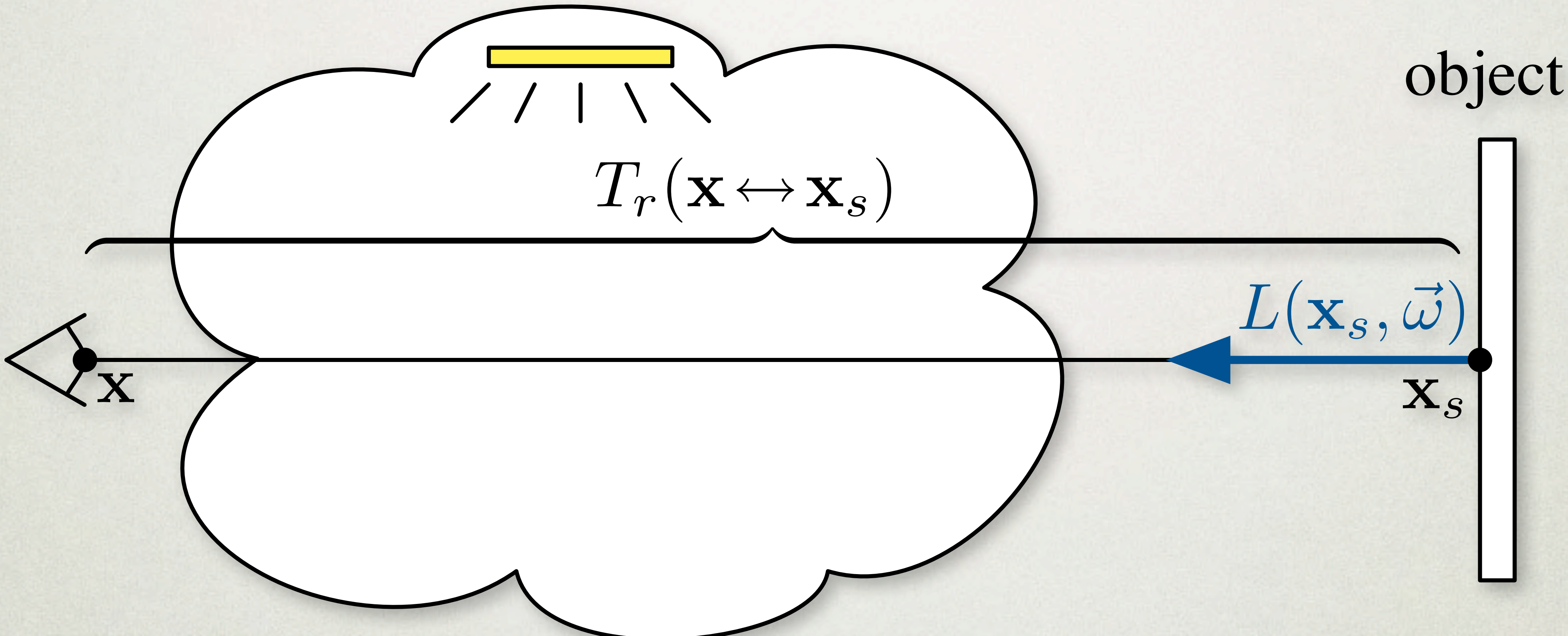

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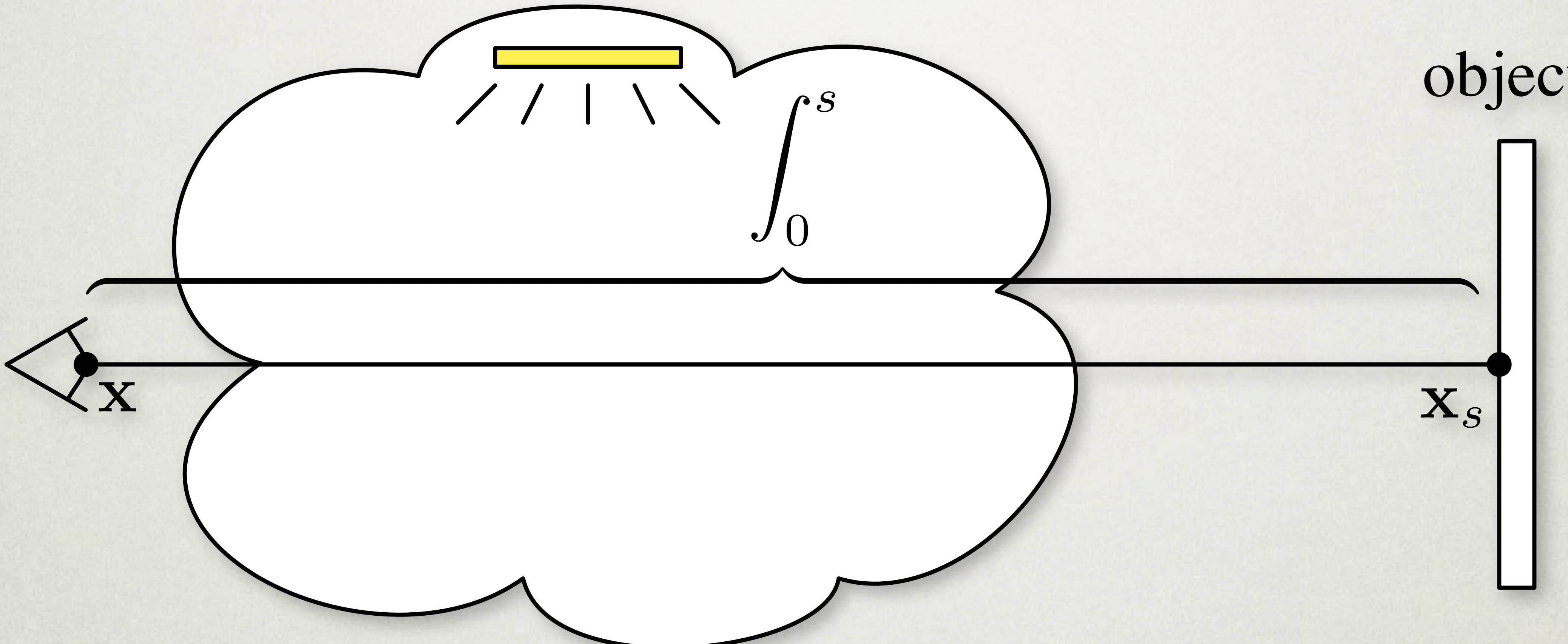
Transmittance: $T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s) = \exp\left(-\int_0^s \sigma_t(\mathbf{x} + t\vec{\omega})\, dt\right)$

Volume Rendering Eqn



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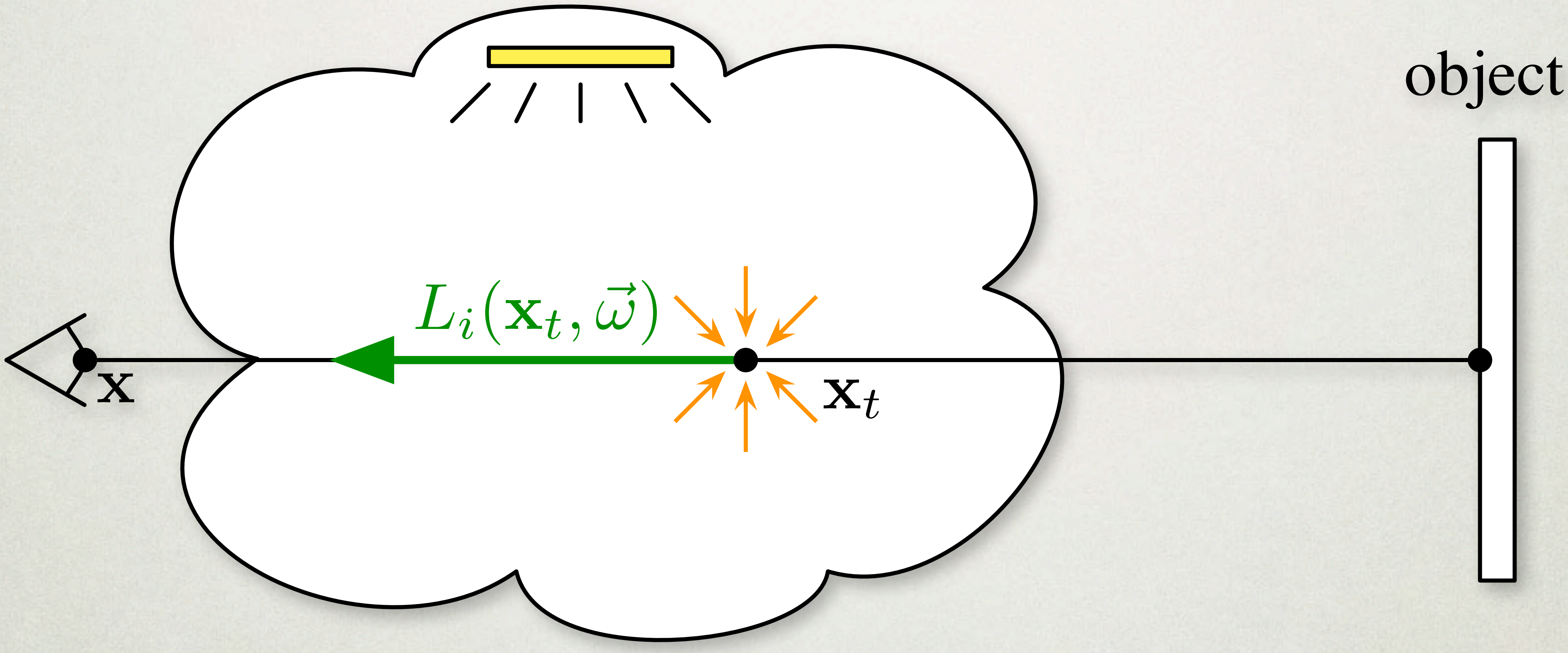
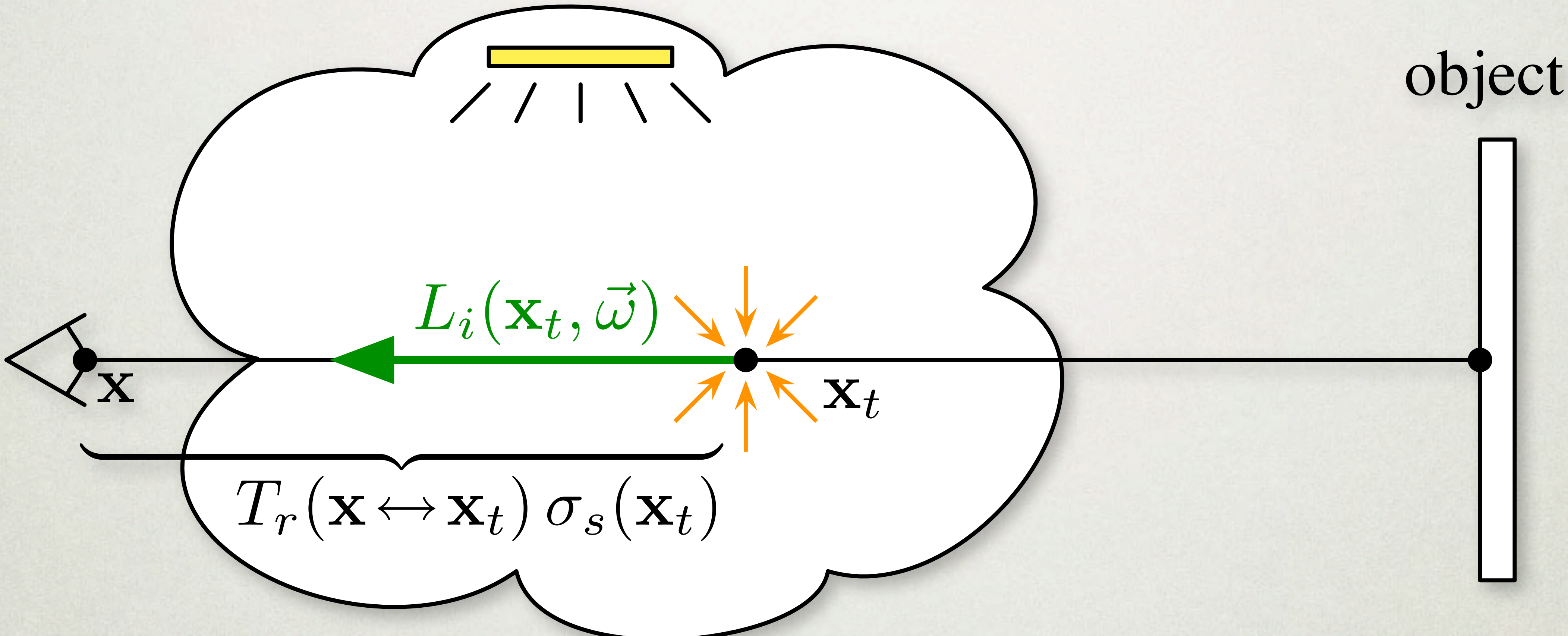


$$L(\mathbf{x}, \vec{\omega}) = \int_0^s T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t)\sigma_s(\mathbf{x}_t) \boxed{L_i(\mathbf{x}_t, \vec{\omega})} \, dt + T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s) L(\mathbf{x}_s, \vec{\omega})$$

$$L_i(\mathbf{x}_t, \vec{\omega}) = \int_{\Omega_{4\pi}} p(\mathbf{x}_t, \vec{\omega}_t, \vec{\omega}) L(\mathbf{x}_t, \vec{\omega}_t) \, d\omega_t$$

Volume Rendering Eqn



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Media Properties

$\sigma_s(\mathbf{x})$: scattering coefficient $[1/m]$

$\sigma_a(\mathbf{x})$: absorption coefficient $[1/m]$

$p(\mathbf{x}, \vec{\omega}', \vec{\omega})$: phase function $[1/sr]$

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Homogeneous
spatially constant

Heterogeneous
spatially varying

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Homogeneous
spatially constant

Heterogeneous
spatially varying

Isotropic
directionally constant

Anisotropic
directionally varying

Derived Properties

$\sigma_t(\mathbf{x}) = \sigma_s(\mathbf{x}) + \sigma_a(\mathbf{x})$: extinction coefficient $[1/m]$

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$\frac{\sigma_s}{\sigma_t}$: scattering albedo $[none]$

$T_r(\mathbf{x}'\leftrightarrow\mathbf{x})$: transmittance $[none]$

$$\exp\left(-\int_0^d \sigma_t(\mathbf{x} + t\vec{\omega})\,dt\right)$$

Outline

- Theoretical background
- Methods for rendering participating media

Available Techniques

Rendering Participating Media

- "Ray Tracing Volume Densities." Kajiya and Herzen. 1984.
- "The Rendering Equation." Kajiya. 1986.
- "The Zonal Method for Calculating Light Intensities in the Presence of a Participating Medium." Rushmeier and Torrance. 1987.
- "Efficient Light Propagation for Multiple Anisotropic Volume Scattering." Max. 1994.
- "Multiple Scattering as a Diffusion Process." Stam. 1995.
- "Rendering Participating Media with Bidirectional Path Tracing." Lafortune and Willems. 1996.
- "Efficient Simulation of Light Transport in Scenes with Participating Media using Photon Maps." Jensen and Christensen. 1998.
- "Metropolis Light Transport for Participating Media." Pauly et al. 2000.
- "Practical Rendering of Multiple Scattering Effects in Participating Media." Premože et al. 2004.
- "Multidimensional Lightcuts." Walter et al. 2006.
- "Radiance Caching for Participating Media." Jarosz et al. 2008.
- "The Beam Radiance Estimate for Volumetric Photon Mapping." Jarosz et al. 2008.

Available Techniques

Rendering Participating Media

- Path tracing
- Ray marching
- Metropolis
- Finite element methods (Radiosity)
- Diffusion
- Interpolation methods (Radiance caching)
- Density estimation methods (Photon mapping)
- VPL methods (Lightcut, Instant radiosity)

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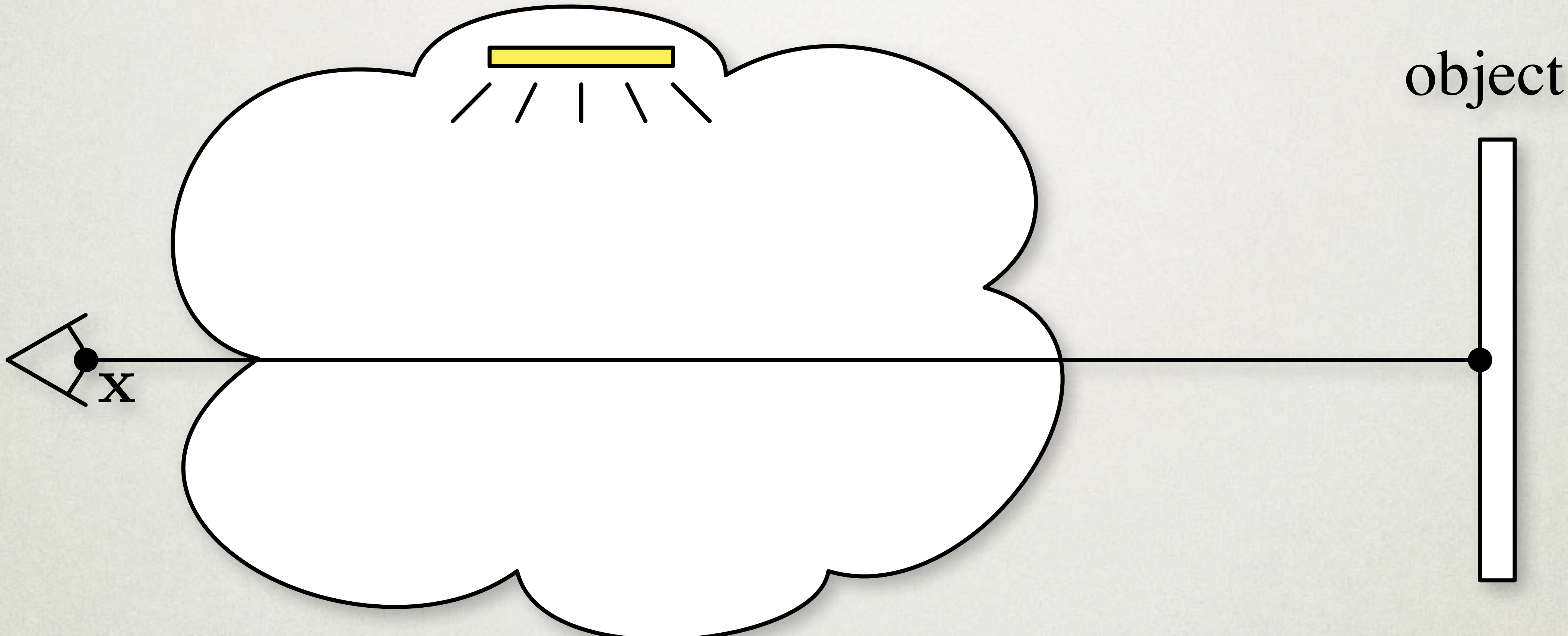
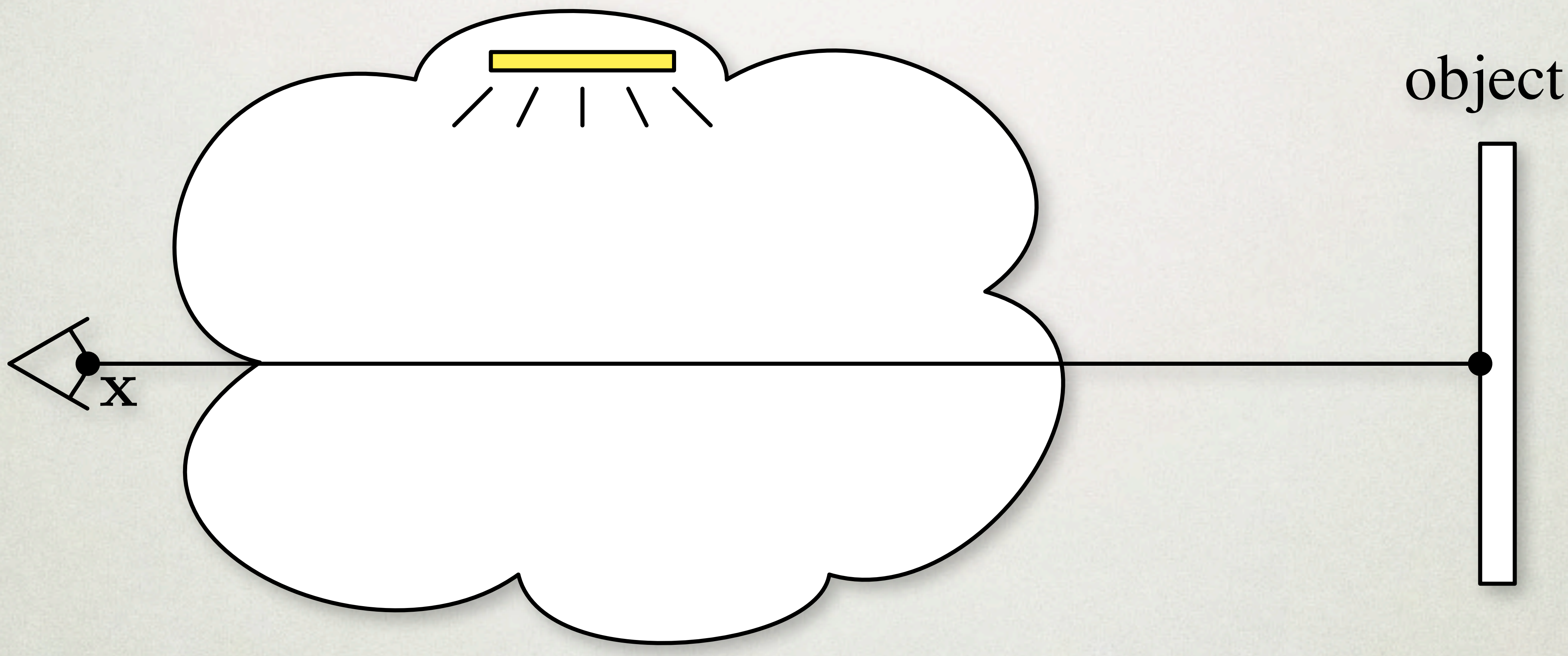
Ray Marching



$$L(\mathbf{x},\vec{\omega}) = \int_0^s T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_i(\mathbf{x}_t,\vec{\omega})\ dt + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

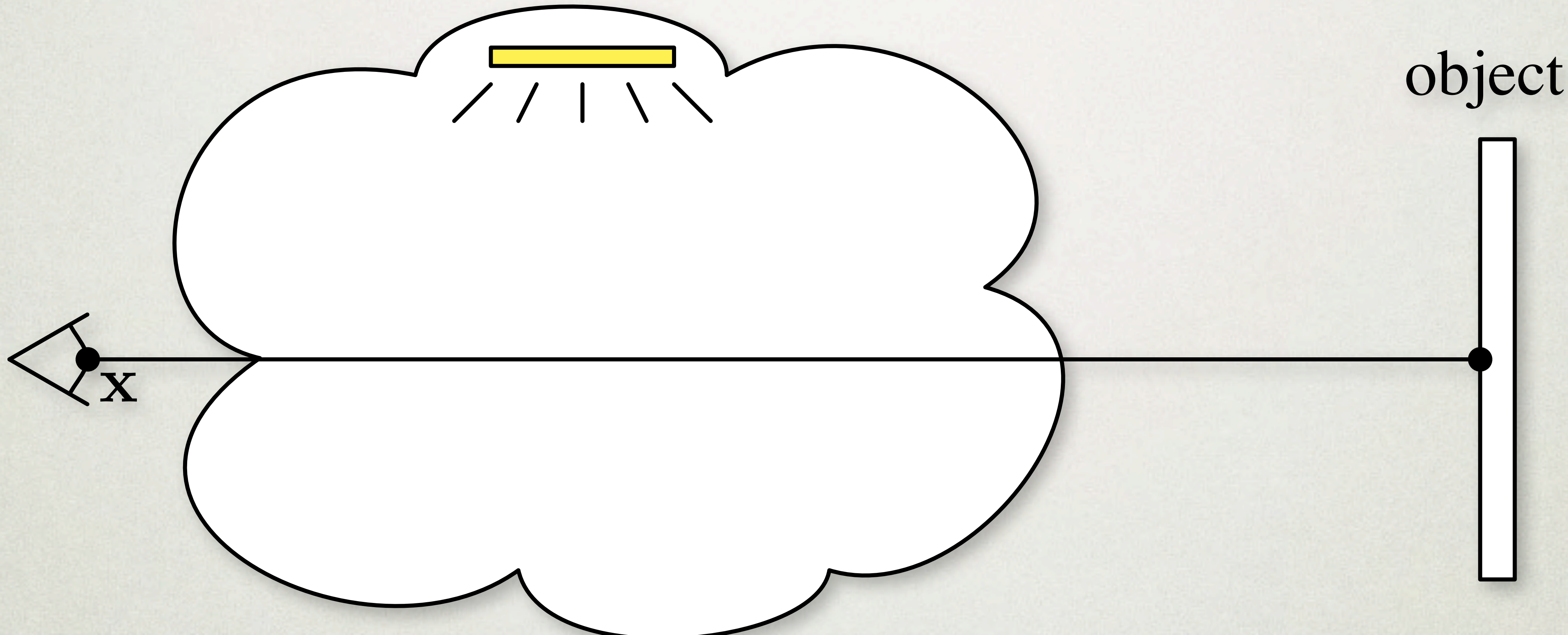
Ray Marching



$$L(\mathbf{x},\vec{\omega}) = \int_0^s T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_i(\mathbf{x}_t,\vec{\omega})\,dt + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

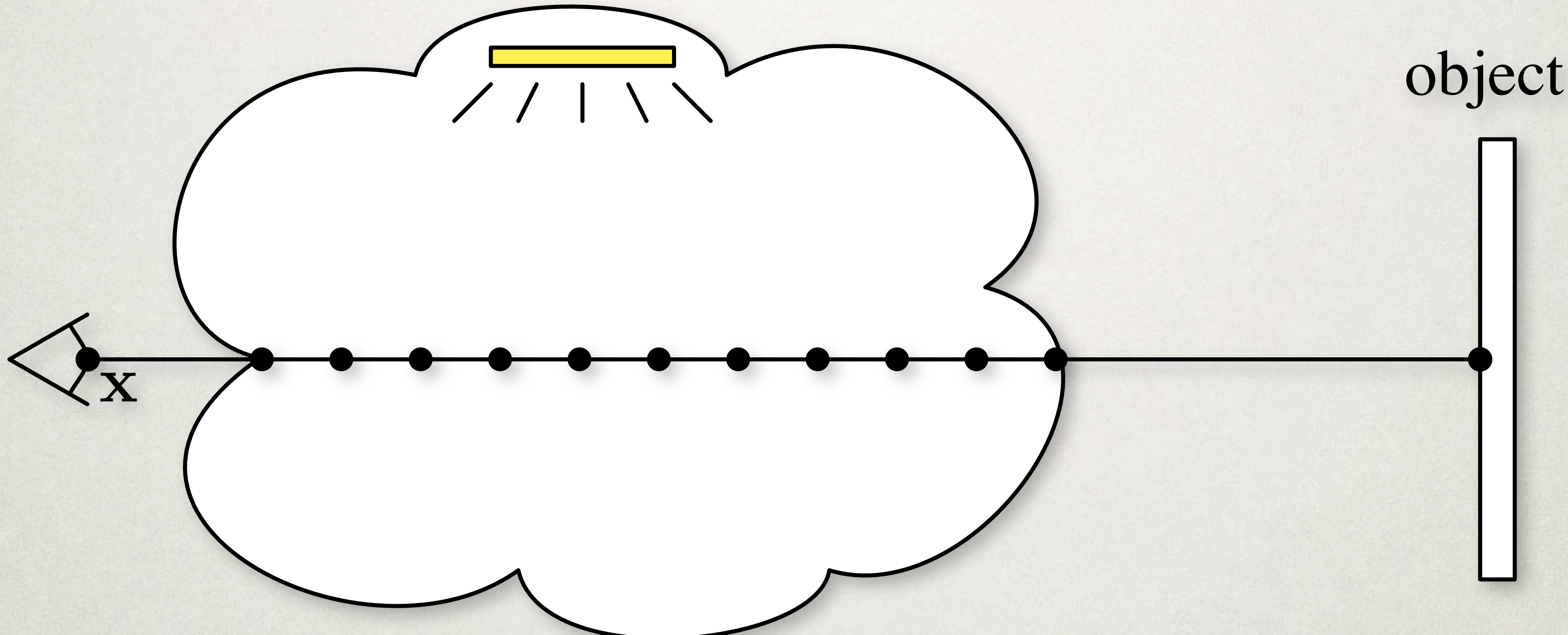
- approximate/compute using Riemann sum

Ray Marching



$$L(\mathbf{x},\vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_i(\mathbf{x}_t,\vec{\omega})\Delta_t + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

Ray Marching



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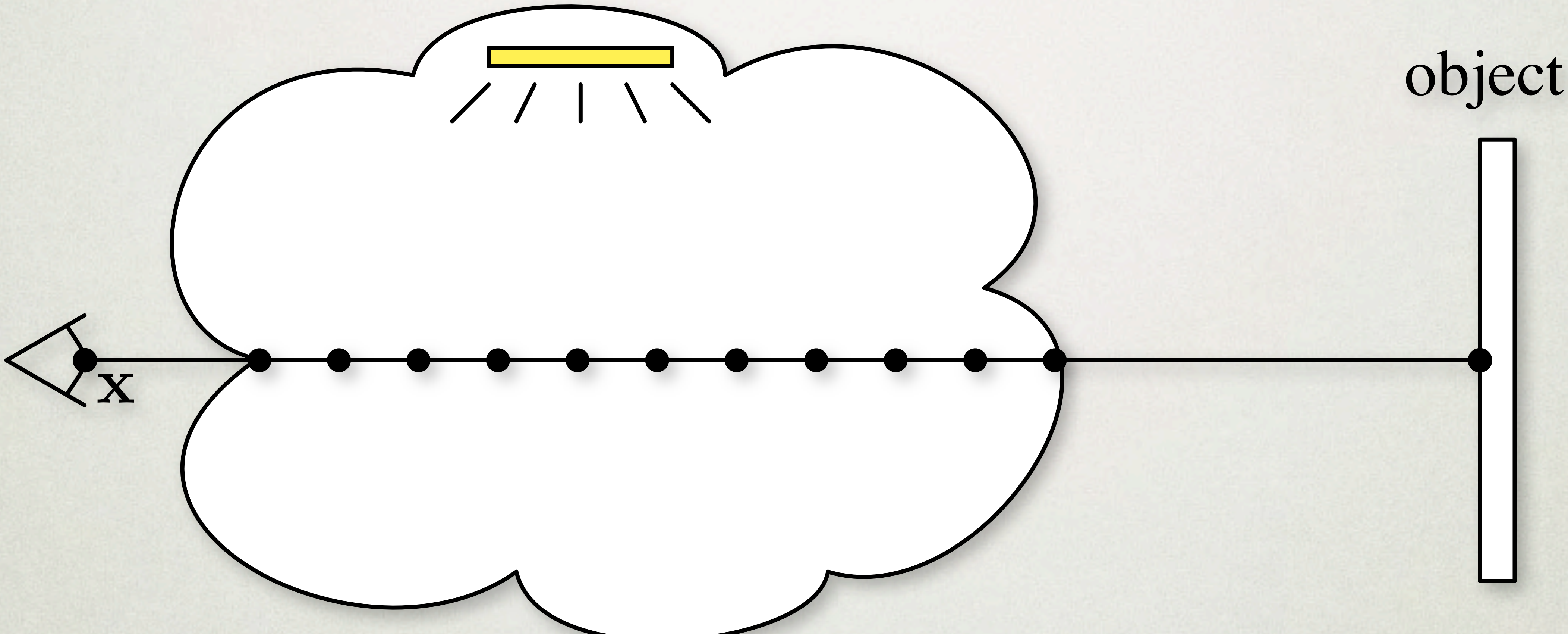
Computing T_R

- In general:

$$T_r(\mathbf{x}' \leftrightarrow \mathbf{x}) = \exp\left(-\int_0^d \sigma_t(\mathbf{x} + t\vec{\omega})\, dt\right)$$

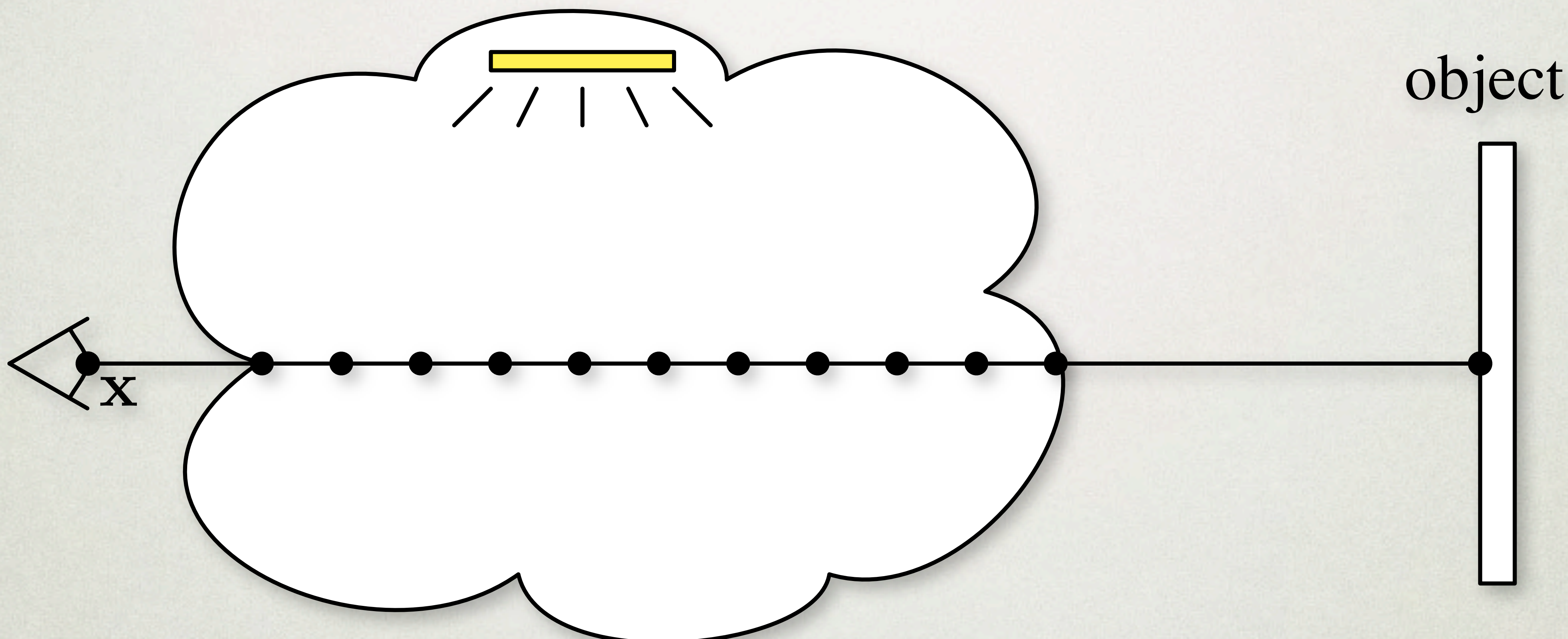
- In homogeneous medium:
  - $T_r(\mathbf{x}' \leftrightarrow \mathbf{x}) = e^{\|\mathbf{x}' - \mathbf{x}\| \sigma_t}$

Ray Marching (Homogeneous Media)



$$L(\mathbf{x}, \vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t) \sigma_s(\mathbf{x}_t) L_i(\mathbf{x}_t, \vec{\omega}) \Delta_t + T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s) L(\mathbf{x}_s, \vec{\omega})$$

Ray Marching (Homogeneous Media)



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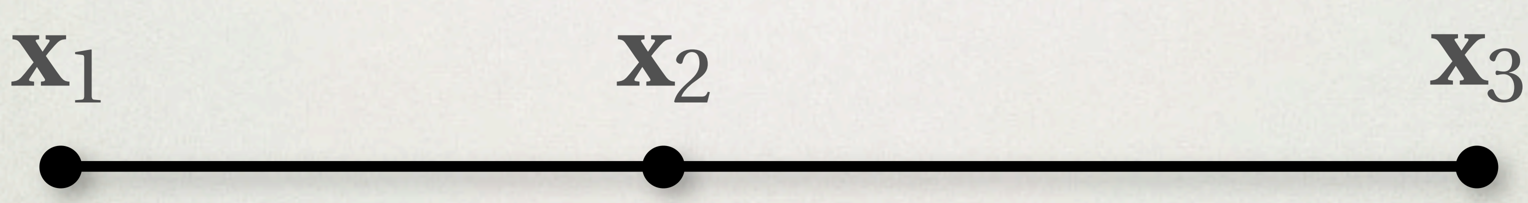
$$\downarrow$$

$$L(\mathbf{x},\vec{\omega}) \approx \sum_{t=0}^{S-1} e^{\|\mathbf{x}_t-\mathbf{x}\|\sigma_t}\sigma_s L_i(\mathbf{x}_t,\vec{\omega})\Delta_t + e^{\|\mathbf{x}_s-\mathbf{x}\|\sigma_t}L(\mathbf{x}_s,\vec{\omega})$$

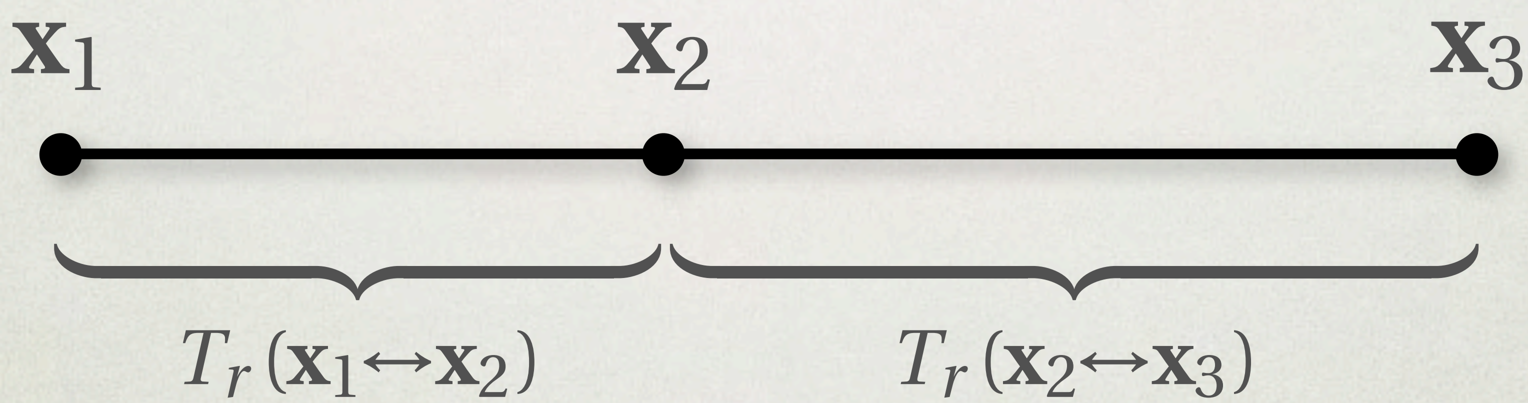
COMPUTING T_R

- In general, if x_1, x_2 and x_3 are collinear,
  - then: $T_r(\mathbf{x}_1 \leftrightarrow \mathbf{x}_3) = T_r(\mathbf{x}_1 \leftrightarrow \mathbf{x}_2)\, T_r(\mathbf{x}_2 \leftrightarrow \mathbf{x}_3)$



Computing T_R

- In general, if x_1, x_2 and x_3 are collinear,
  - then: $T_r(\mathbf{x}_1 \leftrightarrow \mathbf{x}_3) = T_r(\mathbf{x}_1 \leftrightarrow \mathbf{x}_2)\, T_r(\mathbf{x}_2 \leftrightarrow \mathbf{x}_3)$



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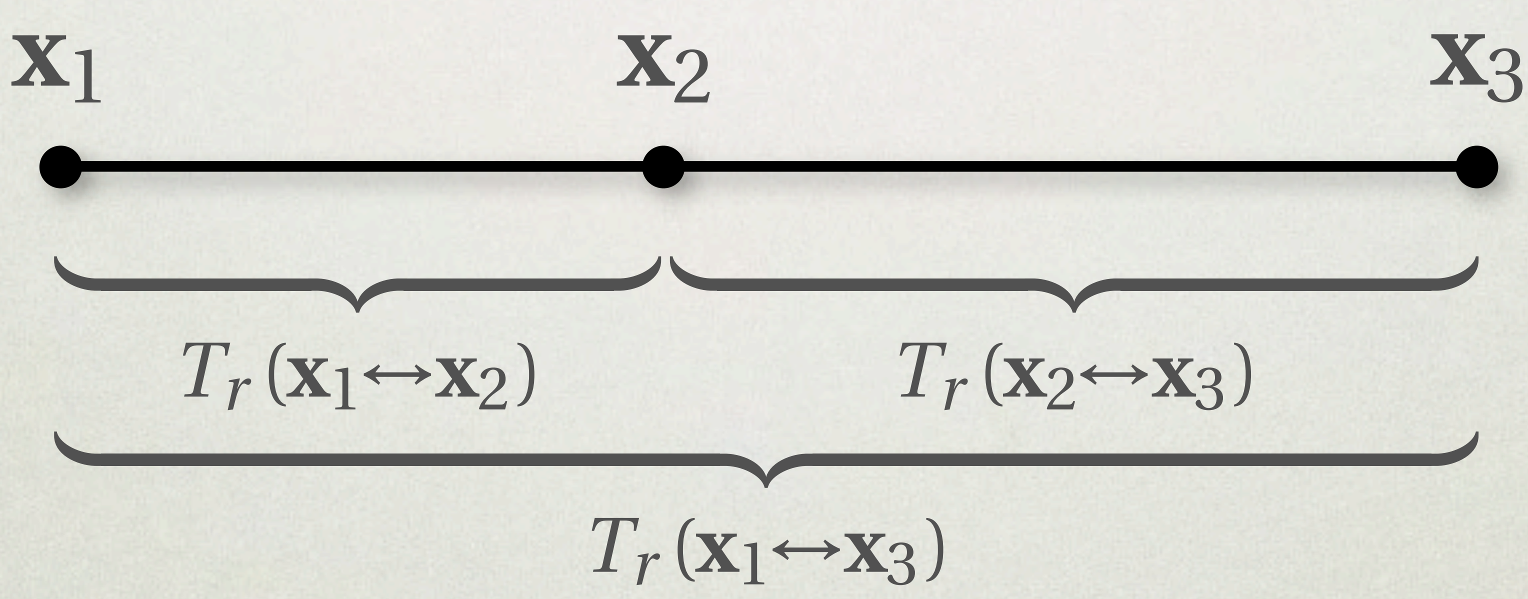
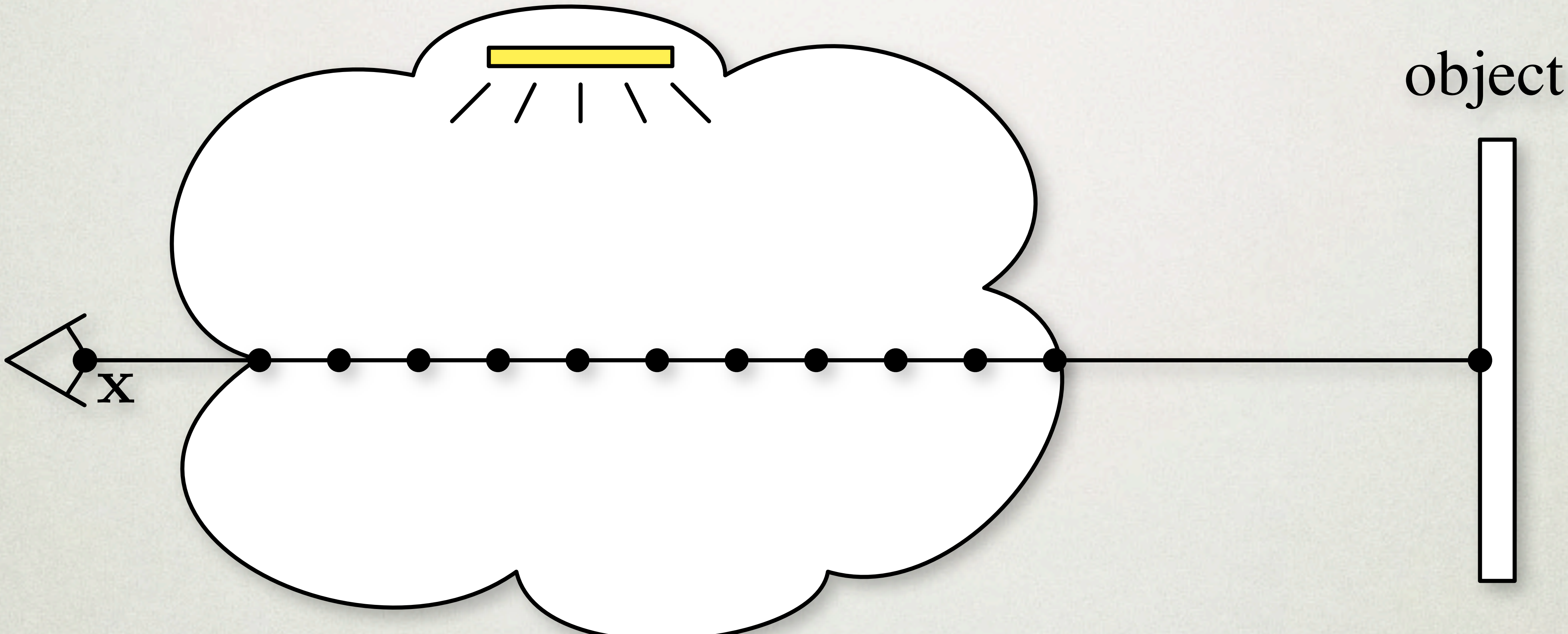


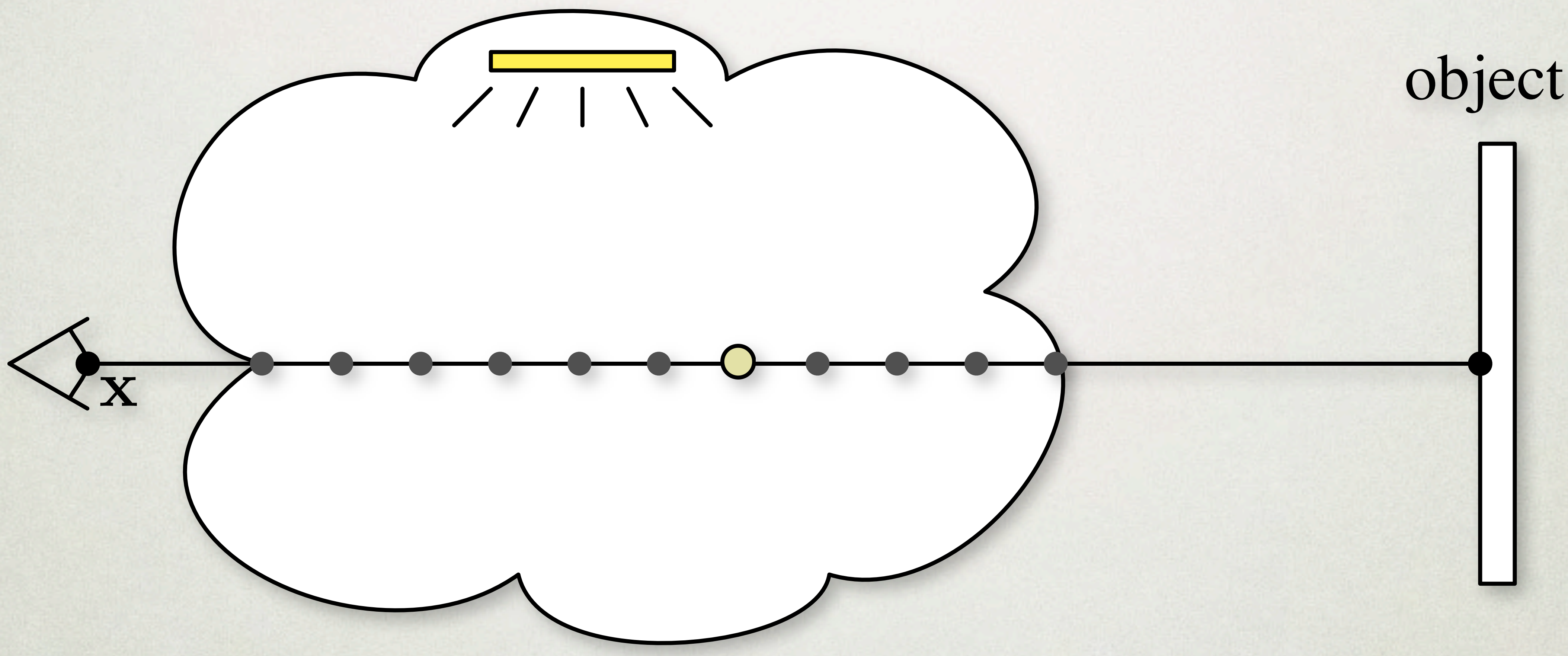
Ray Marching



$$L(\mathbf{x},\vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_i(\mathbf{x}_t,\vec{\omega})\Delta_t + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

- compute T_r incrementally

Ray Marching



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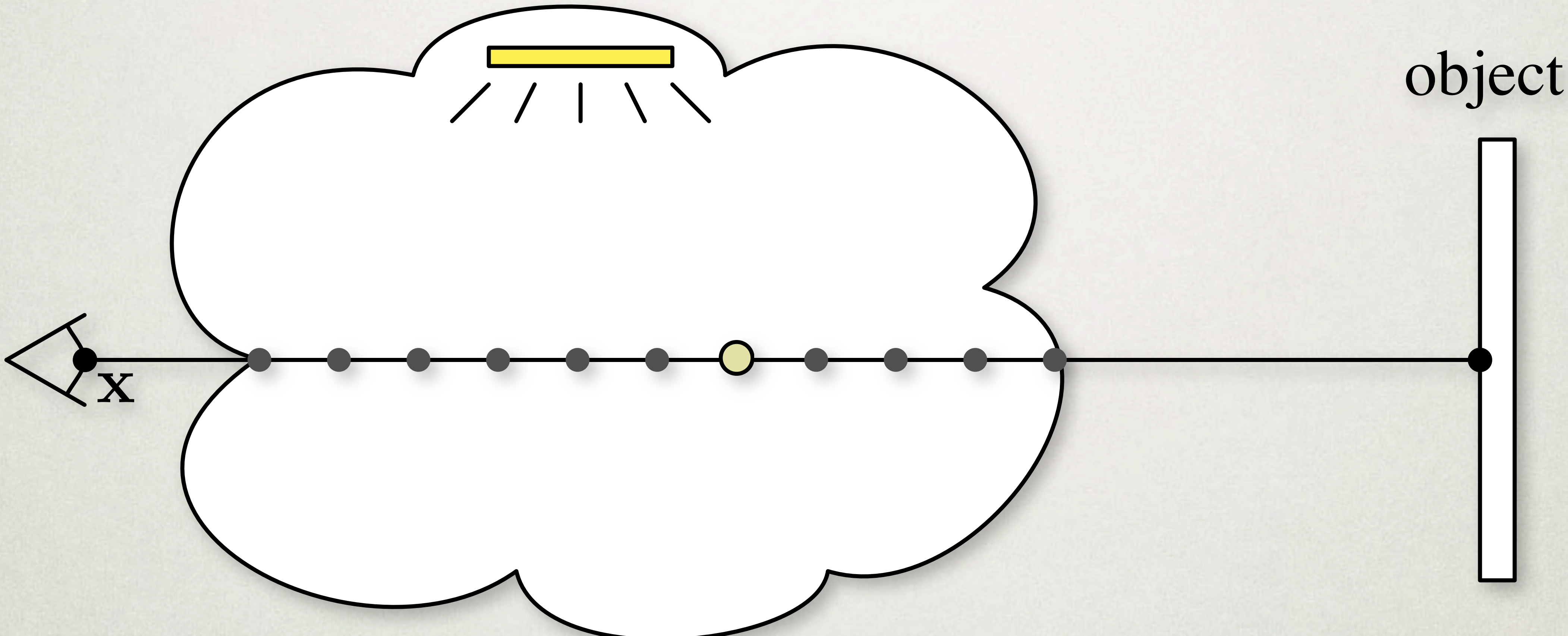
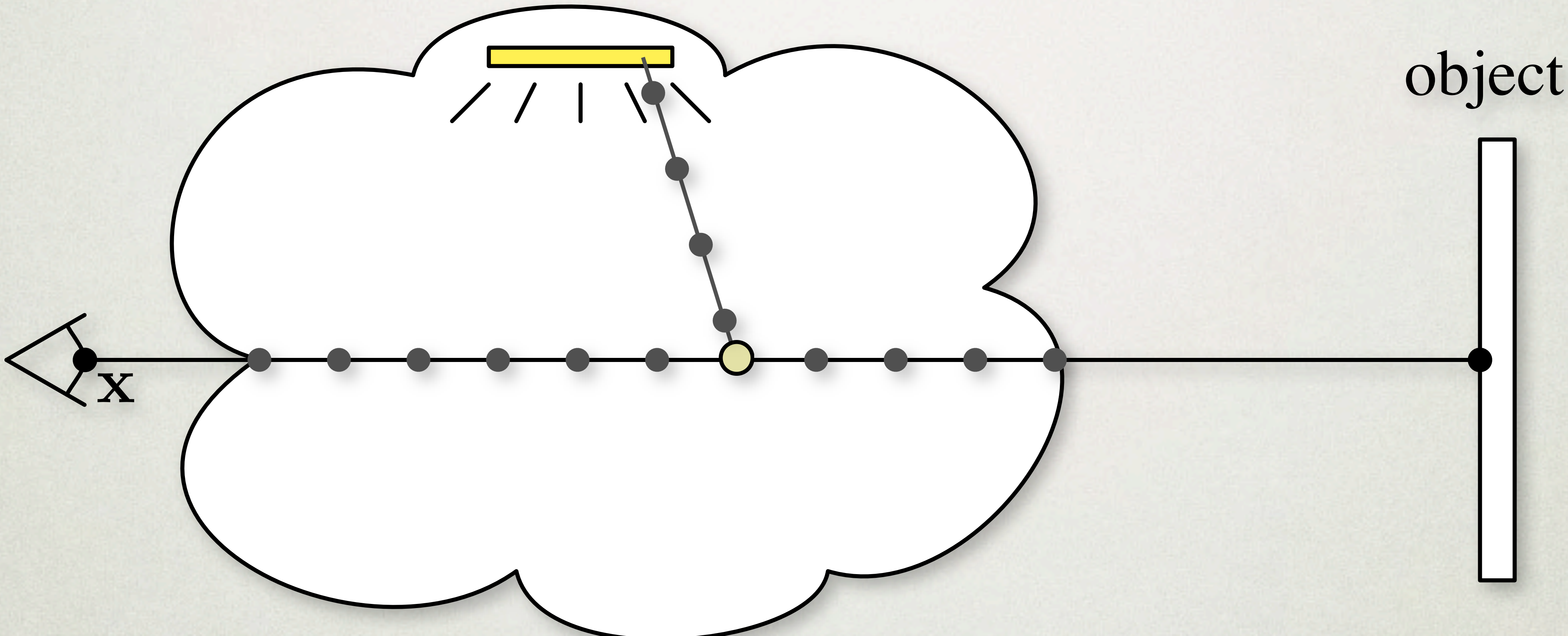
Ray Marching



$$L(\mathbf{x},\vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t) L_i(\mathbf{x}_t,\vec{\omega}) \Delta_t + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s) L(\mathbf{x}_s,\vec{\omega})$$

$$L_i(\mathbf{x}_t,\vec{\omega}) = \int_{\Omega_{4\pi}} p(\mathbf{x}_t,\vec{\omega}_t,\vec{\omega}) L(\mathbf{x}_t,\vec{\omega}_t)\, d\omega_t$$

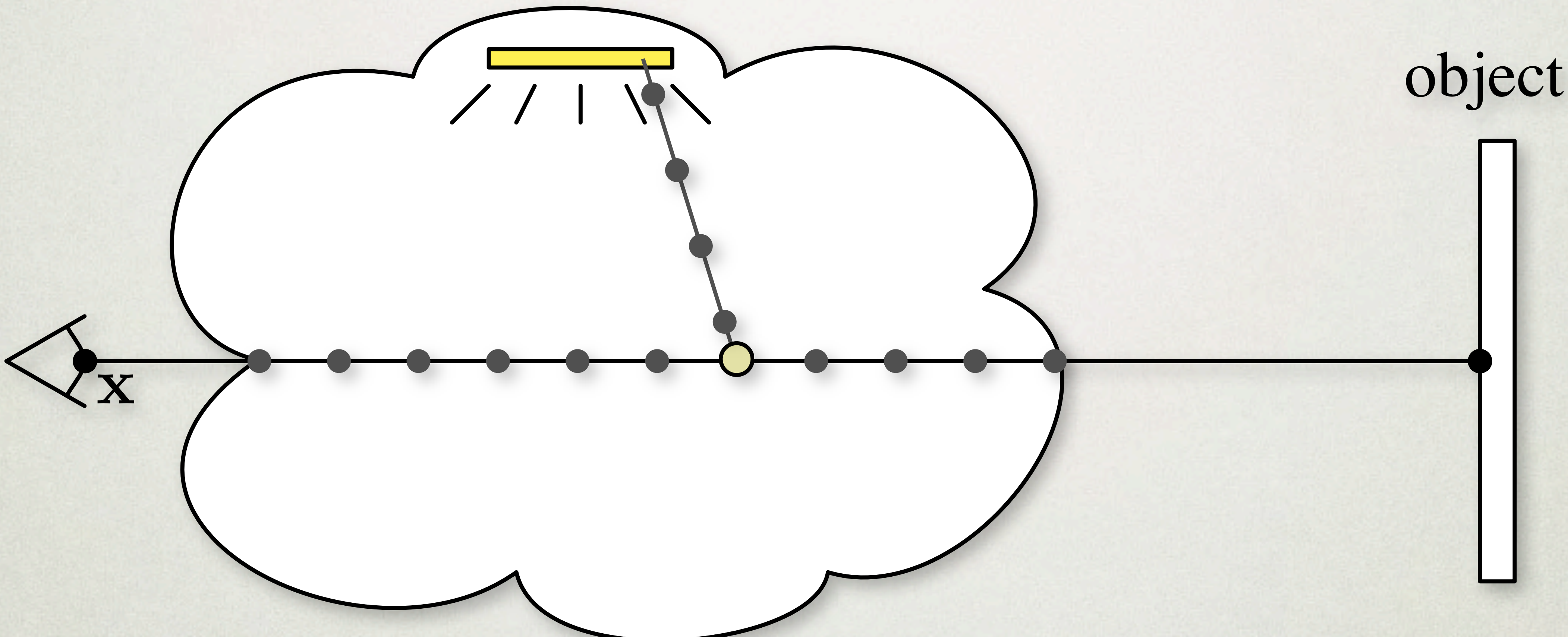
Ray Marching (Single Scattering)



$$L(\mathbf{x},\vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_i(\mathbf{x}_t,\vec{\omega})\Delta_t + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

- assume only single scattering (direct lighting)

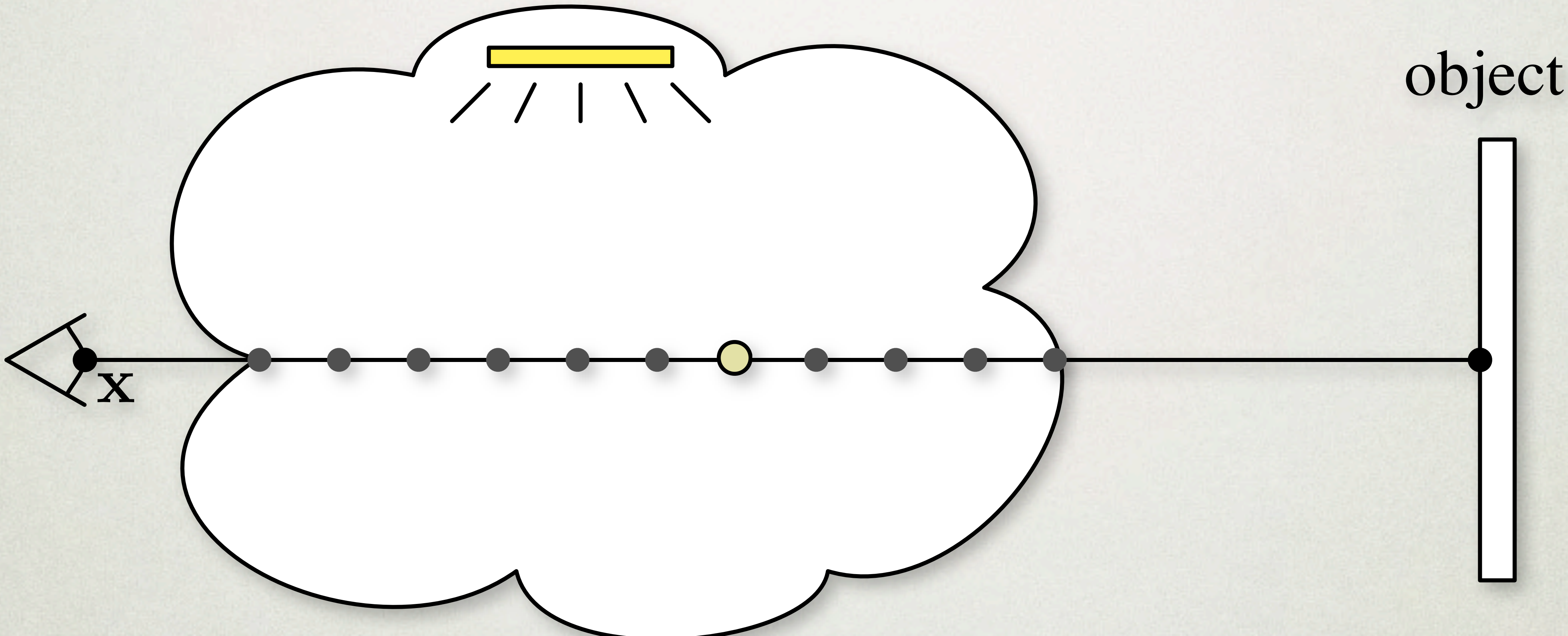
Ray Marching (Single Scattering)



$$L(\mathbf{x},\vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t) L_i(\mathbf{x}_t,\vec{\omega}) \Delta_t + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s) L(\mathbf{x}_s,\vec{\omega})$$

- assume only single scattering (direct lighting)
- trace shadow ray for volumetric shadows

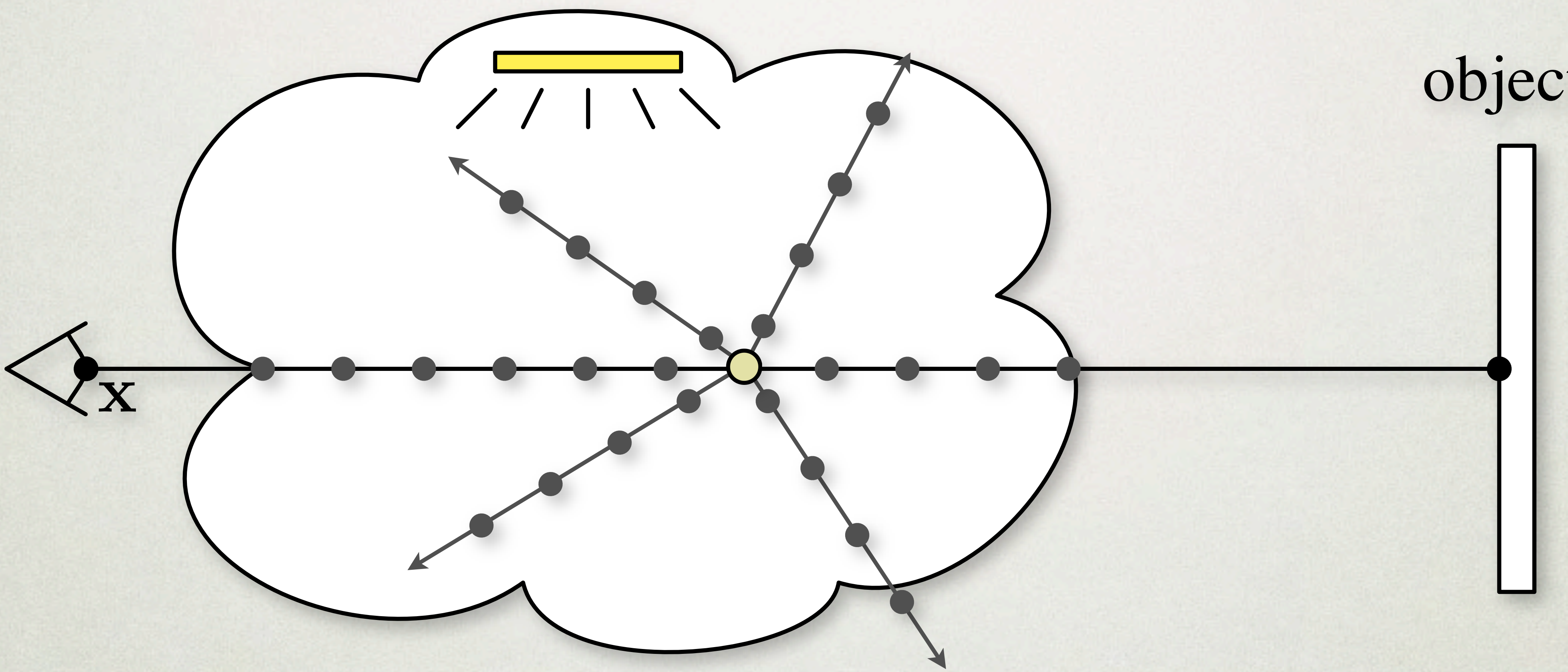
Ray Marching (Multiple Scattering)



$$L(\mathbf{x}, \vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t)\sigma_s(\mathbf{x}_t) L_i(\mathbf{x}_t, \vec{\omega}) \Delta_t + T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s) L(\mathbf{x}_s, \vec{\omega})$$

- recursive ray marching
- exponential growth! expensive!

Ray Marching (Multiple Scattering)



$$L(\mathbf{x},\vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t)\boxed{L_i(\mathbf{x}_t,\vec{\omega})}\Delta_t + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

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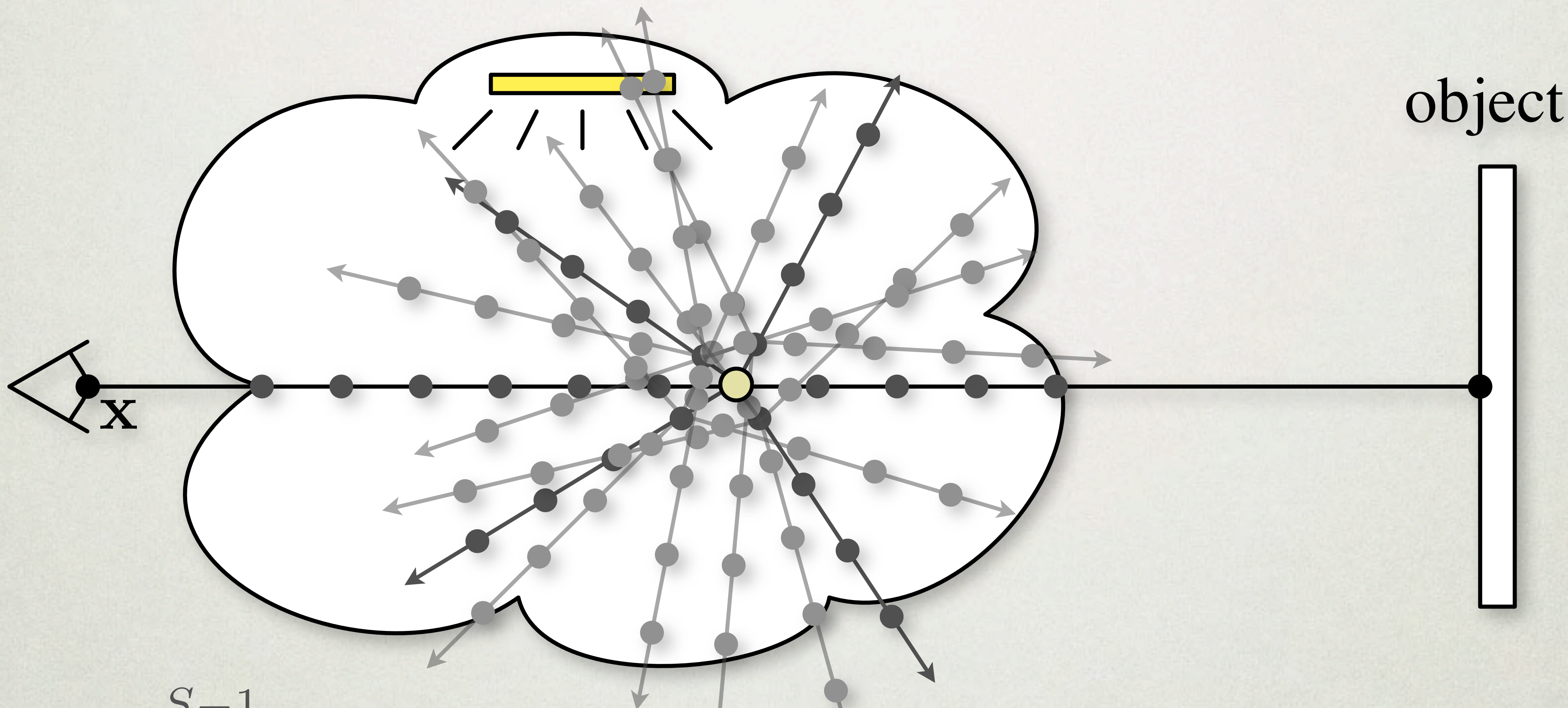

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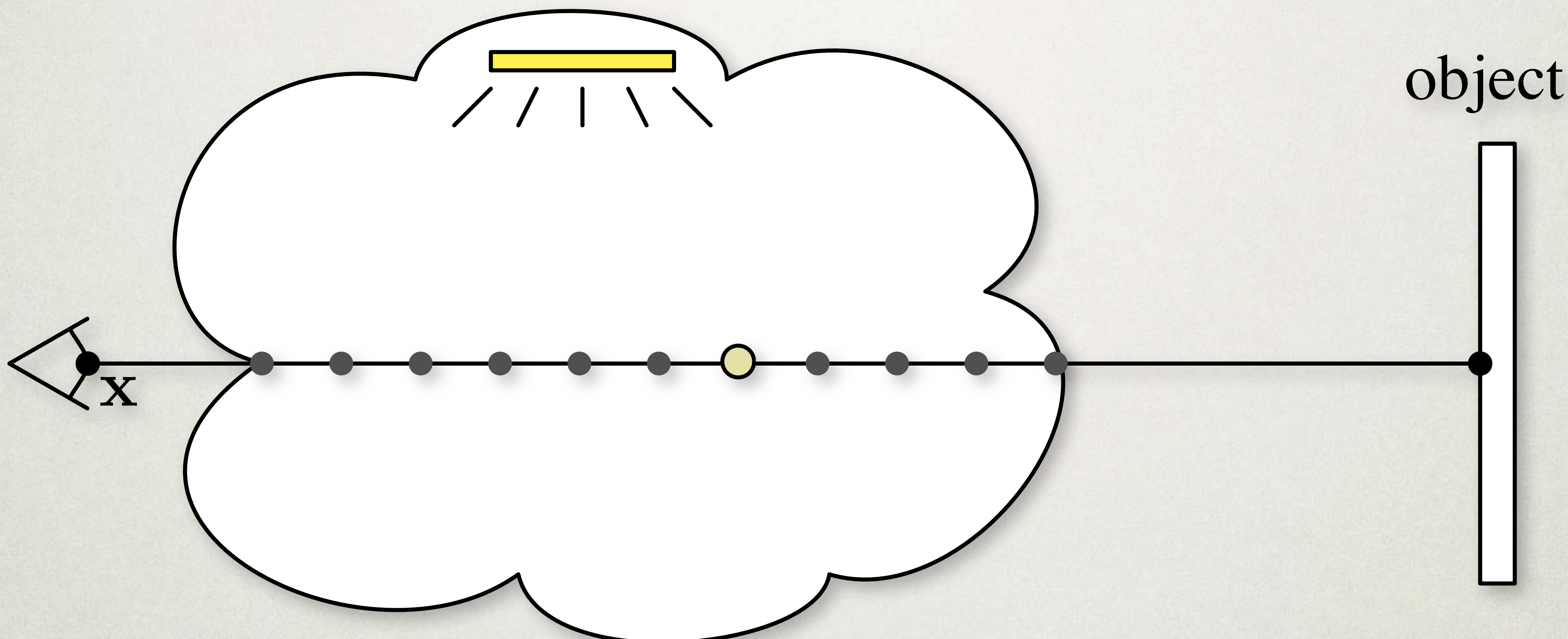
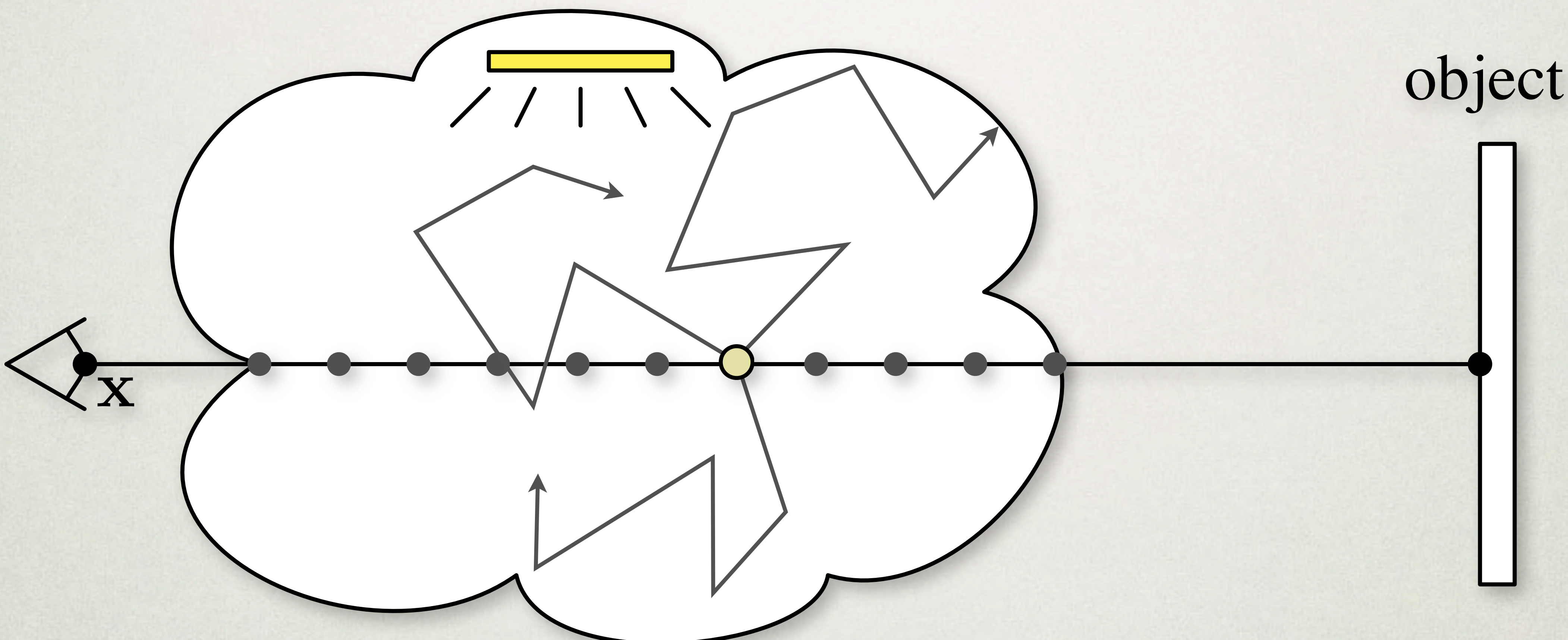


$$L(\mathbf{x},\vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t)\boxed{L_i(\mathbf{x}_t,\vec{\omega})}\Delta_t + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

- random walk sampling (path tracing)
- linear growth, but still expensive.

Ray Marching (Multiple Scattering)



$$L(\mathbf{x}, \vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t)\sigma_s(\mathbf{x}_t) \boxed{L_i(\mathbf{x}_t, \vec{\omega})} \Delta_t + T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s) L(\mathbf{x}_s, \vec{\omega})$$

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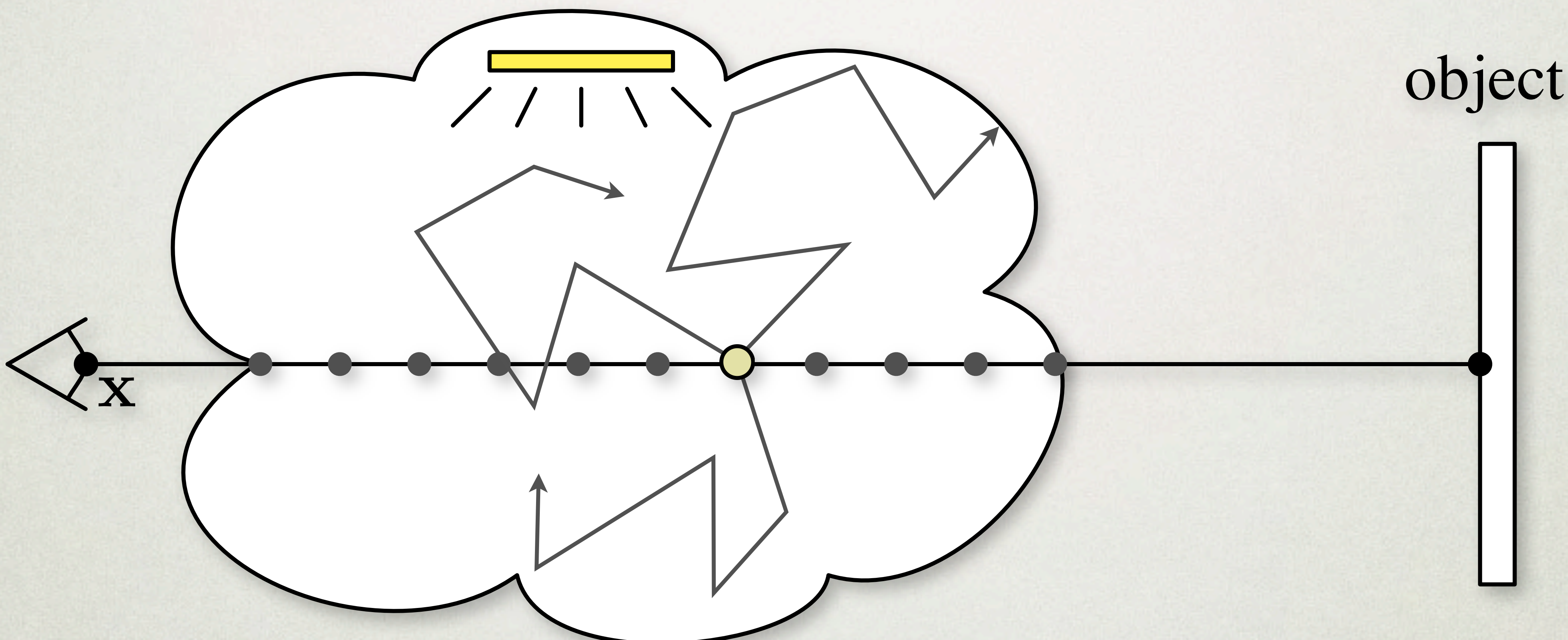
Ray Marching (Multiple Scattering)



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- random walk sampling (path tracing)
- linear growth, but still expensive.

So Far

- Single scattering relatively in-expensive
- Multiple scattering **very** expensive

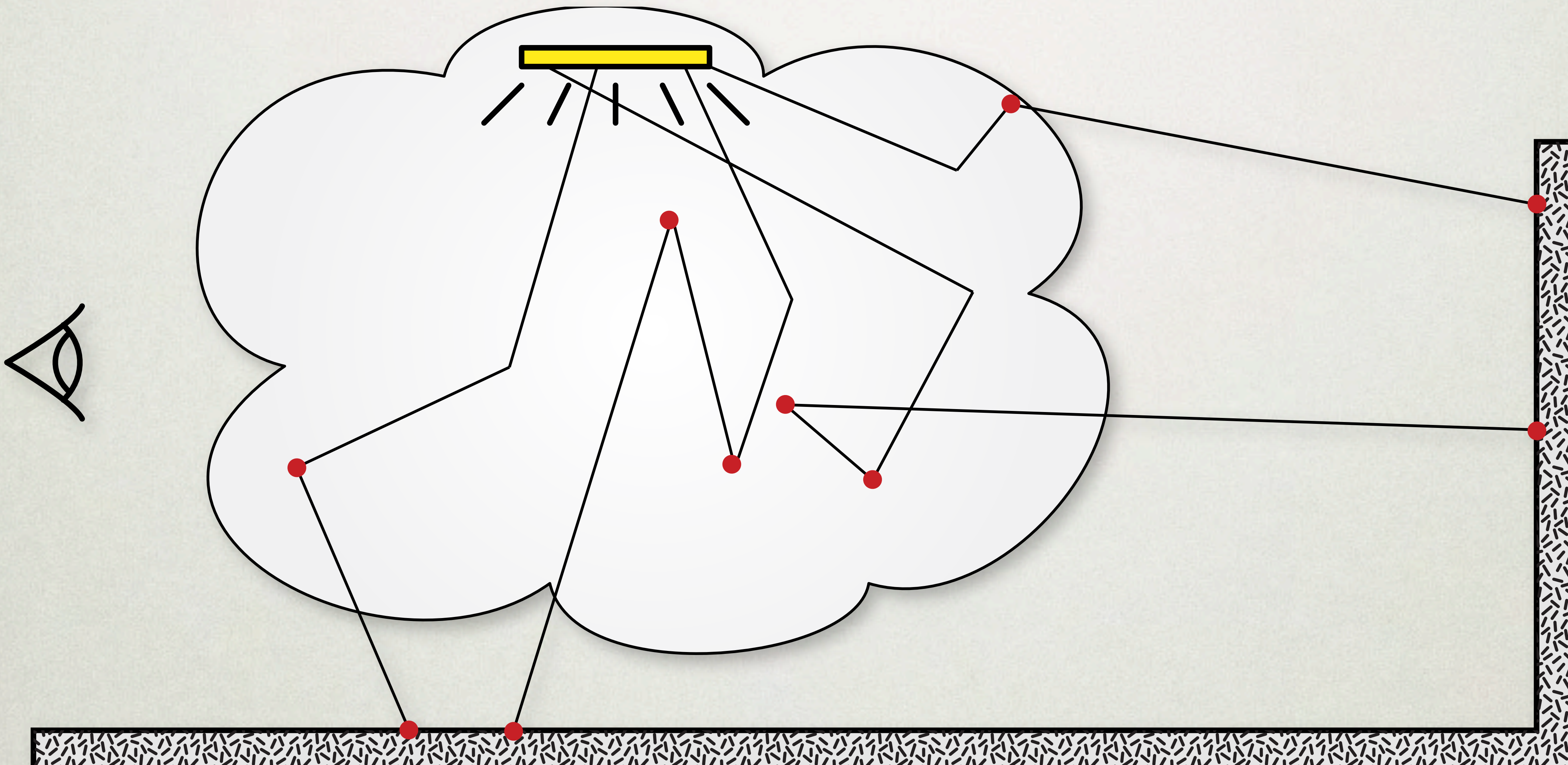
Volumetric Photon Tracing

Two-pass algorithm:

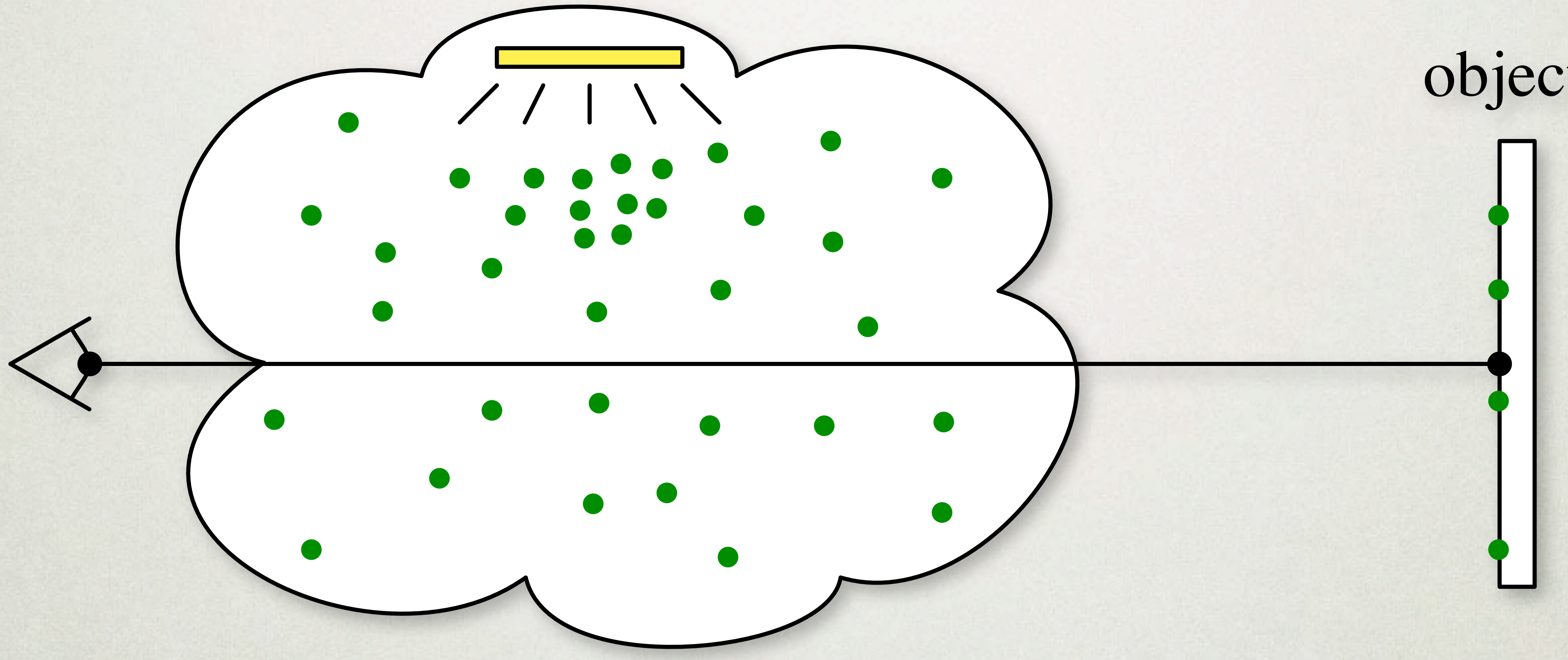
1) Photon tracing
   - Simulate the scattering of photons

2) Rendering
   - Reuse the photons to estimate multiple scattering
     - VPL methods or density estimation

Volumetric Photon Tracing

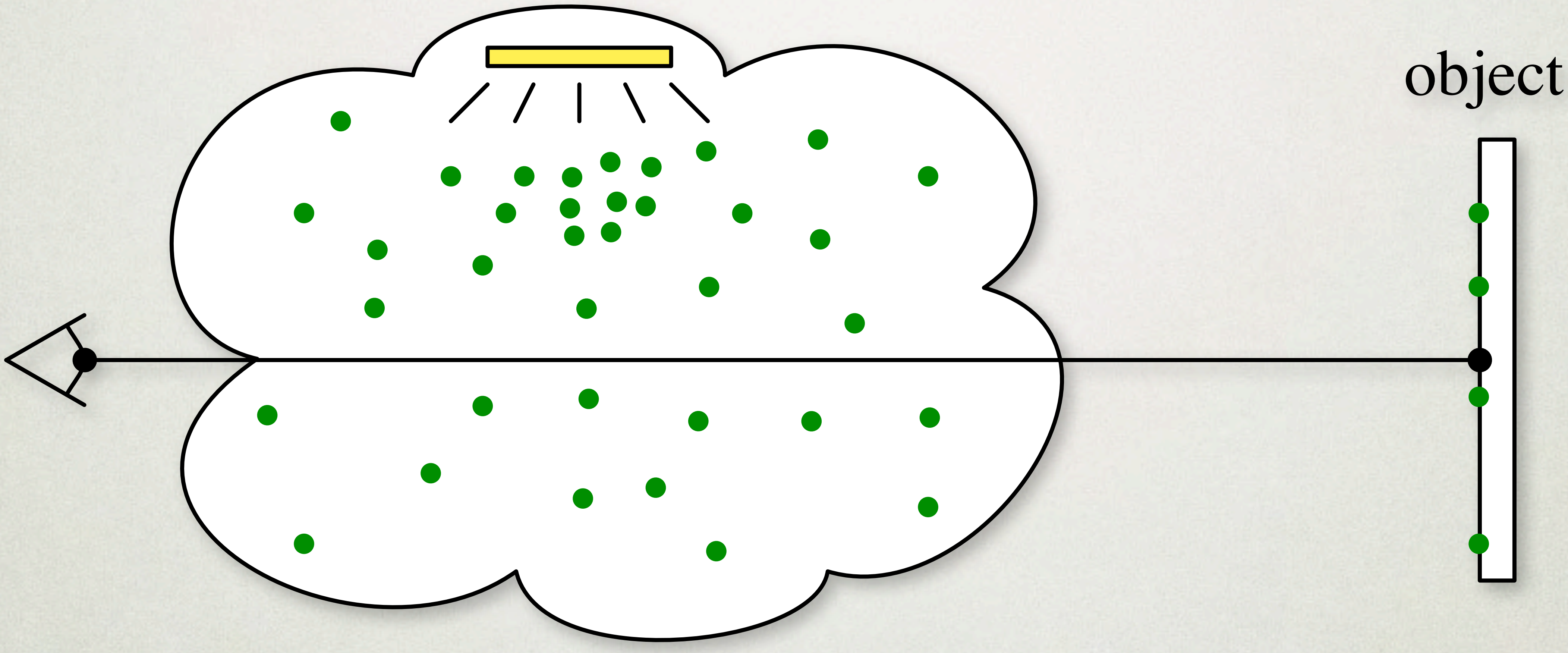


Volume Photon Map



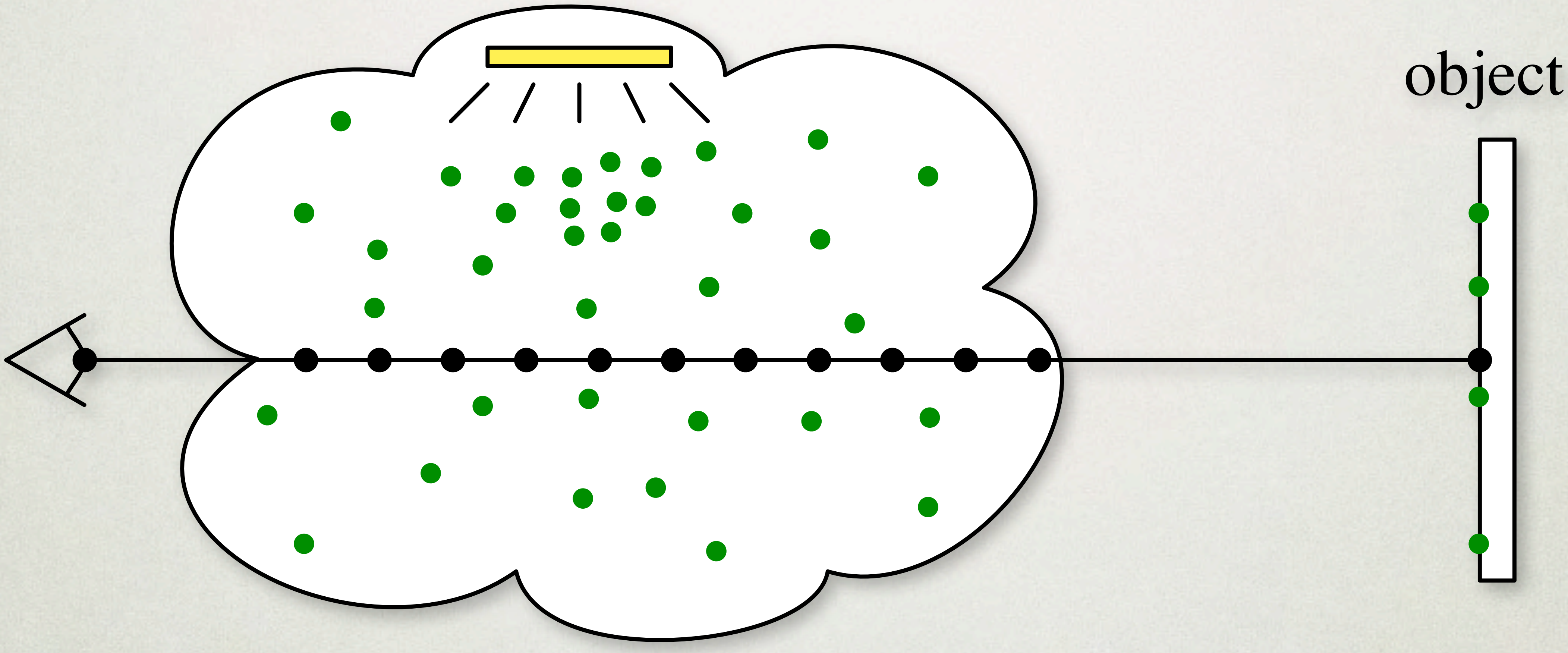
$$L(\mathbf{x},\vec{\omega}) = \int_0^s T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_i(\mathbf{x}_t,\vec{\omega})\,dt + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

VPL (Virtual Point Light) Methods



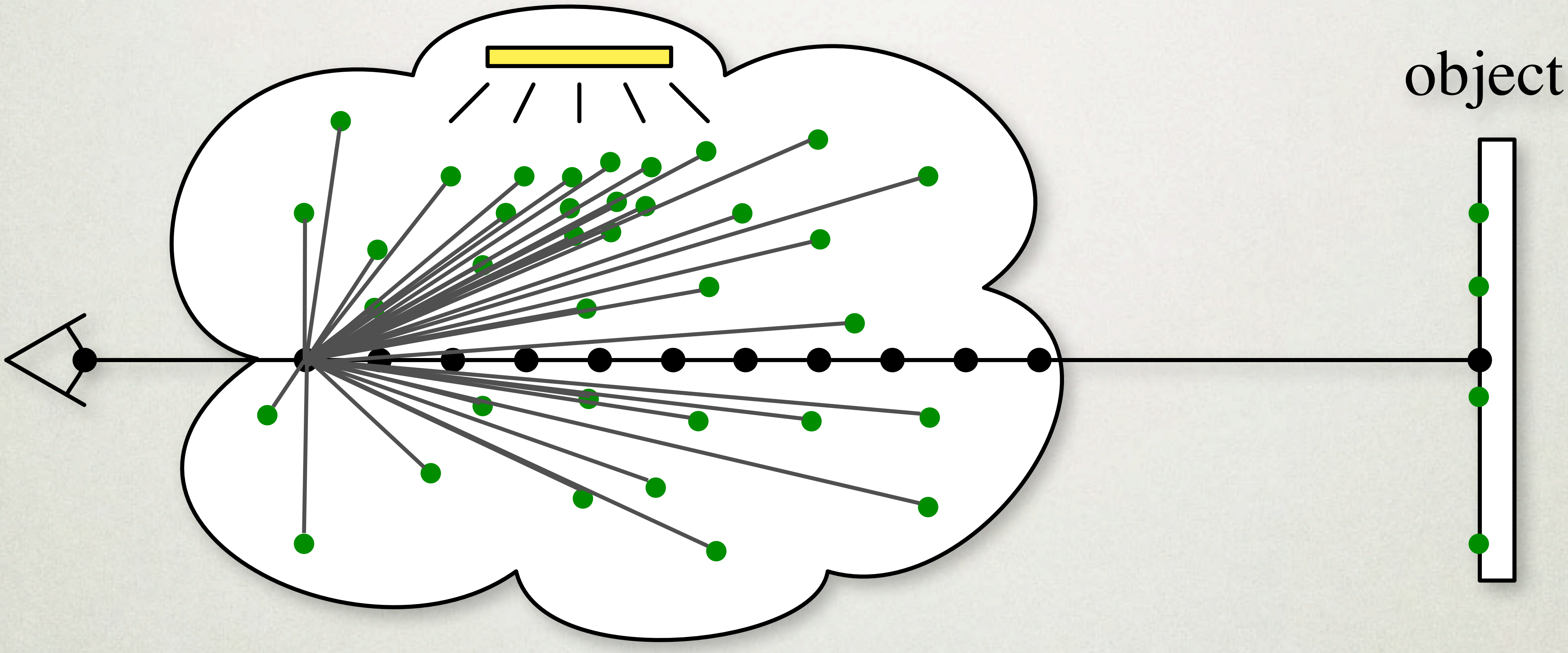
1. Treat each photon as a "virtual point light"

VPL Methods (Instant Radiosity)



1. Treat each photon as a "virtual point light"
2. Perform ray marching
   - At each step: shoot shadow rays to VPLs

VPL Methods (Instant Radiosity)

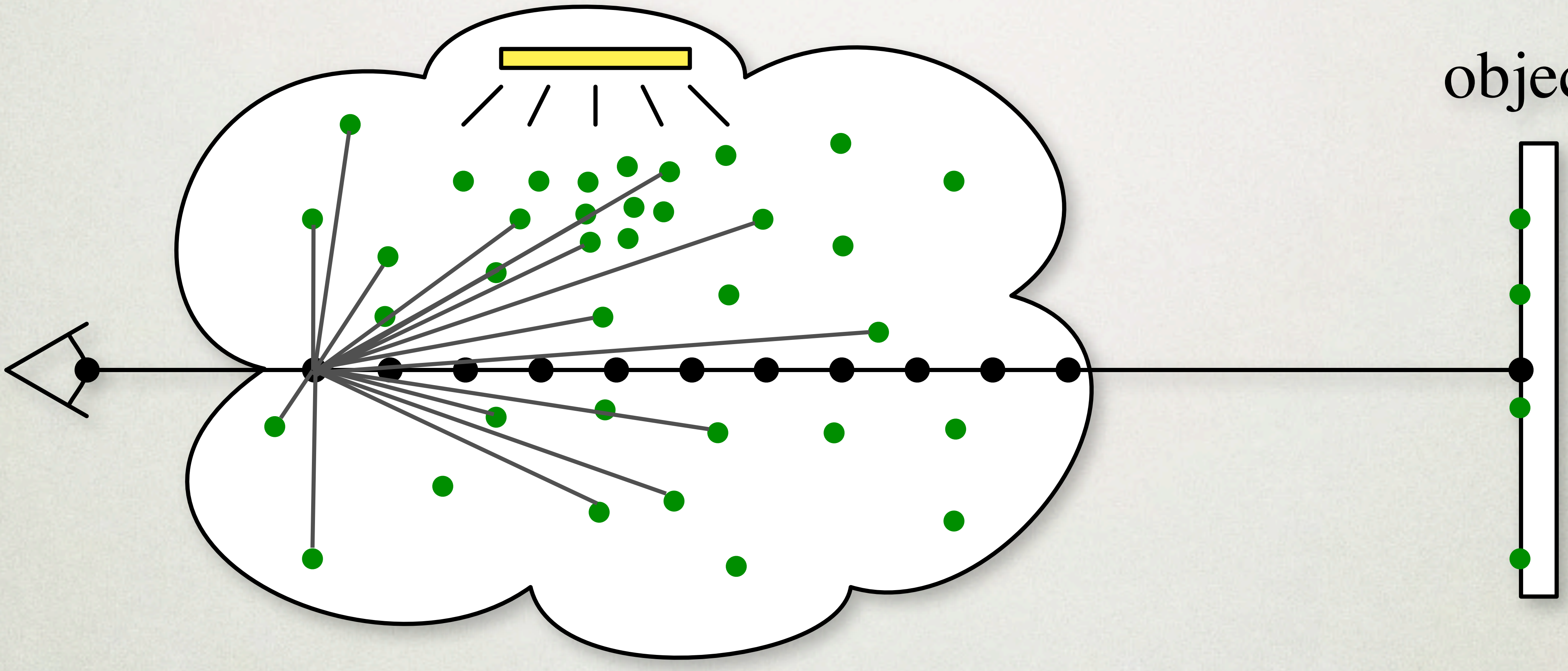


1. Treat each photon as a "virtual point light"
2. Perform ray marching
   - At each step: shoot shadow rays to VPLs

VPL Methods (Instant Radiosity)

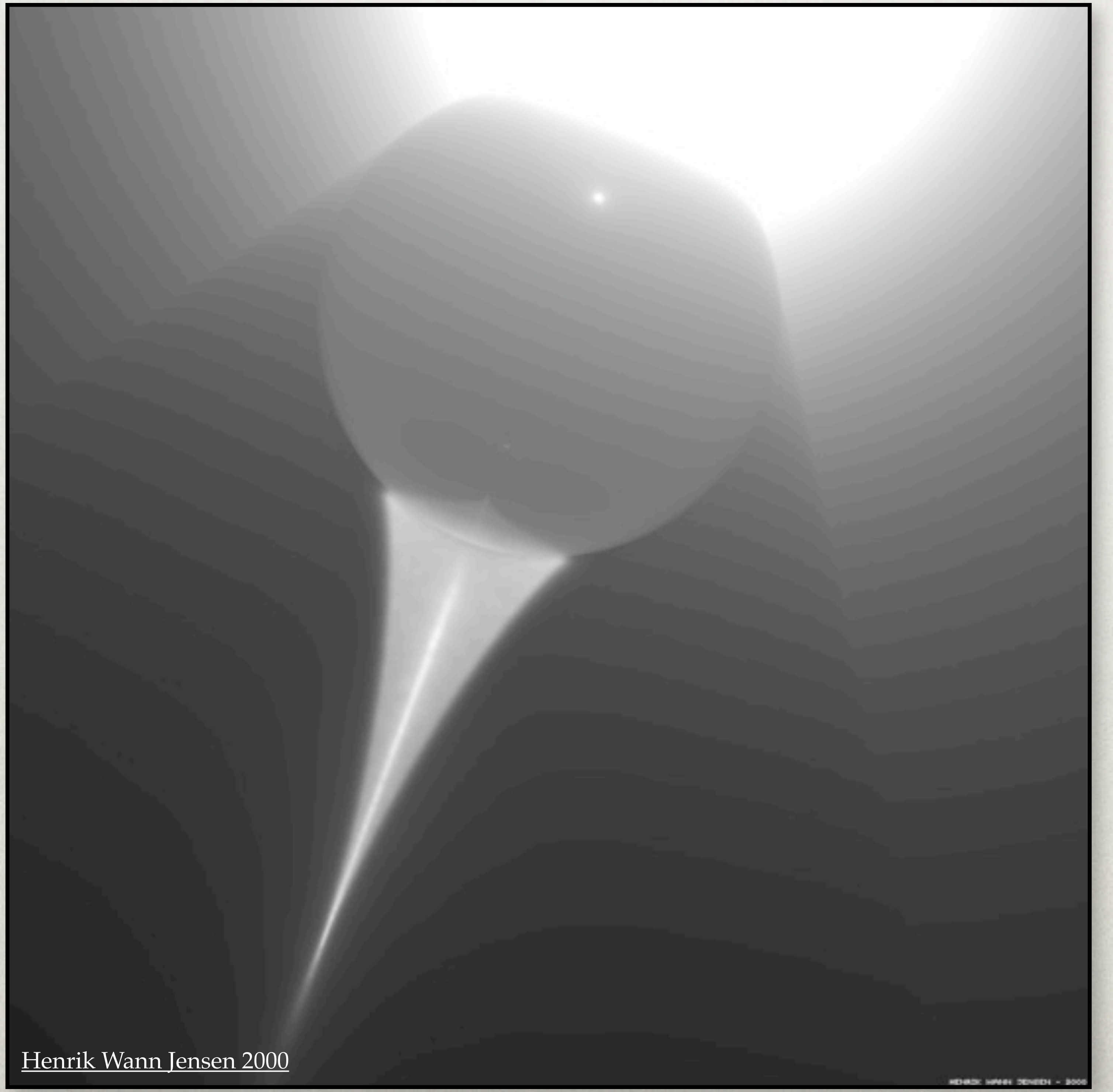
1. Treat each photon as a “virtual point light”
2. Perform ray marching
    - At each step: choose a **subset** of VPLs (faster performance, introduces noise)

VPL Methods (Lightcuts)

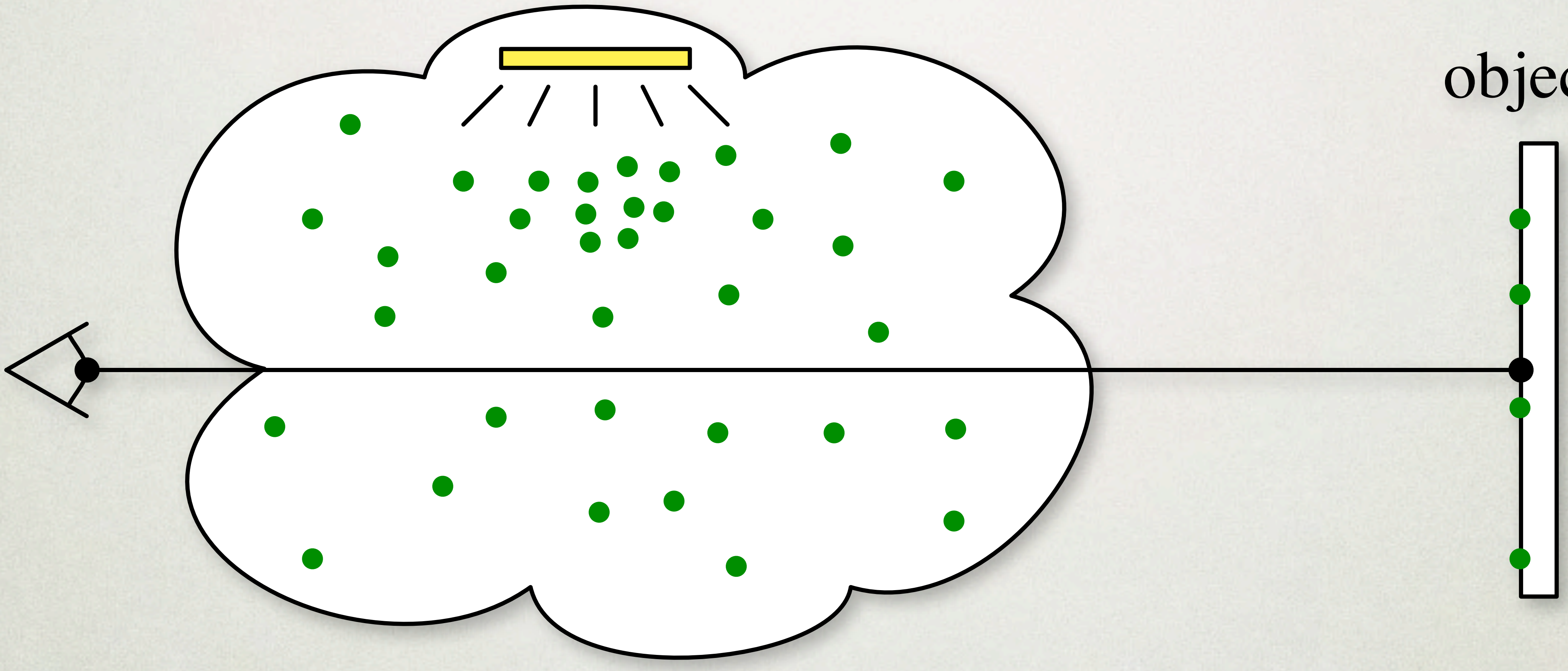


1. Create VPL **hierarchy**
2. Perform ray marching
   - At each step: choose **hierarchical subset** (faster performance, tries to limit noise)

Caustics

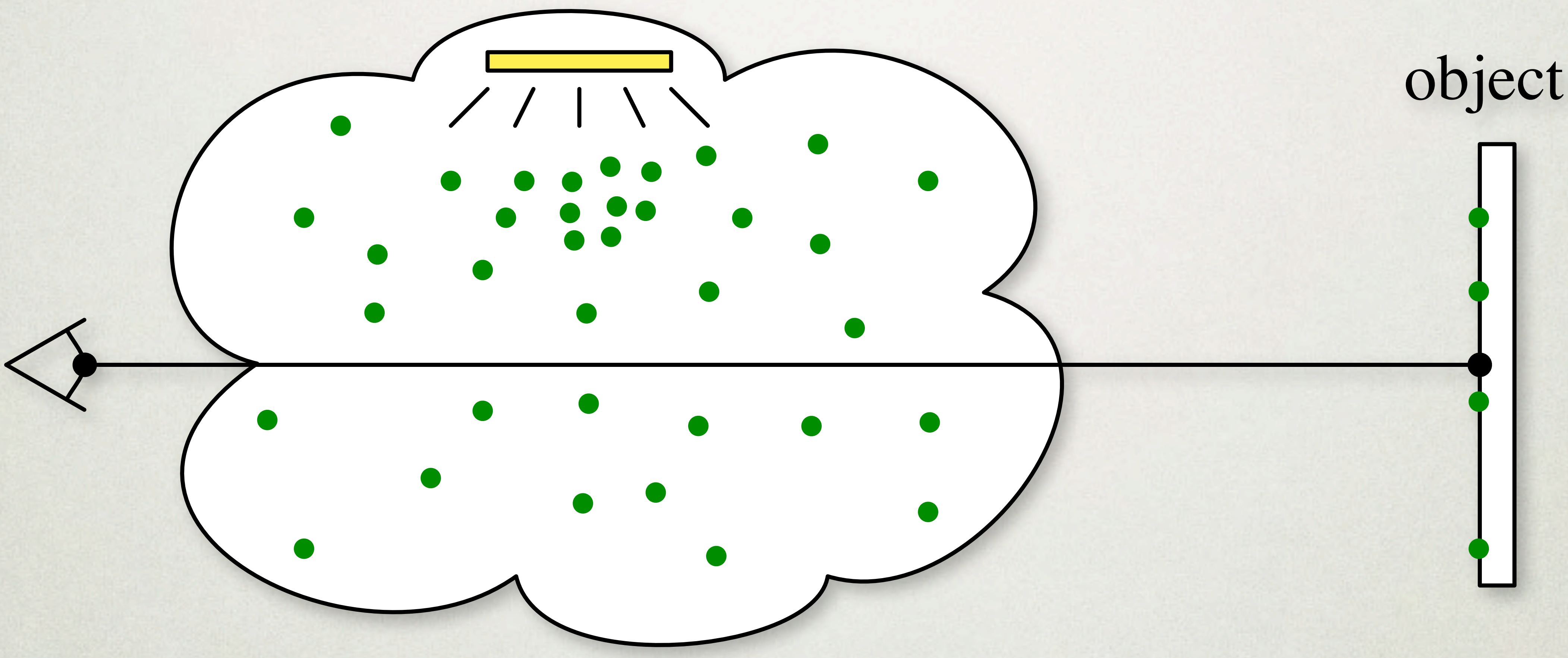

Henrik Wann Jensen 2000

Volumetric Photon Mapping



$$L(\mathbf{x},\vec{\omega}) = \int_0^s T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_i(\mathbf{x}_t,\vec{\omega})\ dt + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

Volumetric Photon Mapping



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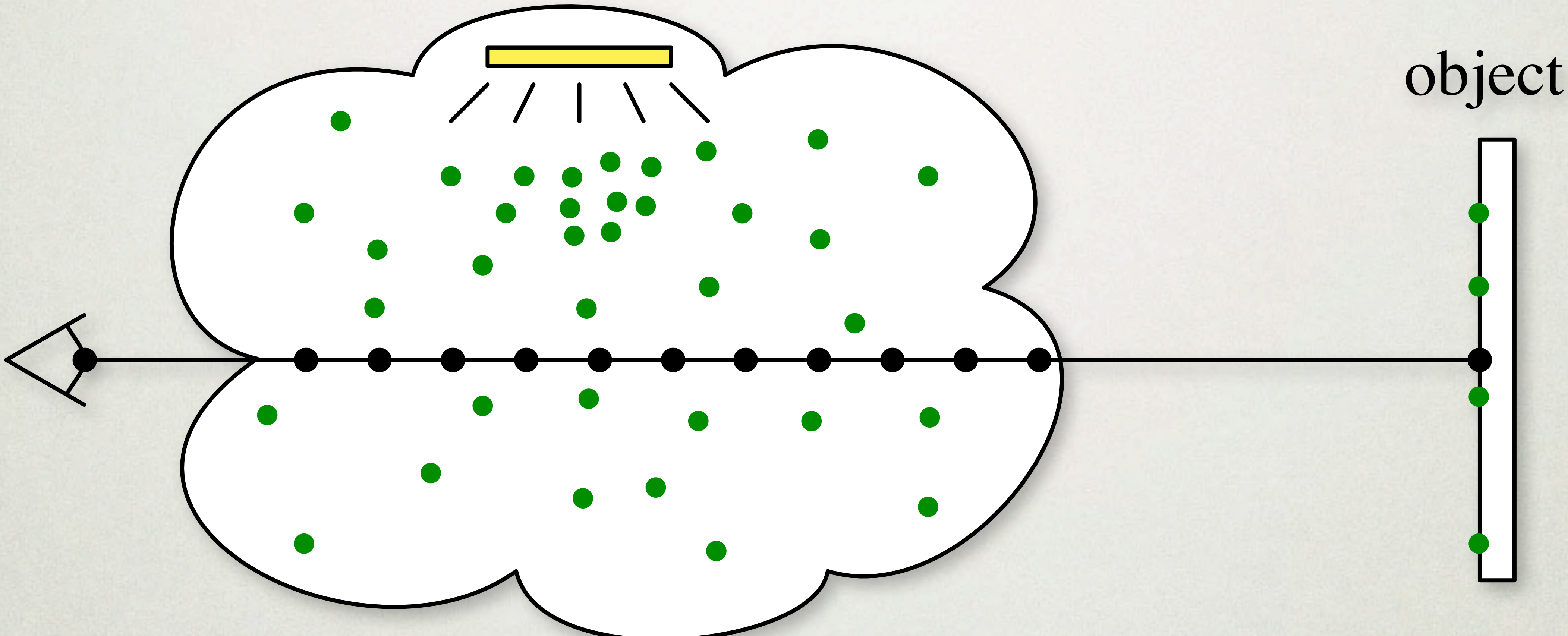
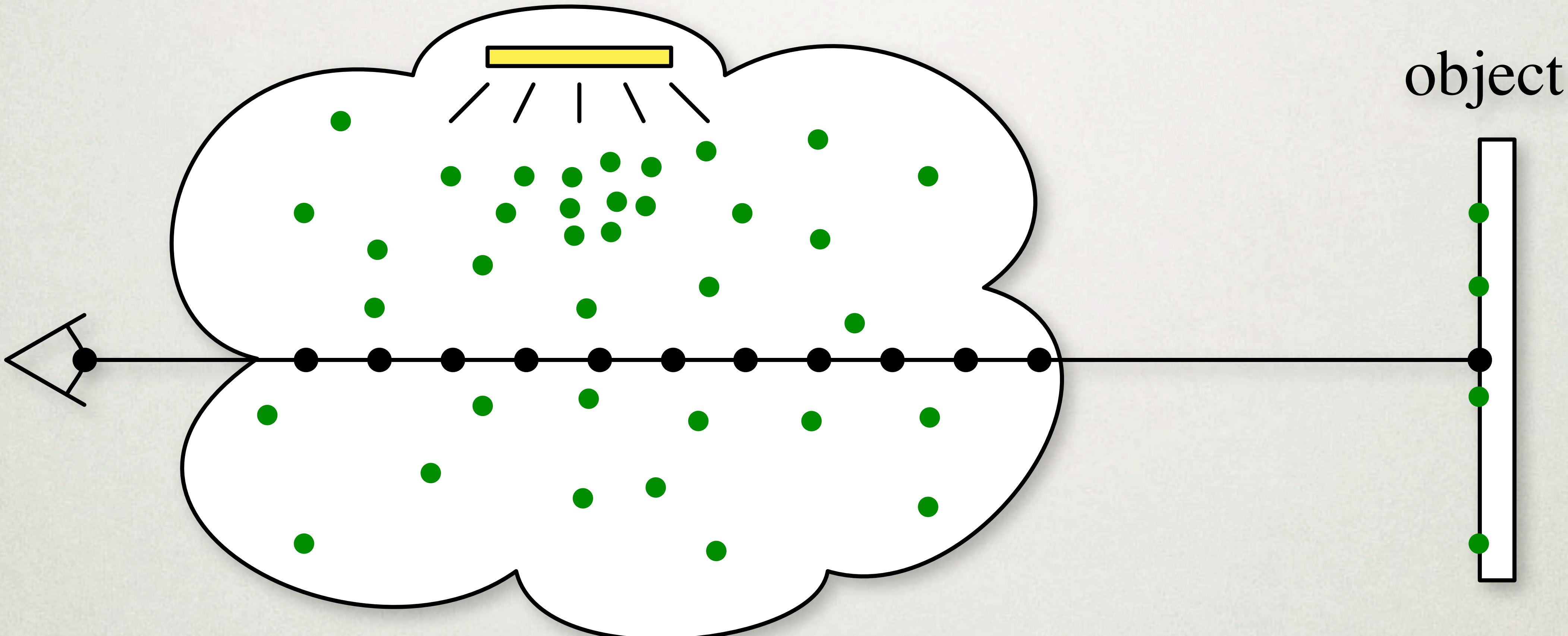
Two approaches: traditional & beam estimation

Traditional Photon Mapping (Ray Marching)



$$L(\mathbf{x},\vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t)L_i(\mathbf{x}_t,\vec{\omega})\Delta_t + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

Traditional Photon Mapping (Ray Marching)



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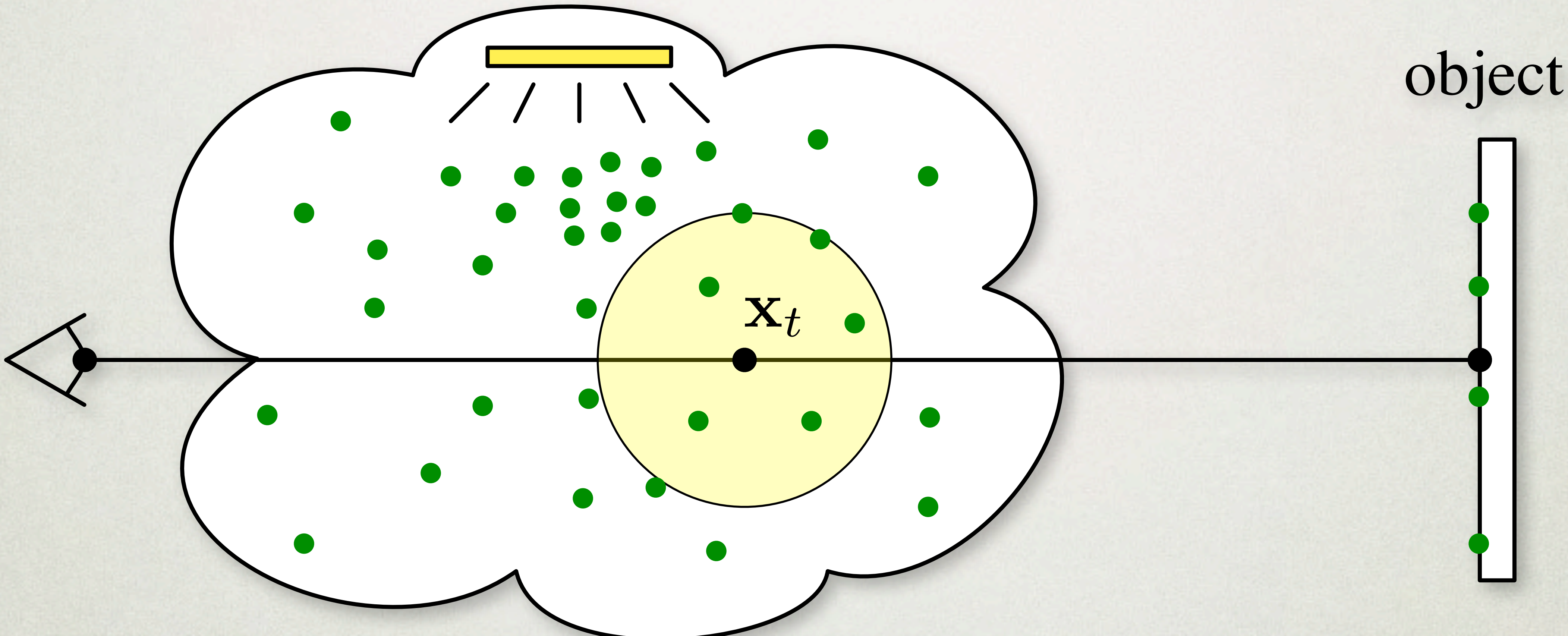
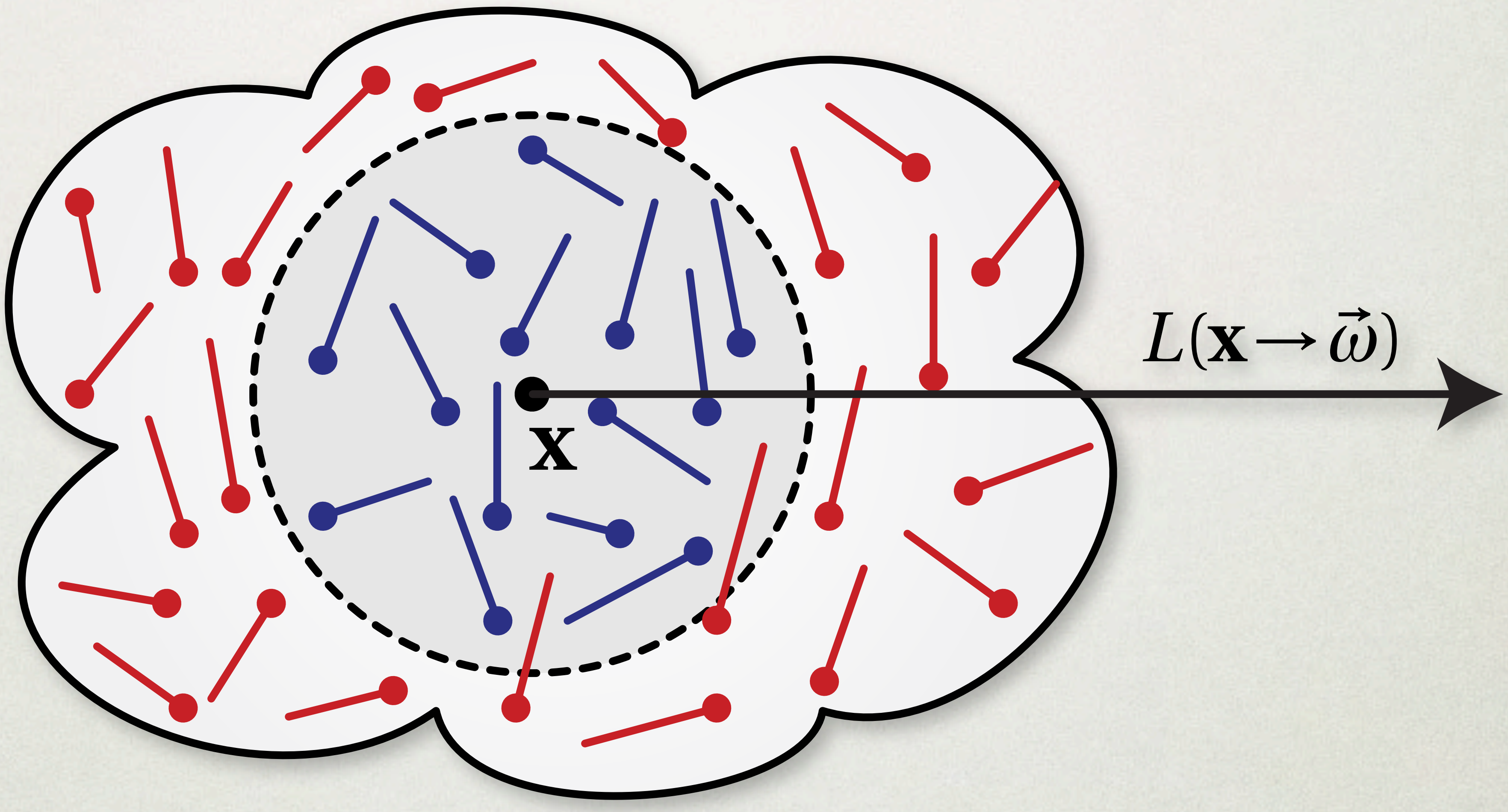
Traditional Photon Mapping (Multiple Scattering)



$$L(\mathbf{x},\vec{\omega}) \approx \sum_{t=0}^{S-1} T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t) \boxed{L_i(\mathbf{x}_t,\vec{\omega})} \Delta_t + T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega})$$

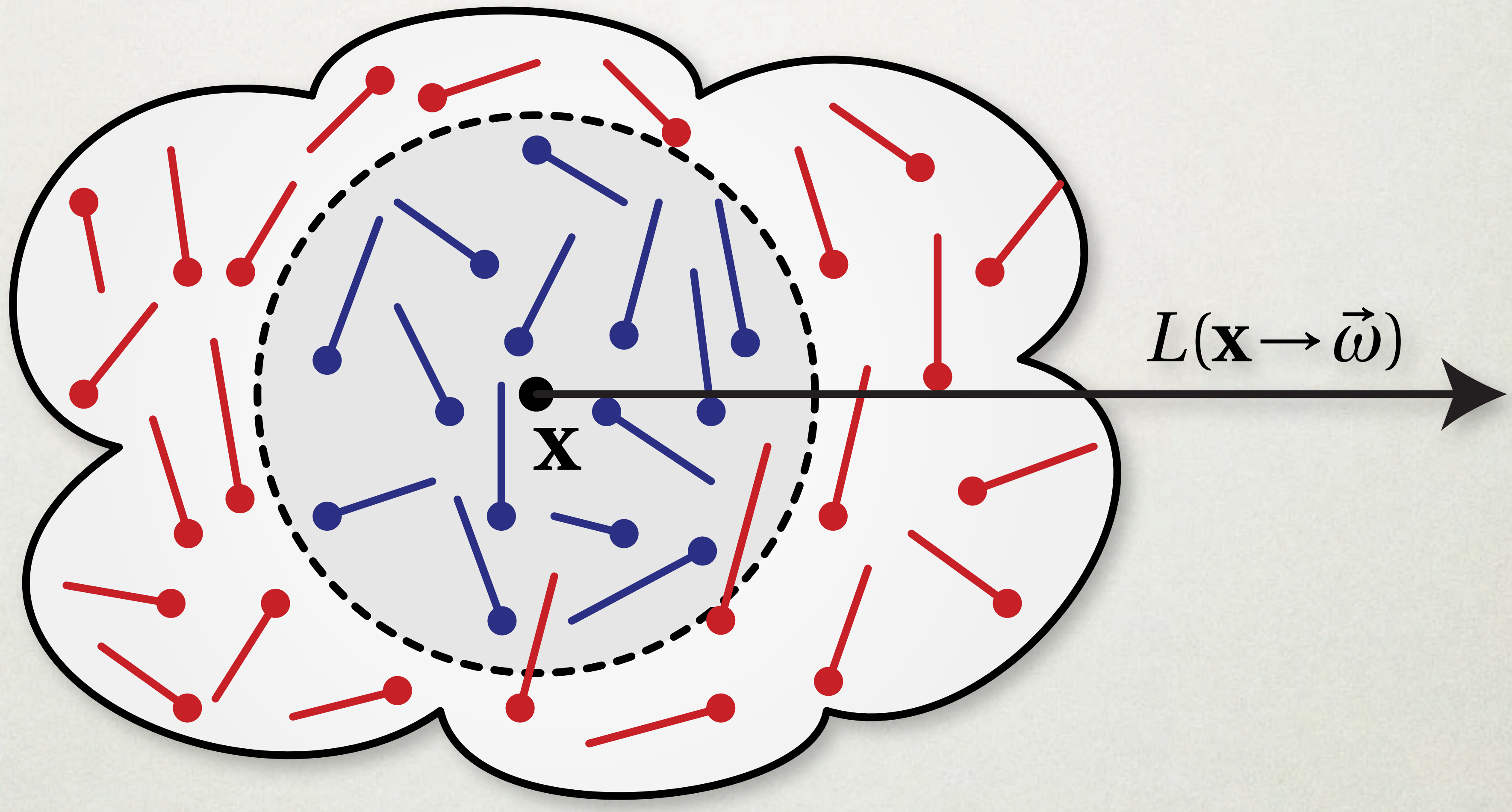
$L_i(\mathbf{x}_t,\vec{\omega})$ photon map for multiple scattering

Volumetric Radiance Estimate



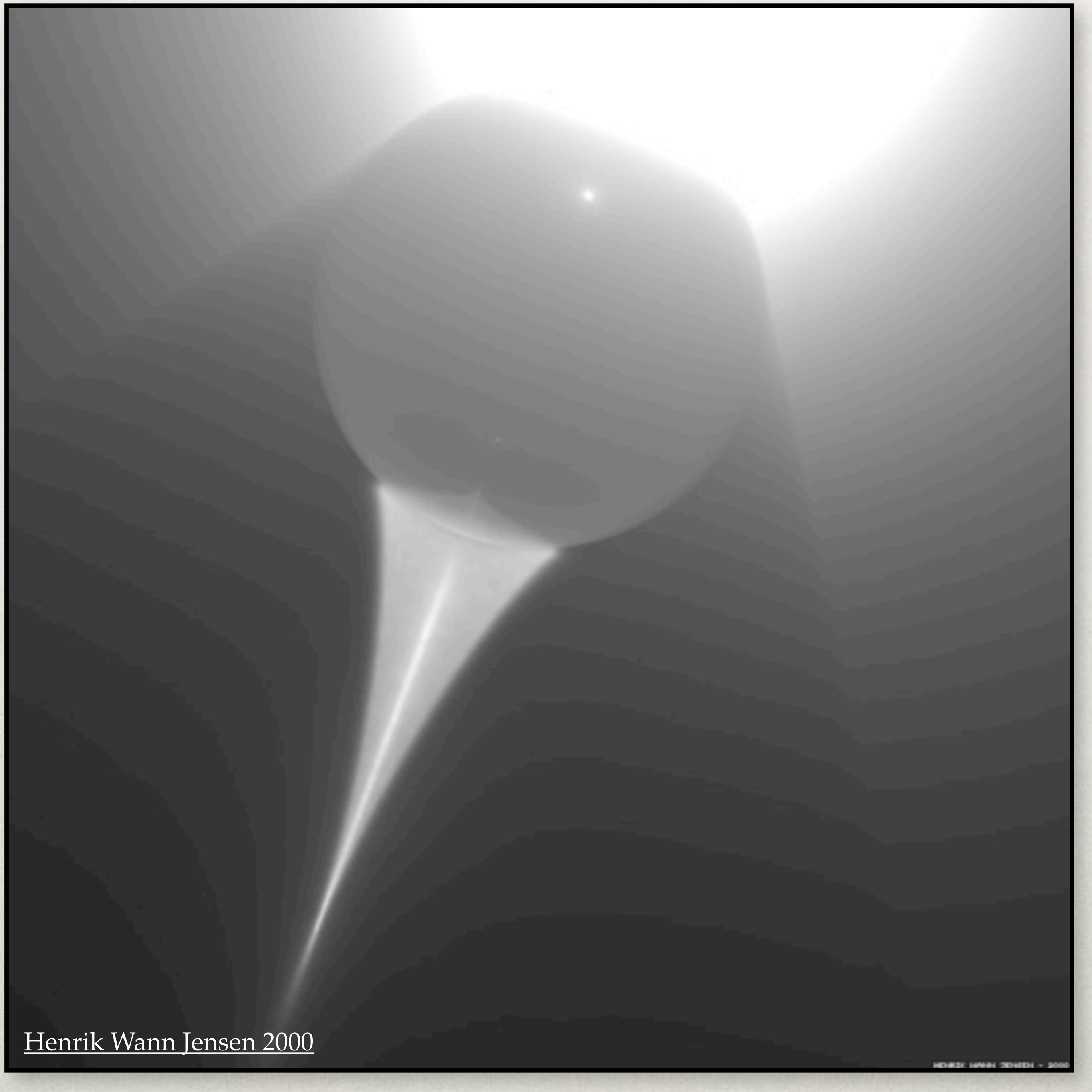
$$L_i(\mathbf{x} \to \vec{\omega}) \approx \sum_{p=1}^{k} p(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \frac{\Phi_p(\mathbf{x}, \vec{\omega}_p)}{\mathcal{V}(\mathbf{x})}$$

Volumetric Radiance Estimate



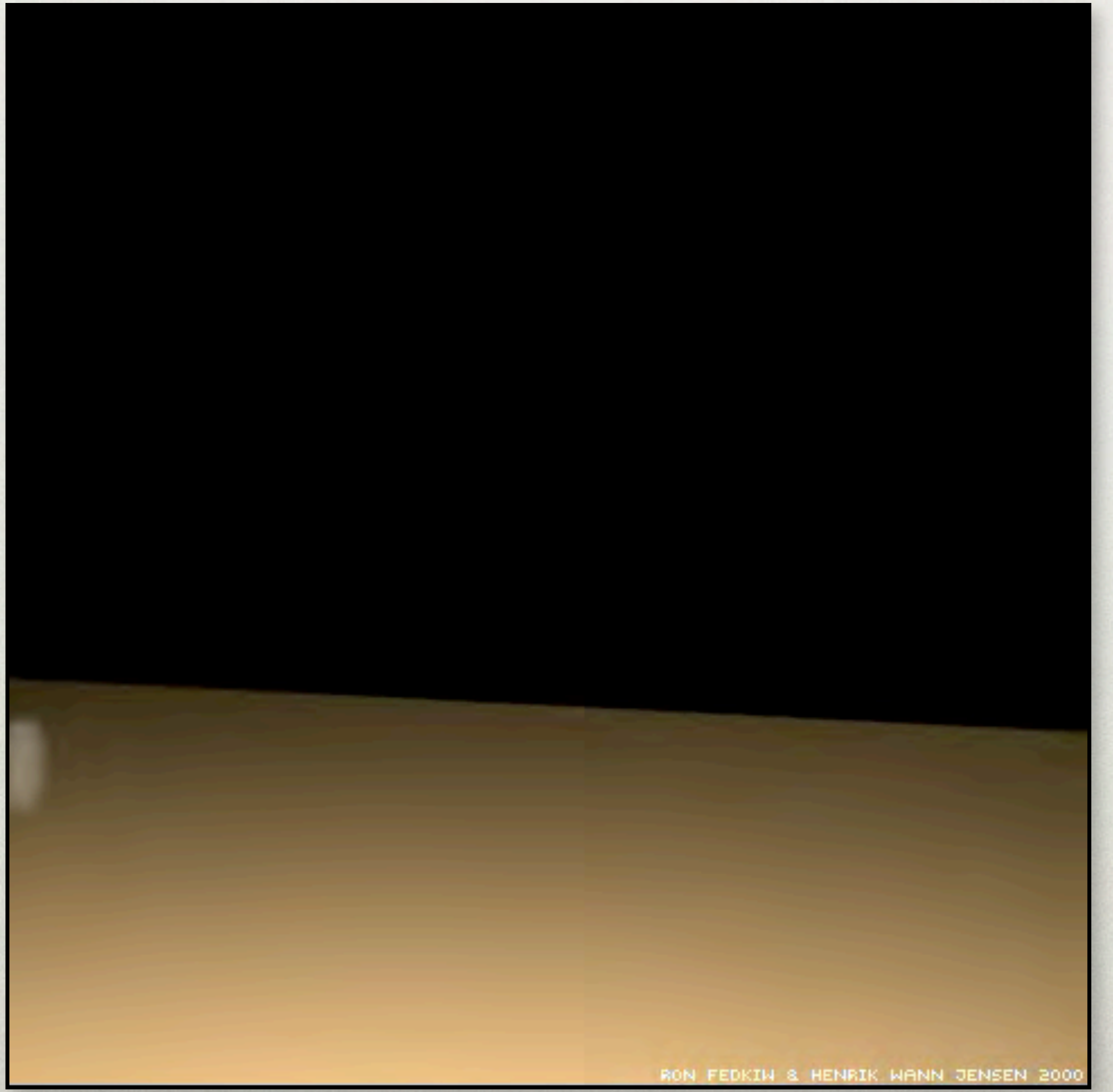
$$L_i(\mathbf{x} \to \vec{\omega}) \approx \sum_{p=1}^{k} p(\mathbf{x}, \vec{\omega}_p, \vec{\omega}) \frac{\Phi_p(\mathbf{x}, \vec{\omega}_p)}{\frac{4}{3}\pi r(\mathbf{x})^3}$$

A Volume Caustic

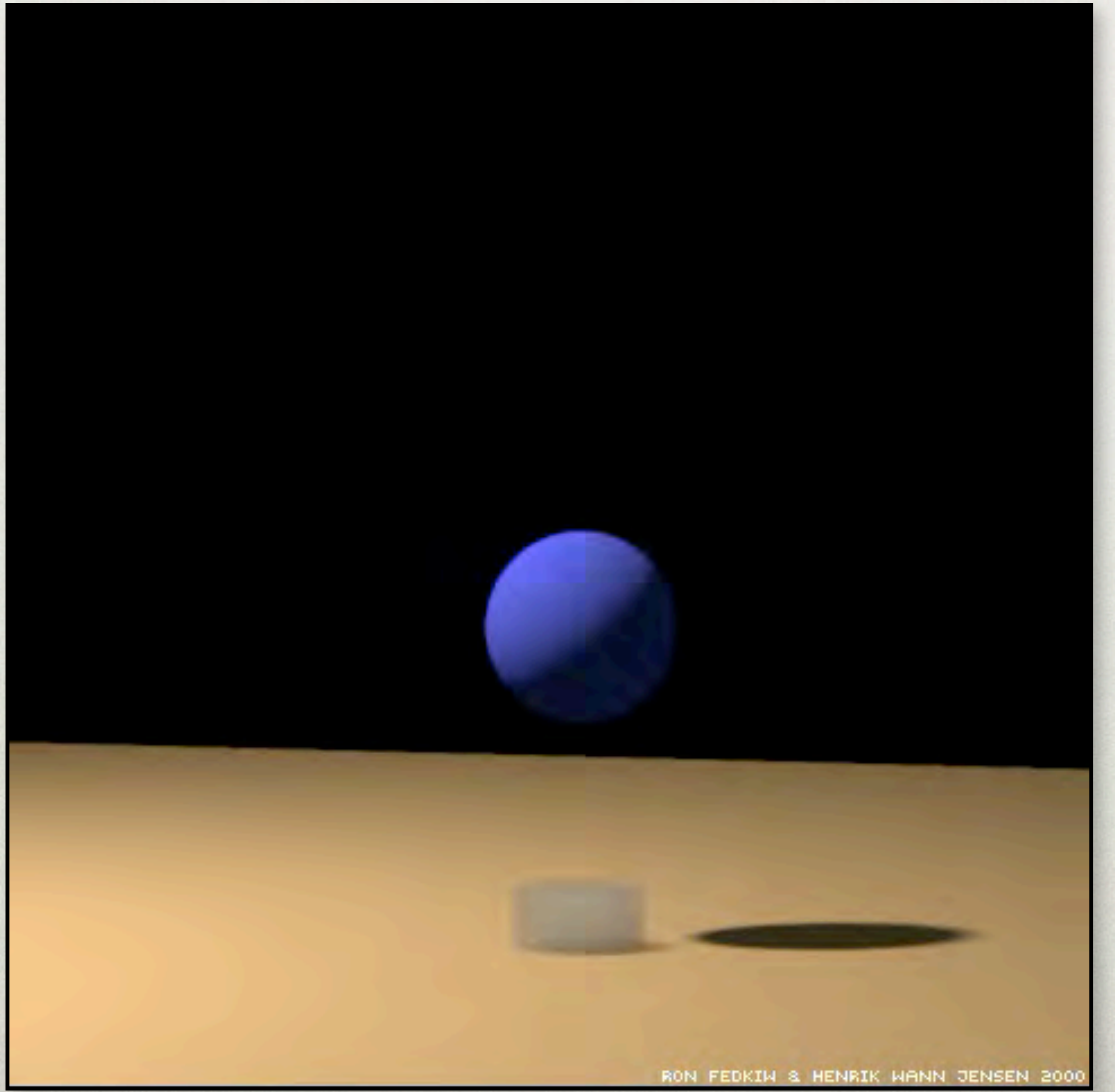


500,000 photons. 1 minute

Rising Smoke

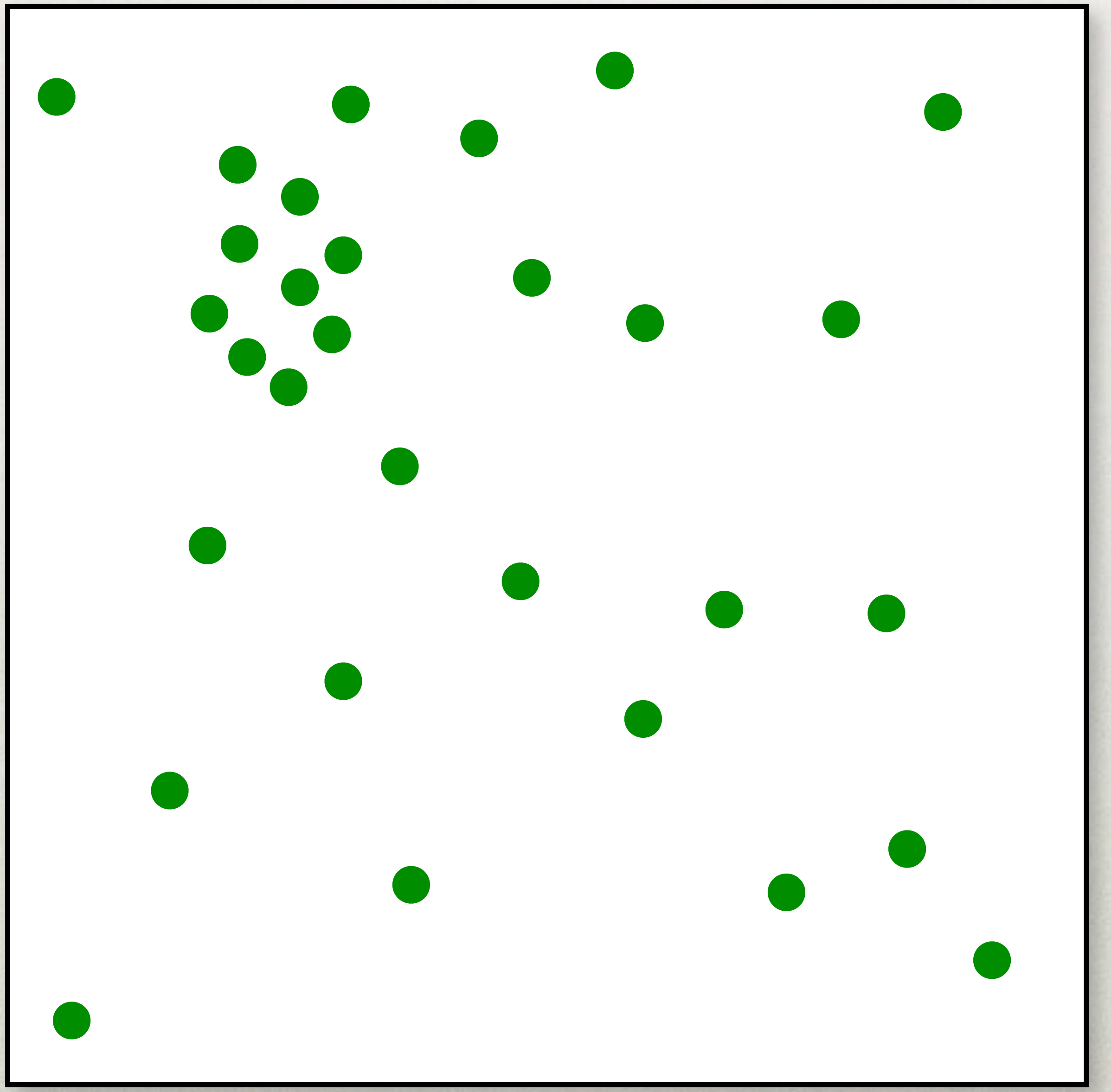


Smoke flowing past a sphere



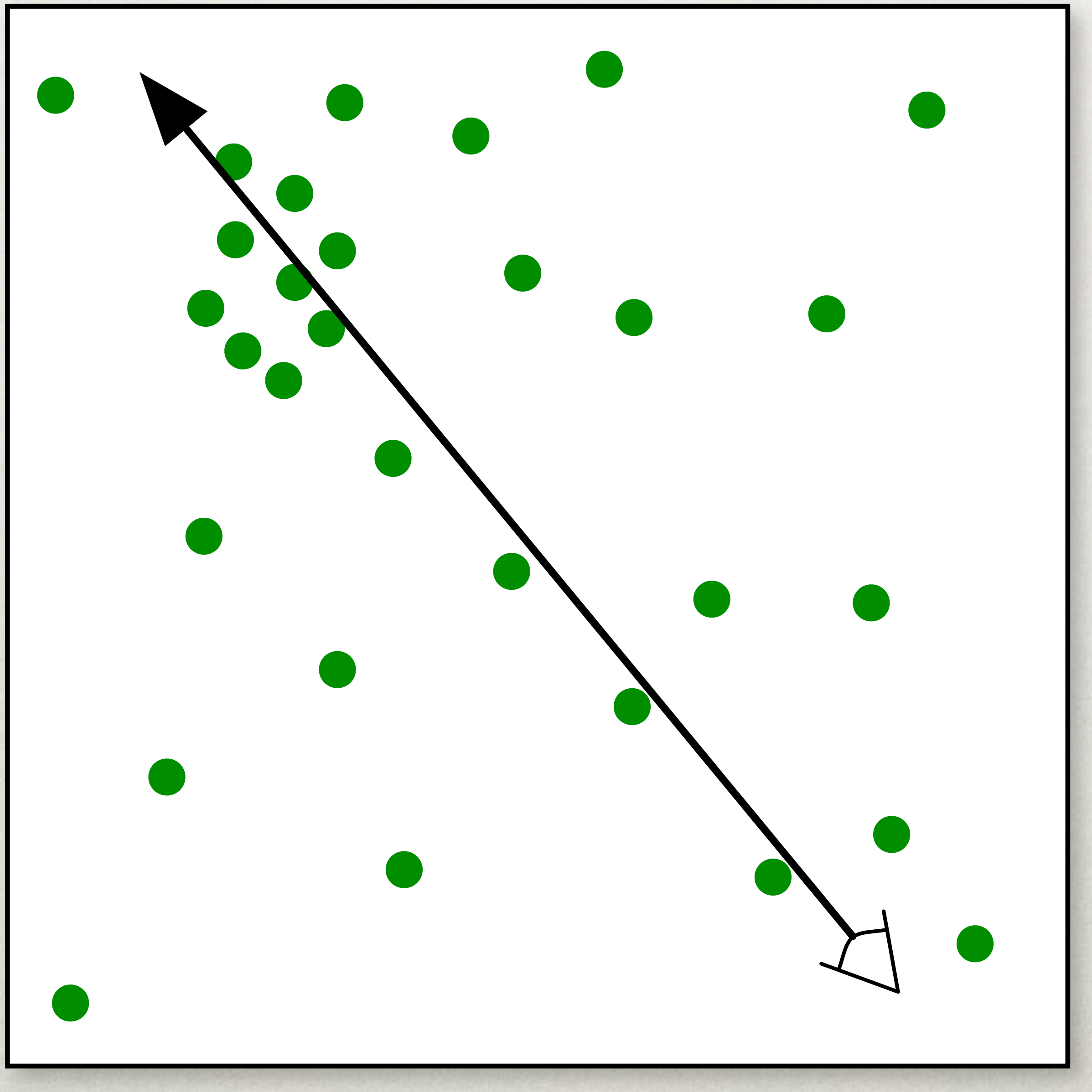
Volumetric Photon Mapping

Conventional Radiance Estimate



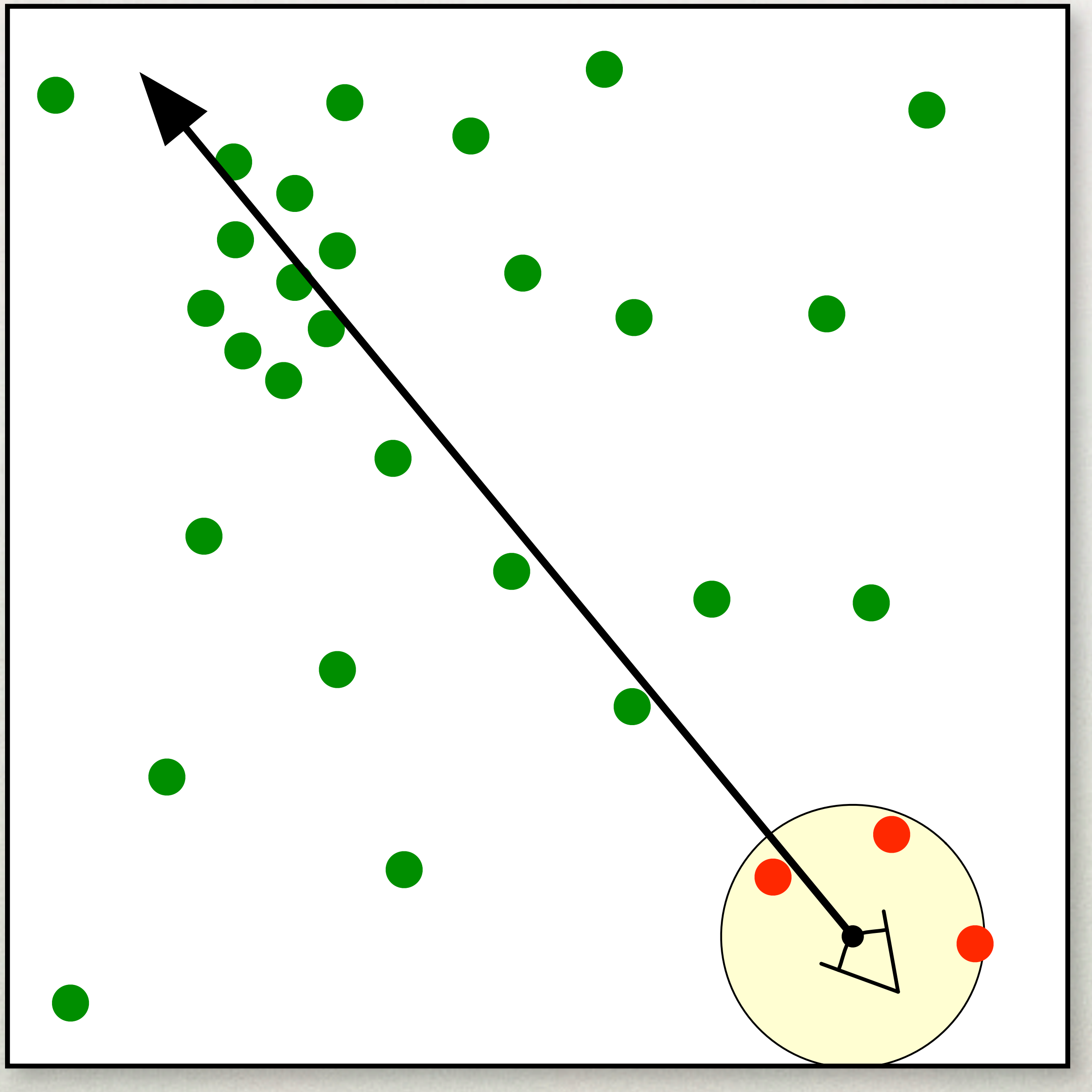
Volumetric Photon Mapping

Conventional Radiance Estimate



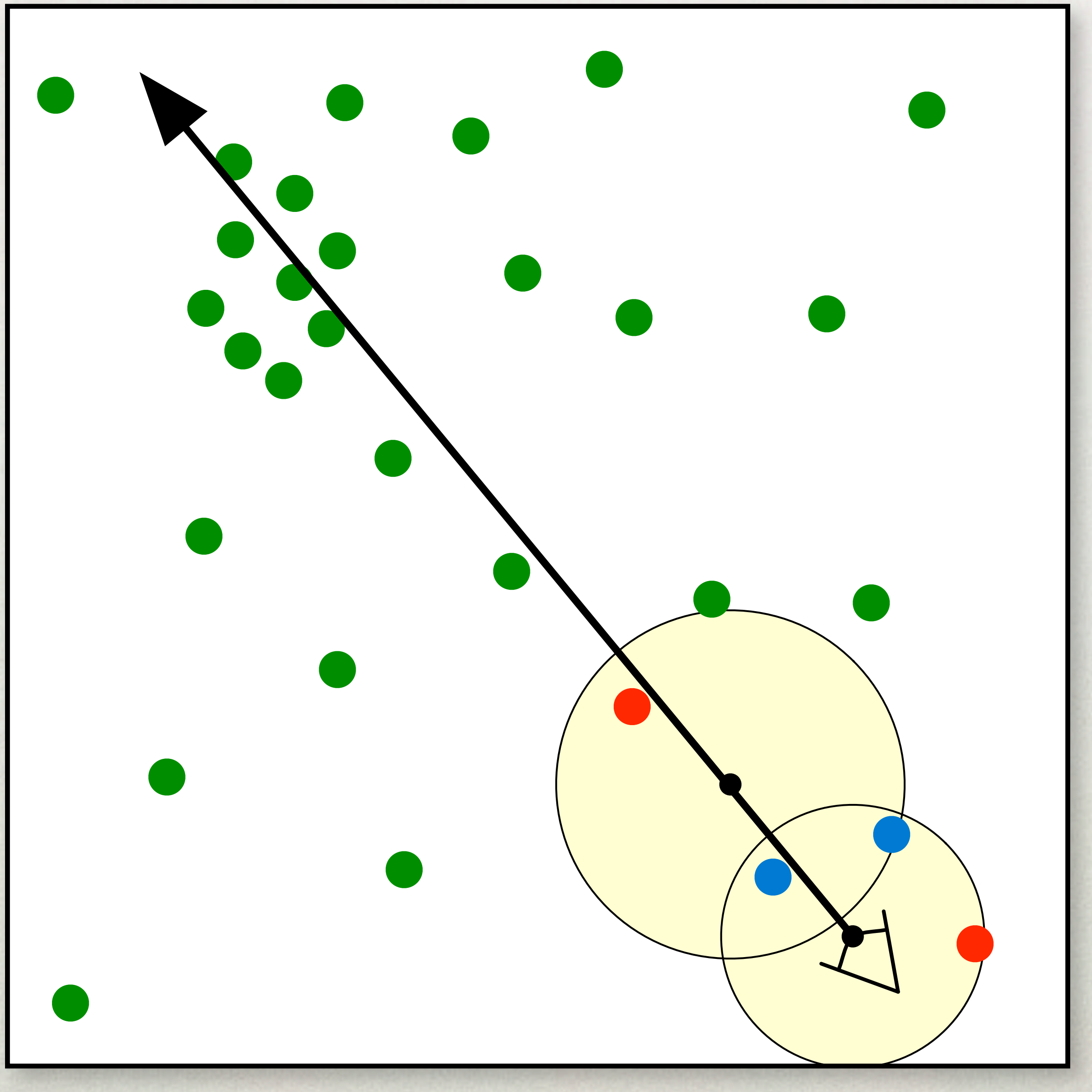
Volumetric Photon Mapping

Conventional Radiance Estimate



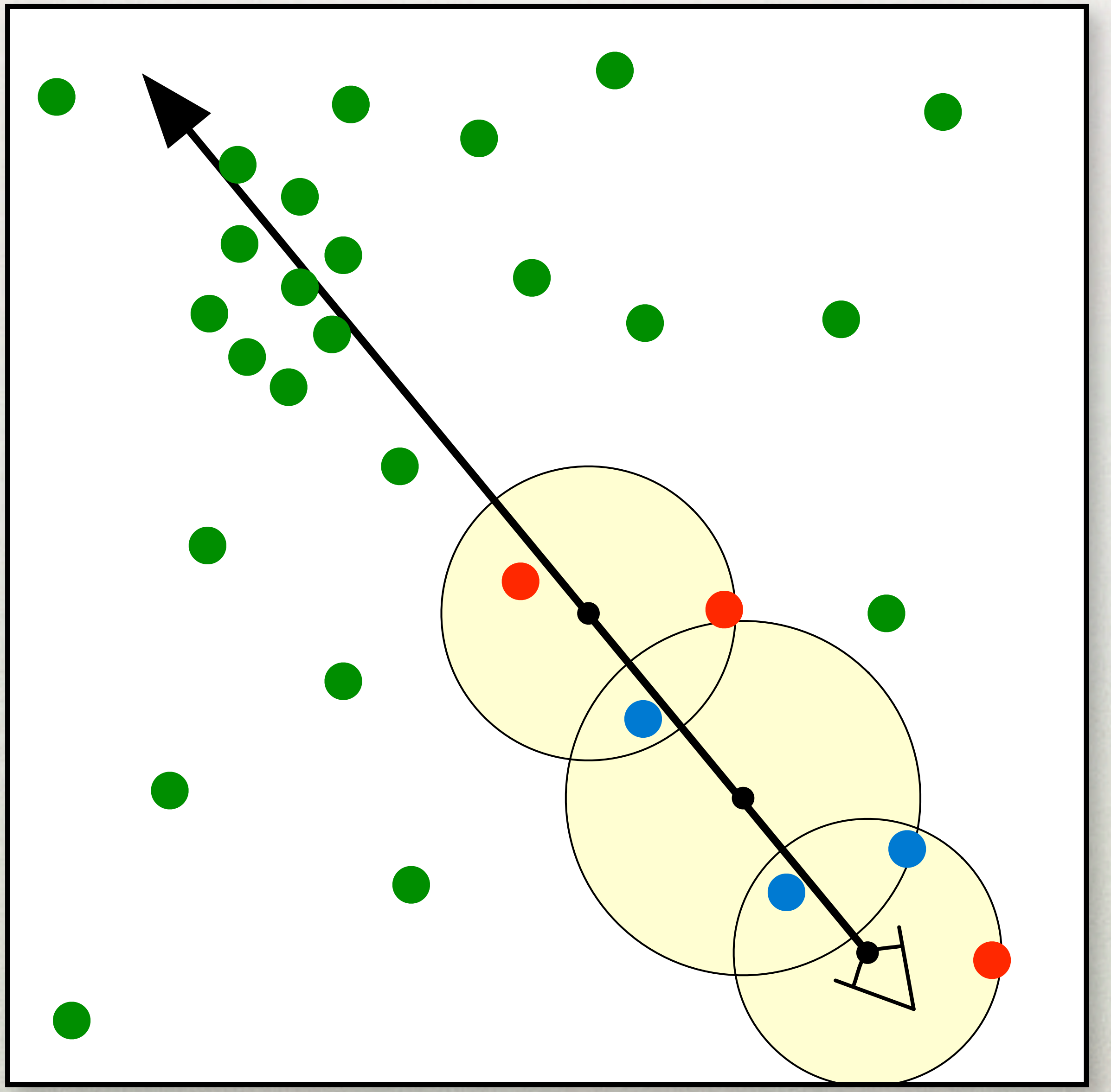
Volumetric Photon Mapping

Conventional Radiance Estimate



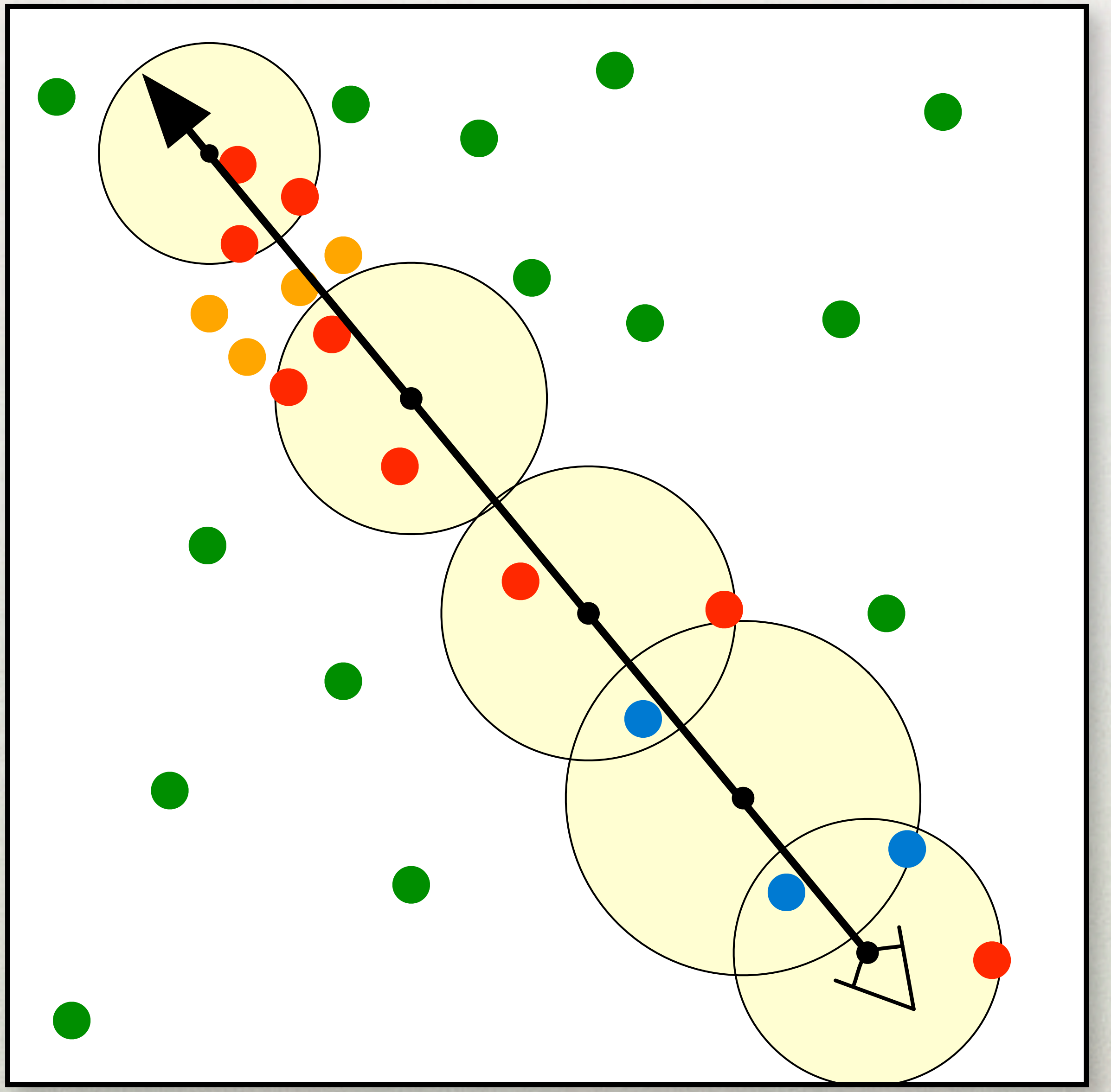
Volumetric Photon Mapping

Conventional Radiance Estimate

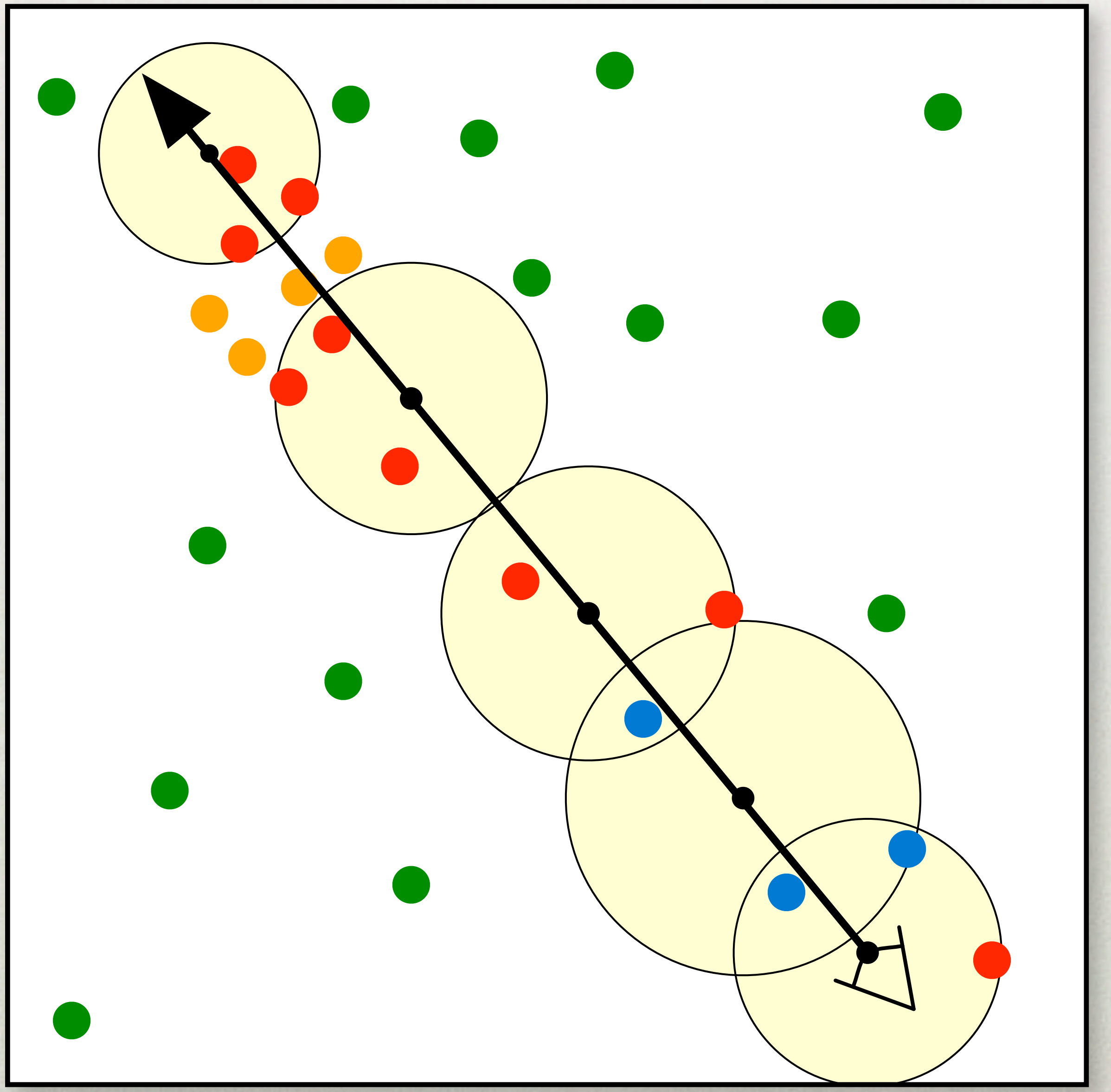


Volumetric Photon Mapping

Conventional Radiance Estimate

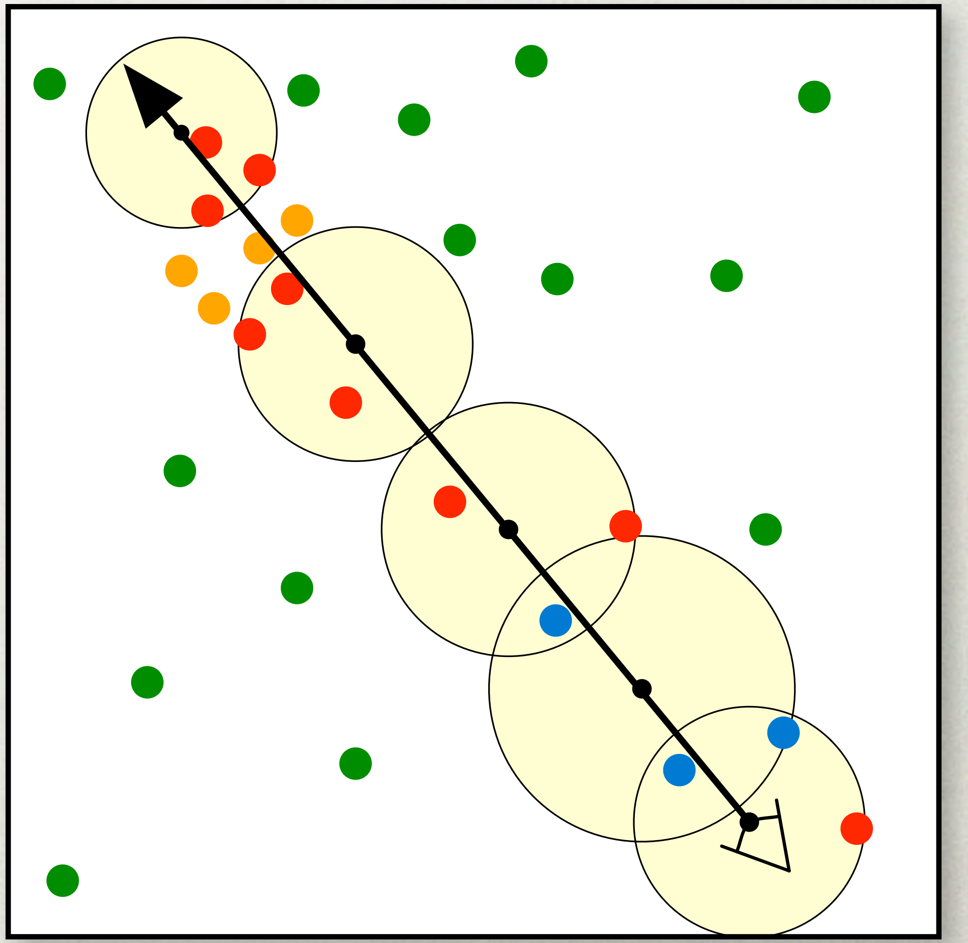


Drawbacks



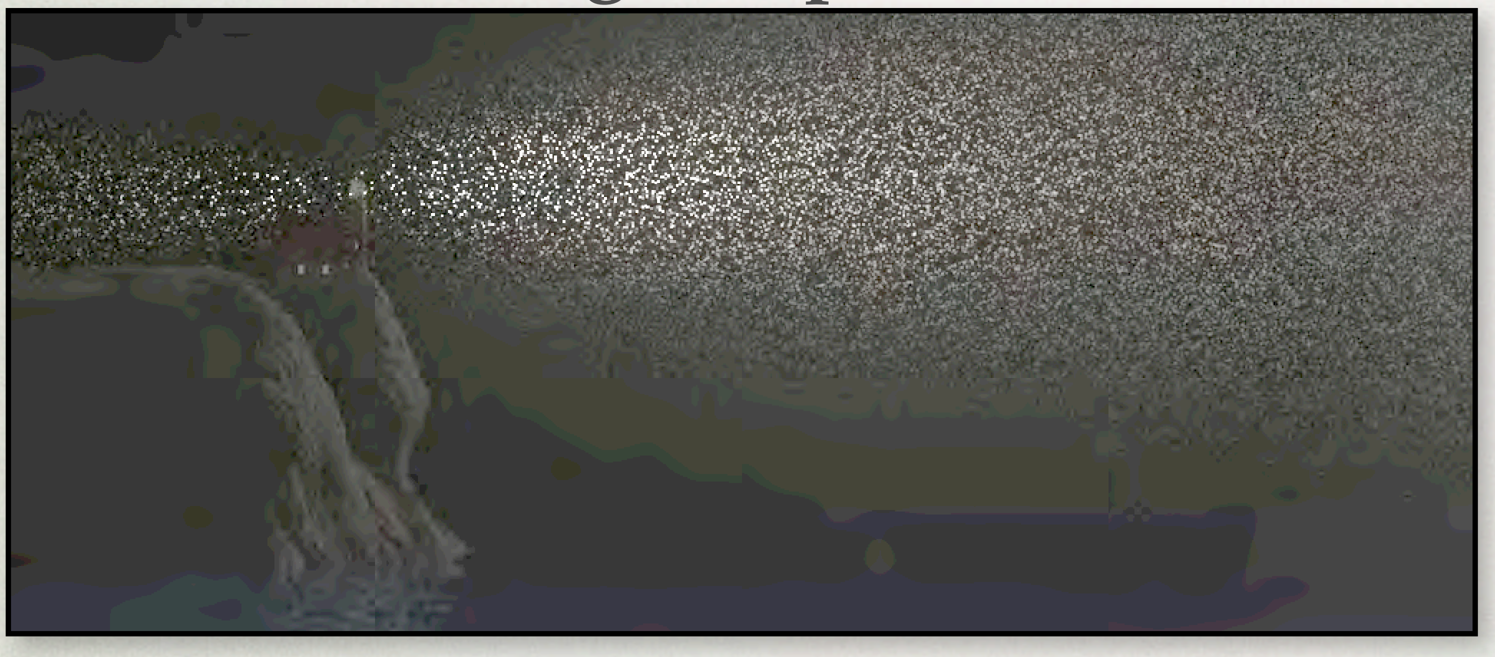
Drawbacks

- Radiance estimation is expensive
  - Requires range search in photon map
  - Performed numerous times per ray



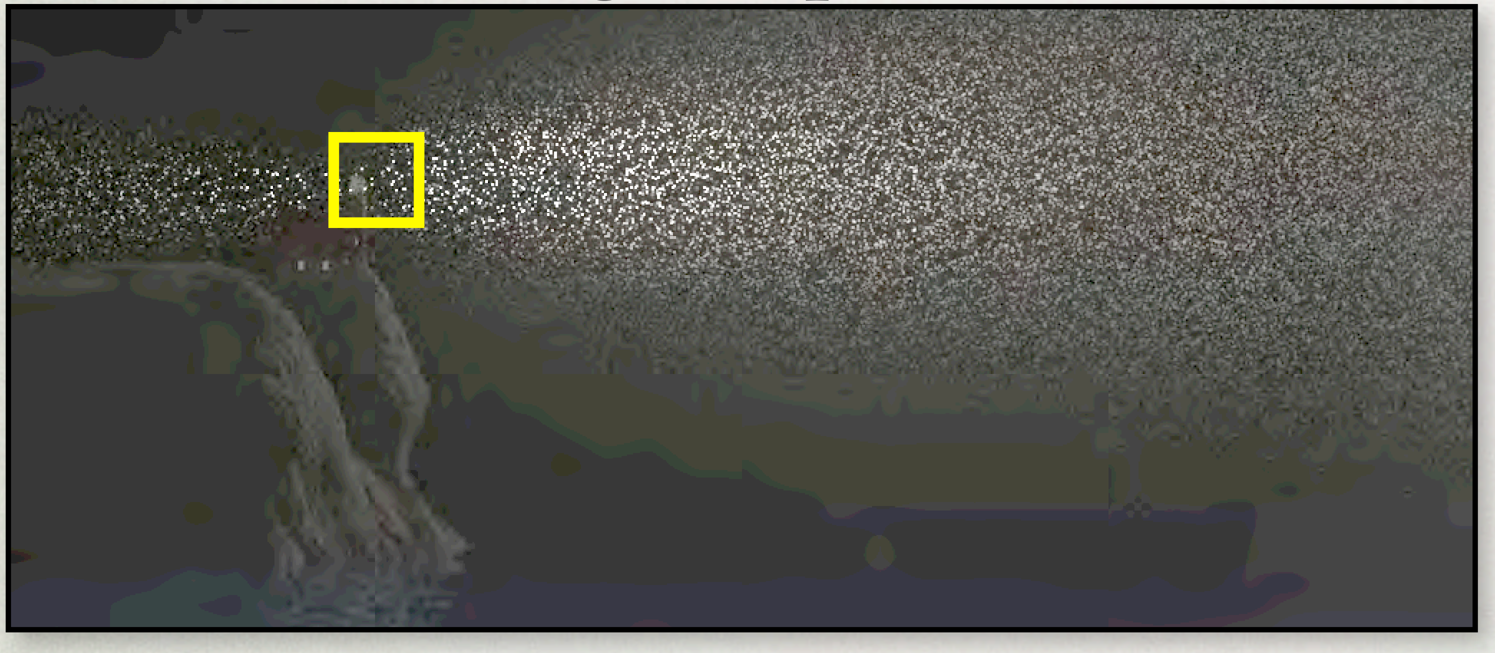
Drawbacks

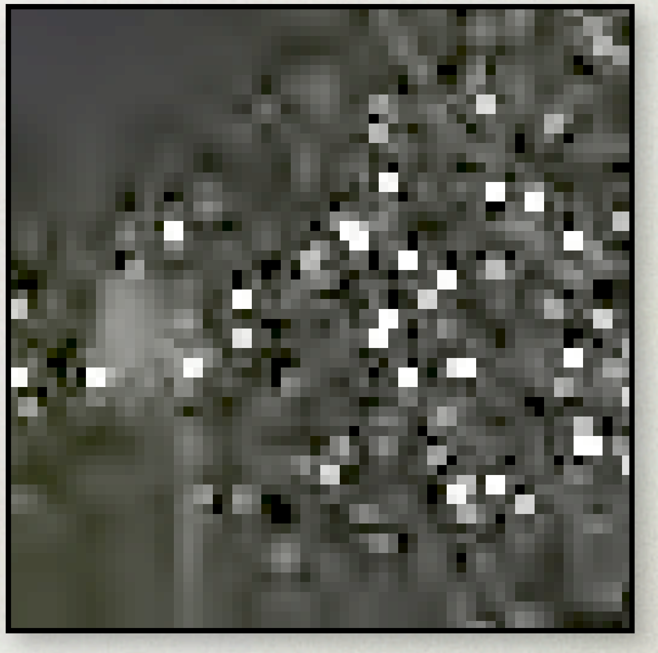
Large Step-size



Drawbacks

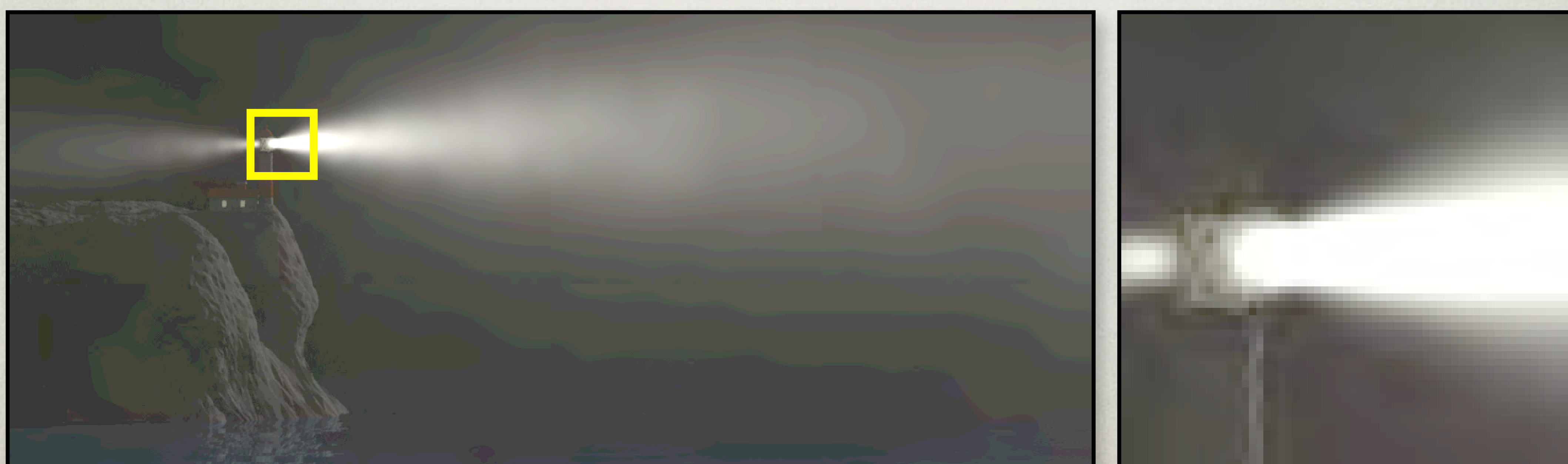
Large Step-size





Drawbacks

Large Step-size

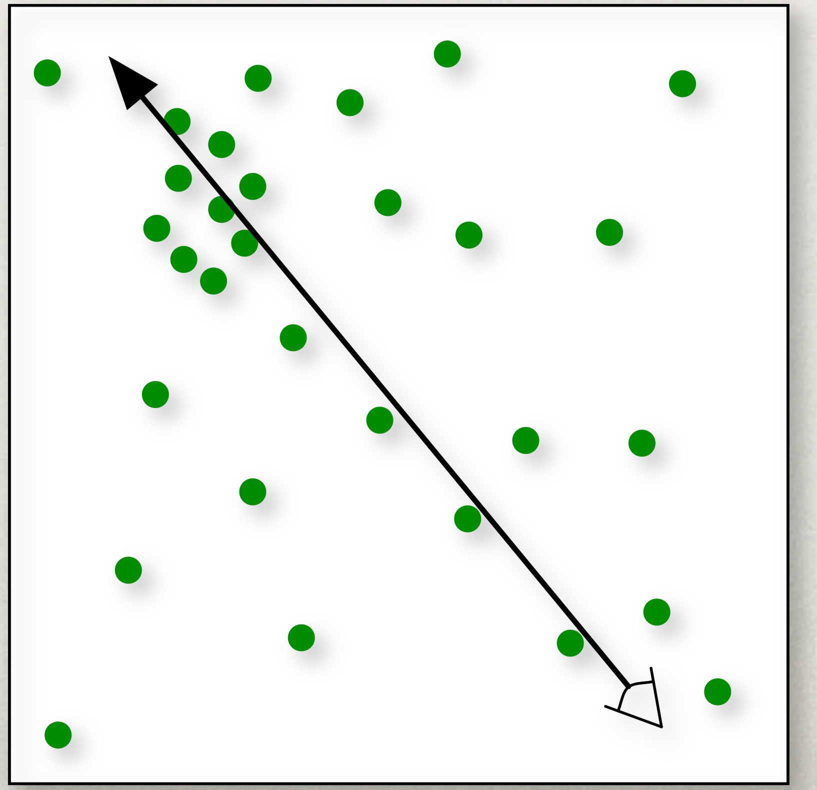


Very Small Step-size

Volumetric Photon Mapping

Conventional Radiance Estimate

$$L(\mathbf{x}, \vec{\omega}) \approx T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s) L(\mathbf{x}_s, \vec{\omega}) + \left(\sum_{t=0}^{S-1} T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t) \sigma_s(\mathbf{x}_t) L_i(\mathbf{x}_t, \vec{\omega}) \Delta_t \right)$$

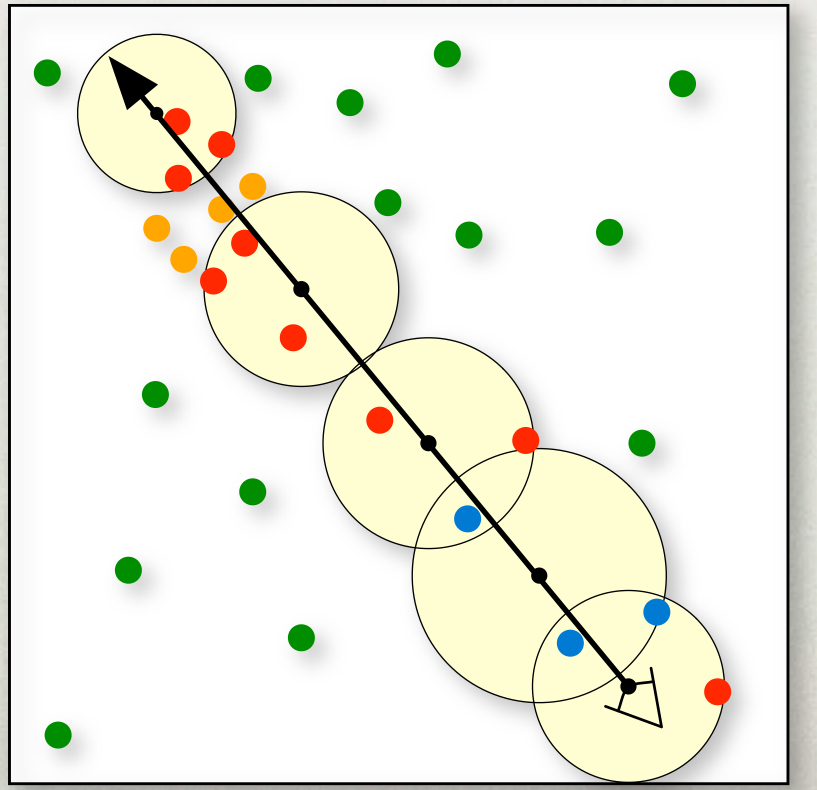


Volumetric Photon Mapping

Conventional Radiance Estimate

$$L(\mathbf{x},\vec{\omega}) \approx T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega}) + \left(\sum_{t=0}^{S-1} T_r(\mathbf{x}\leftrightarrow\mathbf{x}_t)\sigma_s(\mathbf{x}_t) L_i(\mathbf{x}_t,\vec{\omega}) \Delta_t\right)$$

$$L_i(\mathbf{x}_t,\vec{\omega}) \approx \sum_{p=1}^{n} \frac{p(\mathbf{x}_t,\vec{\omega},\vec{\omega}_p)\Delta\Phi_p}{\frac{4}{3}\pi r^3}$$

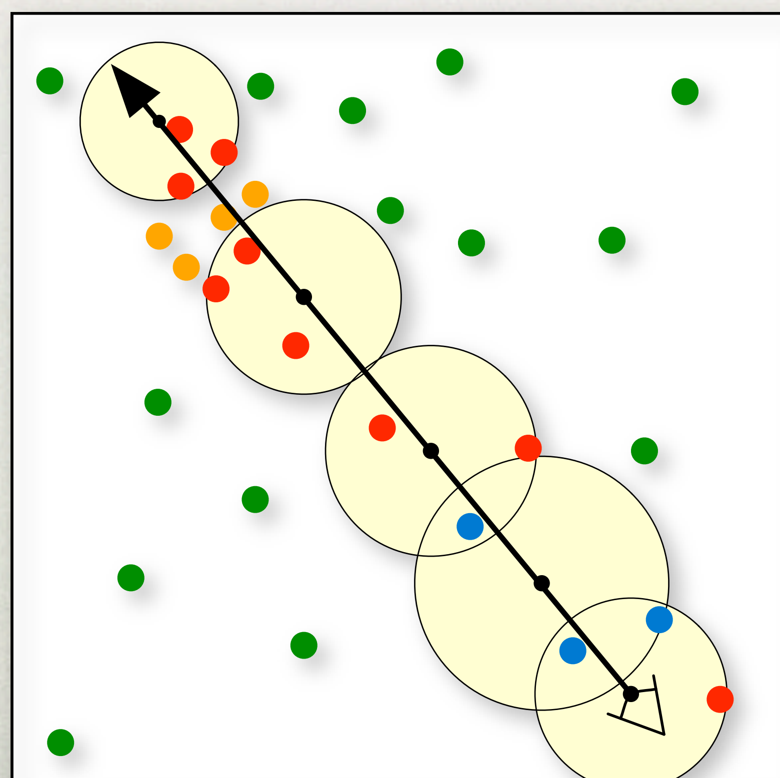


Volumetric Photon Mapping

Conventional Radiance Estimate

$$L(\mathbf{x}, \vec{\omega}) \approx T_r(\mathbf{x} \leftrightarrow \mathbf{x}_s) L(\mathbf{x}_s, \vec{\omega}) +$$

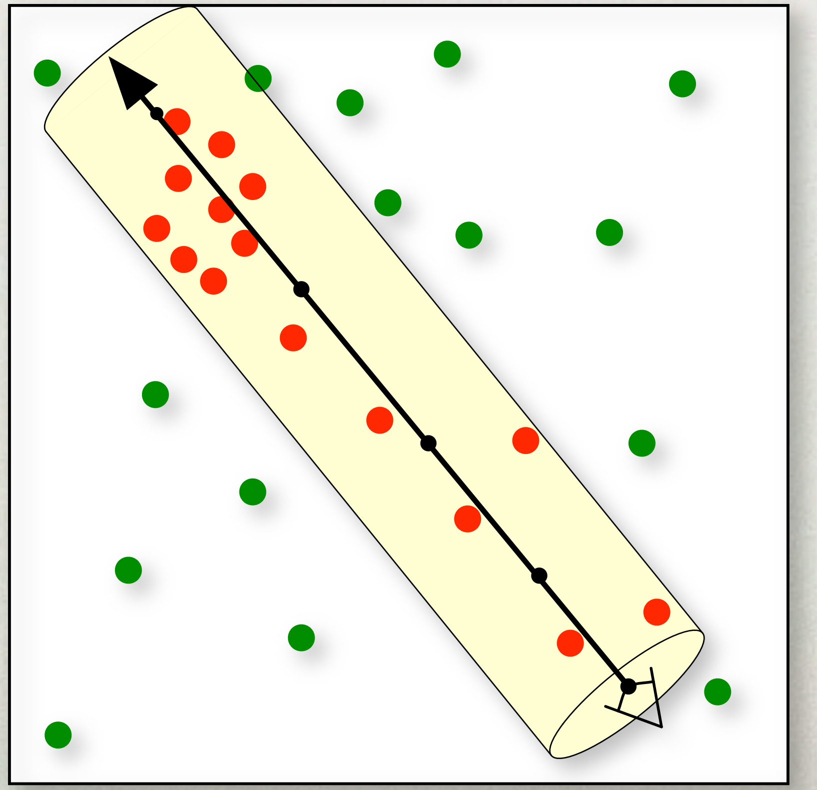
$$\left(\sum_{t=0}^{S-1} T_r(\mathbf{x} \leftrightarrow \mathbf{x}_t) \sigma_s(\mathbf{x}_t) L_i(\mathbf{x}_t, \vec{\omega}) \Delta_t \right)$$



Volumetric Photon Mapping

Beam Radiance Estimate

$$L(\mathbf{x},\vec{\omega}) \approx T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega}) + L_b(\mathbf{x},\vec{\omega})$$

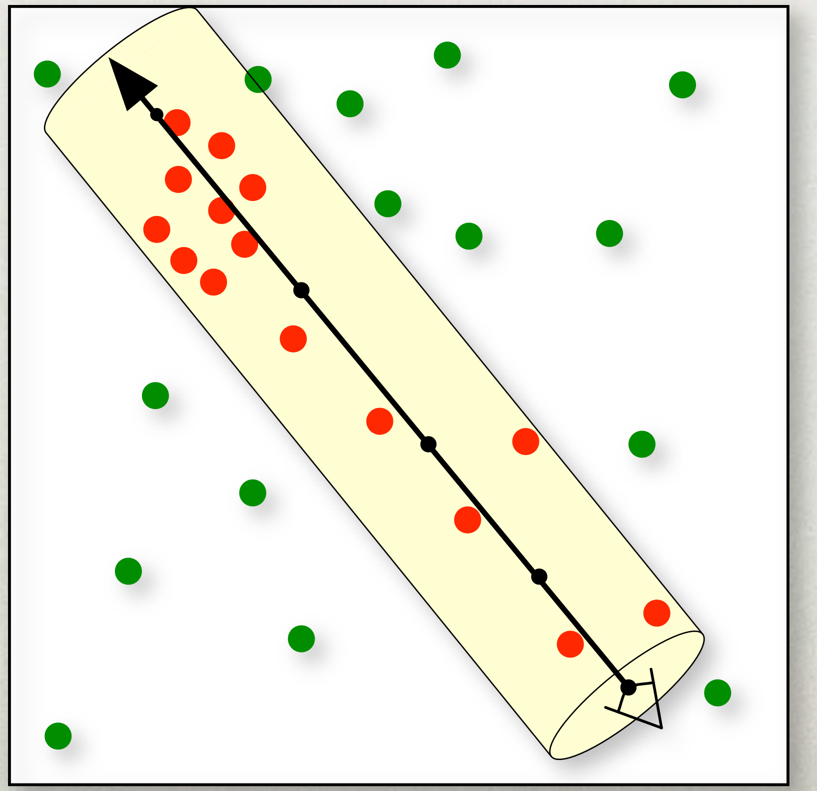


Volumetric Photon Mapping

Beam Radiance Estimate

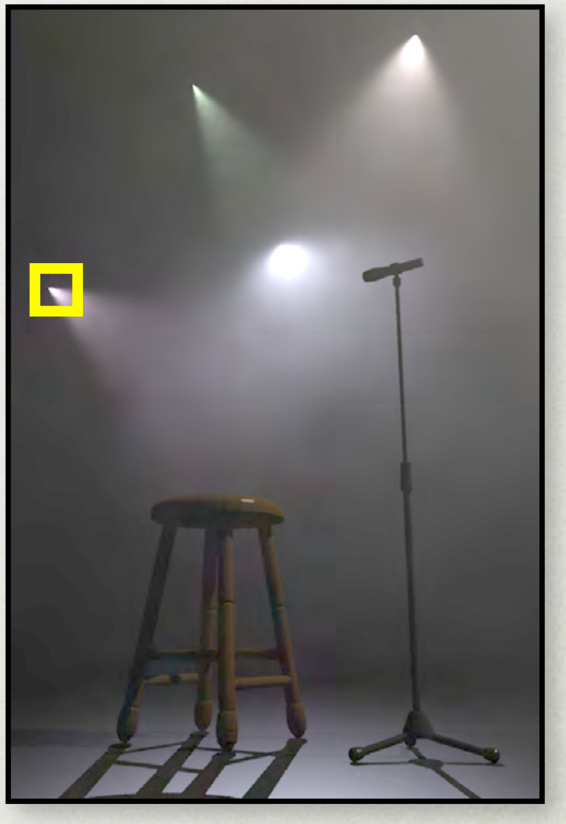
$$L(\mathbf{x},\vec{\omega}) \approx T_r(\mathbf{x}\leftrightarrow\mathbf{x}_s)L(\mathbf{x}_s,\vec{\omega}) + L_b(\mathbf{x},\vec{\omega})$$

$$L_b(\mathbf{x},\vec{\omega}) = \frac{1}{N}\sum_{i=1}^{N} K_i T_r(\mathbf{x}\leftrightarrow\mathbf{x}_i)\sigma_s(\mathbf{x}_i)p(\mathbf{x}_i,\vec{\omega},\vec{\omega}_i)\alpha_i$$



Adaptive Radius Comparison

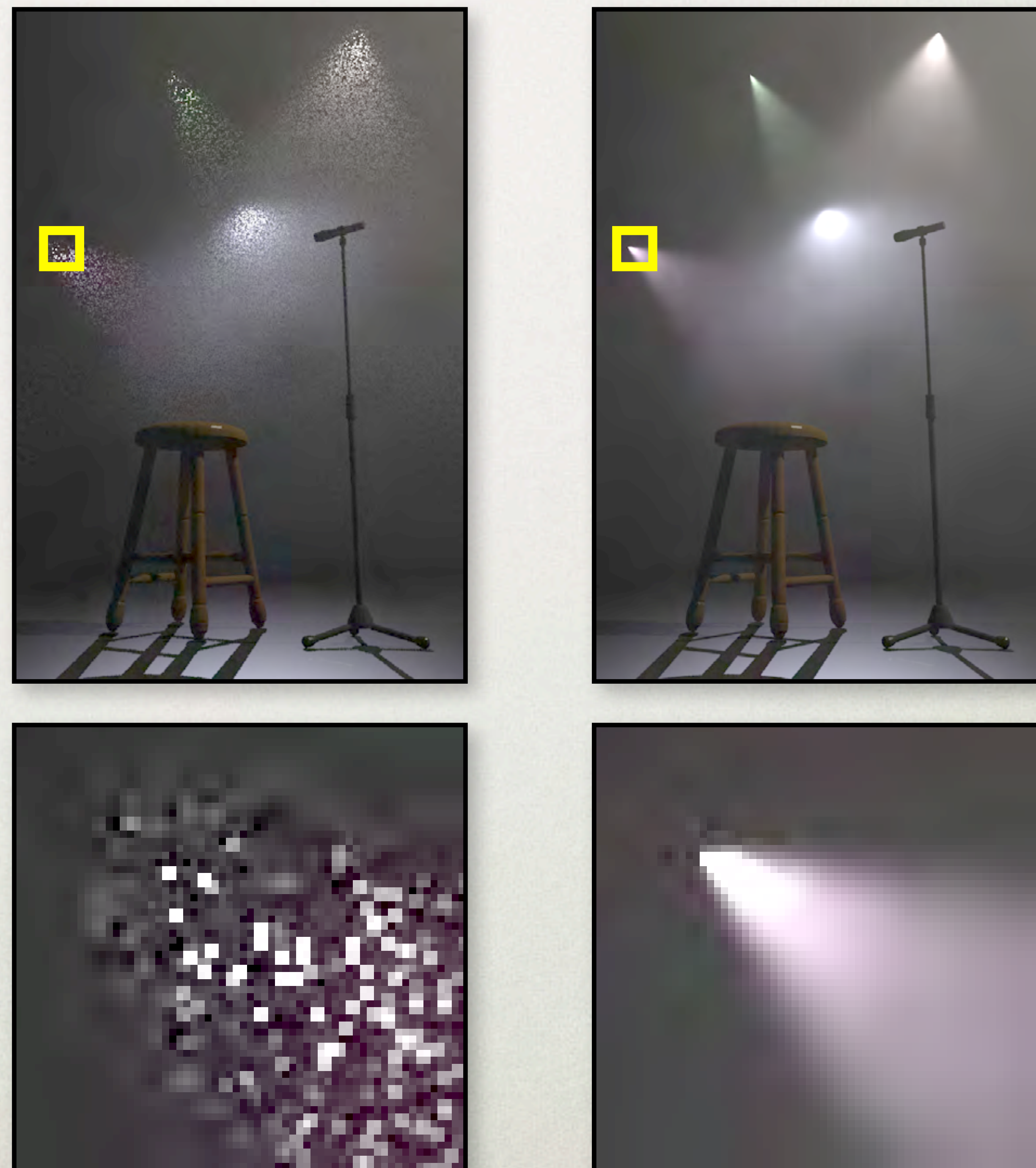
Beam Estimate



(6:22)

Adaptive Radius Comparison

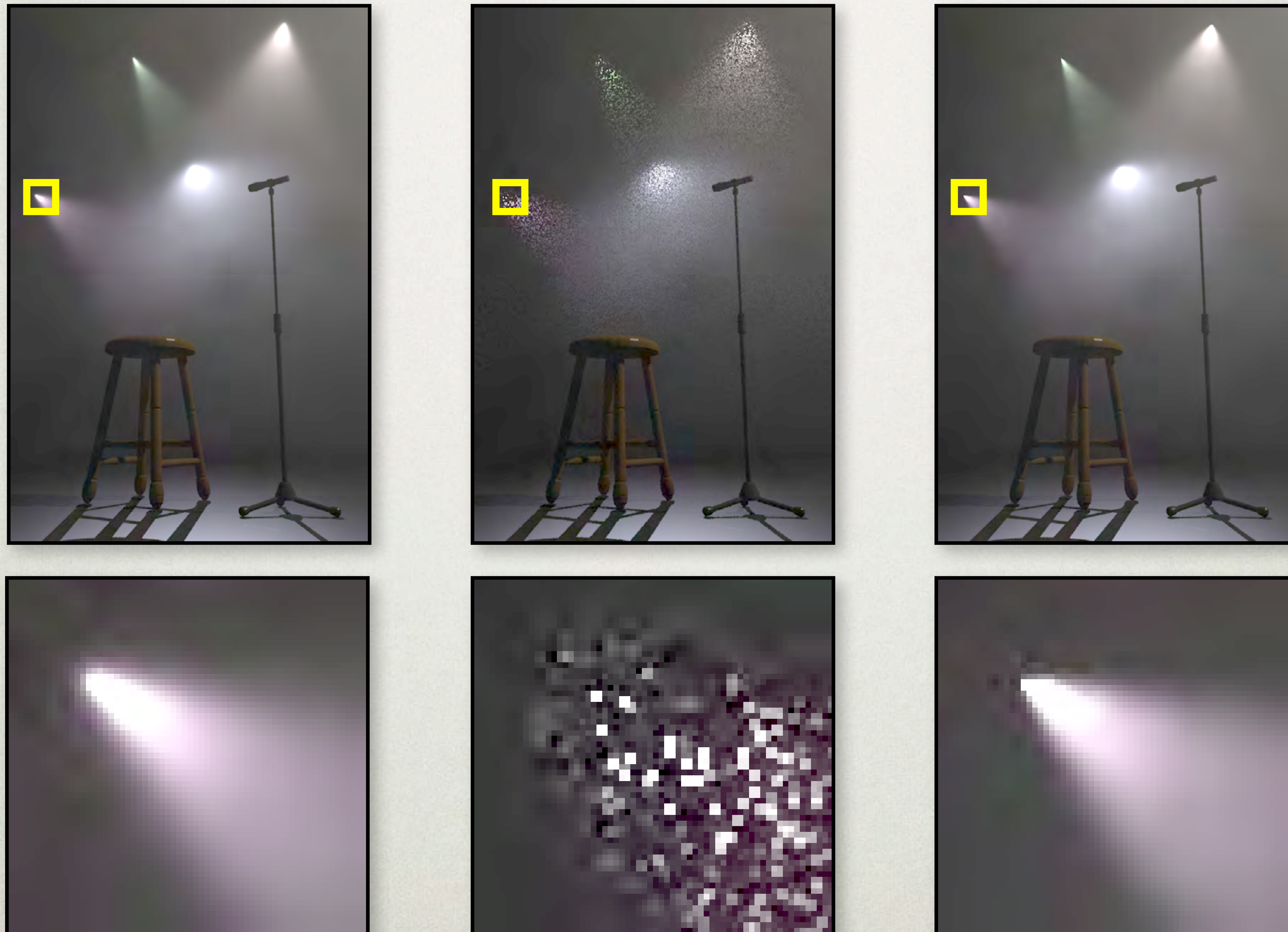
Conv. Estimate Beam Estimate



(6:38) (6:22)

Adaptive Radius Comparison

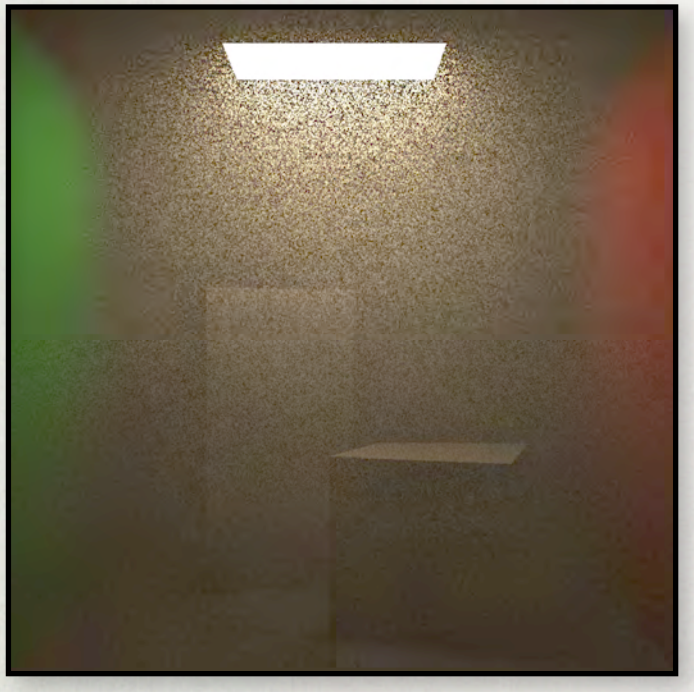
Conv. Estimate | Conv. Estimate | Beam Estimate



(∞) | (6:38) | (6:22)

Smoky Cornell Box

Conv. Estimate



Beam Estimate

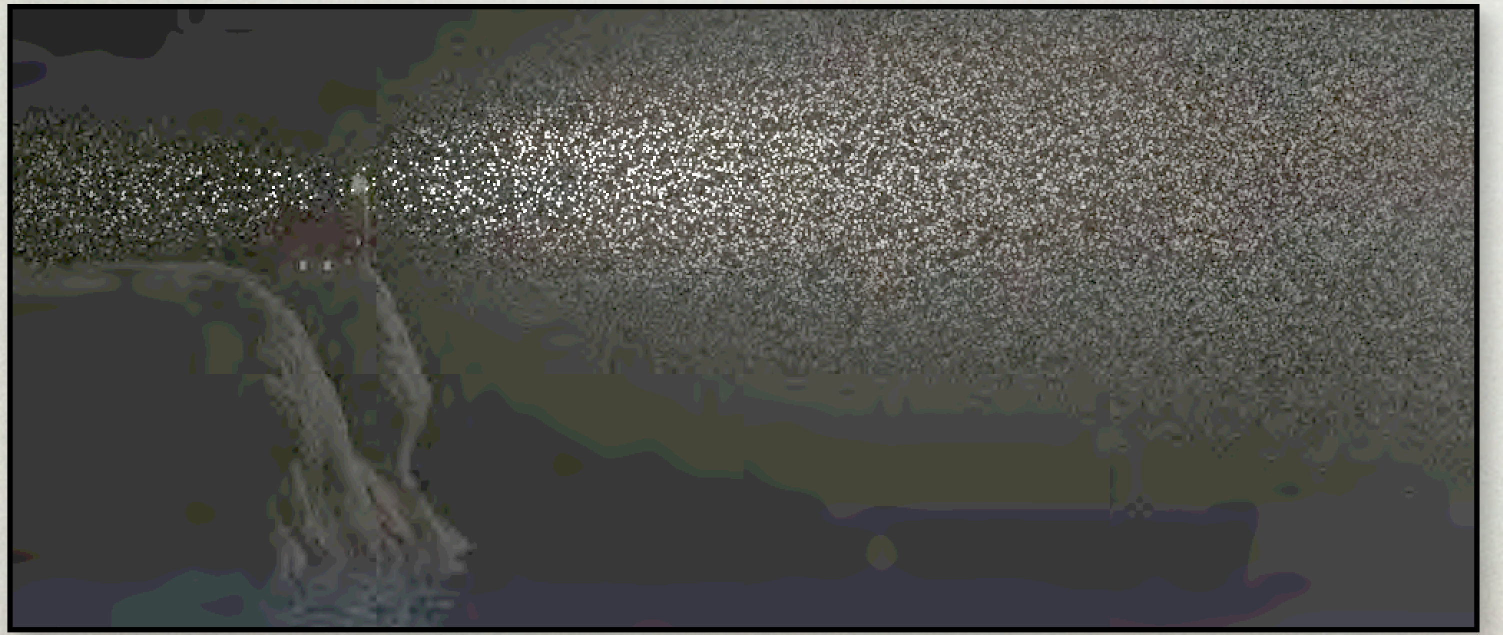
Smoky Cornell Box

Conv. Estimate | Beam Estimate

(4:03) | (3:35)

Lighthouse

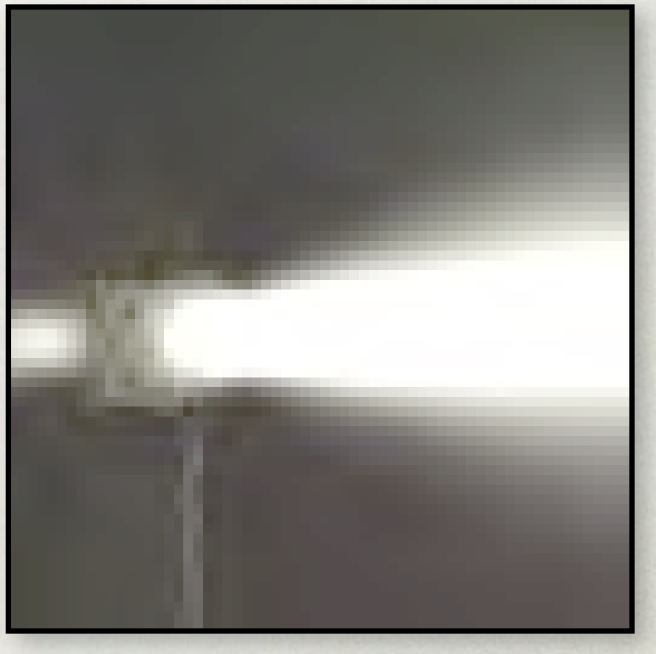
Beam Estimate

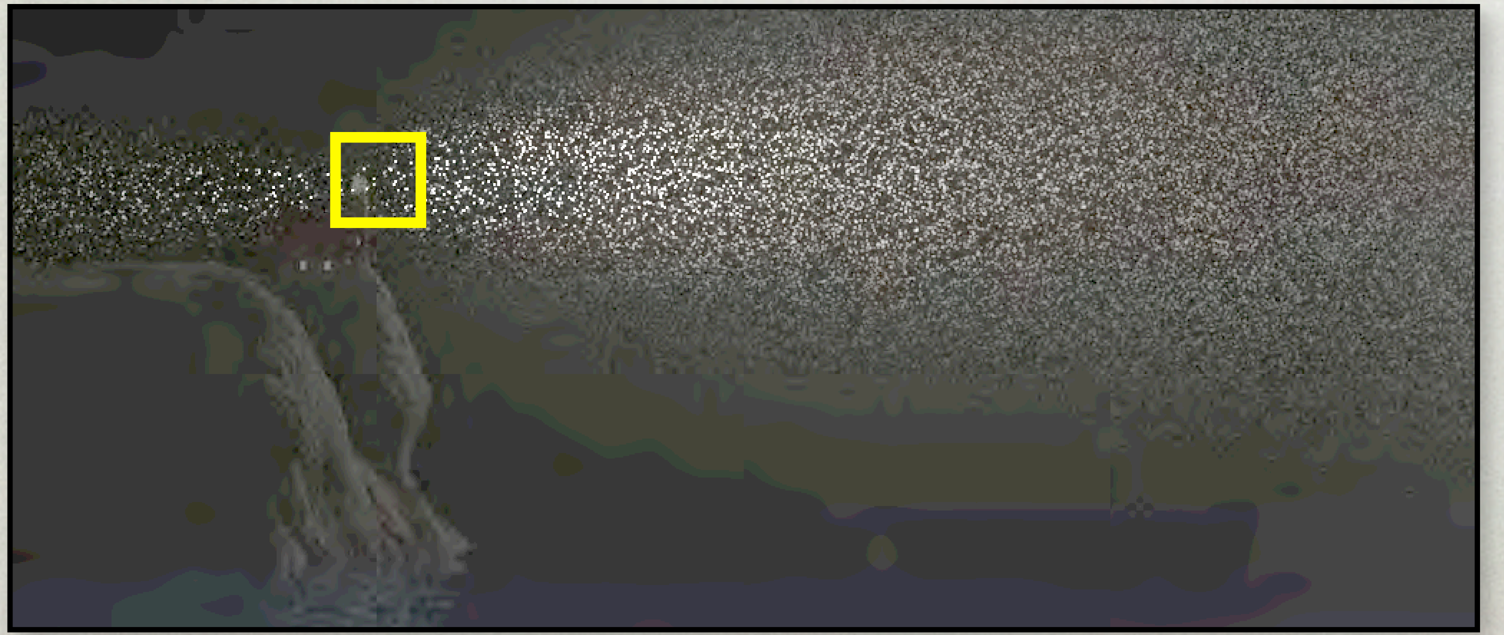


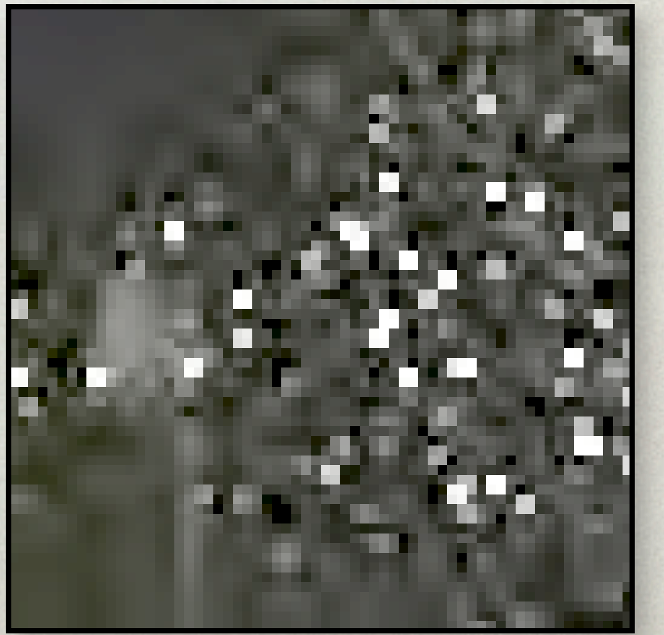
Conventional Estimate

Lighthouse

Beam Estimate (1:05)







Conventional Estimate (1:12)

Cars on Foggy Street

Beam Estimate

Conventional Estimate

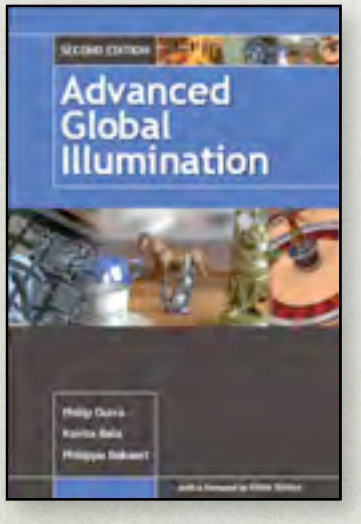
Cars on Foggy Street

Beam Estimate (1:53)

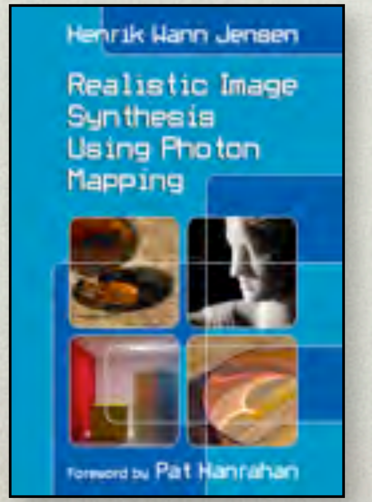
Conventional Estimate (2:02)

For More Information

Physically Based Rendering.
Matt Pharr and Greg Humphreys



Advanced Global Illumination.
Philip Dutre, Kavita Bala, and Philippe Bekaert



Realistic Image Synthesis Using Photon Mapping.
Henrik Wann Jensen

Questions?