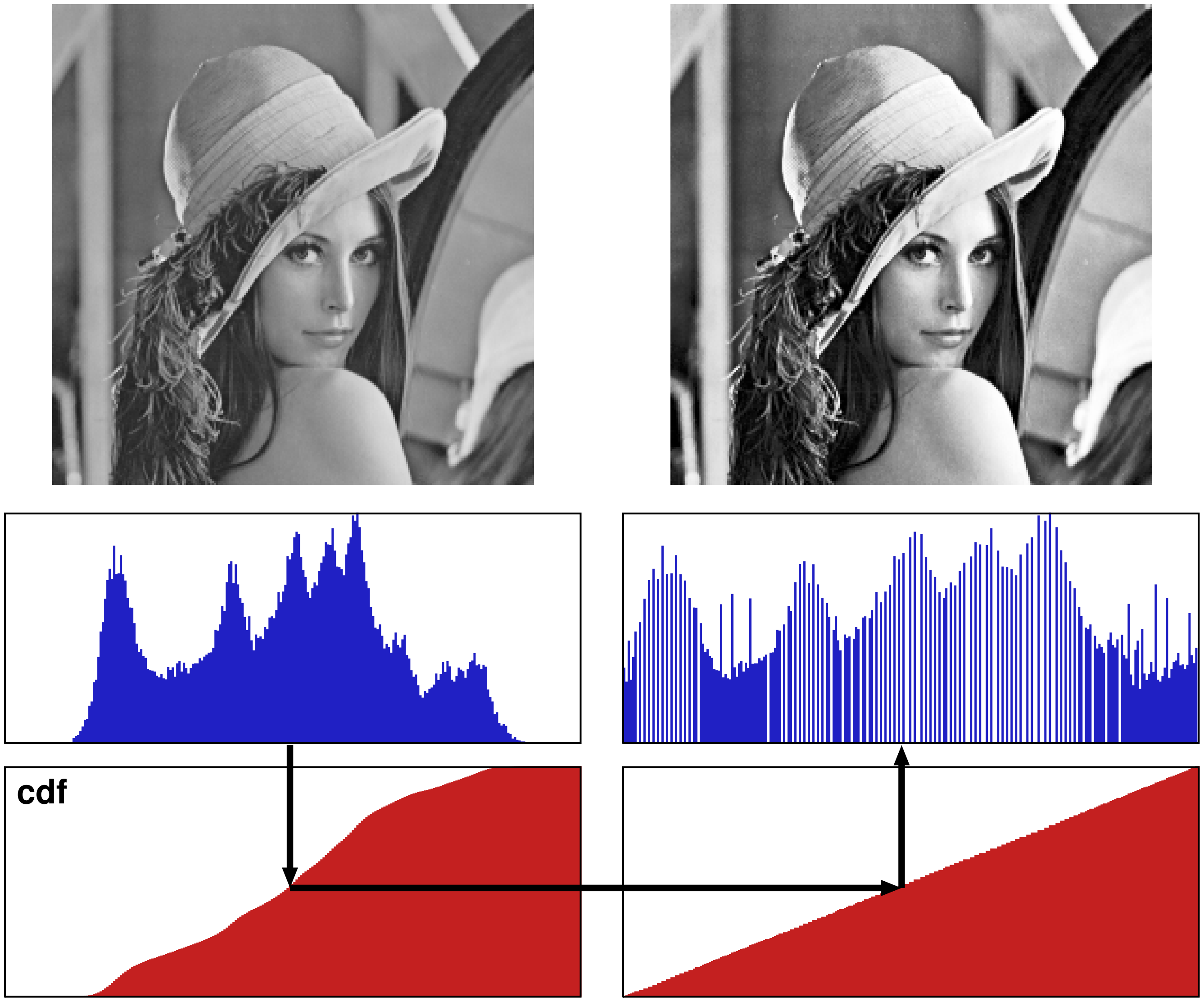
Mid Term Recap

December 4, 2024

Monadic Operations

Equalization: $T(l) = \mathrm{cdf}(l) = \sum_{i=0}^{l} \mathrm{pdf}(i)$



Equalization

I

$I < T_{0.1}$
$I_{eq} < 0.1$

$I < T_{0.3}$
$I_{eq} < 0.3$

$I < T_{0.5}$
$I_{eq} < 0.5$

$I < T_{0.7}$
$I_{eq} < 0.7$

$I < T_{0.9}$
$I_{eq} < 0.9$

Equalization

I

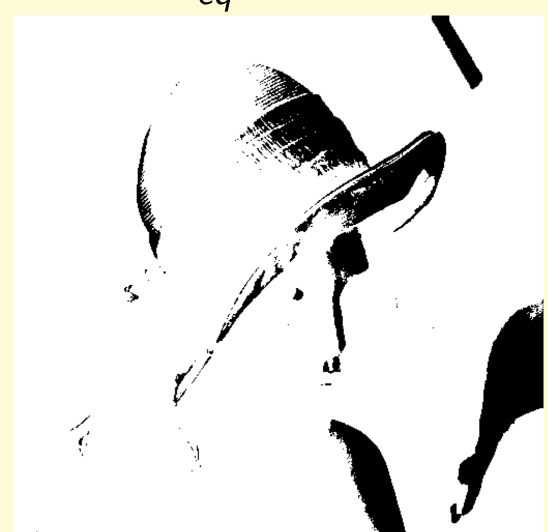
$I < 0.21$
$I_{eq} < 0.1$

$I < 0.39$
$I_{eq} < 0.3$

$I < 0.51$
$I_{eq} < 0.5$

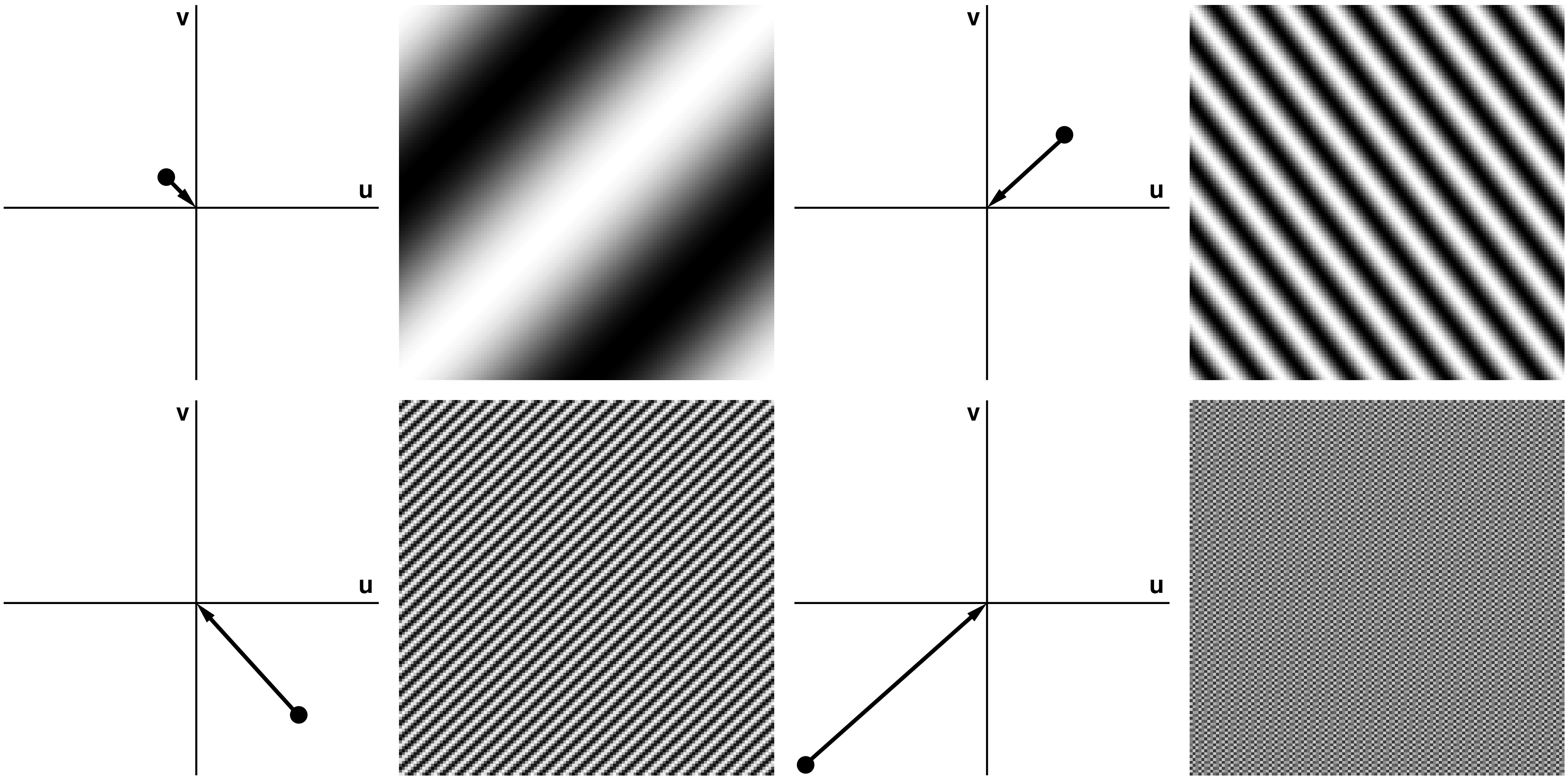
$I < 0.60$
$I_{eq} < 0.7$

$I < 0.75$
$I_{eq} < 0.9$



Fourier Transform (in 2D)

basis functions:



Fourier Basis (in 1D)

$$\phi_k[n] = e^{-i\frac{2\pi kn}{N}}, \quad n, k = 0, 1, \ldots, N-1, \tag{1}$$

- k is the frequency index,
- n is the time sample index,
- N is the total number of samples.

Dot product:

$$\langle \phi_k, \phi_m \rangle = \sum_{n=0}^{N-1} \phi_k[n]\phi_m^*[n] = \sum_{n=0}^{N-1} e^{-i\frac{2\pi kn}{N}} e^{i\frac{2\pi mn}{N}} \tag{2}$$

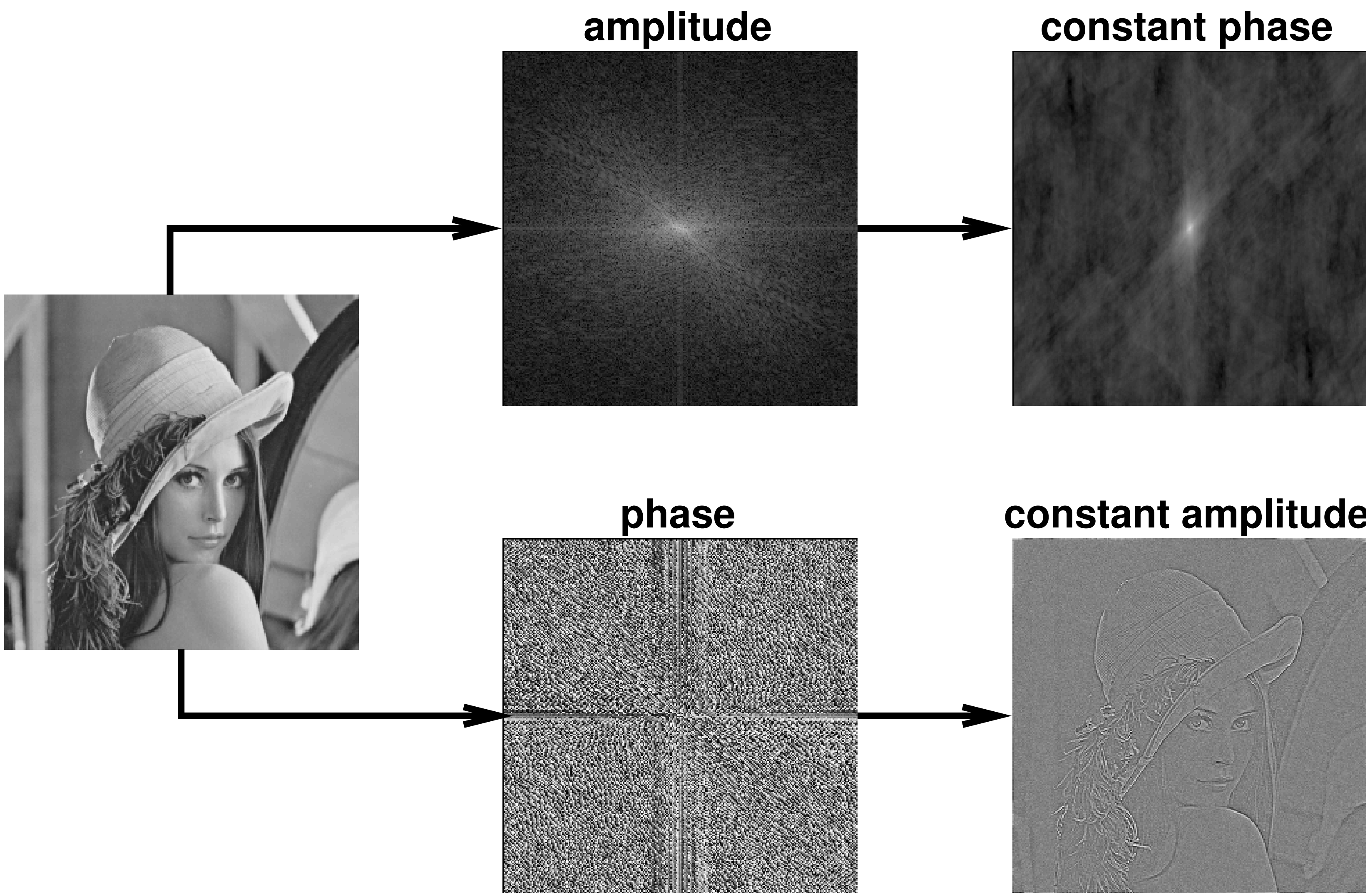
The Fourier Basis is orhogonal:

$$\langle \phi_k, \phi_m \rangle = \begin{cases} N, & \text{if } k = m, \\ 0, & \text{if } k \neq m. \end{cases} \tag{3}$$

Fourier Transform (in 2D)

Edges with orientation $\arctan(v/u)$ and frequency $\sqrt{u^2+v^2}$:

Amplitude $\Rightarrow$ **intensity** **Phase** $\Rightarrow$ **"location"**



Phase (only)

Let us keep the original phase only, and replace the amplitude by $A = f^{-\beta}$, $f = \sqrt{u^2 + v^2}$:

$\beta = 0$

$\beta = 0.4$

$\beta = 0.8$

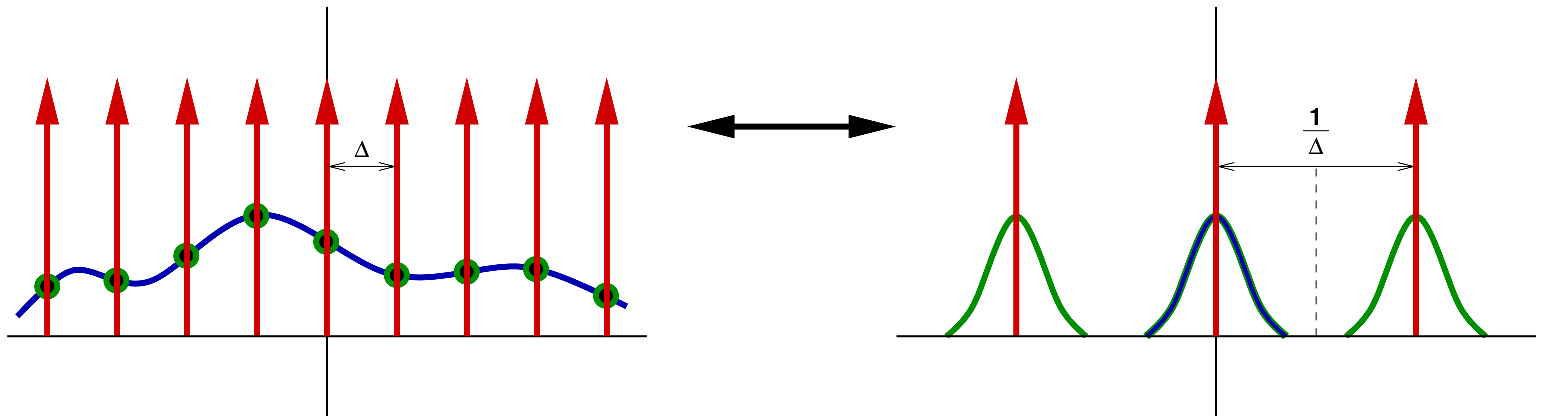
$\beta = 1$

$\beta = 1.5$

$\beta = 2$

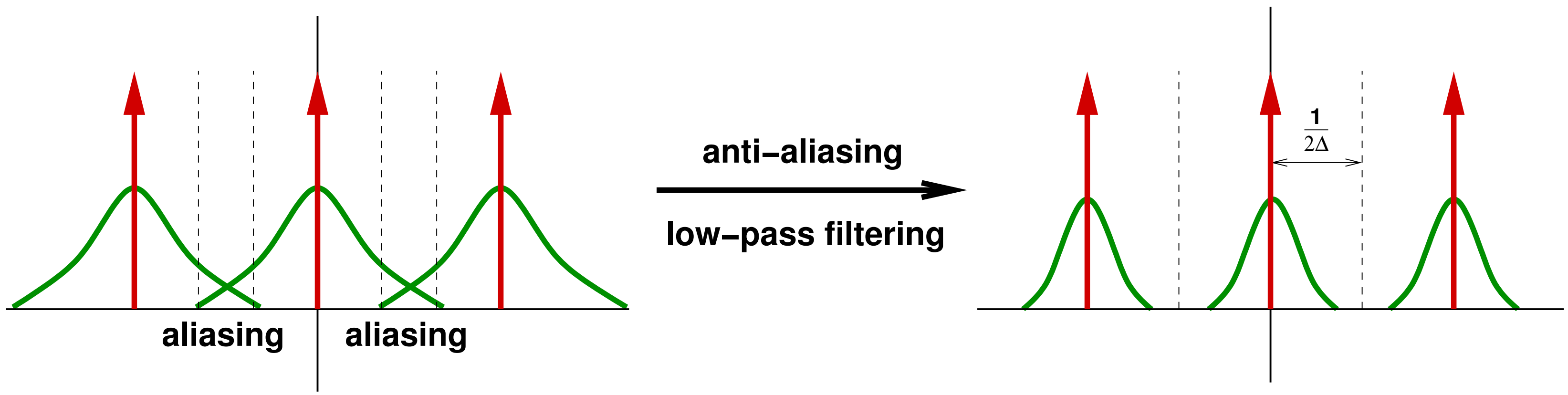
Sampling Theorem

Nyquist frequency: $f_{max} \leq \frac{1}{2\Delta}$



$$s(x) = \sum_k \delta\left(x - k\Delta\right) \iff S(u) = \sum_k \delta\left(u - \frac{k}{\Delta}\right)$$

$$d(x) = f(x) \cdot s(x) \iff D(u) = (F * S)(u)$$



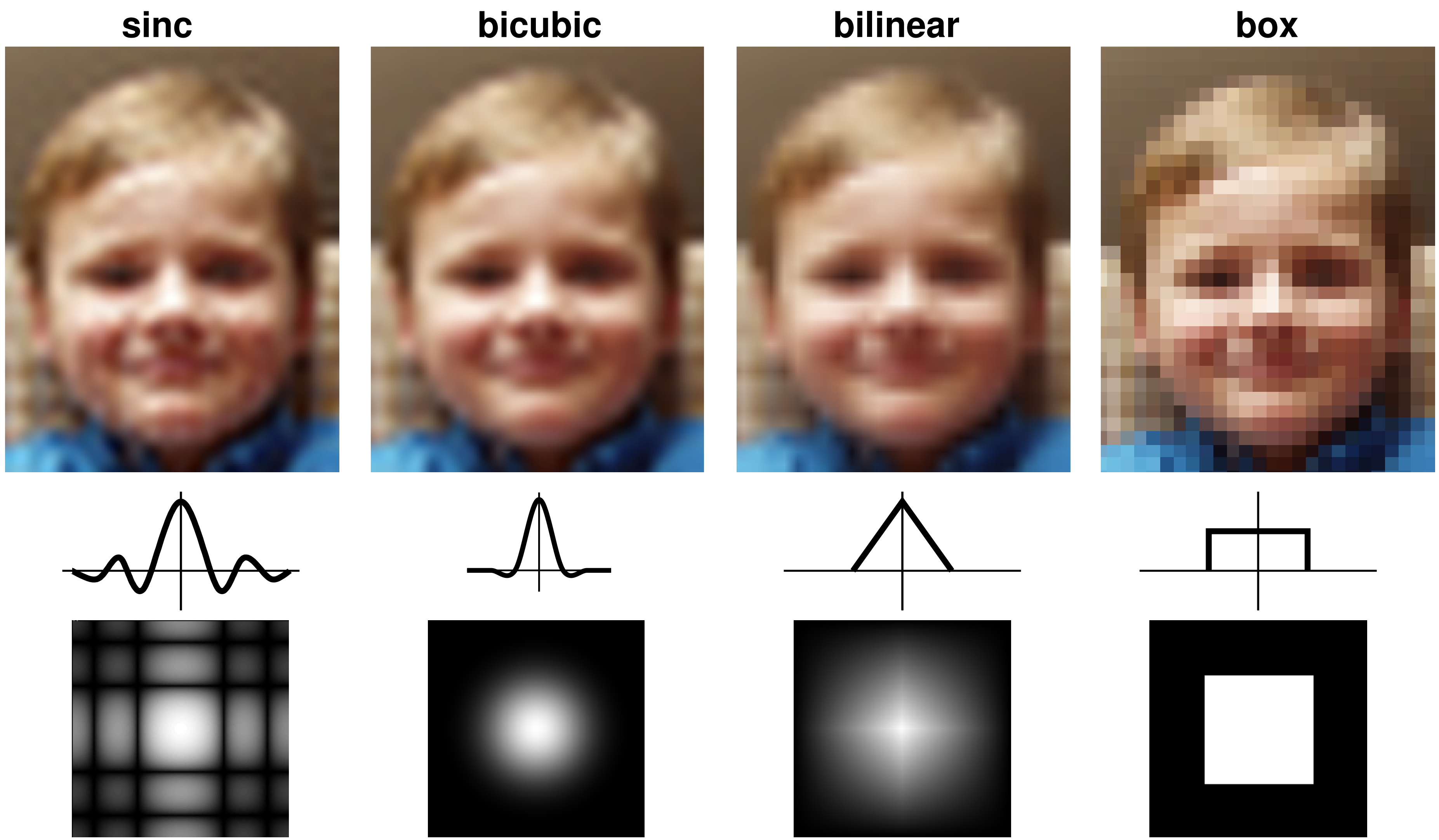
Sampling theorem

- What can I do if sampling rate was too low?
- How about if I have a modulated signal? (e.g. FM radio)

Image Resampling

Problem: convolution with sinc ⇒ time consuming, ringing artifacts.

Sinc approximations with narrow support (bicubic, bilinear, box):



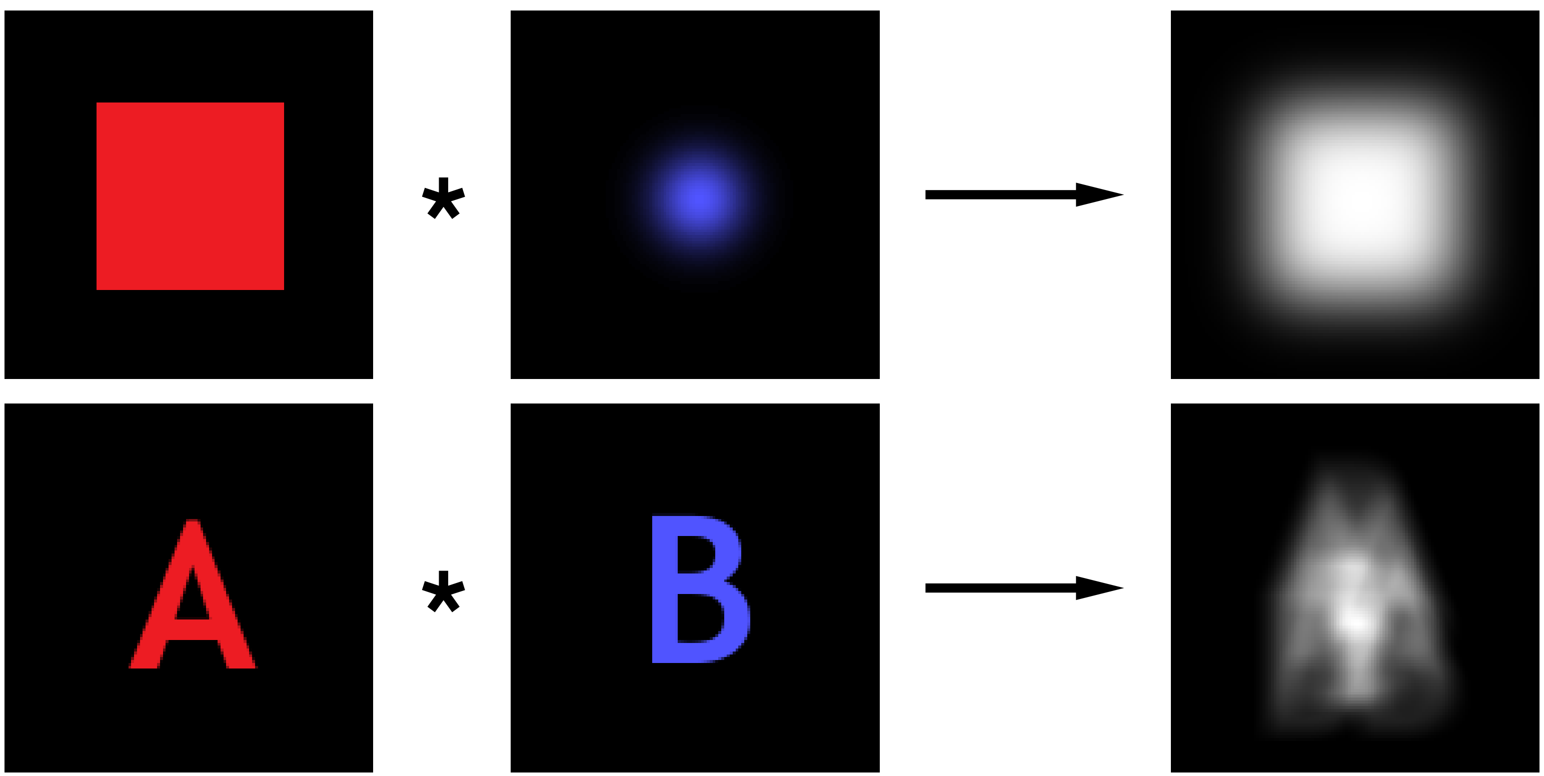
Interpolation

- Does 2D nearest neighbor interpolation (box kernel) produce a result which is piecewise constant?
- Does (2D) bilinear interpolation (pyramid kernel) produce a result which is piecewise linear ($r = ax + by + c$)?

Convolution (in 2D)

Extension of 1D case (image F, convolution kernel G):

$$(F * G)(s,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(x,y)\cdot G(s-x,t-y)\,\mathrm{d}x\mathrm{d}y$$



(Blackboard example – Computing convolution)

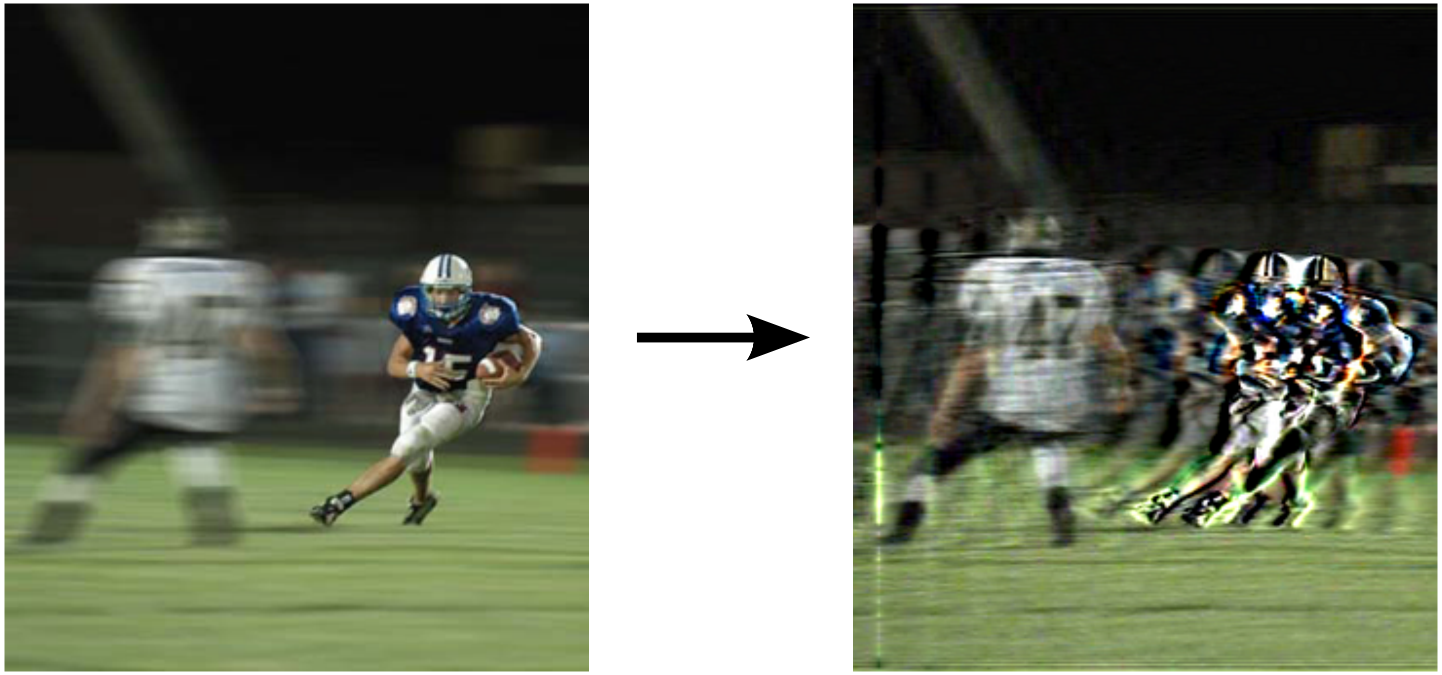
Wiener Filtration

Problem: image f is also degraded by unknown additive noise n.

$$g = h * f + n \quad \Longleftrightarrow \quad G = H \cdot F + N \quad \Rightarrow \quad F = (G - N)/H$$

Solution: find an image $\hat{f}$ such that $||f - \hat{f}||^2$ is minimal.

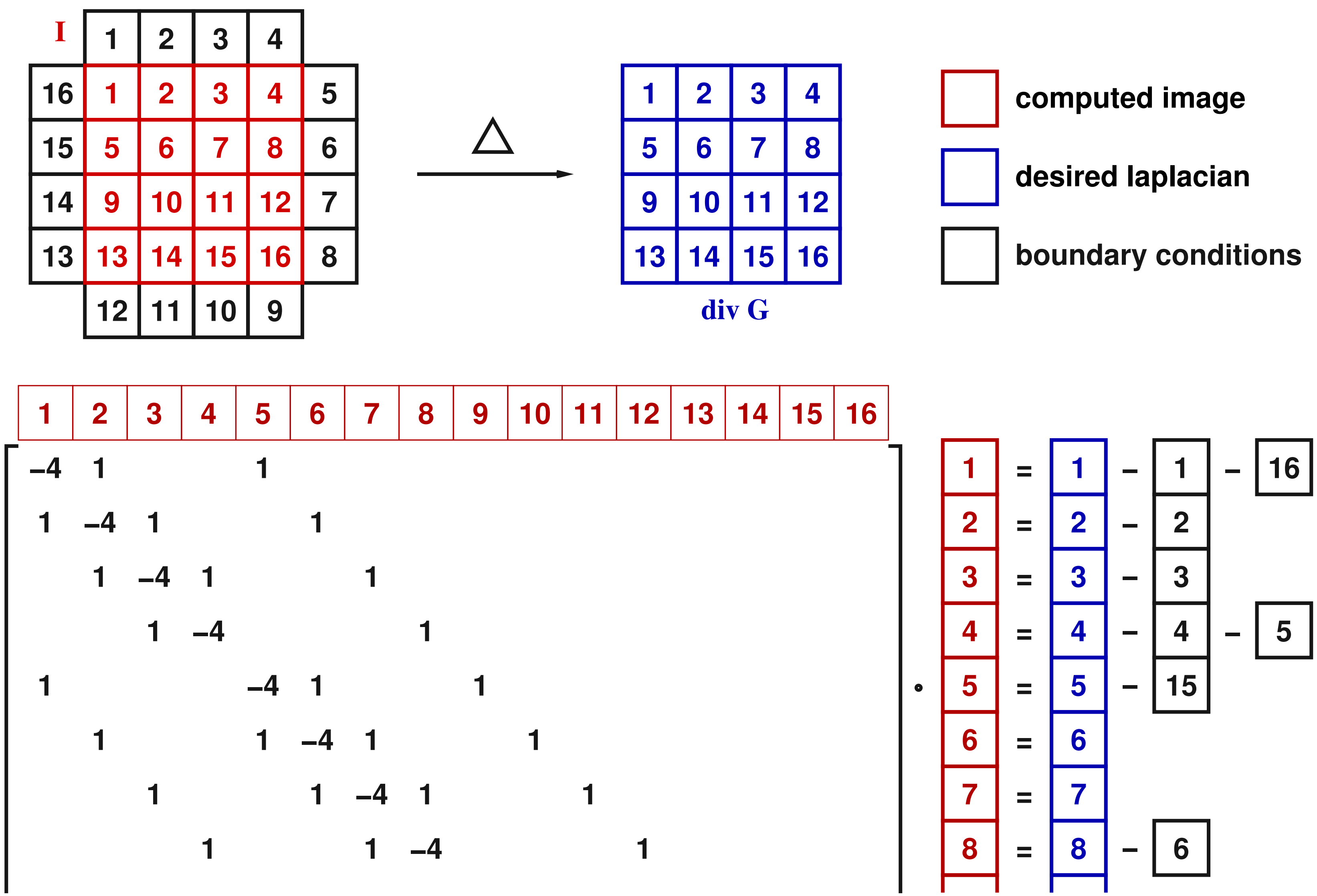
$$\hat{F}(u,v) = \frac{H^*(u,v) \cdot G(u,v)}{||H(u,v)||^2 + \lambda} \qquad \lambda = \mathrm{SNR}^{-1} = \frac{||N(u,v)||^2}{||F(u,v)||^2}$$



(Matlab demo)

Gradient domain

Discretization of Poisson equation:



(Matlab demo and discussion)