Digital Image

(B4M33DZO, Summer 2024)

Lecture 11:

Image Segmentation 1

https://cw.fel.cvut.cz/wiki/courses/b4m33dzo/start

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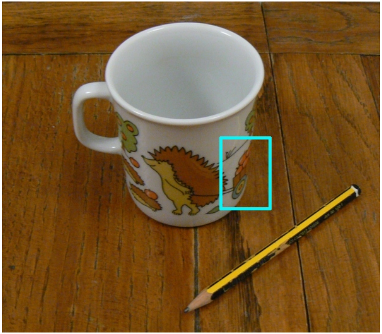
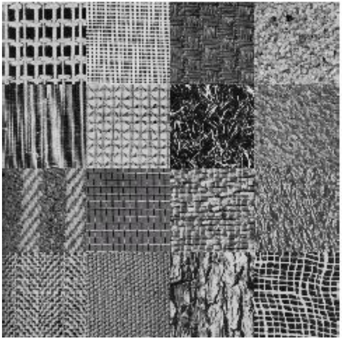
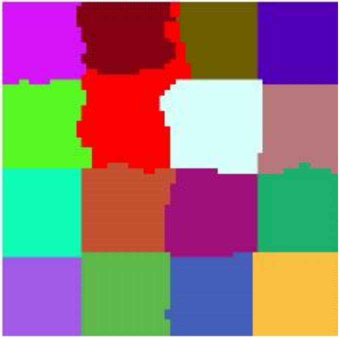
Segmentation

Segmentation



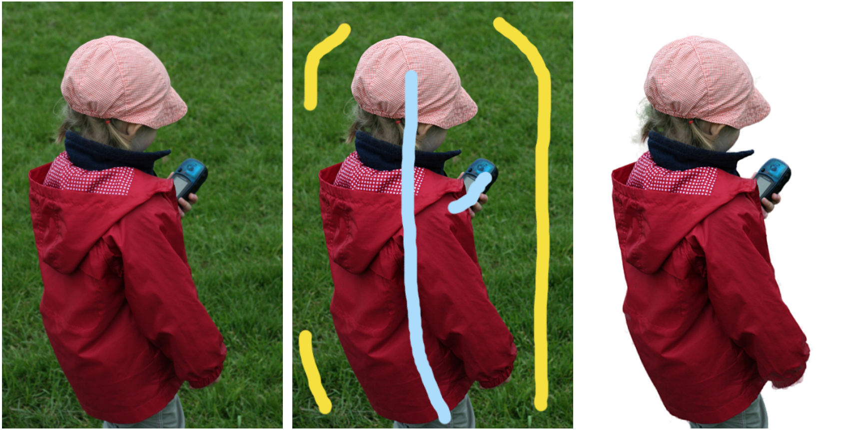
Texture Segmentation Example





User Assisted Segmentation

Interactive image segmentation:



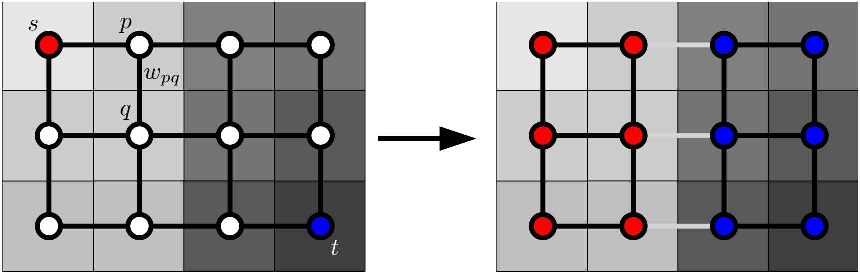
Extract part of the image using a set of user-specified constraints.

Problem formulation

Find an optimal labelling x^* such that:

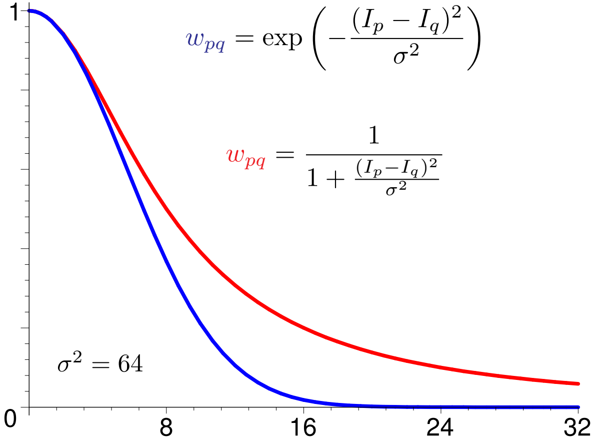
$$x^* = \arg\min_x \sum_{\{p,q\} \in \mathcal{N}} w_{pq} \left| x_p - x_q \right|^{\alpha}$$

subject to: $x_s = 0 \ \wedge \ x_t = 1$



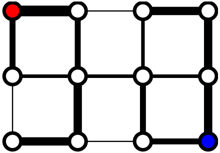
Edge weights

Salient edge ⇒ lower energy:



Similarity metric

$$w_{pq} \left| x_p - x_q \right|^{\alpha}$$



α	**algorithm**	w_{pq}
1	**min-cut**	**capacity**
2	**random walker**	**probability**
∞	**shortest path**	$1/$**length**

Min-cut

Find an optimal labelling $x^* \in \{0, 1\}$ such that:

$$x^* = \arg\min_x \sum_{\{p,q\} \in \mathcal{N}} w_{pq} \, |x_p - x_q|$$

subject to: $x_s = 0 \;\wedge\; x_t = 1$

---

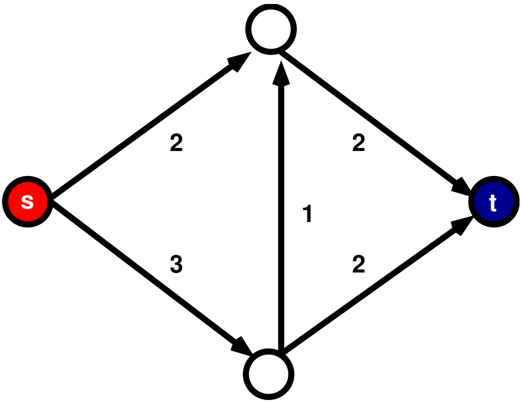
find a subset of edges with minimal total weight whose removal disconnects nodes s and t

$$\Updownarrow$$

find a maximum flow between nodes s and t in a pipe network with capacities equal to edge weights

Max-flow

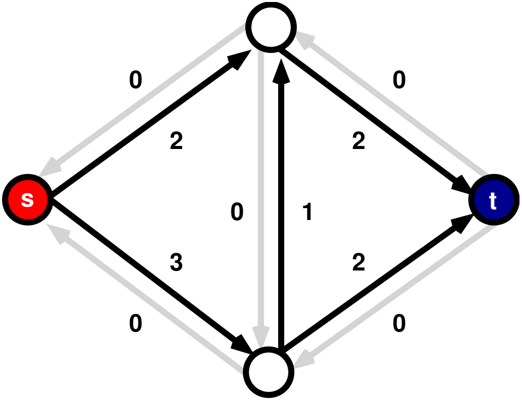
Find a maximum flow from source s to sink t:



[Ford-Fulkerson]

Max-flow

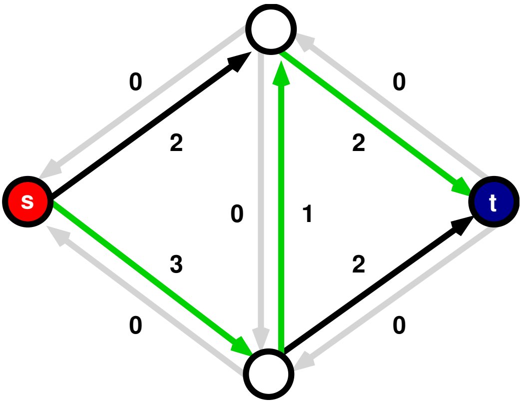
Residual network:



[Ford-Fulkerson]

Max-flow

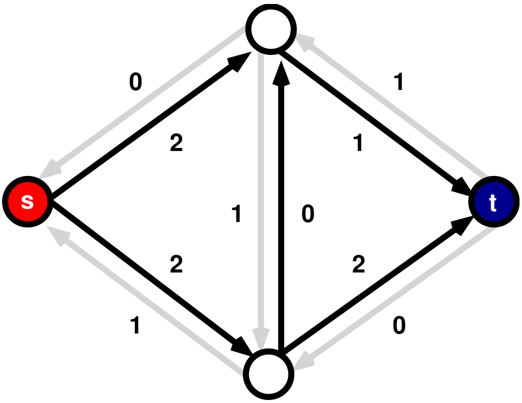
Find any path from node s to node t:



[Ford-Fulkerson]

Max-flow

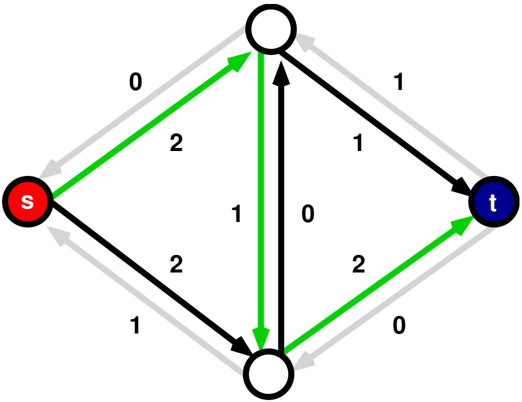
Update residual network according to path capacity:



[Ford-Fulkerson]

Max-flow

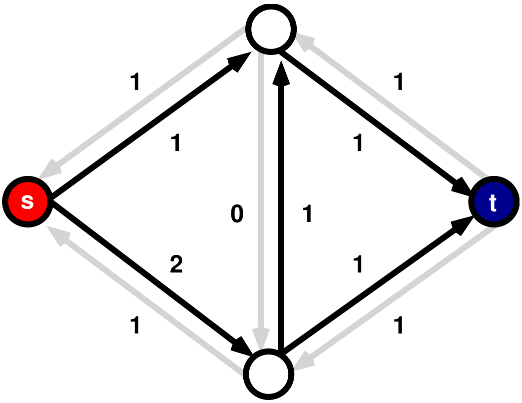
Find another path from node s to node t:



[Ford-Fulkerson]

Max-flow

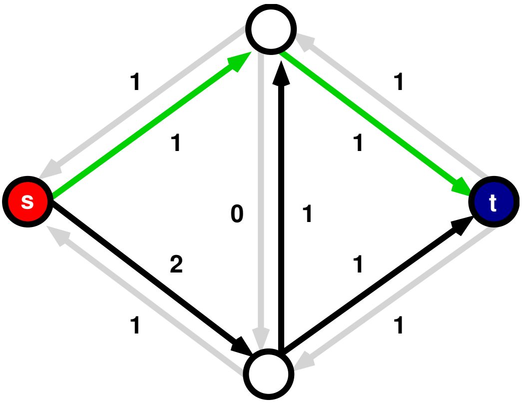
Update residual network according to path capacity:



[Ford-Fulkerson]

Max-flow

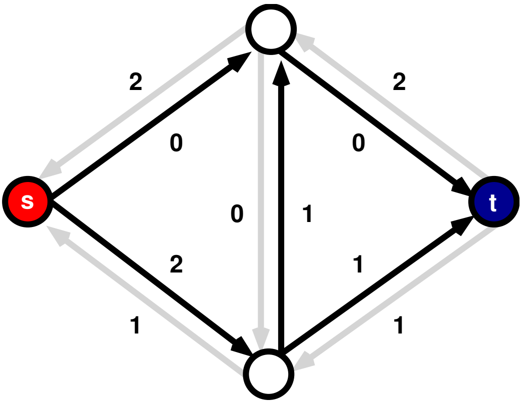
Find another path from node s to node t:



[Ford-Fulkerson]

Max-flow

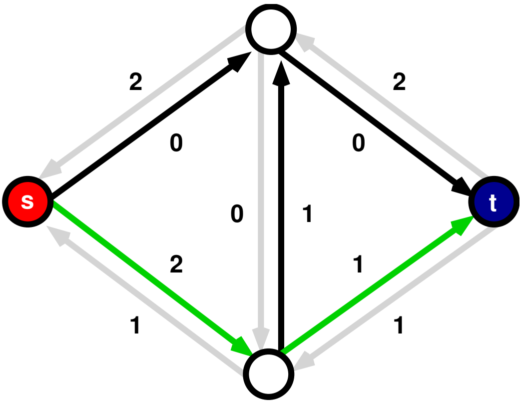
Update residual network according to path capacity:



[Ford-Fulkerson]

Max-flow

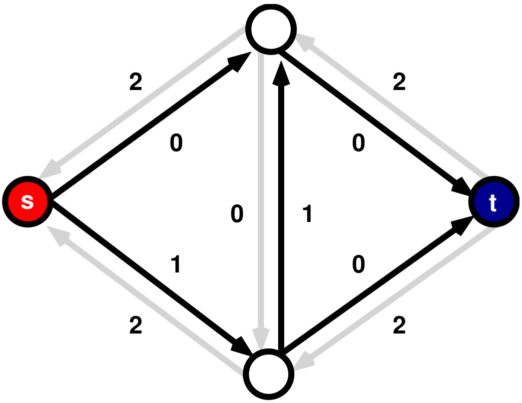
Find another path from node s to node t:



[Ford-Fulkerson]

Max-flow

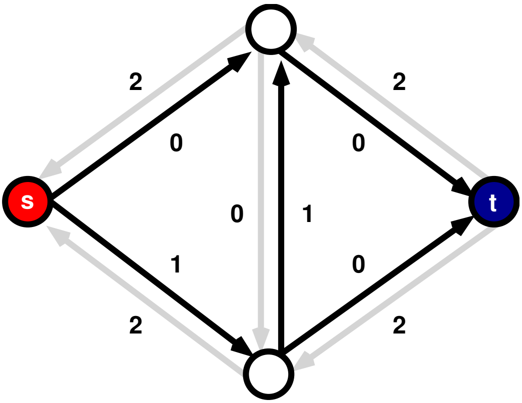
Update residual network according to path capacity:



[Ford-Fulkerson]

Max-flow

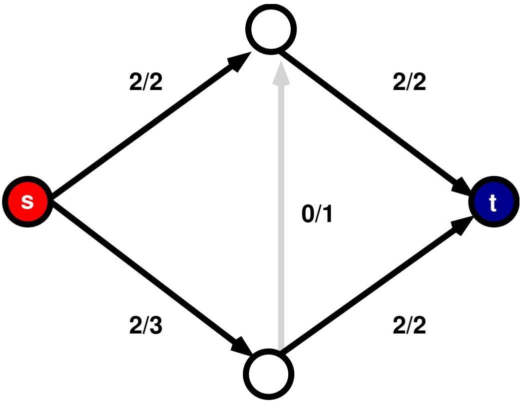
There is no other free path from node s to node t:



[Ford-Fulkerson]

Max-flow

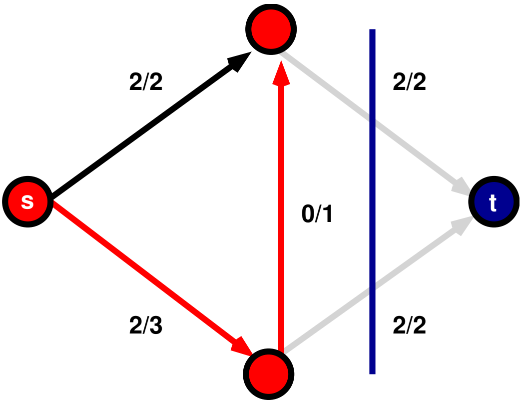
Maximum flow found:



[Ford-Fulkerson]

Max-flow

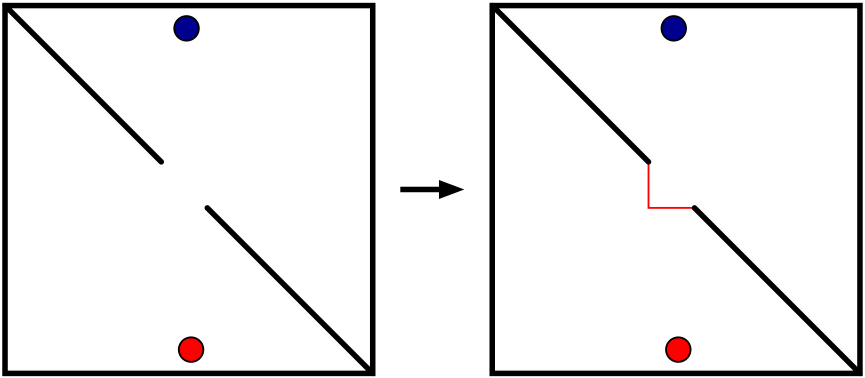
Corresponding minimal cut:



[Ford-Fulkerson]

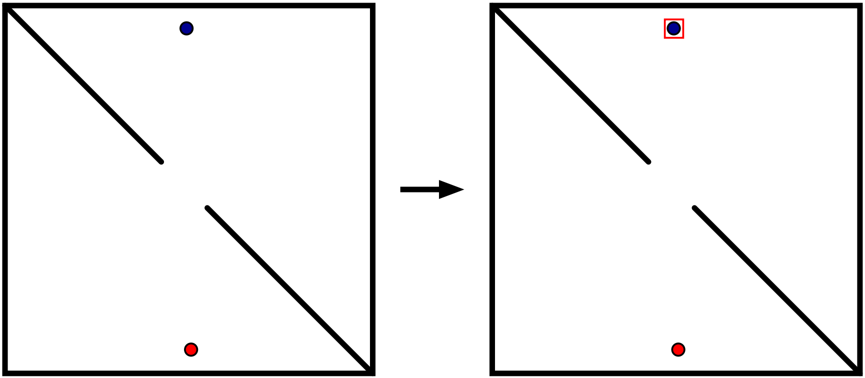
Min-cut

Metrication ($\alpha = 1$):



Min-cut

Shrinking bias ($\alpha = 1$):



Random walker

Find an optimal labelling $x^* \in \langle 0, 1 \rangle$ such that:

$$x^* = \arg\min_x \sum_{\{p,q\} \in \mathcal{N}} w_{pq} \left| x_p - x_q \right|^2$$

subject to: $x_s = 0 \;\wedge\; x_t = 1$

$$E = \sum_{\{p,q\} \in \mathcal{N}} w_{pq} \left| x_p - x_q \right|^2 \quad \Rightarrow \quad E' = 0$$

Laplace equation:

$$\boxed{\Delta x = 0}$$

$$w_{pq} = 1$$

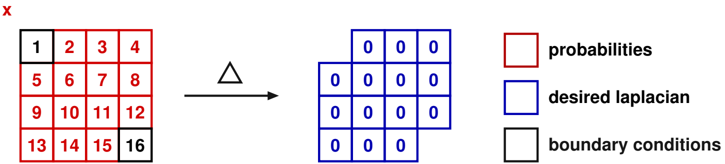
Laplace–Beltrami equation:

$$\boxed{\Delta_w x = 0}$$

$$w_{pq} \in \langle 0, 1 \rangle$$

Random walker

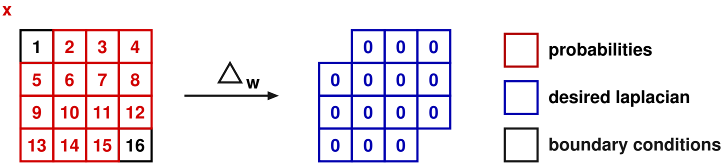
Linear system for Laplace equation:



2	3	4	5	6	7	8	9	10	11	12	13	14	15		x		b		bc
−3	1			1											2	=	0	−	1
1	−3	1			1										3	=	0		
	1	−2				1									4	=	0		
			−3	1			1							•	5	=	0	−	1
1			1	−4	1			1							6	=	0		
	1			1	−4	1			1						7	=	0		
		1			1	−3				1					8	=	0		

Random walker

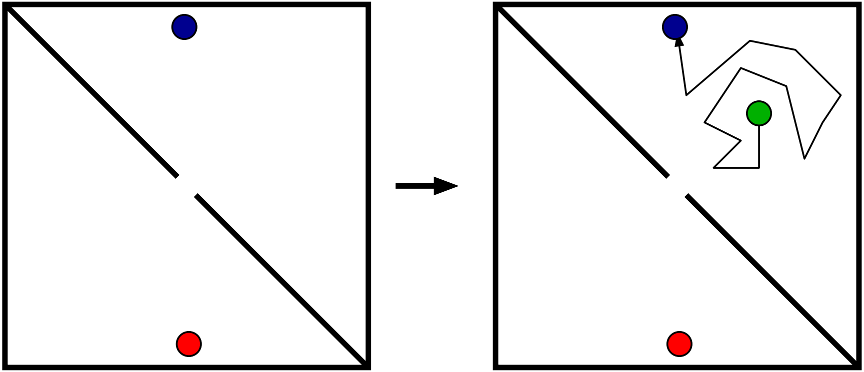
Linear system for Laplace–Beltrami equation:



2	3	4	5	6	7	8	9	10	11	12	13	14	15		x				
–1	0.2			0.5											2	=	0	– 1	* 0.3
0.2	–1	0.3			0.5										3	=	0		
	0.7	–1				0.3									4	=	0		
			–1	0.2			0.2							•	5	=	0	– 1	* 0.6
0.1			0.2	–1	0.3			0.4							6	=	0		
	0.2			0.6	–1	0.1			0.1						7	=	0		
		0.2			0.5	–1				0.3					8	=	0		

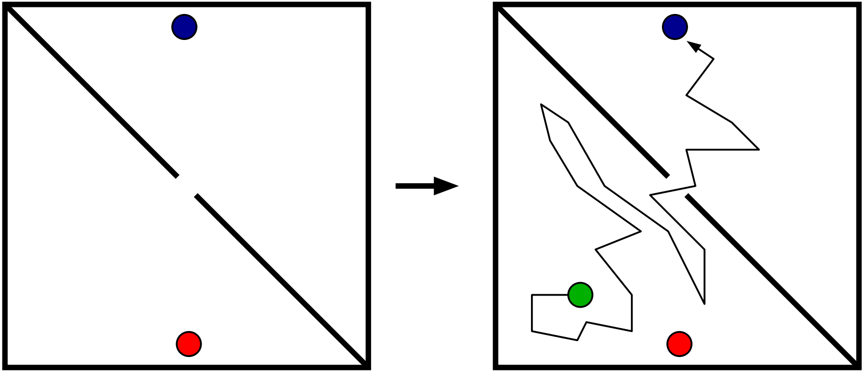
Random walker

What is the probability that random walker reaches blue seed first?



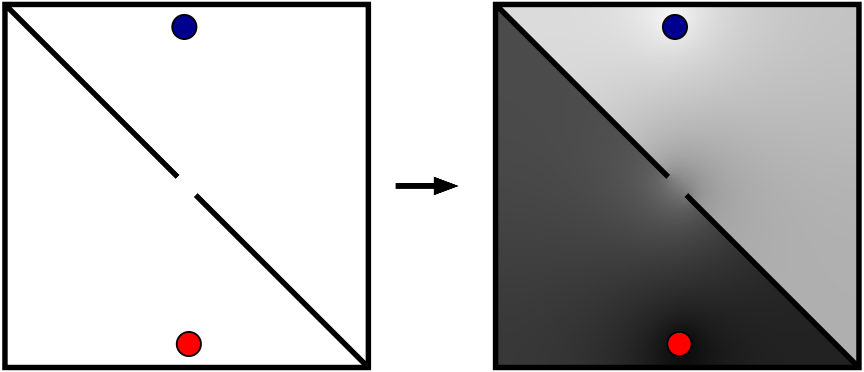
Random walker

What is the probability that random walker reaches blue seed first?



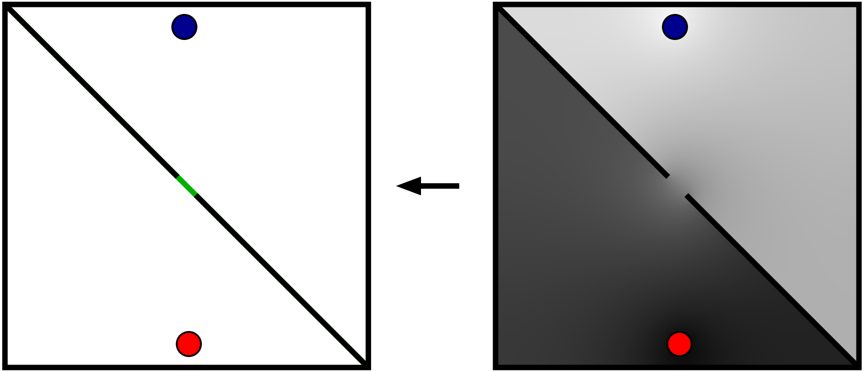
Random walker

What is the probability that random walker reaches blue seed first?



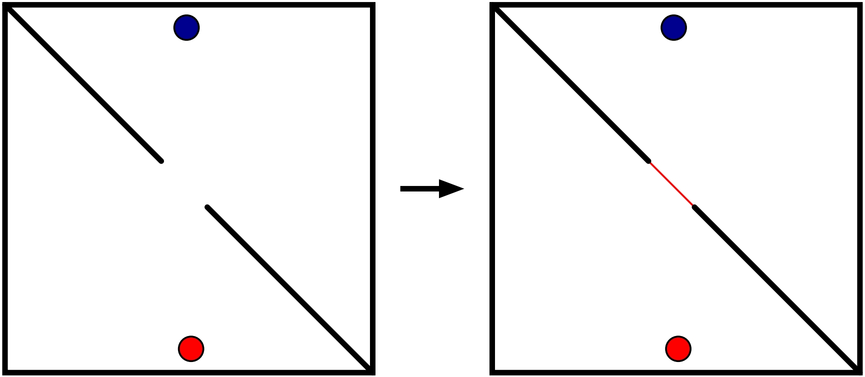
Random walker

The boundary between segments has probability $= \frac{1}{2}$:

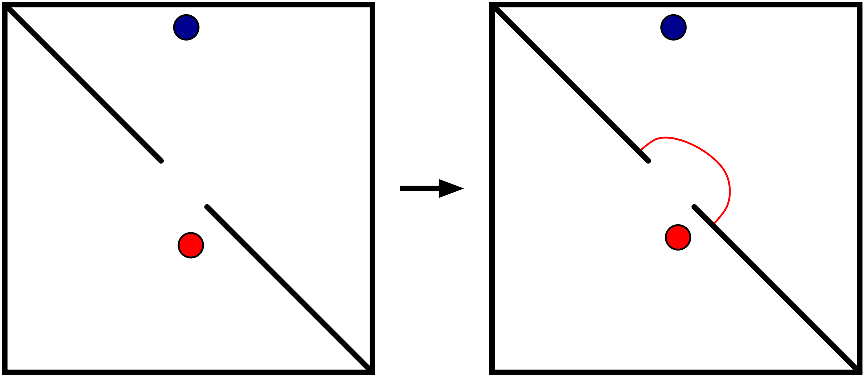


Random walker

Better closure ($\alpha = 2$):



Proximity bias ($\alpha = 2$):



Shortest path

Find an optimal labelling $x^* \in \langle 0, 1 \rangle$ such that:

$$x^* = \arg\min_{x} \lim_{\alpha \to \infty} \sqrt[\alpha]{\sum_{\{p,q\} \in \mathcal{N}} w_{pq} \left| x_p - x_q \right|^{\alpha}}$$

subject to: $x_s = 0 \ \wedge \ x_t = 1$

$$\lim_{\alpha \to \infty} \sqrt[\alpha]{\sum_{\{p,q\} \in \mathcal{N}} w_{pq} \left| x_p - x_q \right|^{\alpha}} \quad \Rightarrow \quad \max_{\{p,q\} \in \mathcal{N}} w_{pq} \left| x_p - x_q \right|$$

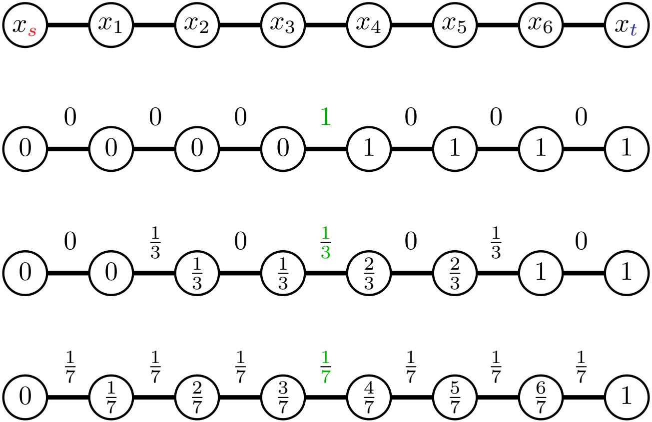
$$\boxed{x^*_p = \min\{l_{pt}/l_{st}, 1\}}$$

$$\{p, q\} \in \mathcal{N} : l_{pq} = 1/w_{pq}$$

l_{pt} = **length of the shortest path between** p **and** t

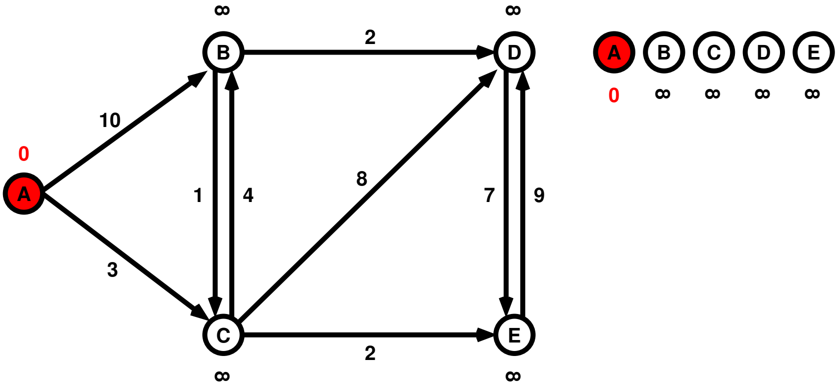
l_{st} = **length of the shortest path between** s **and** t

Shortest path



Shortest path

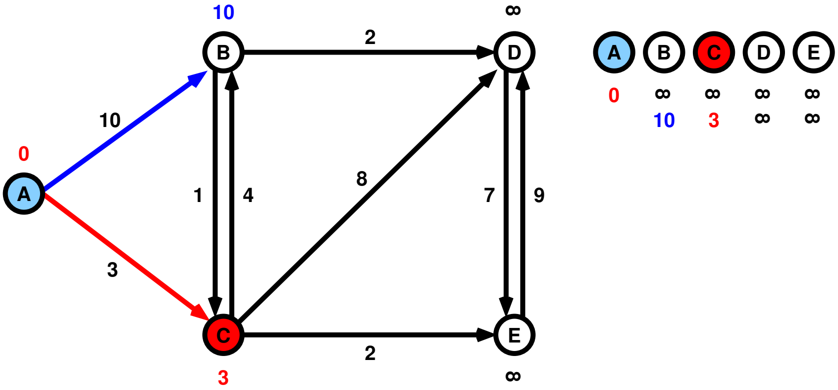
Find shortest paths to all nodes from node A (initial state):



[Dijkstra]

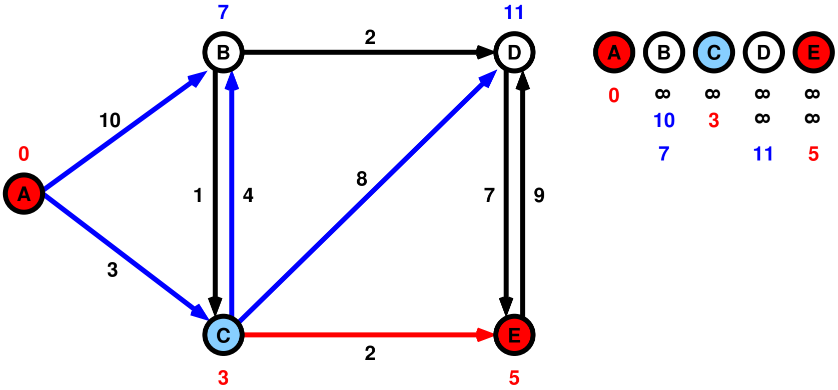
Shortest path

Expand node with shortest distance (A) and relax its neighbors:



[Dijkstra]

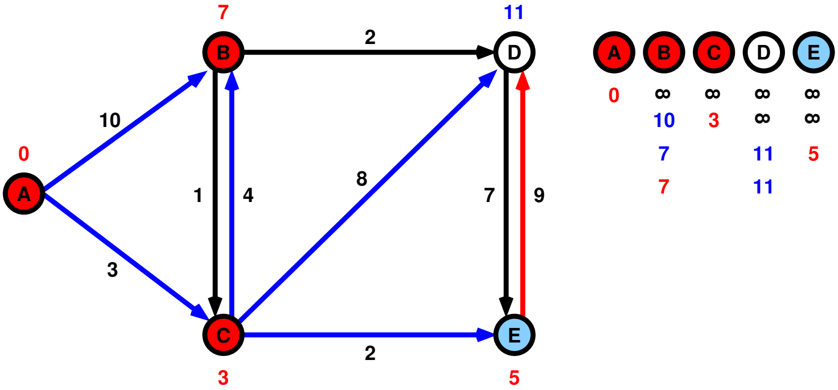
Expand node with shortest distance (C) and relax its neighbors:



[Dijkstra]

Shortest path

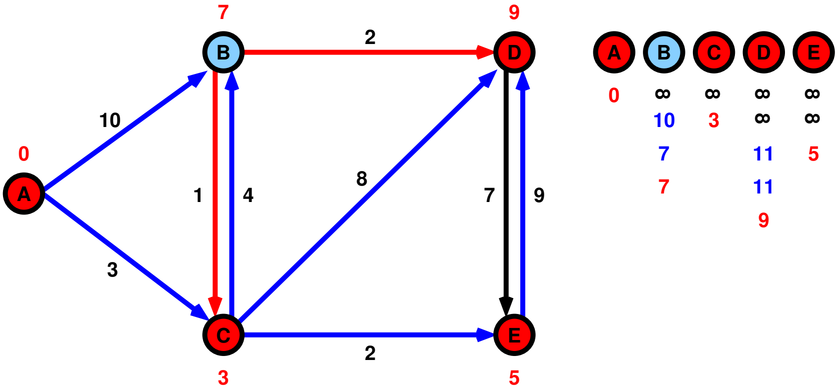
Expand node with shortest distance (E) and relax its neighbors:



[Dijkstra]

Shortest path

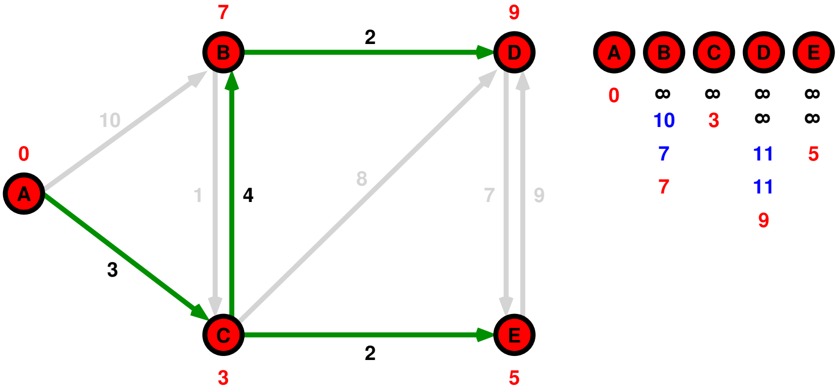
Expand node with shortest distance (B) and relax its neighbors:



[Dijkstra]

Shortest path

Resulting tree of shortest paths from node A to all other nodes:



[Dijkstra]

Shortest path

Leakage ($\alpha \to \infty$):