Digital Image

(B4M33DZO, Summer 2024)

Lecture 10:

Image Registration 3

`https://cw.fel.cvut.cz/wiki/courses/b4m33dzo/start`

Daniel Sýkora & Ondřej Drbohlav

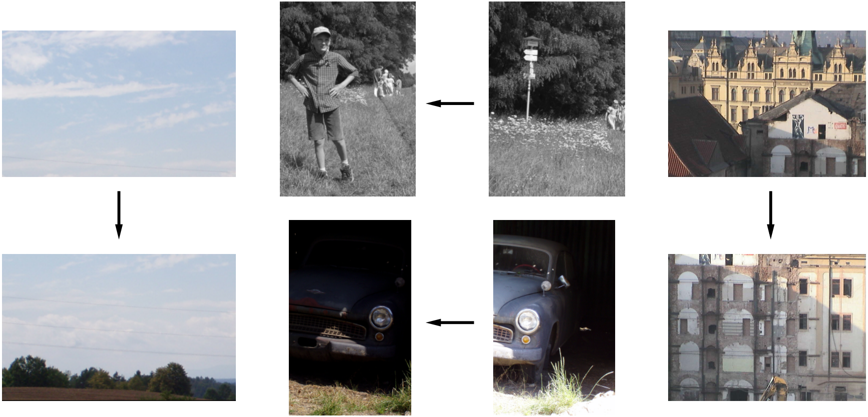
Department of Cybernetics

Faculty of Electrical Engineering

Czech Technical University in Prague

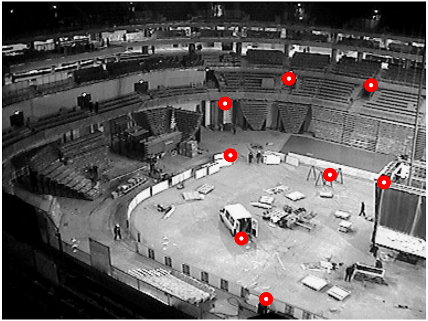
Motivation

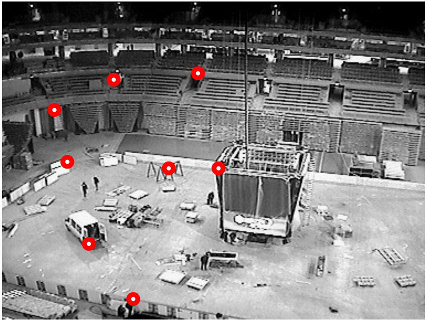
Real world and image registration:



lack of features, occlusion, different exposure, small overlap

Point-based approach





In contrast to image-based approach:

Efficiency: significant problem reduction (e.g. 1k point vs. 1M pixel)

Robustness: occlusion, exposure, noise, does not require initialization

Interest Points

Good localization:

$$\sum_x \sum_y (\mathbf{I}[x,y] - \mathbf{I}[x+s,y+t])^2 > 0 \qquad \forall (s,t) : ||(s,t)|| > 0$$

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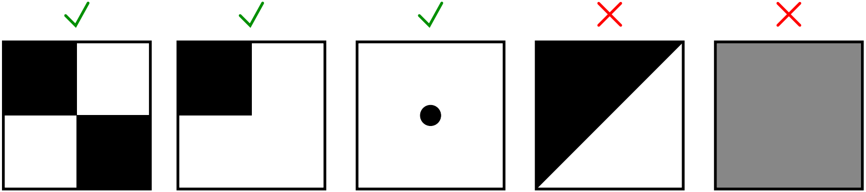

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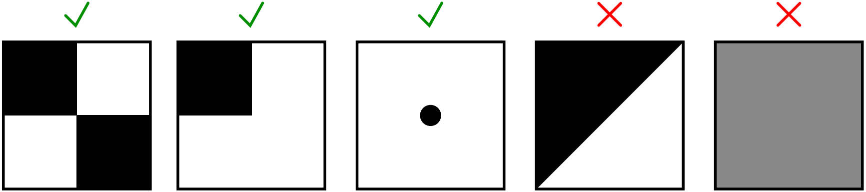
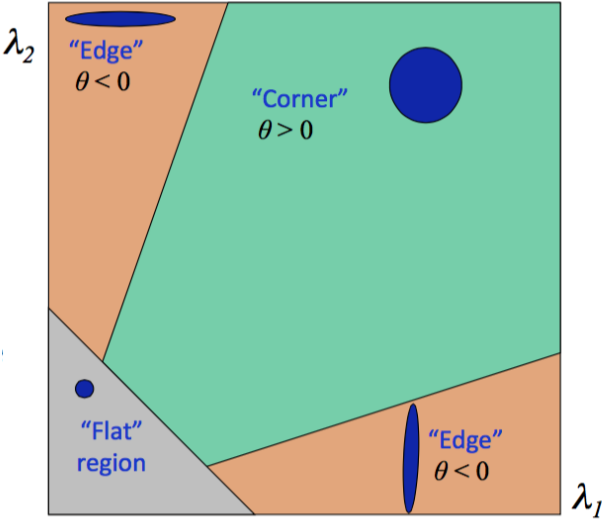


Harris detector:

$$\mathbf{M} = \begin{pmatrix} \sum \left(\frac{\partial \mathbf{I}}{\partial x}\right)^2 & \sum \frac{\partial \mathbf{I}}{\partial x} \frac{\partial \mathbf{I}}{\partial y} \\ \sum \frac{\partial \mathbf{I}}{\partial x} \frac{\partial \mathbf{I}}{\partial y} & \sum \left(\frac{\partial \mathbf{I}}{\partial y}\right)^2 \end{pmatrix} \qquad R = \det(\mathbf{M}) - k\,(\mathrm{trace}(\mathbf{M}))^2$$

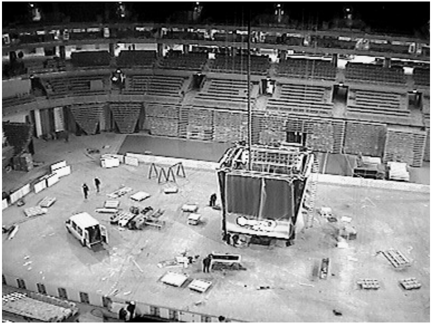
Interest Points



Interest Points

Illustration: Repeat until specified number of points was found:

1. **find pixel (x, y) where $R[x, y]$ has maximal value**
2. **compute centroid (c_x, c_y) in a neighborhood of (x, y)**
3. **store the centroid to the list of interesting points**
4. **remove point by drawing zero circle around (x, y) in R**



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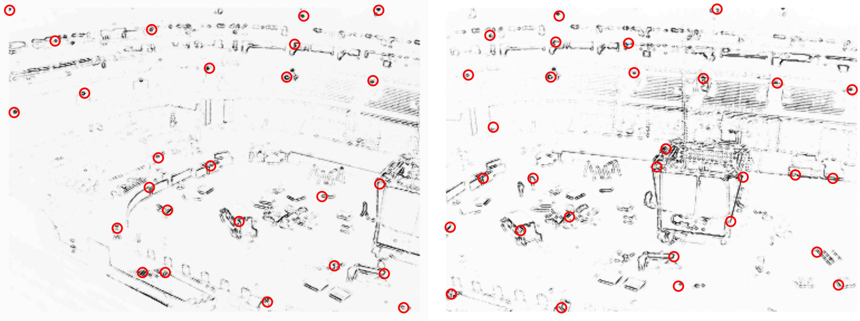
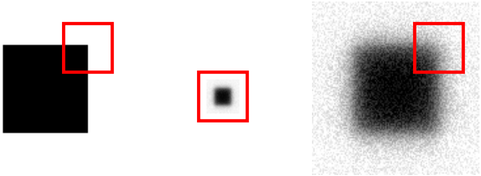


Interest Points

Problem: output of Harris detector depends on the selected scale.



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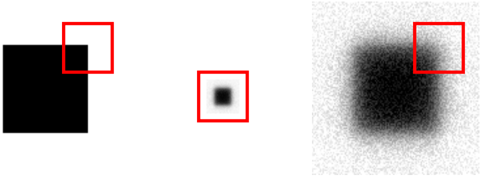

Invariance: e.g. SIFT (Scale-invariant feature transform):

Local extremes of normalized Laplacian-of-Gaussian scale-space.

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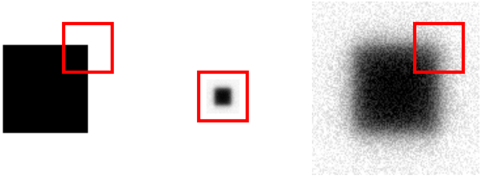
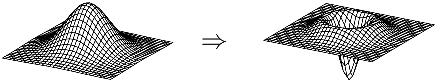


Invariance: e.g. SIFT (Scale-invariant feature transform):

Local extremes of normalized Laplacian-of-Gaussian scale-space.

$$\mathbf{L} \circ \mathbf{G} = \nabla^2 \left(\frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \right) = \frac{1}{\pi\sigma^4} \left(\frac{x^2+y^2}{2\sigma^2} - 1 \right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$



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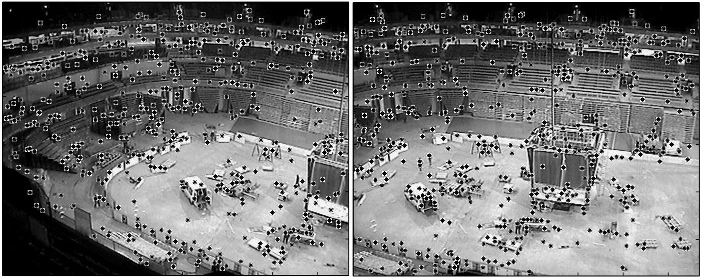


Point descriptor

Normalized cross-correlation:

$$\frac{1}{\sigma_{\mathbf{A}}\sigma_{\mathbf{B}}}\sum_{x}\sum_{y}\left(\mathbf{A}(x,y)-\hat{\mathbf{A}}\right)\left(\mathbf{B}(x,y)-\hat{\mathbf{B}}\right)$$

Point descriptor

Normalized cross-correlation:

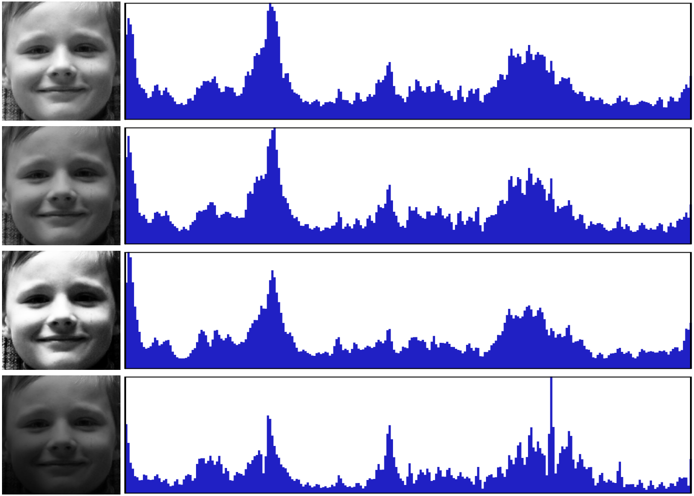
$$\frac{1}{\sigma_{\mathbf{A}}\sigma_{\mathbf{B}}}\sum_{x}\sum_{y}\left(\mathbf{A}(x,y)-\hat{\mathbf{A}}\right)\left(\mathbf{B}(x,y)-\hat{\mathbf{B}}\right)$$

Histogram of gradient orientations (weighted by magnitude):

$$\alpha = \arctan\left(\frac{\mathbf{I} * \mathbf{G}'_y}{\mathbf{I} * \mathbf{G}'_x}\right) \qquad w = \sqrt{(\mathbf{I} * \mathbf{G}'_x)^2 + (\mathbf{I} * \mathbf{G}'_y)^2}$$

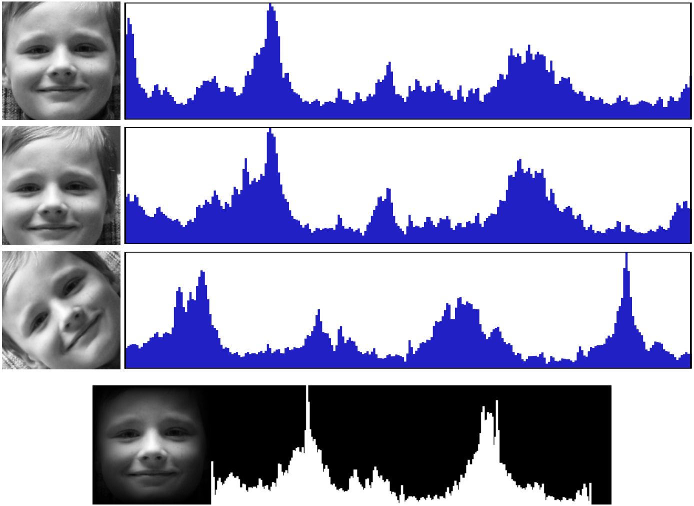
Point descriptor

Histogram of gradient orientations (illumination changes):



Point descriptor

Histogram of gradient orientations (rotation & translation):



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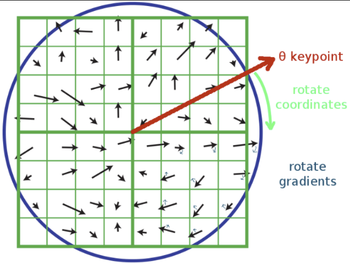

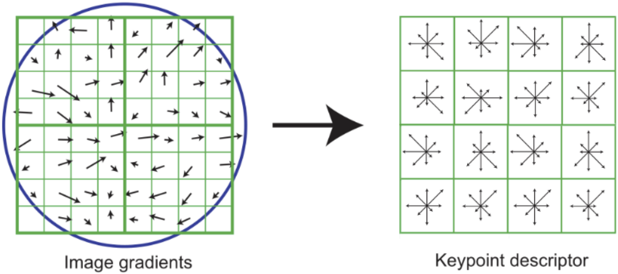

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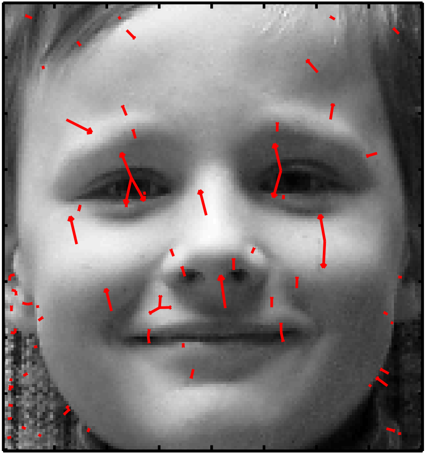

1. **Besides location also orientation and scale are important.**
2. **Single location may contain multiple interesting points.**

Estimating model parameters

Select $\geq M$ best matching point correspondences:

$$x' = \frac{a_{11}x + a_{12}y + x_0}{f_1 x + f_2 y + f_3} \qquad y' = \frac{a_{21}x + a_{22}y + y_0}{f_1 x + f_2 y + f_3}$$

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Build (overdetermined) linear system (solvable by SVD):

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x'_2 x_2 & -x'_2 y_2 & -x'^2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -y'_2 x_2 & -y'_2 y_2 & -y'_2 \\ . & . & . & . & . & . & . & . & \\ . & . & . & . & . & . & . & . & \\ x_M & y_M & 1 & 0 & 0 & 0 & -x'_M x_M & -x'_M y_M & -x'_M \\ 0 & 0 & 0 & x_M & y_M & 1 & -y'_M x_M & -y'_M y_M & -y'_M \end{pmatrix} \cdot \begin{pmatrix} a_{11} \\ a_{12} \\ x_0 \\ a_{21} \\ a_{22} \\ y_0 \\ f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ . \\ . \\ 0 \\ 0 \end{pmatrix}$$

Singular value decomposition

Each matrix $\mathbf{A}_{M \times N}$ can be decomposed into:

$$\mathbf{A} = \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{V}^T$$

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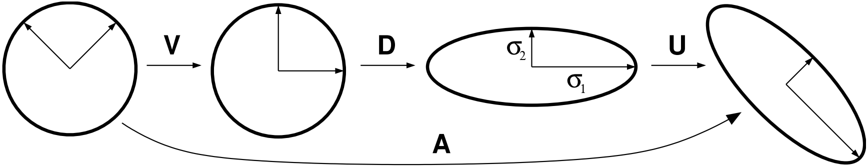

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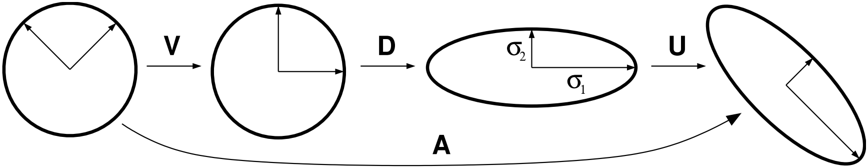

$\mathbf{U}$ & $\mathbf{V}$: $||\mathbf{U}\mathbf{x}|| = ||\mathbf{x}||$ $\quad \mathbf{U}^{-1} = \mathbf{U}^T$ $\quad \mathbf{U}\mathbf{U}^T = \mathbf{U}^T\mathbf{U} = \mathbf{I}$

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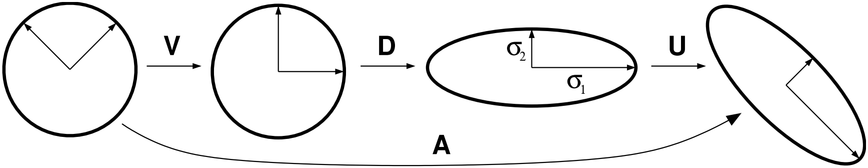

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$$\mathbf{D}_{M \times N} = \begin{pmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{\min(M,N)} \end{pmatrix}$$

$\sigma_i = \sqrt{\lambda_k} \quad \lambda_k \Rightarrow$ **eigenvalue of** $\mathbf{M} = \mathbf{A}^T\mathbf{A} \quad \mathbf{M}\mathbf{x}_k = \lambda_k \mathbf{x}_k$

Singular value decomposition

$\mathbf{w} = \mathbf{v}_{\min(\mathbf{M},\mathbf{N})} \in \mathbf{V}$ **minimizes** $||\mathbf{A}\mathbf{w}||^2$ **subject to** $||\mathbf{w}|| = \mathbf{1}.$

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Last column of $\mathbf{V} \Rightarrow$ **non-trivial solution to** $\mathbf{A}\mathbf{w} = 0$.

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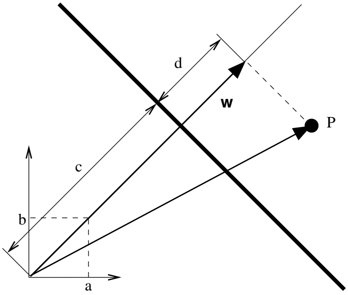
Last column of $\mathbf{V} \Rightarrow$ **non-trivial solution to** $\mathbf{A}\mathbf{w} = \mathbf{0}$.

Example: Fit line $\mathbf{w} \cdot \mathbf{x} + \mathbf{c}$ **to a set of points** $\mathbf{P}$.



$$\mathbf{A} = \mathbf{P} - \mathbf{1}\hat{\mathbf{p}}^T \quad \hat{\mathbf{p}} = \frac{1}{n}\mathbf{P}^T\mathbf{1}$$

$$\mathbf{A} \Rightarrow \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{V}^T \Rightarrow \mathbf{w} = \mathbf{v_2}$$

$$\mathbf{w} = (\mathbf{a}, \mathbf{b})$$

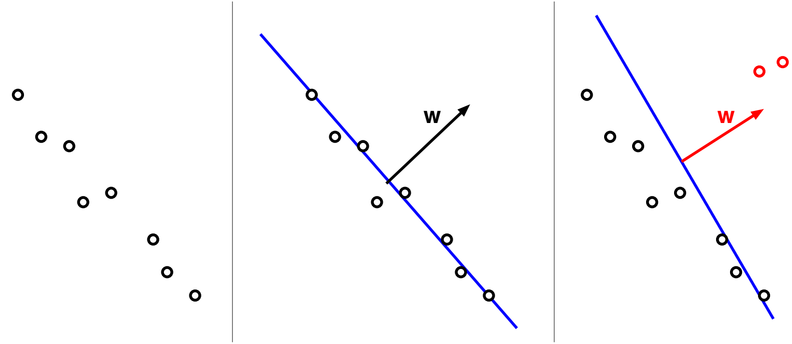
$$\frac{\partial}{\partial c}||\mathbf{P} \cdot \mathbf{w} - \mathbf{c} \cdot \mathbf{1}||^2 = \mathbf{0}$$

$$2(nc - \mathbf{w^T P^T 1}) = \mathbf{0}$$

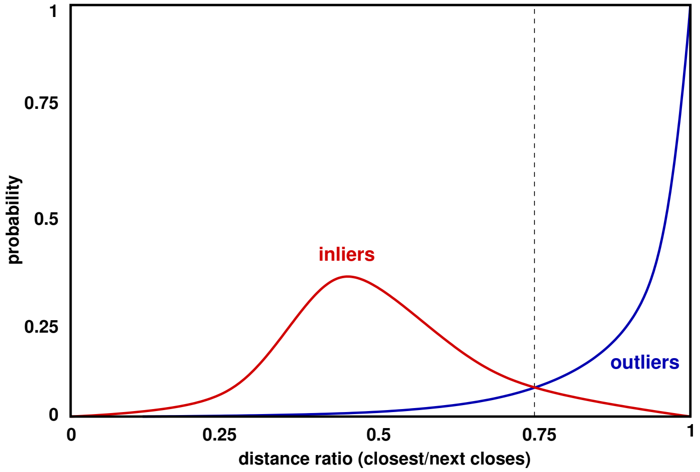
$$\implies \quad c = \hat{\mathbf{p}}^T \mathbf{w}$$

Robustness

Problem: outliers can affect accuracy of parameter estimation.



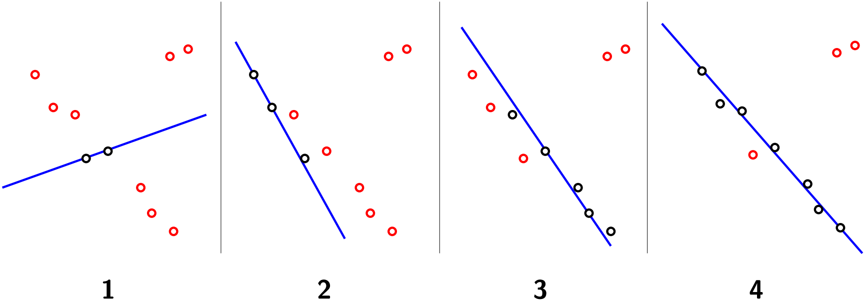
Outlier detection:



Robustness

RANSAC (RANdom SAmple Consensus):

1. **Select M random correspondences.**
2. **Compute exact model parameters $\mathbf{p}$.**
3. **Use $\mathbf{p}$ to get number of outliers N.**
4. **If $N < N_{\min}$ then $N_{\min} = N$ and $\mathbf{p}^* = \mathbf{p}$.**
5. **If $N_{\min} < \epsilon$ then return $\mathbf{p}^*$ else goto 1.**



Applications

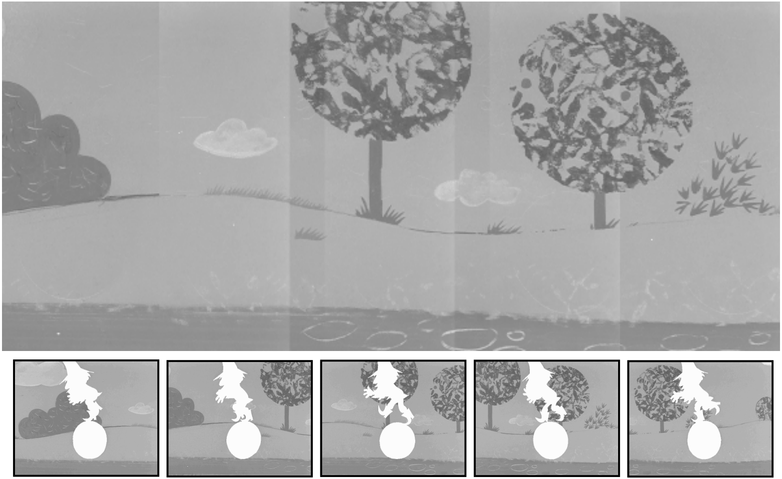
Hyper-resolution panorama stitching:

Applications

Image and object retrieval:

Applications

Recovering background from occluded observations:



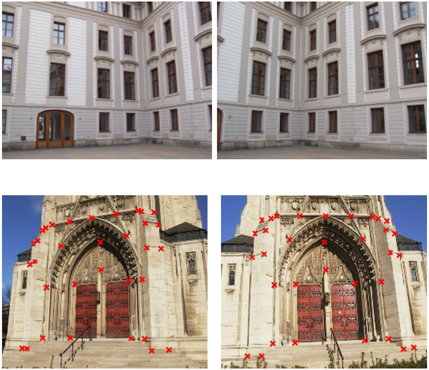
Applications

Augmented reality:

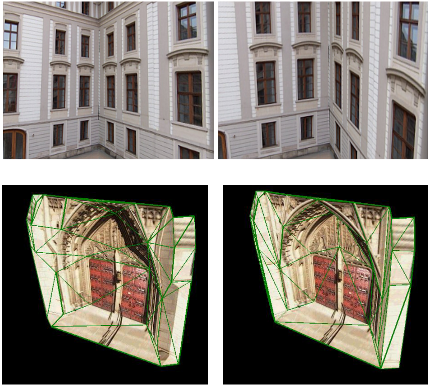
Applications

Structure from motion:

source images



novel view synthesis



Applications

Space carving: