

Homework (A8B17CAS)

Problem Set 2

1 Assignment

Problem 2-A Consider matrix $\mathbf{K} \in \mathbb{R}^{N \times 3}$, containing $\{x, y, z\}$ coordinates of N points. The matrix is generated as

```
c = 1:N;  
r = 1:3;  
K = toeplitz(c, r) - N/2;
```

where N can be chosen freely.

For a given matrix $\mathbf{K}$, calculate vector $\mathbf{n} \in \mathbb{R}^{N \times 1}$ containing 1st norms (called also [Manhattan norm](#)) of all radius vectors pointing from the center of the coordinate system ($x = 0, y = 0, z = 0$) to a corresponding point.

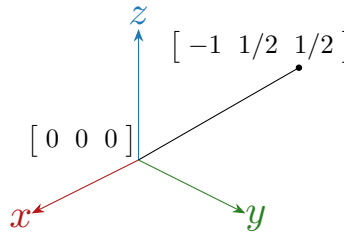


Figure 1: Considering vector in this picture, the Manhattan norm is equal to 2.

Evaluate the ratio r between the biggest and the smallest value in the vector $\mathbf{n}$. (Check: for $N = 6$, we have $r = \max\{\mathbf{n}\} / \min\{\mathbf{n}\} = 3$.)

Implement the above-mentioned functionality into a function with the following header

```
function [n, r] = problem2A(N)
```

(1 point)

Problem 2-B Calculate vector $\mathbf{d}$ containing distances between all consecutive prime numbers starting from 3 to $N = 10^8$ (notice that they are all even numbers). Determine the longest distance $d_{\max} = \max\{d\}$ and the associated prime numbers, p_1 and p_2 , for which it occurs. Finally, determine what distance value d_{most} occurs most often in the vector $\mathbf{d}$ (so-called [the mode](#) of a sample $\mathbf{d}$).

Implement the above-mentioned functionality into a function with the following header

```
[dmax, p1, p2, dmost] = problem2B(N)
```

(2 points)

Problem 2-C Implement a function called `problem2C`, which evaluates Euclidean distances between two sets of points, finds a sphere with a center at the middle point between the two most distant points, and calculates its radius. Finally, verify if all points are inside this sphere.

Imagine two sets of points, $\mathbf{p}_m \in \mathcal{P}$, $m \in \{1, \dots, M\}$ and $\mathbf{r}_n \in \mathcal{R}$, $n \in \{1, \dots, N\}$. Two matrices represent them, $\mathbf{P} \in \mathbb{R}^{M \times 3}$ and $\mathbf{R} \in \mathbb{R}^{N \times 3}$, serving as the sole inputs

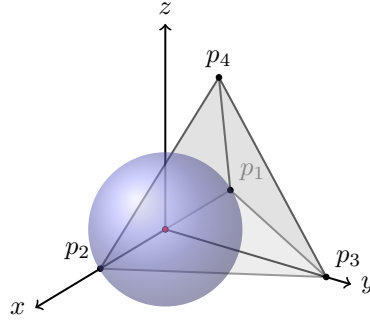


Figure A: An example of point set $\mathbf{P} = \mathbf{R}$ forming a unitary tetrahedron. The distances between all $m \neq n$ points is $d_{mn} = 1$. The radius of a sphere touching the most distant points is $a = 1/2$ and its center non-unique, position $\mathbf{c} = [0 \ 0 \ 0]$ shown here as red circle.

into the function. The function calculates Euclidean distance (2nd norm) between each pair of points, taken one by one from the sets $\mathcal{P}$ and $\mathcal{R}$, as

$$d_{mn} = |\mathbf{p}_m - \mathbf{r}_n|, \quad \mathbf{D} = [d_{mn}] \in \mathbb{R}^{M \times N}. \quad (1)$$

The distance matrix $\mathbf{D}$ is returned as the first output variable. Finally, the function evaluates the center $\mathbf{c}$ of the sphere given as

$$\mathbf{c} = \frac{1}{2} (\mathbf{p}_{m_c} + \mathbf{r}_{n_c}) \quad (2)$$

with boundary points $\mathbf{p}_{m_c}$ and $\mathbf{r}_{n_c}$ found such that

$$m_c, n_c : \quad a = \frac{1}{2} \max_{m,n} \{\mathbf{D}\}, \quad (3)$$

i.e., two points with the largest distance between them. Check at the end if all points from both sets are within this sphere and return `allPtsIn = true` if the answer is yes and `allPtsIn = false` if contrary is the case. To recap, the header of the function `problem2C` reads

```
function [D, a, c, allPtsIn] = problem2C(P, R)
```

For testing purposes, you may use an equilateral tetrahedron with unitary sides

$$\mathbf{P} = \mathbf{R} = \begin{bmatrix} -1/2 & 0 & 0 \\ 1/2 & 0 & 0 \\ 0 & \sqrt{3}/2 & 0 \\ 0 & \sqrt{3}/6 & \sqrt{2/3} \end{bmatrix} \quad (4)$$

with the results

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad (5)$$

$a = 1/2$, and $\mathbf{c} = [0 \ 0 \ 0]$. Notice that the center point $\mathbf{c}$ is, in general, not uniquely defined here; see Figure C. Any valid solution is therefore accepted.

A hint: Check out the function `find()`. You may use it with a syntax like

```
[iRow, iCol] = find(A, 1, 'first'); % the first non-zero entry of A is found
```

(3 points)

Problem 2-D Create a function called `problem2D` which can find all Pythagorean triplets up to the number N and calculates how many of these triplets there are. The header of the function reads

```
function [R, I] = problem2D(N)
```

where $\mathbf{R}$ is the matrix of [Pythagorean triplets](#), described in details below, $\mathbf{I}$ is the number of triplets found, and N is the input variable described below. The function should be reasonably fast, *i.e.*, to calculate all triplets up to $n_I \leq N = 1000$ in terms of seconds. The output variable $\mathbf{R}$ is a matrix $\mathbf{R} \in \mathbb{Z}^{I \times 4}$ with the following structure

$$\mathbf{R} = \begin{bmatrix} n_1 & a_1 & b_1 & c_1 \\ \vdots & \vdots & \vdots & \vdots \\ n_i & a_i & b_i & c_i \\ \vdots & \vdots & \vdots & \vdots \\ n_I & a_I & b_I & c_I \end{bmatrix}, \quad (6)$$

where

$$n_i = a_i + b_i + c_i. \quad (7)$$

A Pythagorean triplet is a set of three natural numbers, $a_i < b_i < c_i$, for which,

$$c_i^2 = a_i^2 + b_i^2. \quad (8)$$

A well-known example of a Pythagorean triplet is $a_1 = 3$, $b_1 = 4$, and $c_1 = 5$ with $n_1 = 12$. As a sanity check, see the first two correct lines of the output variable $\mathbf{R}$

$$\mathbf{R} = \begin{bmatrix} 12 & 3 & 4 & 5 \\ 24 & 6 & 8 & 10 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}. \quad (9)$$

To illustrate how the variable N is used: in case that $N = 15$, there is only one Pythagorean triplet for $n_1 = 12$, see (9), however, for $N = 10$ there is no Pythagorean triplet at all. This problem is freely inspired by the [Project Euler, Problem 9](#).

(4 points)

2 Instructions

The deadline for all assignments is

- December 8, 23:59.

Write your solutions into m-files called `problem2{A-D}.m` and upload them via the [BRUTE system](#). In the case of uploading more files, add them to a ZIP archive.