

Homework 1 (A8B17CAS)

Problem Set 1

October 30, 2024

1 Assignment

For all the following problems, consider N as a positive integer. Please, do not use the `for/while` cycle and/or `if/switch` branching. Do not use functions from the MATLAB Toolboxes.

Problem 1-A Create a matrix $\mathbf{A} \in \mathbb{R}^{N \times 5}$:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0/(N-1) \\ 0 & 1 & 1 & 2 & 1/(N-1) \\ 0 & 1 & 1 & 3 & 2/(N-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 1 & N & (N-1)/(N-1) \end{bmatrix}. \quad (1)$$

Do not enter the numbers element-wise, use the MATLAB functions instead.

(1 point)

Problem 1-B Calculate the norm of the vectors arranged one below the other in matrix $\mathbf{B} \in \mathbb{R}^{N \times 3}$ and normalize them to unitary size. To solve the problem and to verify the solution, use the following matrix:

```
B = reshape( (1:3*N), 3, [] ).'
```

(1 point)

Problem 1-C Find all the elements in the general matrix $\mathbf{C} \in \mathbb{R}^{N \times N}$ greater than or equal to $x = N/2$, return them to vector $\mathbf{u}$ and replace these values in the original matrix $\mathbf{C}$ by 0. The following matrix $\mathbf{C}$ is used to validate the solution:

```
C = magic(N)
```

(2 points)

Problem 1-D Create a matrix $\mathbf{D} \in \mathbb{R}^{N \times N}$ defined as

$$D_{mn} = 2N + 1 - (m + n), \quad (2)$$

where N denotes the size of matrix $\mathbf{D}$, m denotes the row index, and n denotes the column index. Try to find as simple solution as possible.

(2 points)

Problem 1-E Create a matrix $\mathbf{E} \in \mathbb{C}^{2(N+1) \times 2(N+1)}$:

$$\mathbf{E} = \begin{bmatrix} \mathbf{e} + \mathbf{0} & \mathbf{e} - \mathbf{1} & \mathbf{e} - \mathbf{2} & \cdots & \mathbf{e} - \mathbf{N} \\ \mathbf{e} + \mathbf{1} & \mathbf{e} + \mathbf{0} & \mathbf{e} - \mathbf{1} & \cdots & \mathbf{e} - \mathbf{N} + \mathbf{1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{e} + \mathbf{N} & \mathbf{e} + \mathbf{N} - \mathbf{1} & \mathbf{e} + \mathbf{N} - \mathbf{2} & \cdots & \mathbf{e} + \mathbf{0} \end{bmatrix}, \quad (3)$$

such that matrix $\mathbf{e} \in \mathbb{C}^{2 \times 2}$ is a complex matrix

$$\mathbf{e} = \begin{bmatrix} 1 & -j \\ e & \pi \end{bmatrix}, \quad (4)$$

and the remaining matrices are as follows:

$$\mathbf{0} = 0 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad (5)$$

up to

$$\mathbf{N} = N \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}. \quad (6)$$

A hint: Take a look at MATLAB function `repelem`. Remember from the class how to set the Euler's Number $e = \exp(1)$.

(2 points)

Problem 1-F Evaluate matrix $\mathbf{F}$, which is so-called Kronecker tensor product

$$\mathbf{F} = \mathbf{f} \otimes \mathbf{p} \quad (7)$$

of matrices $\mathbf{f}$ and $\mathbf{p}$, respectively, where

$$\mathbf{f} = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \end{bmatrix} \in \mathbb{R}^{2N \times 2N}, \quad (8)$$

and

$$\mathbf{p} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad (9)$$

so that

$$\mathbf{F} = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & \cdots & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 & \cdots & 0 & 0 & -1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & 0 & 0 & \cdots & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & \cdots & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 & \cdots & 0 & 0 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{4N \times 4N}. \quad (10)$$

A hint: Take a look at MATLAB function `kron`.

(2 points)

2 Instructions

The deadline for all assignments is

- November 10, 23:59.

Write your solutions into m-file called `homework1`, each problem is solved within one of the MATLAB code “cell”s (use syntax: `%%`). Upload all files via [BRUTE system](#). In the case of uploading more files, add them to a ZIP archive.