Magnetic resonance imaging

Part 2

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based on lectures 2008-2023 by J.Kybic, J.Hornak[1], M.Bock, J.Hozman, P.Doubek
Department of Cybernetics, FEE CTU

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[1] http://www.cis.rit.edu/htbooks/mri/

MRI excitation sequence

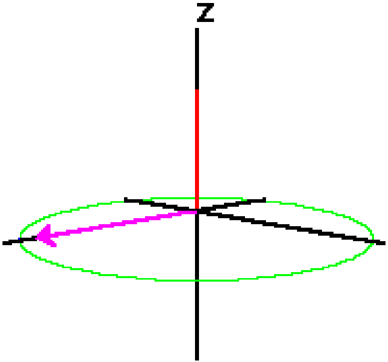
Time sequence

- radio frequency pulses
- magnetic field changes
- signal acquisition intervals

for signal or image acquisition

90° Free induction decay
(FID)

- 90° pulse flips **M** to *xy* plane



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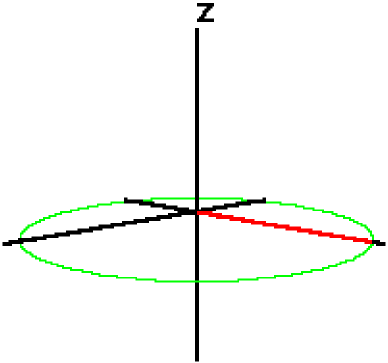

90° Free induction decay (FID)

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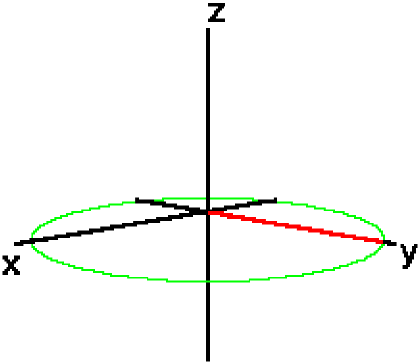

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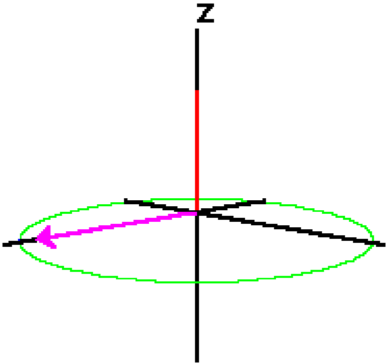

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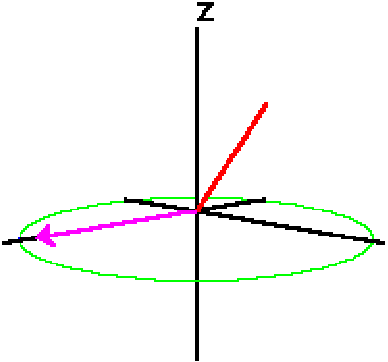

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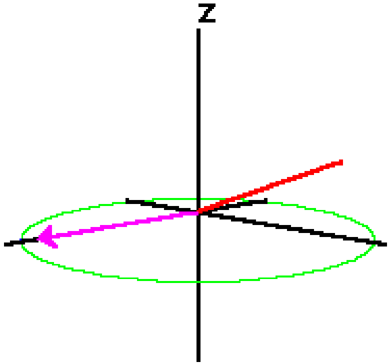

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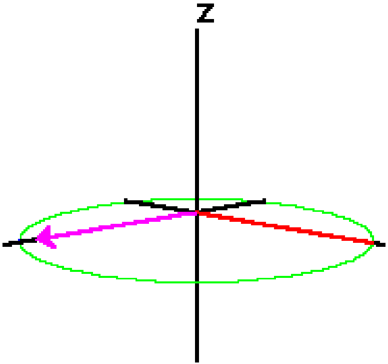

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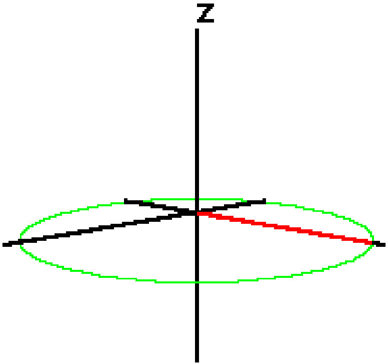

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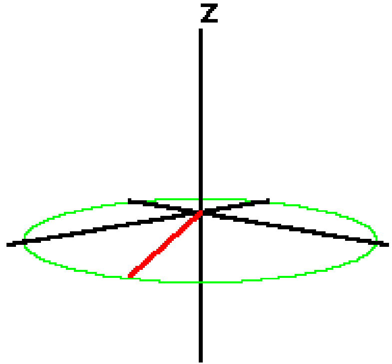

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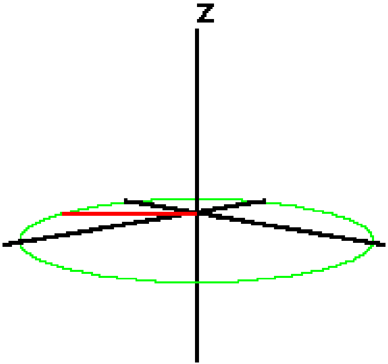

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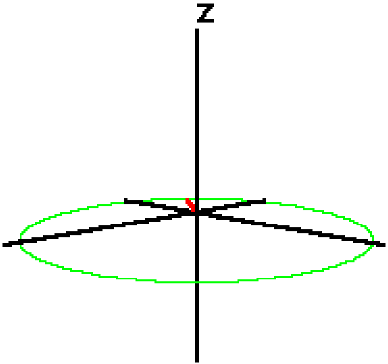

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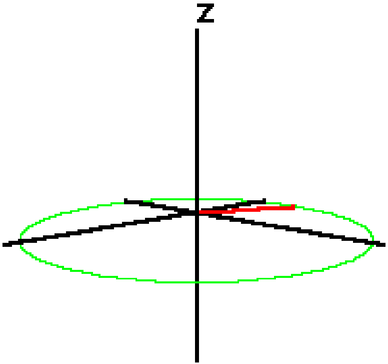

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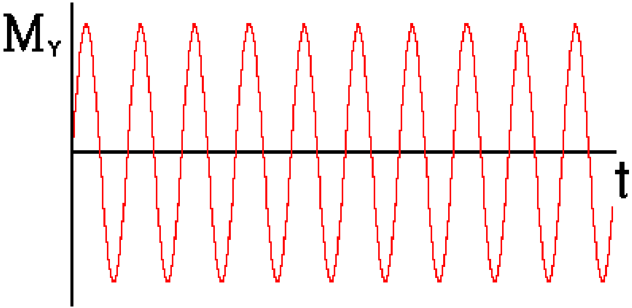

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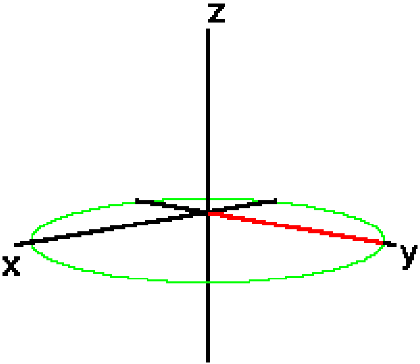

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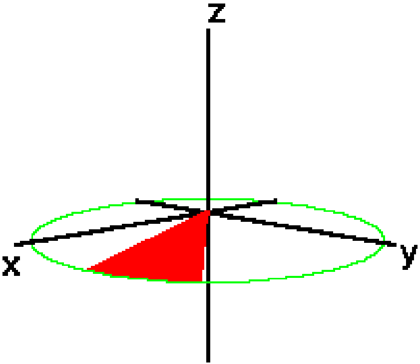

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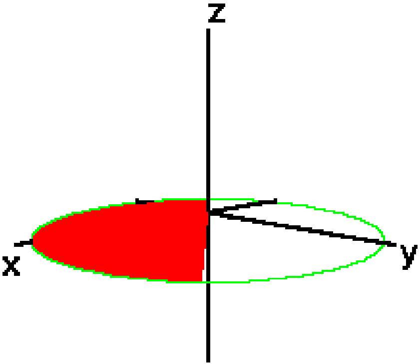

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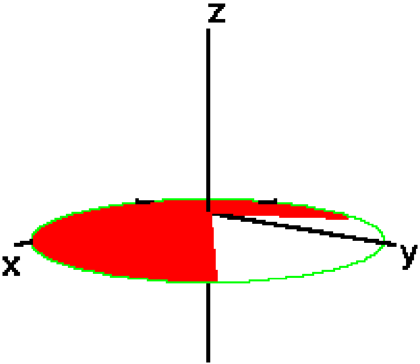

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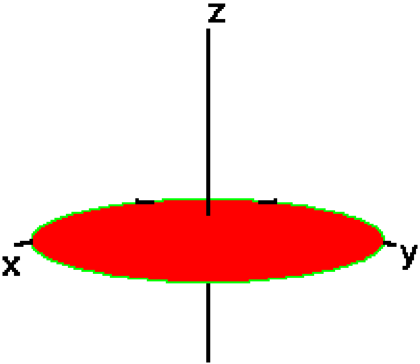

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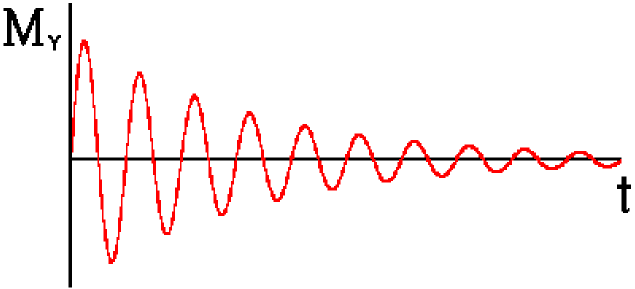

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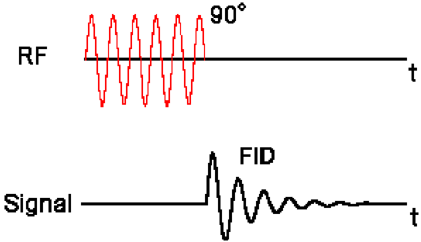
- Time diagram / Excitation sequence



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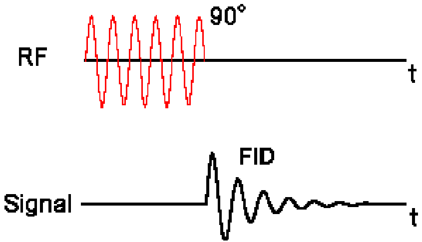
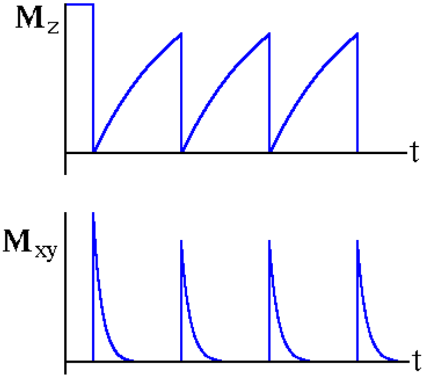


Sequence is repeated with period T_R (repetition time).

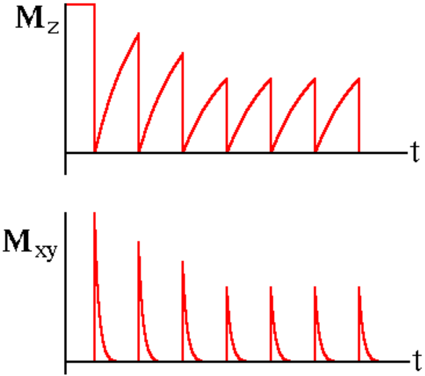
Complete and partial relaxation

- For maximum signal wait until complete T_1 relaxation ($T_R > T_1 \approx 1\,\mathrm{s}$) — long acquisition
- Shorter $T_R \rightarrow$ only partial relaxation, smaller $M_z \rightarrow$ smaller M_{xy}



Complete and partial relaxation

- For maximum signal wait until complete T_1 relaxation ($T_R > T_1 \approx 1\,\mathrm{s}$) — long acquisition
- Shorter $T_R \rightarrow$ only partial relaxation, smaller $M_z \rightarrow$ smaller M_{xy}
- Calibration cycles before each slice acquisition.



90° Free induction decay (2)

Signal intensity after excitation

$$S \propto \varrho(1 - \mathrm{e}^{-\frac{T_R}{T_1}})$$

depends on M_z, which depends on T_R — time from the previous excitation.

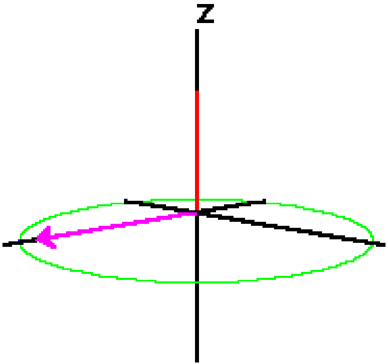
S — signal amplitude

ϱ — spin density

T_R — repetition time ($T_R > T_2$)

Spin-echo sequence

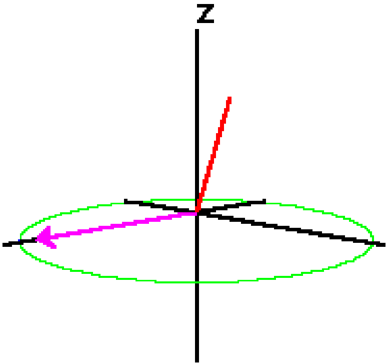
- 90° pulse



Erwin Hahn, 1949

Spin-echo sequence

- 90° pulse



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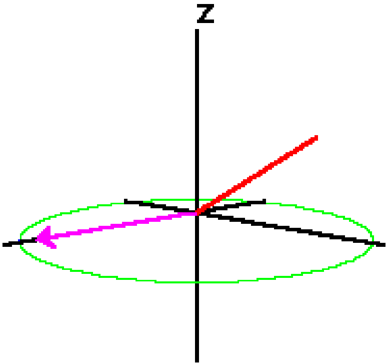

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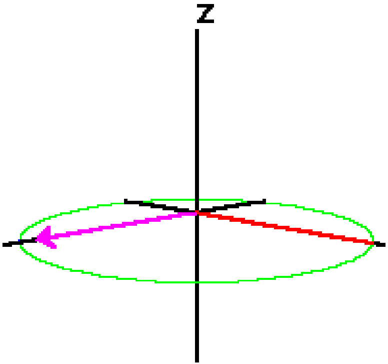
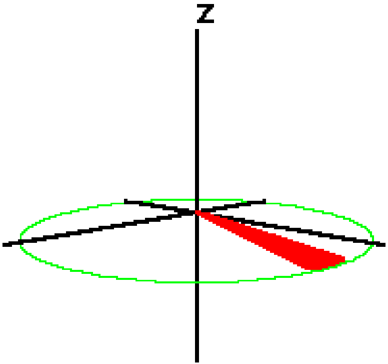


Erwin Hahn, 1949

Spin-echo sequence

- 90° pulse
- Spins start to desynchronize



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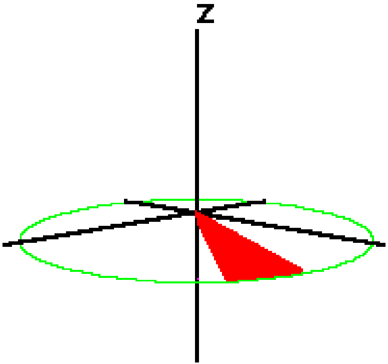

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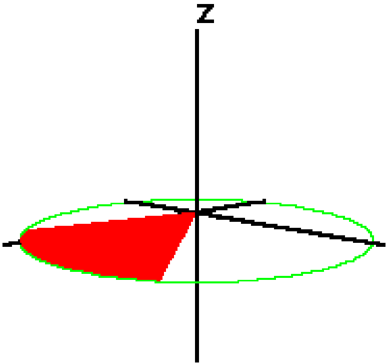

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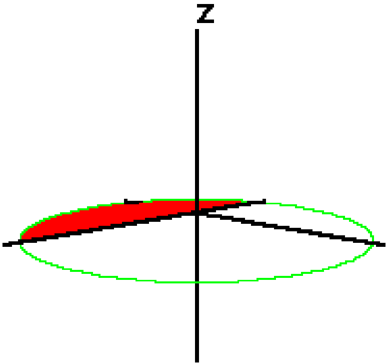

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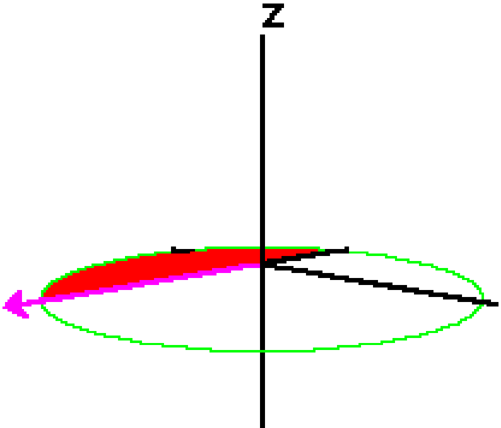
Spin-echo sequence (2)

- 180° pulse — rotation around x'



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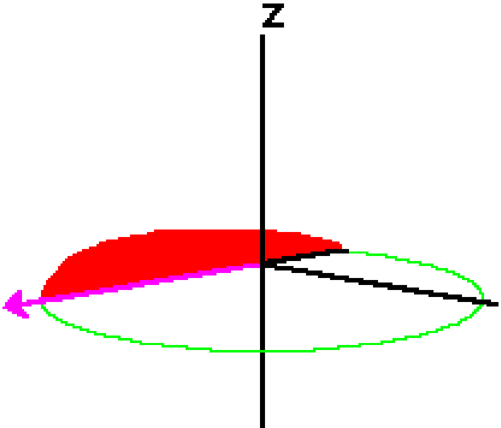

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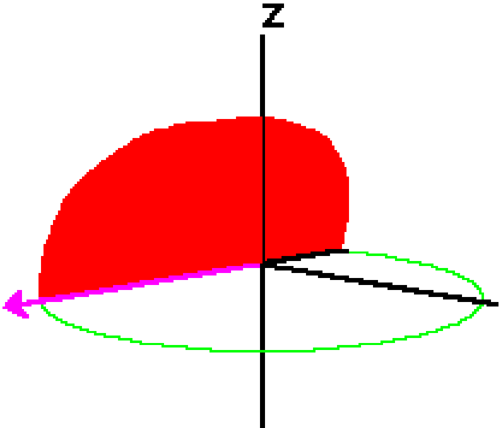

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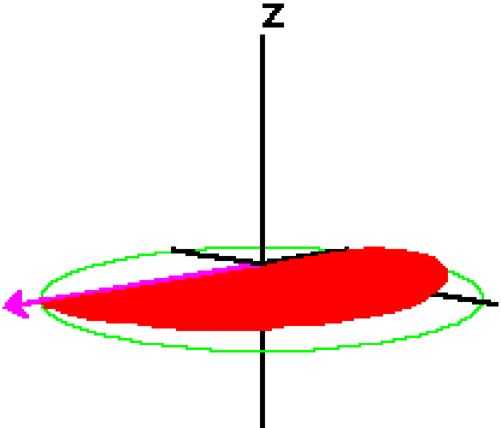
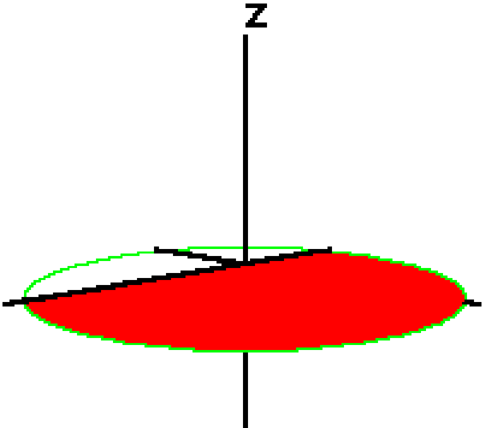


Spin-echo sequence (2)

- 180° pulse — rotation around x'
- Resynchronization (slower spins will be ahead and vice versa)



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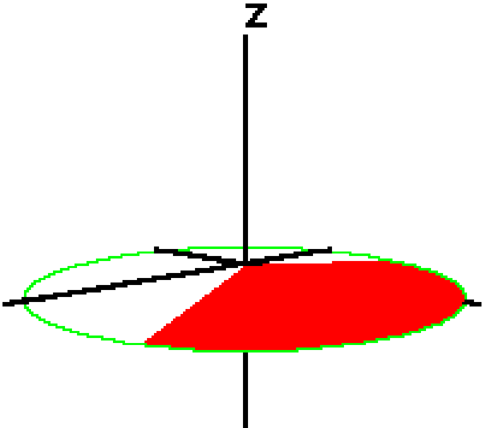

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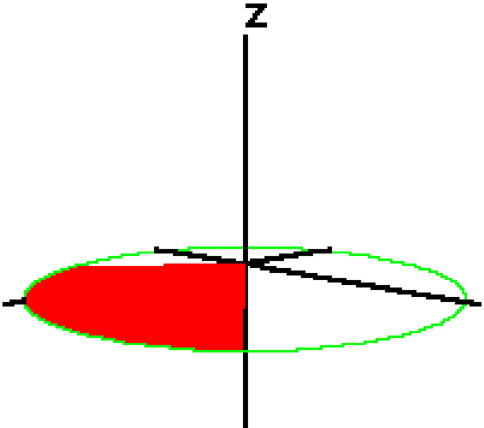

Spin-echo sequence (2)

- 180° pulse — rotation around x'
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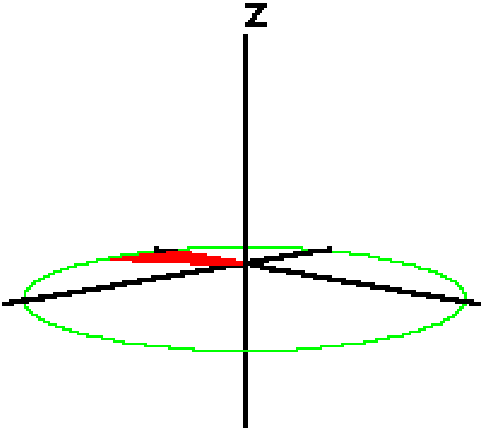
- Echo signal appears



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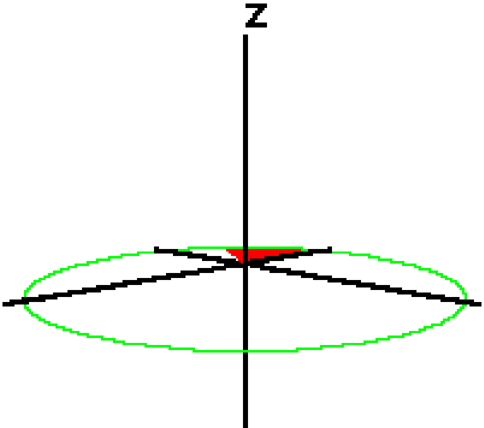

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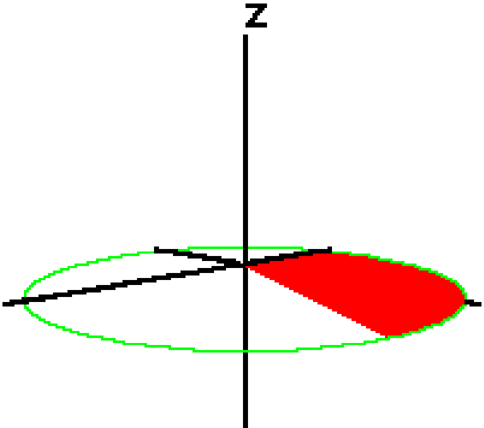

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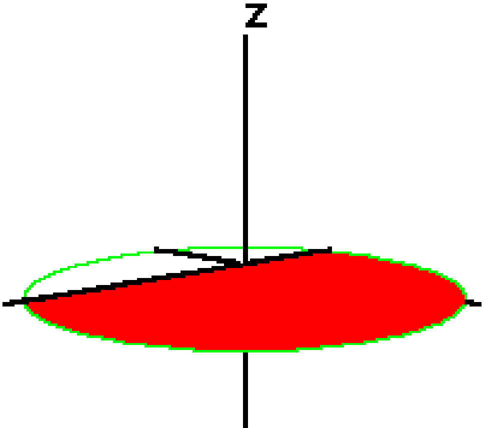

Spin-echo sequence (2)

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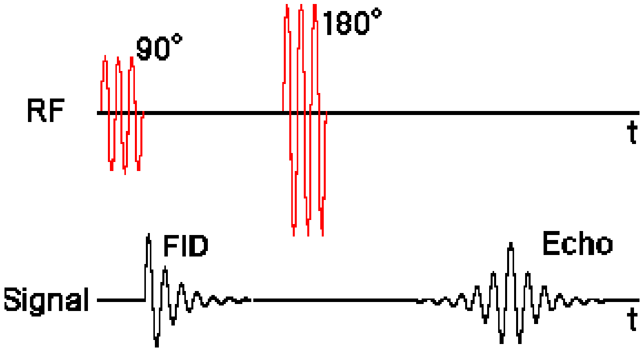
- Echo signal appears
- Time diagram



Spin-echo sequence (3)

Signal intensity

$$S \propto \varrho(1 - \mathrm{e}^{-\frac{T_R}{T_1}})\mathrm{e}^{-\frac{T_E}{T_2}}$$

- S — signal amplitude
- ϱ — spin density
- T_R — repetition time
- T_E — echo time (time between the 90° pulse and readout)
- T_1 — spin-lattice relaxation time
- T_2 — spin-spin relaxation time

changing T_R a T_E determines the influence of T_1 and T_2

Spin-echo sequence — T_2^{inhom} compensation

T_2^* relaxation is caused by spin-spin interactions (T_2) and field inhomogeneity (T_2^{inhom})

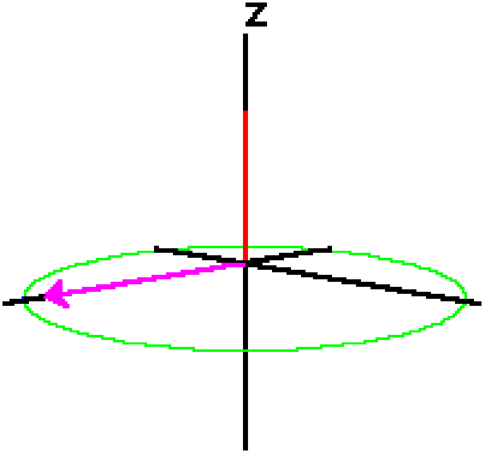
$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2^{\text{inhom}}}$$

resynchronization compensates the inhomogeneity (T_2^{inhom}) to measure T_2

- homogeneous samples: $T_2^{\text{inhom}} \gg T_2 \rightarrow T_2^* \approx T_2$
- real tissues: $T_2^{\text{inhom}} < T_2 \rightarrow T_2^* < T_2$

Inversion recovery sequence

- 180° pulse → magnetization $\mathbf{M} \to -z$



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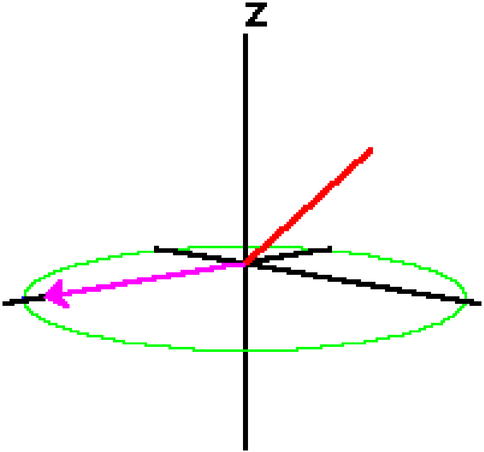

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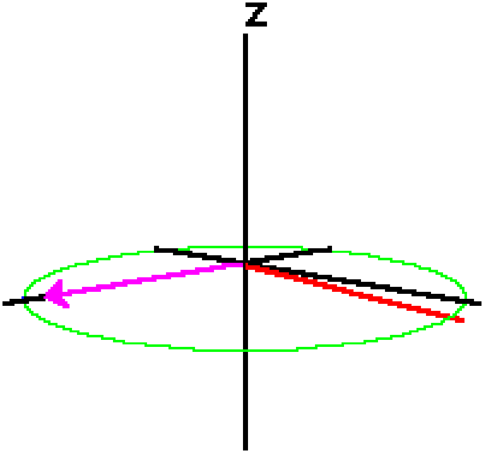

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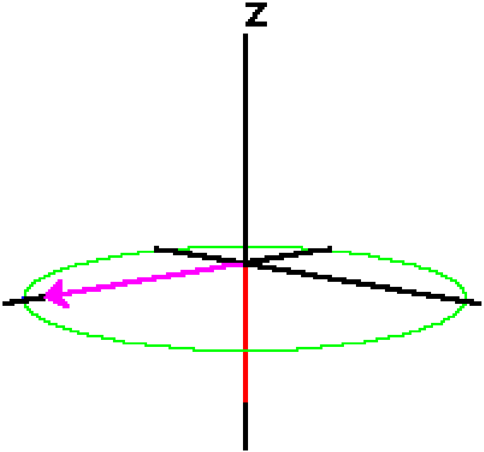

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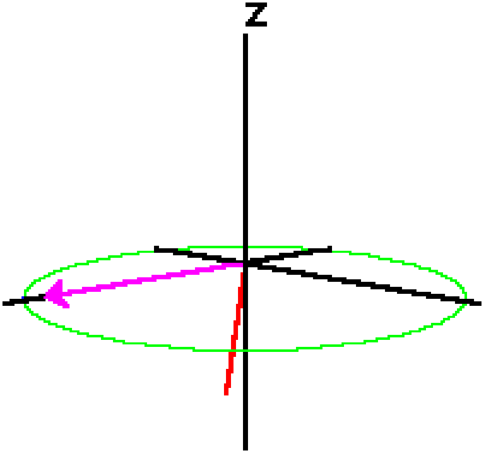
- 180° pulse → magnetization **M** → $-z$
- Before equilibrium, 90° pulse → precession around z



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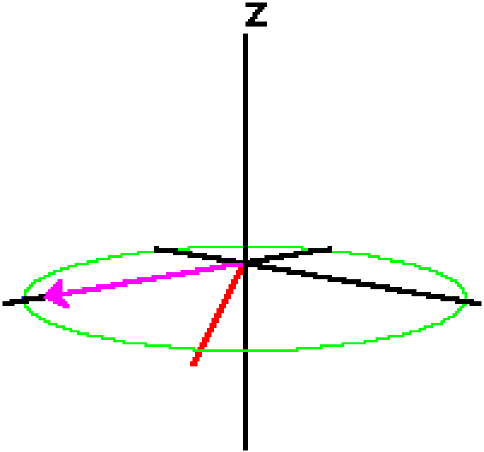

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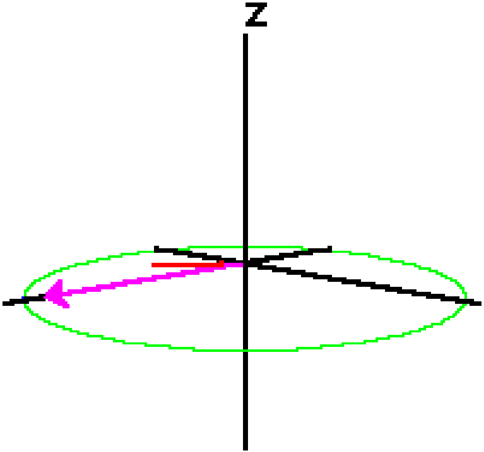

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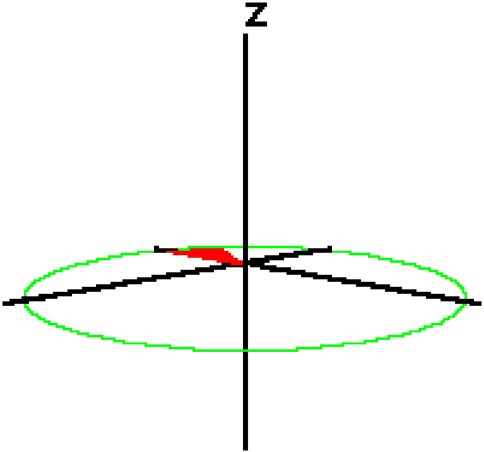

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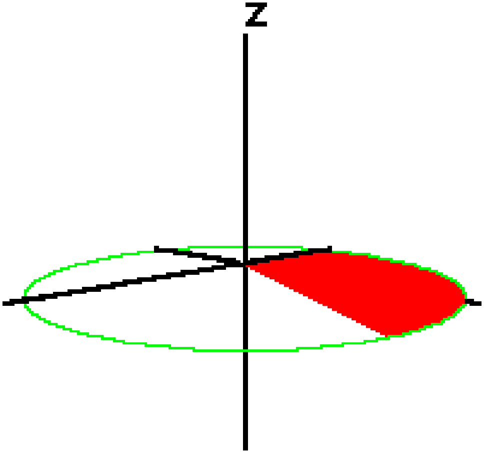
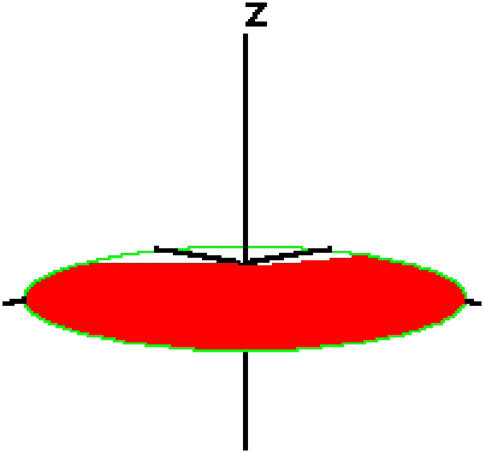


Inversion recovery sequence

- 180° pulse → magnetization **M** → $-z$
- Before equilibrium, 90° pulse → precession around z



Inversion recovery sequence

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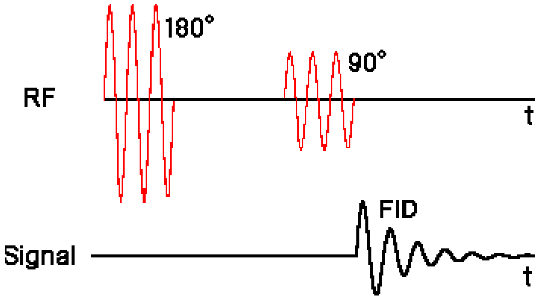
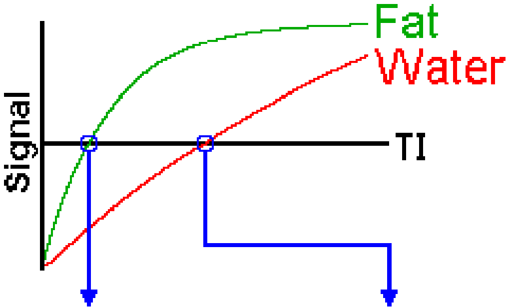
- Time diagram



Inversion recovery (2)

- Good choice of T_I suppresses tissue with specific T_1
- RF impuls when $M_z = 0 \rightarrow$ no signal



Inversion recovery sequence (2)

Signal amplitude after the 90° pulse after one repetition

$$S \propto \varrho(1 - 2\mathrm{e}^{-\frac{T_I}{T_1}})$$

Signal amplitude after many repetitions

$$S \propto \varrho(1 - 2\mathrm{e}^{-\frac{T_I}{T_1}} + \mathrm{e}^{-\frac{T_R}{T_1}})$$

S — signal amplitude

ϱ — spin density

T_R — repetition time

T_E — echo time (between the 90° pulse and readout)

T_1 — spin-lattice relaxation time

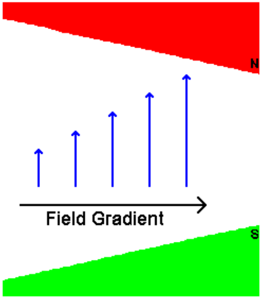
T_I — inversion time (between the 90° and 180° pulses)

Magnetic field gradient

$$f = \gamma B$$

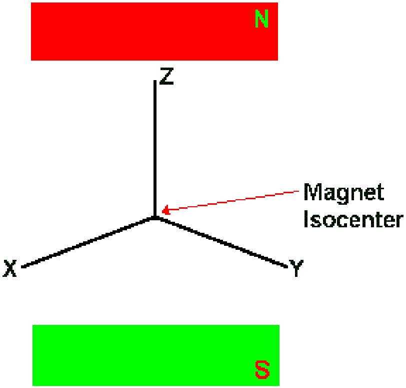
- spatially dependent B
- $\rightarrow$ spatially dependent f

$$B_z = B_0 + xG_x + yG_y + zG_z$$



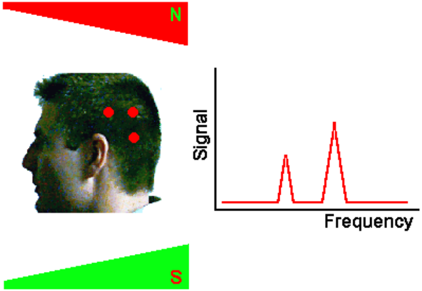
Magnet isocenter

In the origin $(0,0,0)$, field $B_z = B_0$



Frequency encoding

Magnetic field: $B_z = B_0 + xG_x$

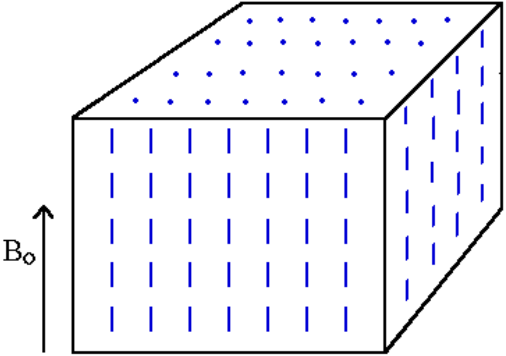


Frequency: $f = \gamma(B_0 + xG_x)$

Slice selection

- Gradient G_z together with RF pulse with frequency f
- Only spins at the resonance frequency are excited

$$\gamma(B_0 + zG_z) = f$$



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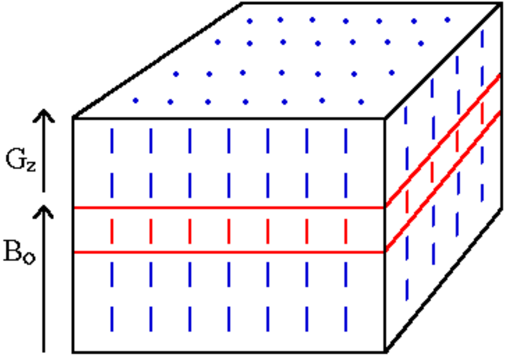
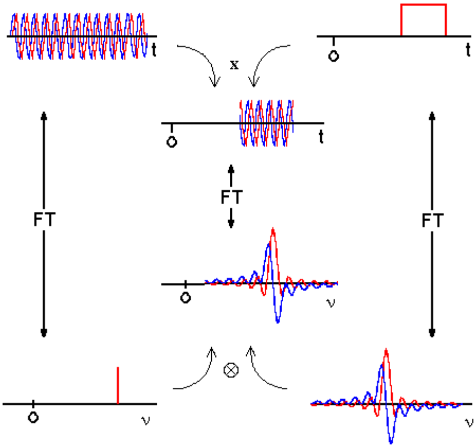


RF pulse envelope shape

- Rectangular 90° pulse $\mathrm{rect}(t)\sin(2\pi ft)$
- ... sinc in the frequency domain ($\mathrm{sinc}(x) = \sin(x)/x$)



RF pulse envelope shape

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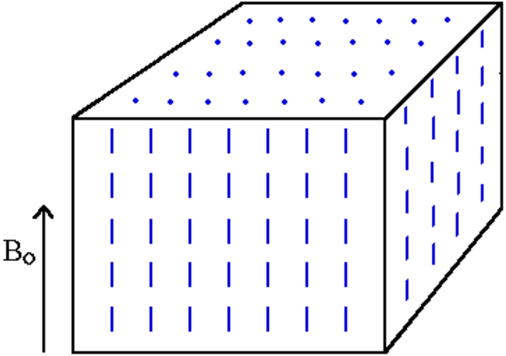
- $\rightarrow$ excitation profile is not rectangular



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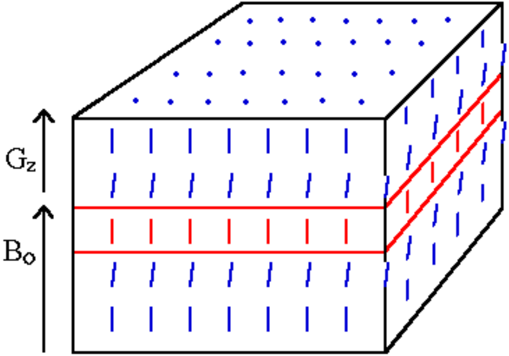

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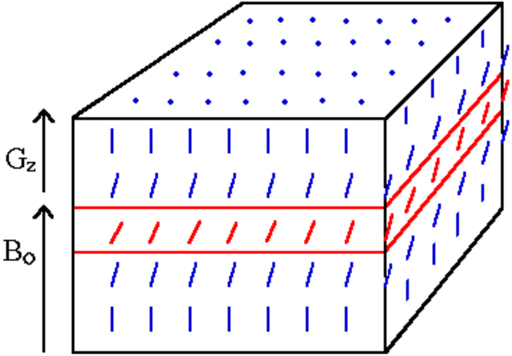

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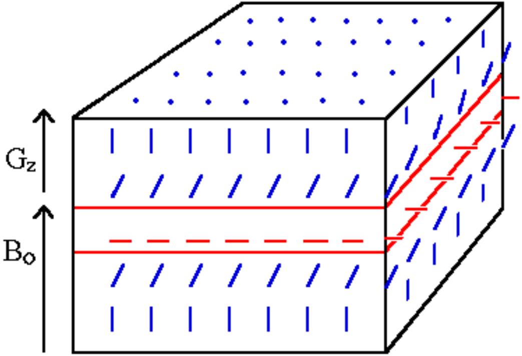
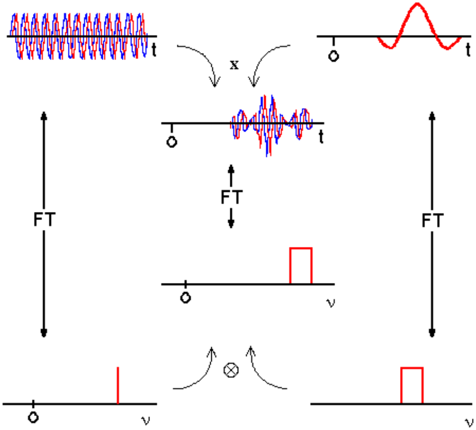


RF pulse envelope shape (2)

- sinc-shaped 90° pulse sinc $\frac{t-t_0}{\tau}$ $\sin(2\pi ft)$
- ...rectangle in the ve frequency domain



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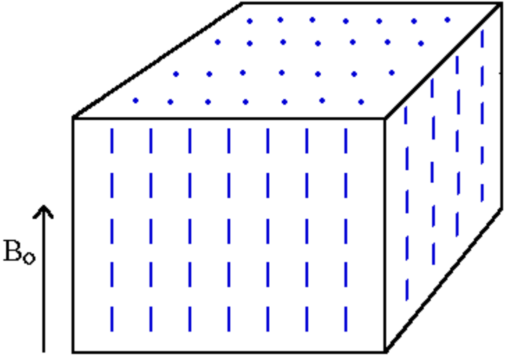
- $\rightarrow$ rectangular excitation profile



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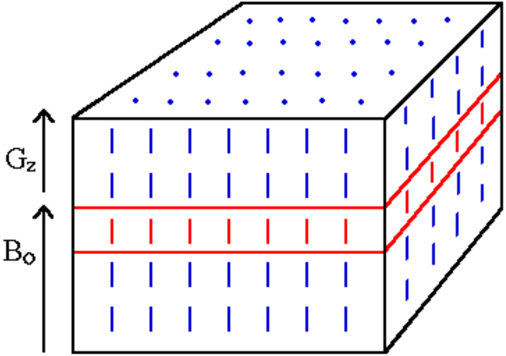

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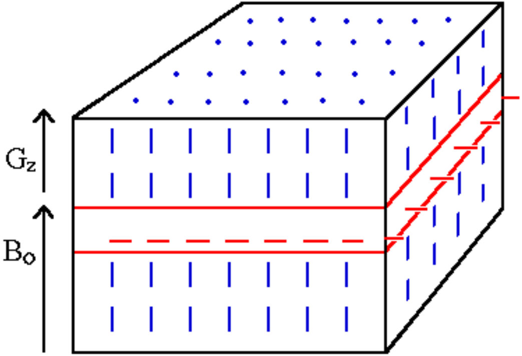

RF pulse envelope shape (3)

- The shorter the RF pulse (in time)
- $\rightarrow$ the wider in the frequency domain
- $\rightarrow$ the wider the excited slice
- . . . and vice versa

Slice thickness:

$$d = \frac{2\Delta f_{\text{RF}}}{\gamma G_{\text{slice}}}$$

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Slice thickness:

$$d = \frac{2\Delta f_{\mathrm{RF}}}{\gamma G_{\mathrm{slice}}}$$

Typical values

- $G_z = 4\,\mathrm{mT/m}$
- bandwidth $\Delta f = 1\,\mathrm{kHz}$
- slice thickness 11.7 mm

Encoding gradients

Gradients of B_z

- Slice selection gradient
- Frequency encoding gradient

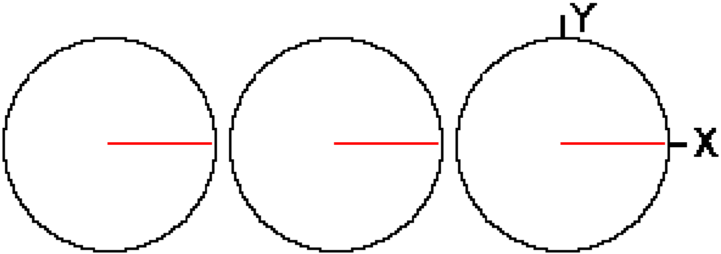
Encoding gradients

Gradients of B_z

- Slice selection gradient
- Frequency encoding gradient
- Phase encoding gradient

Phase encoding gradient

- In constant **B**, the same f



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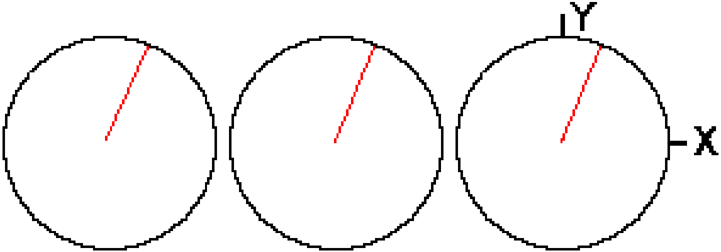

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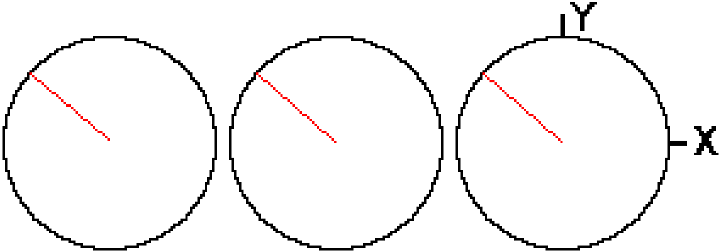
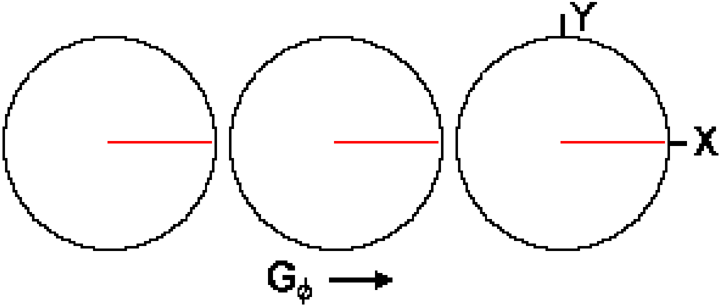


Phase encoding gradient

- In constant **B**, the same f
- Gradient G_φ on $\rightarrow$ different f



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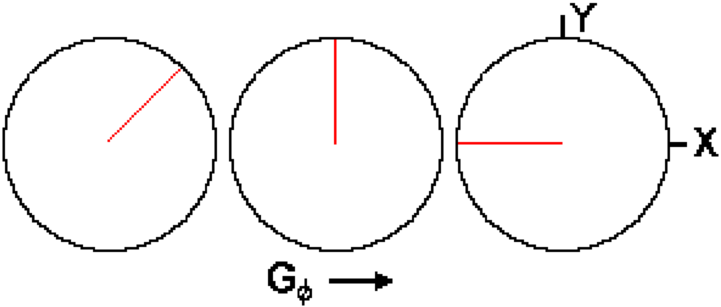

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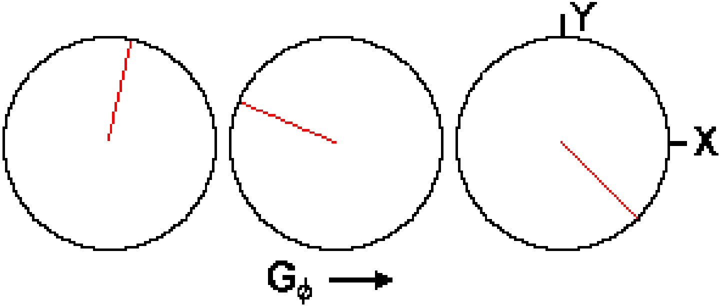
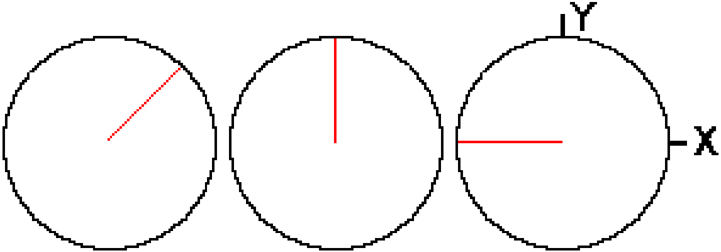


Phase encoding gradient

- In constant **B**, the same f
- Gradient G_{φ} on $\rightarrow$ different f
- Gradient G_{φ} off $\rightarrow$ same f but different phase



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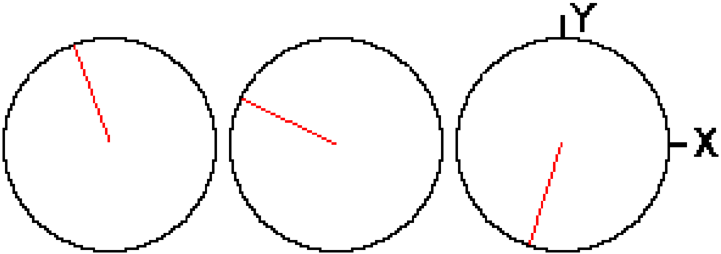

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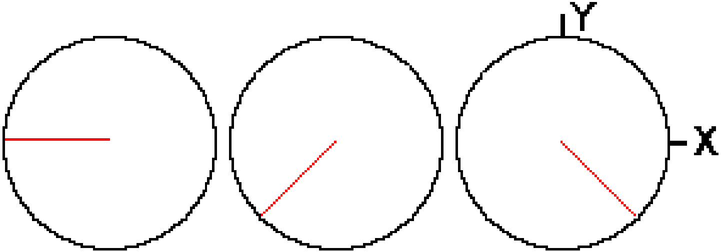

Phase encoding gradient

- In constant **B**, the same f
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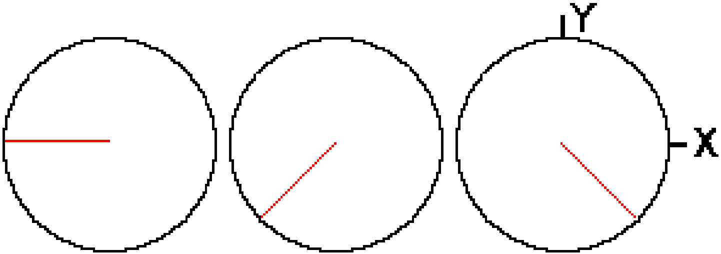
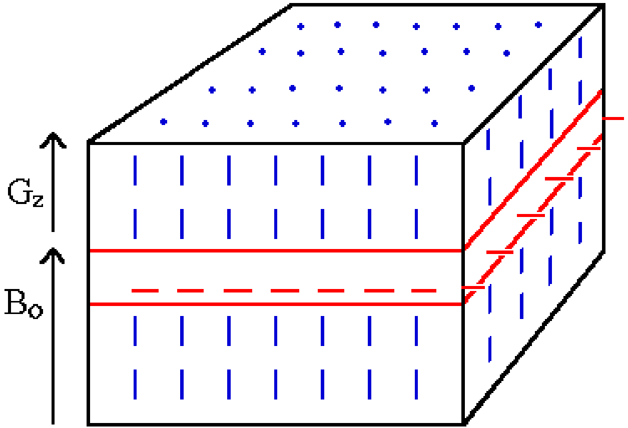
- Gradient G_φ off $\rightarrow$ same f but different phase
- $\rightarrow$ phase encodes position



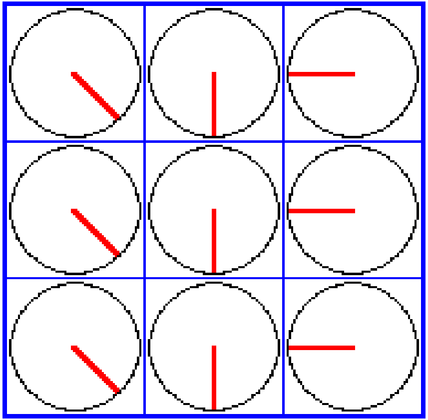
Macroscopic view

- Slice excitation



Macroscopic view

- Slice excitation
- After phase and frequency gradient
  - phase is a function of x
  - frequency is a function of y



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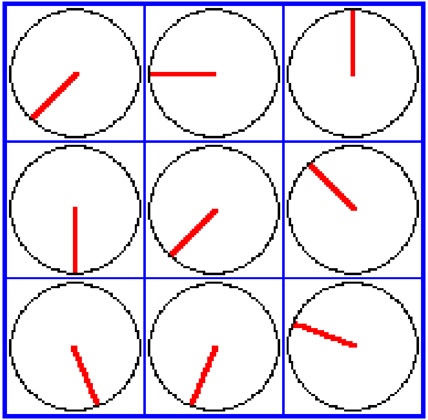

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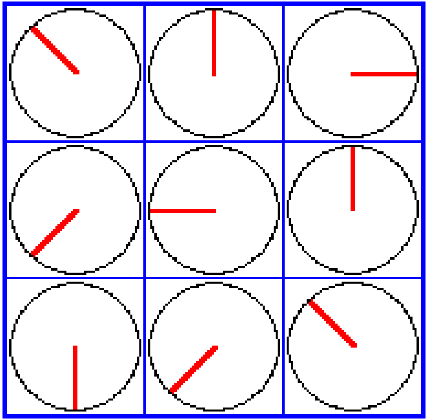

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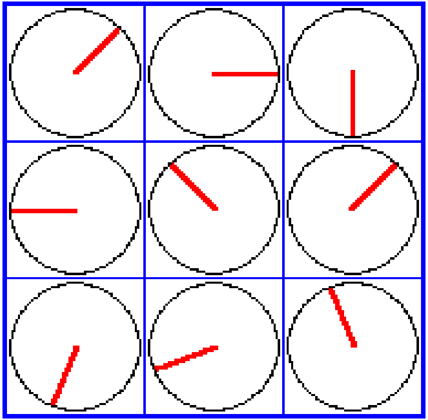

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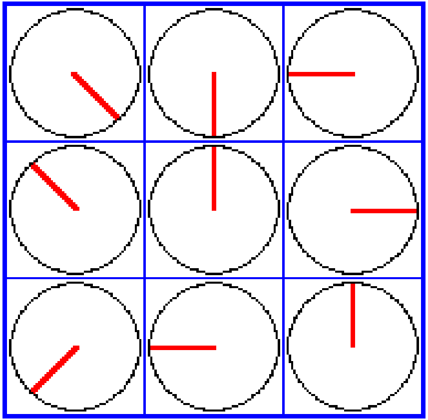

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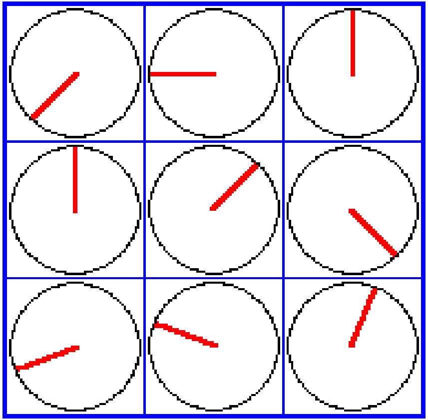

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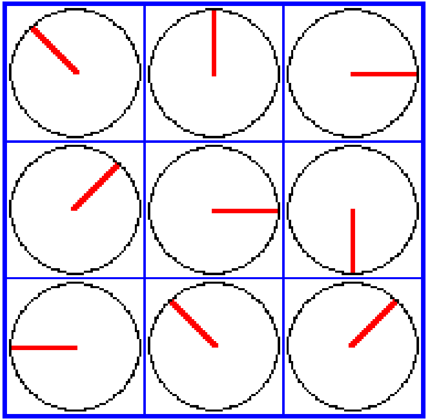
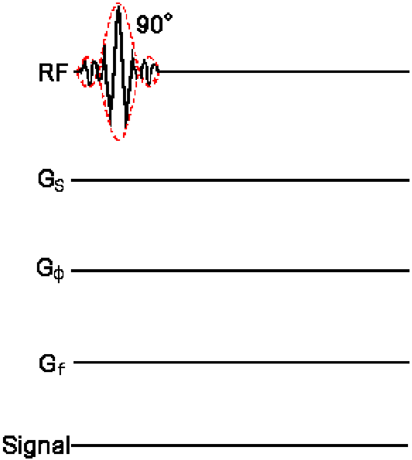


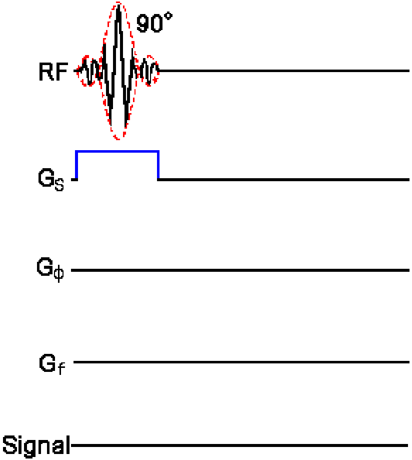
Fourier MRI sequence

- RF pulse



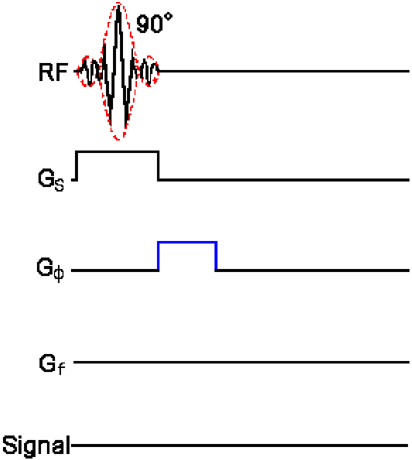
Fourier MRI sequence

- Slice selection gradient



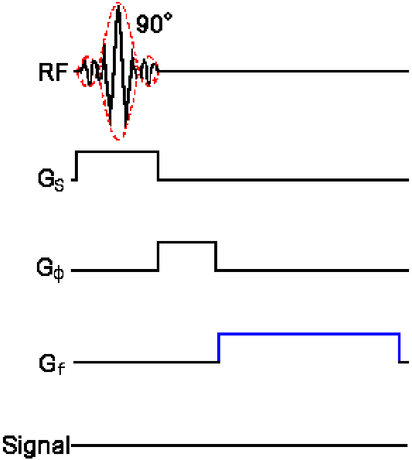
Fourier MRI sequence

- Phase encoding gradient (before readout)



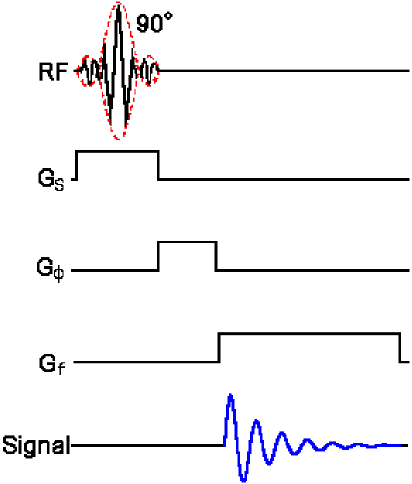
Fourier MRI sequence

- Frequency encoding gradient (during readout)



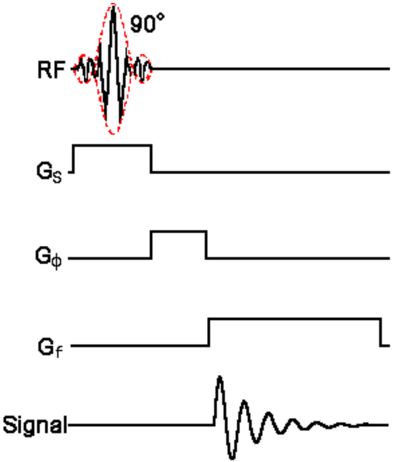
Fourier MRI sequence

- Readout



Multiple excitations

- To acquire a 2D slice $128 \sim 512$ excitations are needed
- Repetition time T_R
- Phase encoding intensity G_ϕ varies ($\pm$)



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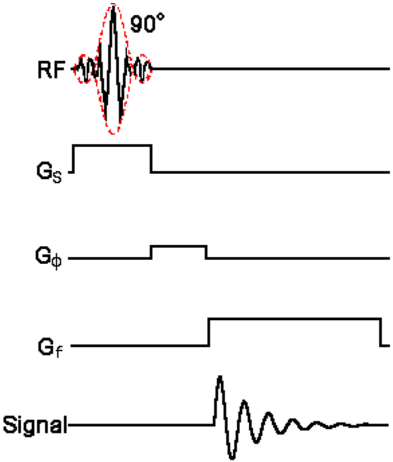

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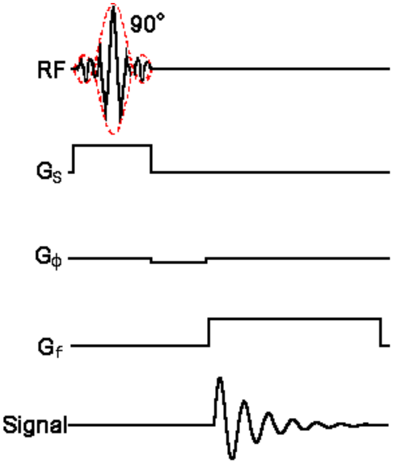

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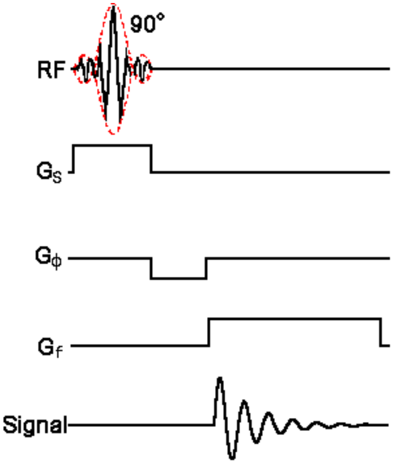
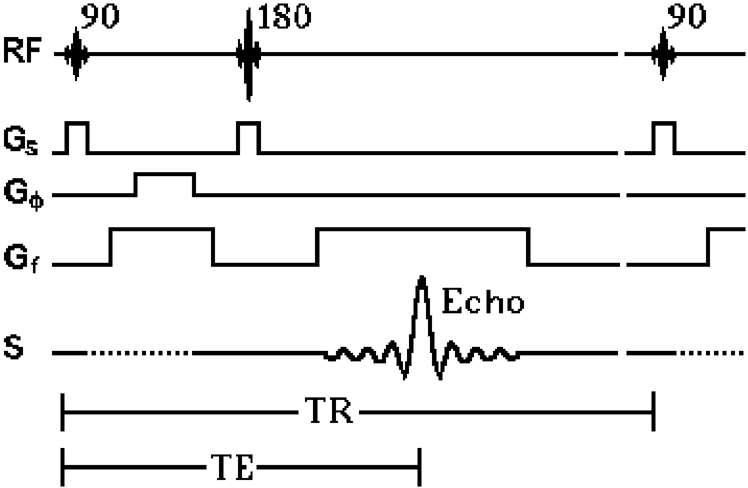


Spin echo — optimized sequence

Time diagram:



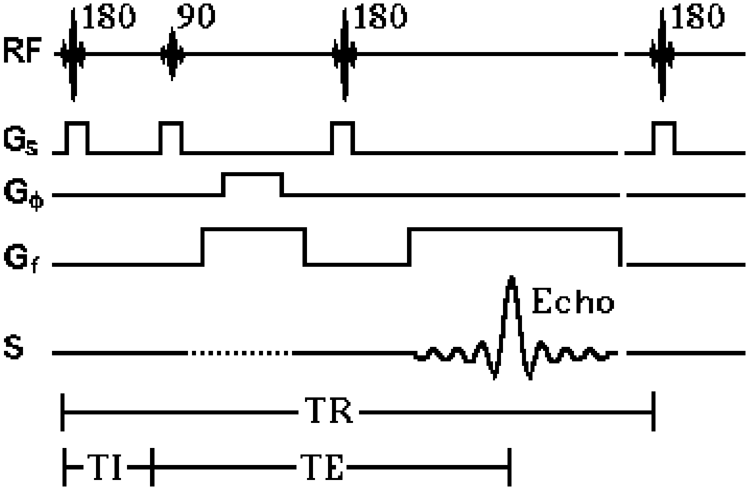
Spin echo — optimized sequence (2)

Note:

- G_ϕ between 90° and 180° pulses $\longrightarrow$ shorter T_E
- FID signal not used
- Desynchronization G_f together with G_ϕ . . .
- . . . $\longrightarrow$ maximum synchronization in the center of the readout window
- Sequence repeated for all G_ϕ

Inversion recovery — optimized sequence

Time diagram:



Inversion recovery — optimized sequence (2)

Note:

- All RF pulses are selective (applied together with G_s)
- G_ϕ cannot be after the first 180° pulse (no transversal magneticazation) ...
- ... applied after the 90° pulse
- starting from the 90° pulse = spin-echo sequence, including desynchronization G_f

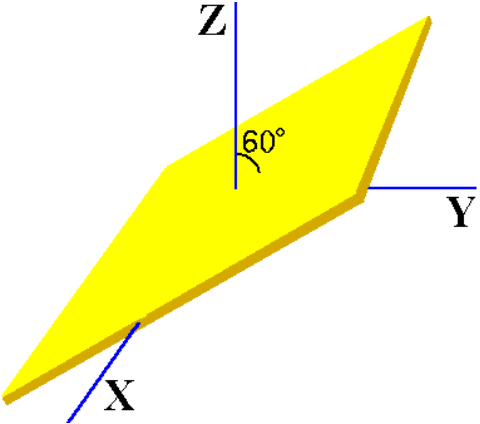
Gradient orientation

Gradient along direction φ is a linear combination

$$G_x = G_f \sin \varphi$$
$$G_y = G_f \cos \varphi$$

Slice orientation

- Slice orientation can be arbitrary — xy,yz,xz, or oblique
- All gradients change B_z. Using linear combination



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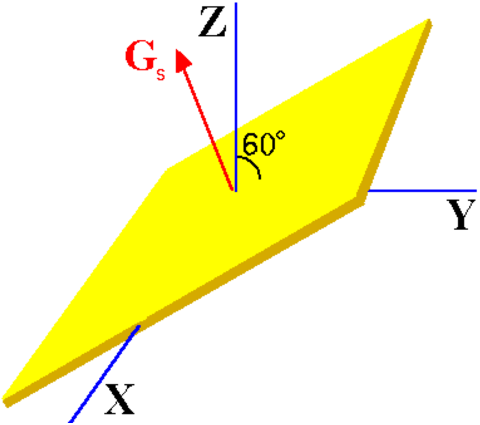
- Slice orientation can be arbitrary — xy,yz,xz, or oblique
- All gradients change B_z. Using linear combination
- Slice selection gradient perpendicular to the slice plane



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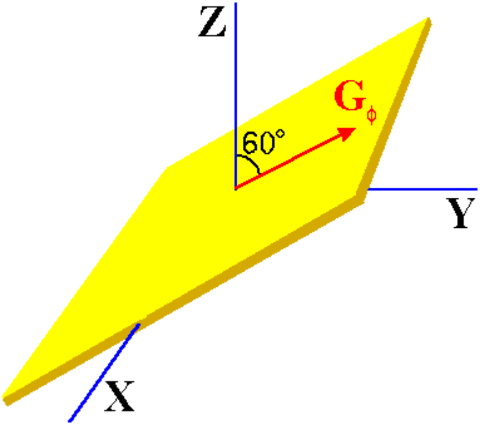
- Phase encoding gradient in the slice plane



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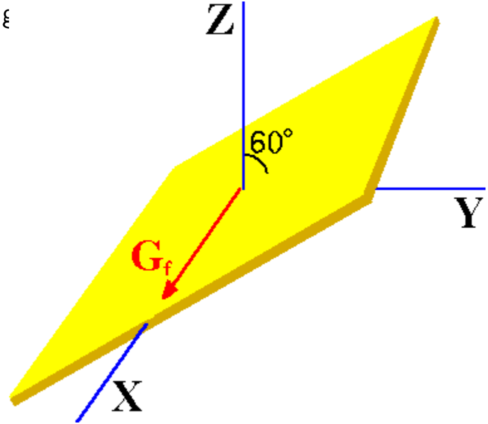
- Frequency encoding



Slice orientation

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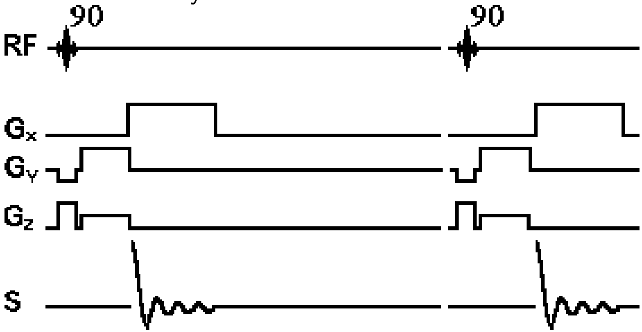
- All gradients change B_z. Using linear combination
- Slice selection gradient perpendicular to the slice plane
- Phase encoding gradient in the slice plane
- Frequency encoding gradient in the slice plane
- Gradient coils simultaneously on.



Spin packet signal

Received (complex) signal:

$$s(t) = M_x(t) + jM_y(t) \propto \mathrm{e}^{-j\phi(t)}$$
$$\text{with phase} \quad \phi(t) = 2\pi f t$$

substituting $f = \gamma B$:

$$\phi(t) = 2\pi\gamma B t$$

Time-dependent magnetic field

Received (complex) signal:

$$s(t) \propto \mathrm{e}^{-j\phi(t)}$$

for stationary field B:

$$\phi(t) = 2\pi\gamma B t$$

for time dependent field $B(t)$:

$$\phi(t) = 2\pi\gamma \int B(t)\,\mathrm{d}t$$

Effects of phase encoding

$$\phi(t) = 2\pi\gamma \int B(t)\,\mathrm{d}t$$

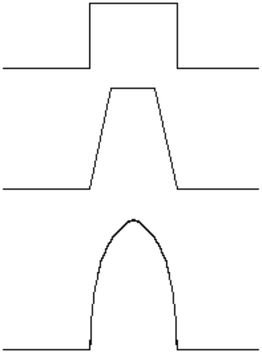
$$B(t) = B_0 + G_\phi(t)y, \qquad \phi(t) = 2\pi\gamma \int B_0 + G_\phi(t)y\,\mathrm{d}t$$

phase shift due to a gradient :

$$\Delta\phi = 2\pi\gamma y \int G_\phi(t)\,\mathrm{d}t$$

Effects of phase encoding (2)

Only the integral of $G_\phi(t)$ matters, not the shape:



For rectangular pulse G_ϕ with duration τ_ϕ:

$$\Delta\phi = 2\pi\gamma y G_\phi \tau_\phi$$

Phase and frequency encoding

After phase encoding :

$$s(t) \propto \mathrm{e}^{-2\pi j\gamma \int B_0 + G_\phi(t) y \, \mathrm{d}t}$$

$$s(t) \propto \mathrm{e}^{-2\pi j\gamma (B_0 t + G_\phi \tau_\phi y)}$$

After phase and frequency encoding :

$$s(t) \propto \mathrm{e}^{-2\pi j\gamma (B_0 t + G_\phi \tau_\phi y + G_f t x)}$$

Quadrature Detector

- **Input:** RF coil signal
- **Output:** signals correspoding to magnetization $M_{x'}$, $M_{y'}$
- x', y' is the rotating frame of reference

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- We can determine *phase*, not only *amplitude* (as in standard AM detector)
- Output is $s(t) = M_{x'} + jM_{y'}$ is considered a *complex signal*

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How?

- *product mixer* with a reference signal f_0

Product mixer

Doubly Balanced Mixer (DBM)

- *Input:* $g_a = \cos(at), \quad g_b = \cos(bt)$
- *Output:* $g = g_a g_b = \frac{1}{2}\cos((a+b)t) + \frac{1}{2}\cos((a-b)t)$
- Signal $\cos((a+b)t)$ can be filtered (low-pass filter)
- Difference frequence signal $\cos((a-b)t)$

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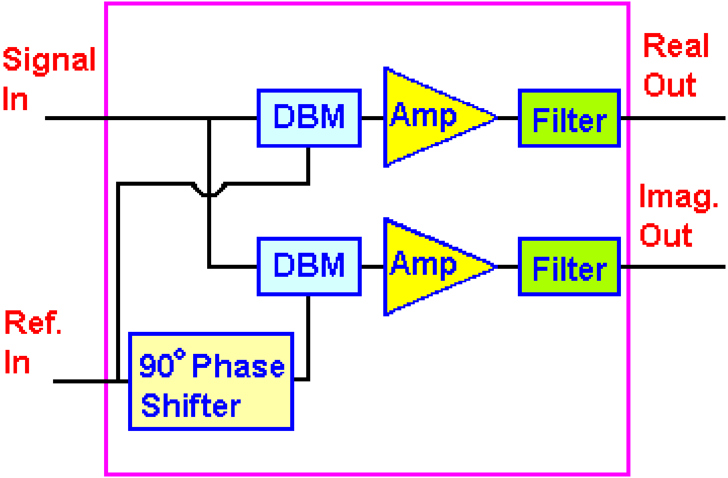
- Signal $\cos((a+b)t)$ can be filtered (low-pass filter)
- Difference frequence signal $\cos((a-b)t)$
- For a 'complex' signal $x\cos(at) + y\sin(at)$, multiplication with $\cos(bt)$ and $\sin(bt)$ recovers x and y.

Quadrature detector (2)



Quadrature detector in Fourier imaging

Signal

$$s(t) \propto \mathrm{e}^{-2\pi j \gamma (B_0 t + G_\phi \tau_\phi y + G_f t x)}$$

Quadrature demodulation with $f_0 = \gamma B_0$ is like using the rotating coordinate system:

$$s(t) \propto \mathrm{e}^{-2\pi j \gamma (G_\phi \tau_\phi y + G_f t x)}$$

k-space

Demodulated signal

$$s(t) \propto \mathrm{e}^{-2\pi j \gamma (G_\phi \tau_\phi y + G_f t x)}$$

Substition

$$k_x(t) = \gamma \int G_f(t) \mathrm{d}x = \gamma G_f t \qquad k_y(t) = \gamma \int G_\phi(t) \mathrm{d}x = \gamma G_\phi \tau_\phi$$

$$s(t) \propto \mathrm{e}^{-2\pi j (k_x(t) x + k_y(t) y)}$$

k-space, slice signal

Demodulated signal from one point:

$$s(t) \propto \mathrm{e}^{-2\pi j(k_x(t)x + k_y(t)y)}$$

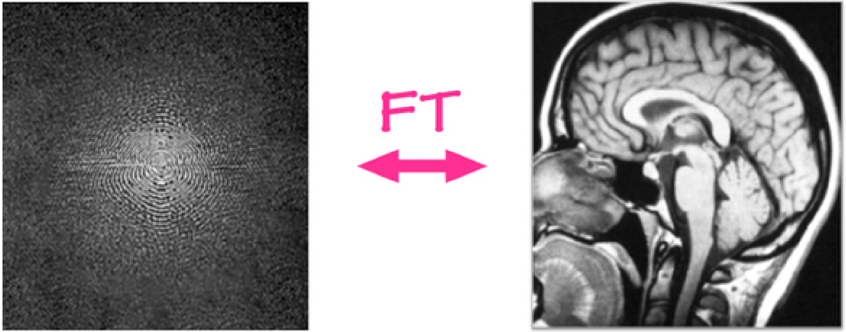
Signal from the whole slice:

$$s(t) \propto \int_{(x,y)\in \text{slice}} \rho(x,y)\mathrm{e}^{-2\pi j(k_x(t)x+k_y(t)y)}\,\mathrm{d}x\mathrm{d}y$$
$$s(t) = S(k_x(t), k_y(t))$$

where $\rho(x, y)$ is the spin density.

Received signal $S(k_x, k_y)$ is a 2D Fourier transform of $\rho(x, y)$

k-space example

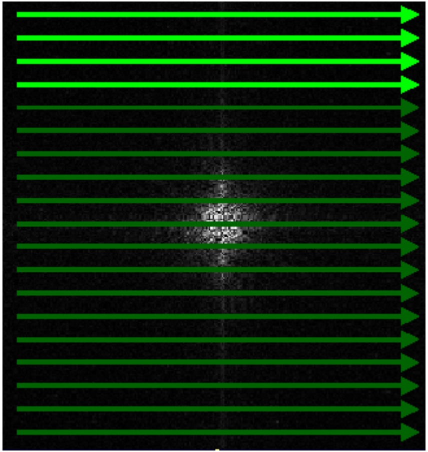


- $S(k_x, k_y)$ is a 2D Fourier transform of $\rho(x, y)$.
- Trajectory $(k_x(t), k_y(t))$ controlled by gradients
- We sample $S(k_x, k_y)$ at points $(k_x(t), k_y(t))$ to get samples from a 1D signal $s(t) = (k_x(t), k_y(t))$.

k-space sampling

k-space acquisition line by line

One line — one excitation



Other trajectories are possible and often used (e.g. spiral)

Field of view
(FOV)

- Sampling step in k-space

$$\Delta k_x = \gamma G_f t_{\text{samp}} \qquad \Delta k_y = \gamma \Delta G_\phi \tau_\phi$$

- Shannon/Nyquyist/Whittaker/Kotelnikov sampling theorem $\rightarrow$ imaged object must be smaller than

$$\text{FOV}_x = \frac{1}{\Delta k_x} = \frac{1}{\gamma G_f t_{\text{samp}}}$$
$$\text{FOV}_y = \frac{1}{\Delta k_y} = \frac{1}{\gamma \Delta G_\phi \tau_\phi}$$

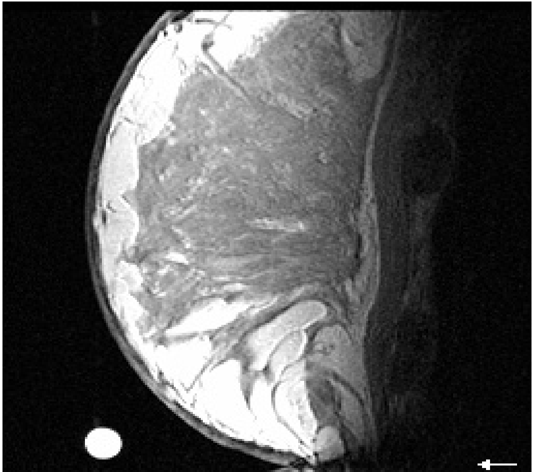
(quadrature detector $\rightarrow$ complex sampling $\rightarrow$ factor 2)

- if the object is larger, aliasing (folded object)

Aliasing

(Wrap Around Effect)

- Part of the object outside of FOV will appear elsewhere
- Object too big, FOV too small

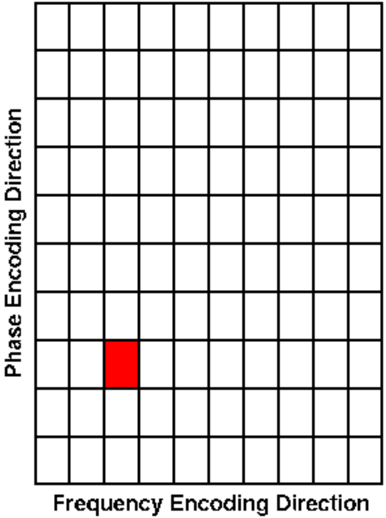


Aliasing (2)

- Aliasing in frequency encoding direction can be suppressed by:
  - Using higher f_{samp}, e.g. 2 MHz instead of 16 kHz. This reduces SNR.
  - Suppressing signal outside of FOV (e.g. using a smaller coil)
- Aliasing in phase encoding direction can be suppressed by:
  - reducing $\Delta k_y \longrightarrow$ increase of the number of phase encoding steps (longer acquisition) or decreasing spatial resolution
  - Suppressing signal outside of FOV (e.g. using a smaller coil)
  - Changing the phase encoding direction

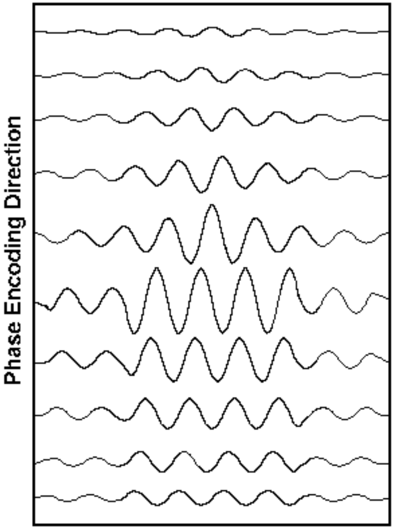
Slice reconstruction

- One active pixel



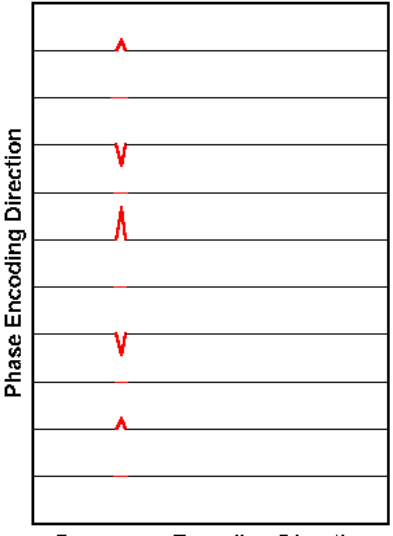
Slice reconstruction

- 10 excitations with different G_ϕ



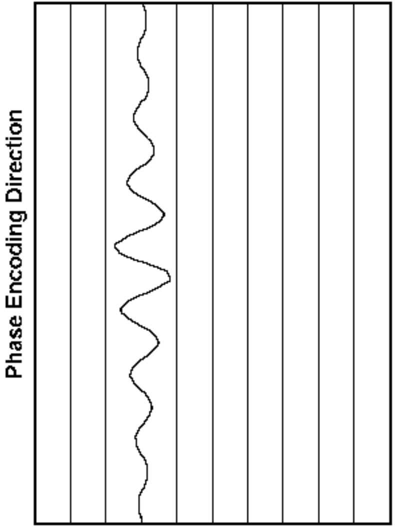
Slice reconstruction

- FT along x



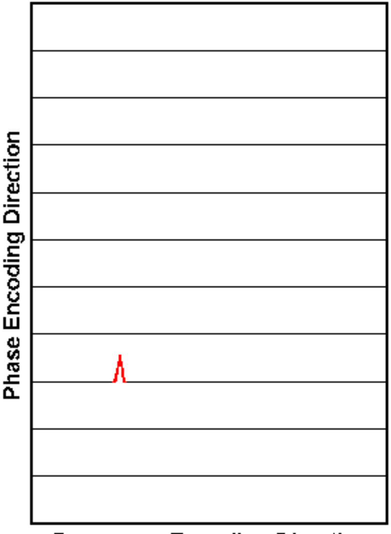
Slice reconstruction

- Finer sampling



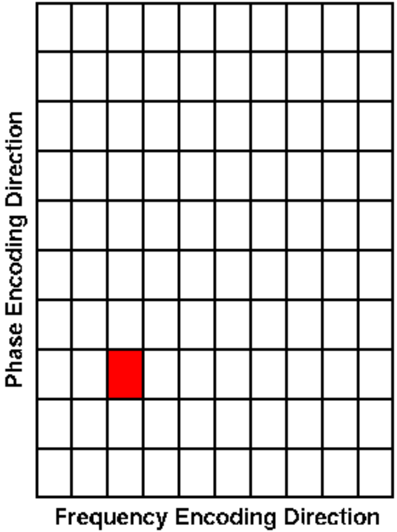
Slice reconstruction

- FT along y



Slice reconstruction

- original



Visualization

- Show amplitude of the 2D FT signal as a grayscale image