

# Chapter 3

## Physics of Projection Radiography

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This chapter is based largely on [1, Ch. 3].

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Introduction

**Projection radiography** refers to the many “conventional” X-ray studies (*e.g.*, dental, chest, broken bones...) in which X-ray photons transmitted through the body are recorded on film or detected electronically. Typically the films and digital images are viewed directly by the radiologist, usually with no or minimal additional processing. This is in contrast to computed tomography where the “raw data” is nearly uninterpretable, and sophisticated computer algorithms are needed to form cross-sectional images from the measurements.

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**Overview**

- What type of energy is used? X-ray photons.
- Source of that energy? X-ray tube.
- How does energy interact with object? Absorbed and scattered.
- What object property is imaged? Attenuation coefficient, or projection from 3D to 2D thereof.
- How is energy measured?
  - Usually phosphor screen converts X-ray photons to light photons, followed by a light-sensitive detector such as film, video camera, photodiode array, etc.
  - Direct conversion of X-ray photons into electrical signals is also used [2].
- What is the form of the PSF? Depends on source size and spreading of light in detector.
- Noise? Poisson
- Artifacts? Scatter, spatial distortion due to point-source geometry (no lens to focus X-rays)

The object (body) absorbs (or scatters) many of the X-ray photons, a process called **attenuation**. The attenuation varies with tissue type. For example, the fluid within bone fractures attenuates less than the bone itself, so the fracture is visible on an X-ray image. We would like X-ray images to be “representative reproductions” of the attenuation characteristics of the body. For interpretation of such images, one needs to know how attenuation relates to material properties. We saw in introduction (Fig. 1.1) that transmission depends on energy  $\mathcal{E}$ , so we first study X-ray sources.

The attenuation properties of different tissues are often insufficiently distinct to make good images, so often we inject a **contrast agent**, *e.g.*, Iodine, to enhance the differences between tissue types.

Radiographs are called **projection** images because 3d object characteristics are projected onto a 2d image.

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**Outline**

- basic physics of X-ray source
- basic physics of X-ray attenuation
- Beer’s law, photoelectric absorption, Compton scatter
- Parallel-beam geometry

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**Useful Constants**

- $1 \text{ \AA} = 10^{-8} \text{ cm} = 10^{-10} \text{ m} = 100 \text{ pm}$
- $c = 299792.5 \cdot 10^3$  (speed of light in m/s)
- $h = 6.6262 \cdot 10^{-27}$  (Planck’s constant in erg·sec)
- $1.60207 \cdot 10^{-12} \text{ erg} = 1 \text{ eV}$  (electron volt)
- $1 \text{ Joule} = 10^7 \text{ ergs} = 1 \text{ Watt second} = 1 \text{ Newton meter} = (\text{kg m} / \text{s}^2) \text{ m}$

$$\mathcal{E} = h\nu = h \frac{c}{\lambda}$$

To convert wavelength  $\lambda$  in meters to photon energy  $\mathcal{E}$  in eV, use  $\mathcal{E} = h(c/\lambda)/(1.60207 \cdot 10^{-12})$

So  $0.01 \text{ \AA} = 1 \cdot 10^{-12} \text{ m} = 1 \text{ pm}$  is associated with a photon energy of about 1.24 MeV, and

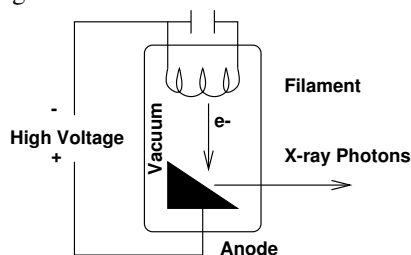
$\lambda = 0.5 \text{ \AA} = 5 \cdot 10^{-11} \text{ m} = 50 \text{ pm}$  is associated with a photon energy of about 25 keV.

## 3.1

**X-ray source**

(What is the distribution of X-ray photon energies?) Excellent review chapter: [4].

The most common X-ray source is the Coolidge Tube.



A high voltage accelerates free electrons from the heated filament toward the anode **target**, typically made of tungsten (W). These electrons interact with the target, in ways described shortly, producing X-ray photons.

Design parameters include the following.

- voltage (controls upper energy limit)
- current (controls intensity, and thus dose and SNR)
- target material (cooling is a challenge for high intensity systems)
- filters (thin sheets of material to remove “soft” or low-energy X-rays)
- exposure time (affects dose, SNR, and motion artifacts)
- shape of focal spot (affects spatial resolution)

The accelerated electrons interact with the target atoms in two distinct ways, producing two photons by two different mechanisms. Though they have different physical origins, we call both types of photons “X-rays” because they are indistinguishable to a detector.

- **continuum spectrum**: Electrons are decelerated by the atoms of the anode resulting in **Bremsstrahlung** (braking) radiation.
- **line spectrum**: accelerated electron ejects an inner-shell electron of a target atom, leading to X-ray fluorescence.

3.1.1

**Continuum spectrum** (Electron-nucleus interactions)

The **continuum spectrum** is the direct result of decelerating highly energetic electrons by the nucleus of the tungsten atoms. The deceleration of electrons is caused by the Coulomb force between the W nucleus and the electrons. This force is proportional to  $F_c \propto zZe^2$  where

- $z$  = electron number  $\approx 1$  (i.e., charge of electron =  $ze$ )
- $Z$  = atomic number (i.e., charge of nucleus =  $Ze$ ) or  $a = F_c/m \propto zZe^2/m$ .

From the theory of EM radiation, the intensity of radiation from a charged particle is simply proportional to the deceleration. Thus,

$$I_{X\text{-ray}} \propto Z \equiv \text{Atomic Number}$$

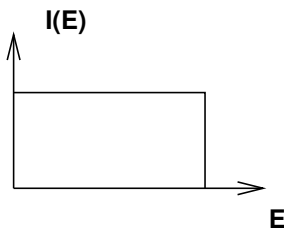
and the most efficient X-ray generators are high atomic number targets (e.g., tungsten,  $Z = 74$ ).

What determines the upper limit on the X-ray photon energy?

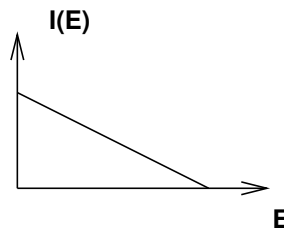
An electron with kinetic energy  $\mathcal{E}$  cannot produce an X-ray with energy  $\mathcal{E}_{X\text{ray}} = h\nu$  greater than  $\mathcal{E}$ . That is,  $\mathcal{E}_{X\text{ray}} \leq \mathcal{E}$ .

- For an infinitesimally thin target, quantum electrodynamics predicts that the intensity of X-rays produced by a decelerating charge is constant up to the electron energy.
- For a thick target, the spectrum can be modeled as the superposition of a number of thin layers, where the electronic energy decreases to zero through the target.
- The total X-ray flux from the continuum spectrum of a thick target is proportional to  $Z\mathcal{E}^2$ .

**Thin Target**

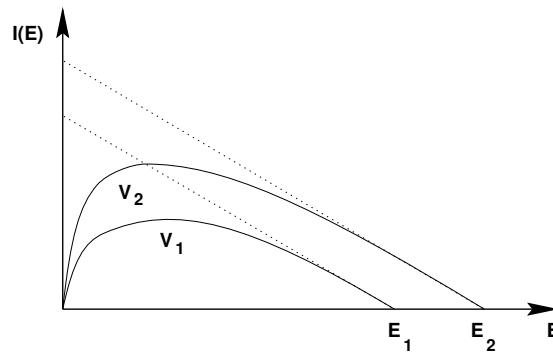


**Thick Target**



(Height proportional to  $Z$ )

Many X-ray photons, especially those at lower energies, are absorbed before they leave the X-ray tube. They are filtered in the target itself, the glass envelope, etc. Thus the continuum spectrum of the photons emitted from the source has the following form.



Because of this filtering, the output power increases at a rate greater than  $\mathcal{E}^2$ .

## 3.1.2

**X-ray line spectra** (Electron-electron interactions)

An accelerated electron can “collide” with and eject an inner-shell electron of a target atom. Another electron (from higher shell) will fill the vacancy, and loss in potential energy is radiated as an X-ray photon. This process is called **X-ray fluorescence**.

X-ray photons originate from electrons, whereas  $\gamma$  photons originate from the nucleus of atoms. (The range of energies overlap between two types; the detector cannot tell the difference between an X-ray and a  $\gamma$ -ray, the name only distinguishes how the photons originated.)

An energetic electron incident upon an atom can transfer an appreciable fraction of its kinetic energy to an atomic electron only in a close collision with that electron. Consider the Coulombic potential energy between two electrons.  $V(r) = e^2/r$ . Using this formula, an energy transfer on the order of 100 eV occurs only in collisions in which the distance of closest approach is less than 0.01 nm (0.1 Å). This distance is about 10% of an atomic “diameter.”

To produce X-rays at energies greater than 100 eV (*i.e.*,  $\lambda < 12\text{nm}$  (120 Å)) through electron-electron interactions, the inner shell electrons of an atom must be involved.

In heavy atoms, inner shell electrons are tightly bound at energies comparable to those needed to produce short-wavelength, hard X-rays. Consequently, a highly energetic collision between a free electron and a tightly bound inner electron can induce level transitions giving rise to X-rays of very specific energies.

The energy of an electron in the innermost shell of  $\text{W}^{74}$  is about  $7 \cdot 10^4$  eV or 70 keV [5].

These higher energy transitions can be induced by scattering with a high energy free electron resulting in the total X-ray intensity from a tungsten target.

**Fig. 3.2** shows the energy spectrum of a thick target, including effects of filtering and characteristic radiation. Note that the K edge is approximately  $\lambda = 0.1$  nm (1.0 Å).

**Compton scatter**

Generated X-ray photons can Compton scatter in the source materials, further spreading out the spectrum.

**Summary**

The main point is that X-ray source is **polychromatic**, or **polyenergetic**, *i.e.*, it emits photons having a wide spectrum of energies. As we will see, this is somewhat undesirable. A **monochromatic** or **monoenergetic** X-ray source would simplify analysis, and perhaps interpretation, of X-ray imaging, just as narrowband pulses simplified analysis in ultrasound imaging. (Nearly monochromatic X-ray sources are made at synchrotron facilities, like the Brookhaven National Labs light source, but these systems are large and expensive.)

Typically less than 1% of the electron energy is converted into X-rays; the rest is heat, ultraviolet, etc. It is impractical to try to create a monoenergetic source by using a polyenergetic source and then by filtering out all but a narrow range of photon energies.

## 3.2

## Attenuation relationship

X-ray photons passing through the body either interact with a particle of mass or pass unaffected. Any interaction removes the photon from the beam, either through scattering or absorption. The loss of photons is called **attenuation**. The term refers to the beam, not to individual photons. An interaction between an X-ray photon and a particle does not affect the other photons in the beam.

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**Beer's Law** (or Beer-Lambert Law, or Lambert-Beer Law, or Beer-Lambert-Bouguer Law)

Suppose  $N_{\text{in}}$  X-ray photons impinge on a homogeneous material of thickness  $\Delta z \approx 0$ . Let  $N_{\text{out}}$  denote the *expected* number of photons that pass through the material without interacting. The (negative of the) expected number of photons  $\Delta N = N_{\text{out}} - N_{\text{in}}$  that interact and are removed from the X-ray beam is given by

$$\Delta N = -\mu N_{\text{in}} \Delta z,$$

where  $\mu$  is called the **linear attenuation coefficient**.

What are the units of  $\mu$ ? Inverse length.

(In chemistry, this property was discovered by Bouguer circa 1729, and this law is studied by passing light through liquid dyes. Doubling the concentration of the dye (*i.e.*, doubling  $\mu$ ) and halving the thickness of the material ( $z$ ) leaves unchanged the amount of light that passes through. Whether the name refers to the original experimenter or to the liquid involved is unknown to me.)

In differential form:

$$dN = -\mu N dz \quad \text{or} \quad \frac{dN}{dz} = -\mu N \quad \text{or} \quad \frac{dN}{N} = -\mu dz.$$

In integral form:

$$\int_0^l \frac{dN(z)}{N(z)} dz = - \int_0^l \mu dz, \quad \text{so} \quad \log N(l) - \log N(0) = \log N_{\text{out}} - \log N_{\text{in}} = -\mu l.$$

Thus for a monoenergetic beam passing through a uniform material of thickness  $l$  with linear attenuation coefficient  $\mu$ :

$$N_{\text{out}} = N_{\text{in}} e^{-\mu l}.$$

Thus larger thickness  $l$  or higher attenuation  $\mu$  means fewer photons get through (on average, because these are expected values). Note that because  $\exp(x) \approx 1 + x$  for  $x \approx 0$ , we have for  $l \approx 0$ :  $N_{\text{out}} \approx N_{\text{in}}(1 - \mu l)$  or  $N_{\text{out}} - N_{\text{in}} = \Delta N \approx N_{\text{in}} \mu l$ .

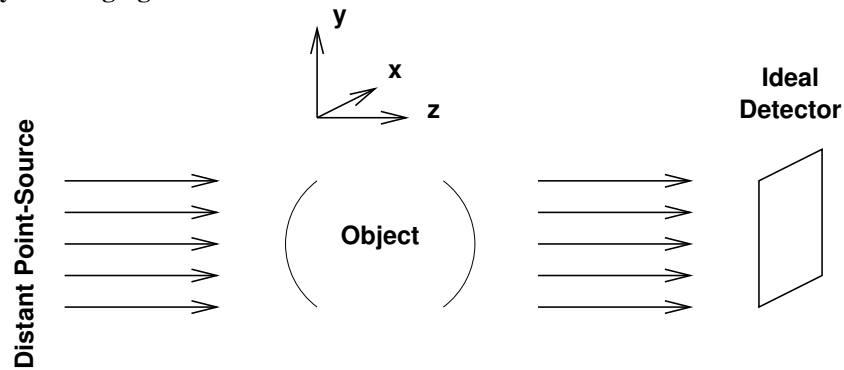
The above expressions are for a homogeneous material (with constant  $\mu$ ). For an inhomogeneous object  $\mu(z)$ , we need a more general solution to the differential equation:

$$N_{\text{out}} = N_{\text{in}} e^{-\int_0^l \mu(z) dz}.$$

In the introduction (**Fig. 1.1**) we saw that  $\mu = \mu(\mathcal{E})$  is a function of X-ray wavelength, or energy  $\mathcal{E}$ . Thus, more precisely:

$$N_{\text{out}}(\mathcal{E}) = N_{\text{in}}(\mathcal{E}) e^{-\int_0^l \mu(z;\mathcal{E}) dz}.$$

### Parallel-beam geometry for imaging



In practice, the X-ray source provides a **flux** of photons impinging on the object. In the **parallel-beam geometry** illustrated above, the flux over an  $x-y$  plane is the same for any  $z$  position preceding the object.

To quantify this flux, we define  $I_0(x, y; \mathcal{E})$  to be the incident X-ray **intensity**. The (expected) total number of photons incident on a region  $A$  in an  $x-y$  plane preceding the object is

$$N_{\text{in}}(A) = \int_0^{\mathcal{E}_{\text{max}}} \int_A I_0(x, y; \mathcal{E}) dx dy d\mathcal{E}.$$

What are the units of  $I_0(\mathcal{E})$ ? Photons per unit area per unit electron energy, *e.g.*, photons / (m<sup>2</sup> keV)

In the parallel-beam geometry, usually we assume the photon flux is spatially uniform, called **uniform illumination**. In this case we write simply  $I_0(\mathcal{E})$ .

More generally the attenuation  $\mu = \mu(x, y, z; \mathcal{E})$  varies spatially within the object.

Redoing the integral equation above in more generality then yields the following relationship between the incident intensity and the exit intensity:

$$I_{\text{out}}(x, y; \mathcal{E}) = I_0(x, y; \mathcal{E}) \exp\left(-\int_0^l \mu(x, y, z; \mathcal{E}) dz\right).$$

Most X-ray detectors cannot discriminate between photons of different energies (more on this later), so the (ideal) recorded intensity  $I_d$  is proportional to the superposition (over  $\mathcal{E}$ ) of the contributions of photons impinging on the detector:

$$I_d(x, y) = \int_0^{\mathcal{E}_{\text{max}}} \eta(\mathcal{E}) I_{\text{out}}(x, y; \mathcal{E}) d\mathcal{E},$$

where  $\eta(\mathcal{E})$  is the **quantum efficiency** of the detector at energy  $\mathcal{E}$ .

Thus we have the following general relationship between the incident energy spectrum  $I_0(\mathcal{E})$ , the object attenuation properties  $\mu(x, y, z; \mathcal{E})$  and the detected intensity:

$$I_d(x, y) = \int_0^{\mathcal{E}_{\text{max}}} \eta(\mathcal{E}) I_0(\mathcal{E}) \exp\left(-\int \mu(x, y, z; \mathcal{E}) dz\right) d\mathcal{E},$$

where the range of integration over  $z$  covers the entire object.

This is the “signal equation” for X-ray imaging via projection radiography.

$I_d(x, y)$  is called a **projection image** of  $\mu(x, y, z; \mathcal{E})$ .

For **rays** where the line integrals through  $\mu$  are larger (more attenuation), the output intensity  $I_d(x, y)$  will be smaller.

**Transmissivity** 

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Is projection radiography a linear imaging method? No, because there is a nonlinear relationship between  $I_d$  and  $\mu$ . However, if we define the **transmissivity** of the object to be:

$$t(x, y; \mathcal{E}) \triangleq \exp\left(-\int \mu(x, y, z; \mathcal{E}) dz\right) \in (0, 1],$$

then we can rewrite the detected intensity as:

$$I_d(x, y) = \int_0^{\mathcal{E}_{\max}} \eta(\mathcal{E}) I_0(\mathcal{E}) t(x, y; \mathcal{E}) d\mathcal{E},$$

which is a linear relationship between  $I_d$  and  $t$ .

**What does X-ray imaging image?** 

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Ideally we would like the system to provide a complete description of  $\mu(x, y, z; \mathcal{E})$ . Because generally we acquire only one projection image with little energy selectivity because the detector cannot discriminate energies (much), what we measure is a weighted average of the transmissivity (weighted by the product of the source spectrum and the detector efficiency). Hopefully this measurement is approximately related to  $\int \mu(x, y, z; \mathcal{E}_0) dz$  for some effective energy  $\mathcal{E}_0$ .

To be able to interpret images of  $\mu$ , one should understand its properties.

How does  $\mu$  relate to the physical properties of the material?



## 3.3

Attenuation coefficient  $\mu$ 

The **linear attenuation coefficient**  $\mu$  of a material depends on the photon energy of the beam  $\mathcal{E}$ , and the atomic numbers  $Z$  of the elements that compose the material, and the material density  $\rho$ .

Because the mass of the material itself provides the attenuation, attenuation coefficients are often characterized by  $\mu/\rho$ , where  $\rho$  is the material density.

In the diagnostic range below 200 Kev, three mechanisms dominate the attenuation:

- Coherent (Rayleigh) scatter
- Photoelectric absorption
- Compton scatter.

A figure below shows the relative strengths of these interactions (and their total) over the diagnostic range of energies for *water*.

The effect of the first two mechanisms is related to atomic number  $Z$ . Body is mostly H, O, C, which are all low  $Z$  elements.

Other mechanisms of interaction [3, p. 4]

- pair production: when a high-energy photon ( $> 1$  MeV) interacts with an atomic nucleus, producing an electron and a positron.
- photodisintegration: when an extremely high-energy photon interacts with an atomic nucleus and causes it to enter an excited state that immediately decays into two or more daughter nuclei, *e.g.*, dislodging a proton or neutron from a nucleus.

## 3.3.1

**Coherent scatter aka Rayleigh scatter**

**Coherent scattering** arises from the periodic (or correlated) potential seen by an X-ray at an atomic scale. In coherent scattering the photons interact with the material and are then “remitted” at the **same wavelength** (*i.e.*, the *same energy*) but (possibly) in a *different direction*. For example, Bragg scattering in a crystal is a result of coherent scattering. This form of scattering is strongest at X-ray wavelengths comparable to atomic and molecular dimensions in a material. As seen in the figure below, for low  $Z$  materials, *Rayleigh scattering is not a major contributor to the attenuation over the diagnostic energy range*. For very high  $Z$  materials, coherent scattering can produce a non-negligible contribution over the diagnostic range.

## 3.3.2

**Photoelectric absorption**

The **photoelectric effect**, first described in quantum terms by Einstein in 1905, is the absorption of a photon by a **bound or orbital electron**. For X-ray and  $\gamma$ -ray energies, electrons in the K shell are predominantly involved. The kinetic energy of the electron is increased by the energy of the absorbed photon. *At X-ray energies, the excited electron overcomes the binding energy and is ejected from the atom (*i.e.*, is **ionized**)*. The kinetic energy of the ejected electron is dissipated in the material. Often an electron in a outer orbital falls into the inner orbital to fill the hole left behind, releasing energy in the form of a photon, the energy of which equals the energy difference of the two orbitals. The emitted radiation has energies characteristic of the atoms present. This phenomenon is called **fluorescence** because absorption of higher-energy radiation leads to the re-emission of lower-energy radiation. Usually the lower energy photons are again absorbed (or scattered) in tissue, possibly with further **ionizations**.

Again, tightly bound electrons in high  $Z$ -materials are more likely to be involved in photoelectric absorption because the binding energies are closer to X-ray energies.

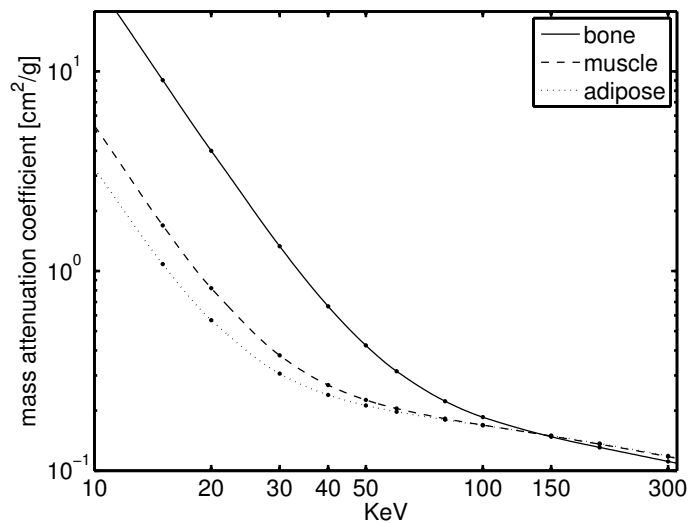
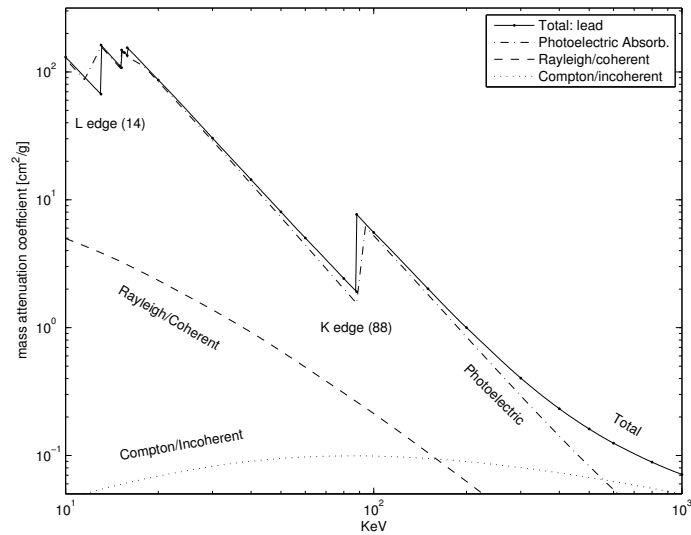
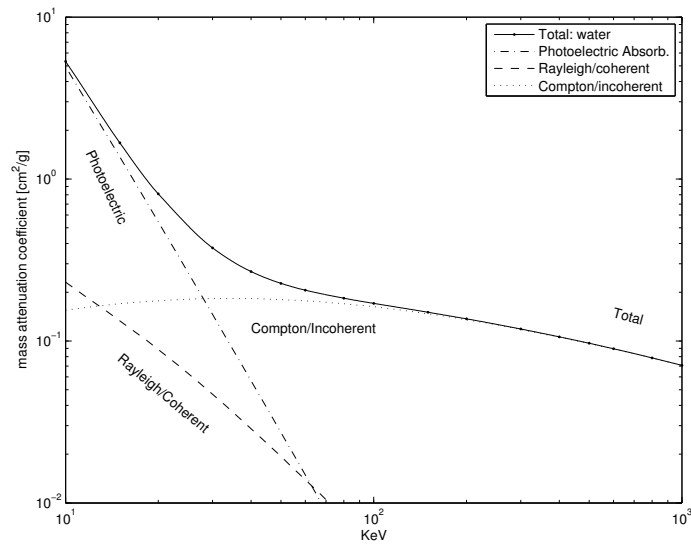
Photoelectric absorption is a random phenomena, and the probability of an absorption is higher when the photon energy is close to the binding energy of the electron. This phenomena is called **resonance absorption**, and occur at energies corresponding to specific transitions between tightly bound levels.

Some properties:

- mass attenuation coefficient  $\propto Z^3$  (approximately)
- linear attenuation coefficient  $\propto Z^4$  (approximately)
- **K-edge** of Pb about 88 KeV (resonance absorption), see figure below.
- (for lower  $Z$  materials, the K-edge is below diagnostic spectrum)

Over the diagnostic range of energies, only high  $Z$  materials (*i.e.*, not soft tissue) have significant photoelectric absorption, *e.g.*, calcium in bone ( $Z = 20$ ), because for the lower atomic numbers the binding energies for the K shells are lower.

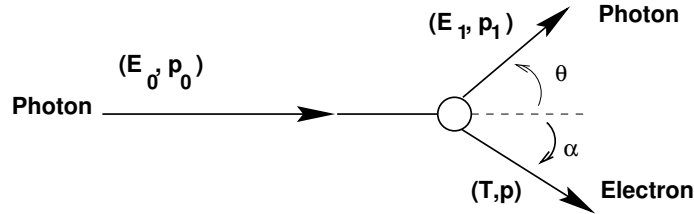
The following figures used the command `xray_atten_interp` test in the Image Reconstruction Toolbox, and are based on NIST data [6].



## 3.3.3

**Compton scattering**

The most significant source of tissue attenuation in the diagnostic region is **Compton scattering**. The **Compton effect** [7] describes a collision between an X-ray photon (or  $\gamma$ -ray photon) and a **free electron** or a **loosely bound electron** in an outer shell. Compton scattering is an **ionizing** event, where the loosely bound electron is scattered and exits at velocity  $v$  (*i.e.*, with momentum  $p$ ), and the scattered photon exits with energy  $\mathcal{E}_1$  and momentum  $p_1$ .



We follow the original treatment of Compton [7] that considers a free electron that is at rest. For a more complete derivation that includes the effects of a bound electron with momentum, which leads to **Doppler broadening**, see [8]. This topic remains an active research area [9]. (See also [10].)

Compton events can be described using momentum and energy conservation. The two components for momentum conservation are:

$$\begin{aligned} p_0 &= p_1 \cos \theta + p \cos \alpha \\ p_1 \sin \theta &= p \sin \alpha. \end{aligned}$$

With some algebra we obtain

$$p^2 \cos^2 \alpha = (p_0 - p_1 \cos \theta)^2$$

$$p^2 \sin^2 \alpha = p_1^2 \sin^2 \theta$$

$$\boxed{p^2 = p_0^2 + p_1^2 - 2p_0 p_1 \cos \theta.}$$

Similarly, assuming the electron is initially at rest, energy conservation requires that:

$$\mathcal{E}_0 + m_0 c^2 = \mathcal{E}_1 + T + m_0 c^2,$$

where  $m_0$  is the rest mass of electron, so

$$\mathcal{E}_0 - \mathcal{E}_1 = T \quad \text{and thus because } \mathcal{E} = cp: \quad c(p_0 - p_1) = T.$$

But from the definition of momentum (relativistic):

$$(T + m_0 c^2)^2 = \mathcal{E}^2 = c^2 p^2 + (m_0 c^2)^2,$$

thus

$$p^2 = T^2/c^2 + 2Tm_0$$

$$\boxed{p^2 = (p_0 - p_1)^2 + 2m_0 c(p_0 - p_1).}$$

Equating the two boxed equations, we obtain

$$p_0^2 + p_1^2 - 2p_0 p_1 \cos \theta = (p_0 - p_1)^2 + 2m_0 c(p_0 - p_1),$$

which reduces to

$$2p_0 p_1 (1 - \cos \theta) = 2m_0 c(p_0 - p_1)$$

or

$$\frac{1}{p_1} - \frac{1}{p_0} = \frac{1}{m_0 c} (1 - \cos \theta).$$

Thus the change in wavelength is:

$$\Delta\lambda = \lambda_1 - \lambda_0 = \lambda_C(1 - \cos\theta),$$

where  $\lambda_C$  is the **Compton wavelength**:

$$\lambda_C = \frac{h}{m_0c} = 0.0025 \text{ nm } (0.025\text{\AA}) (2.5 \text{ pm}).$$

Using  $\mathcal{E} = cp$ , an alternative final expression is:

$$\frac{1}{\mathcal{E}_1} = \frac{1}{\mathcal{E}_0} + \frac{1}{511\text{keV}}(1 - \cos\theta),$$

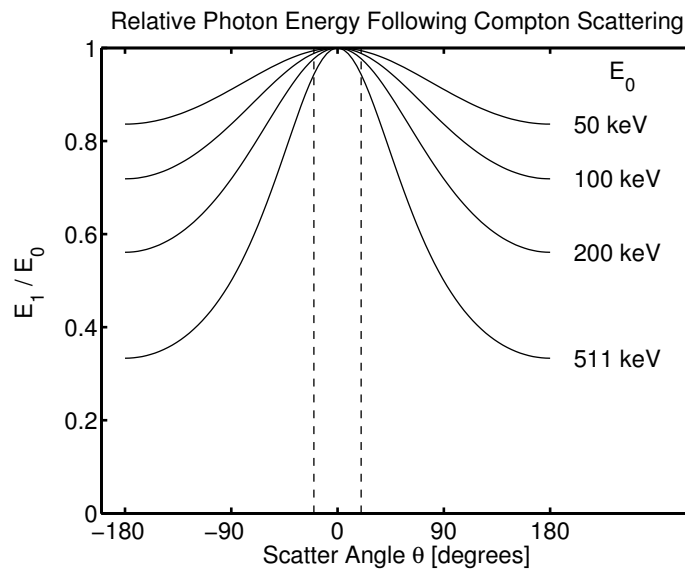
where  $\mathcal{E} = m_0c^2 = 511 \text{ keV}$  is the “equivalent energy” of an electron at rest.

Thus the scattered photon is left with somewhat longer wavelength, or *lower energy*, because  $\mathcal{E} = hc/\lambda$ .

Clearly, the change in wavelength is significant only at relatively high energies, where the wavelength is smaller so  $\Delta\lambda$  has a greater effect. ( $\lambda = 0.02 \text{ nm} = 20\text{pm}$  at 50 Kev).

Thus, the scattered photons in the diagnostic range are in the same energy range as the incident photons.

This is a *major complication* in X-ray imaging.



At lower energies, where the fractional change in wavelength is negligible, the **scattering angle distribution** is nearly **isotropic**. Again, this is a *major complication*.

Because Compton scattering is related to outer shell, *i.e.*, loosely bound electrons, it is relatively *independent* of atomic number  $Z$ , and weakly a function of energy  $\mathcal{E}$ . This is shown in Fig. 3.4 for lead (Pb), where Compton scattering is relatively unimportant.

The relation between photoelectric and Compton events is summarized in Fig. 3.6.

- For low energies and high atomic number, photoelectric absorption dominates.
- For high energies and low atomic number, Compton scattering dominates.

The line shows the values of  $Z$  and photon energy  $h\nu$  for which the photoelectric and Compton effects are equal.

## 3.4

**Attenuation coefficients of tissue**

In most mixtures (*e.g.*, heterogeneous tissue types) a **mass attenuation coefficient** can be used given by

$$\frac{\mu}{\rho} = \sum_i w_i \left( \frac{\mu_i}{\rho_i} \right),$$

where  $w_i$  is the fraction by weight of the  $i$ th element in the material. Thus, mass attenuation coefficients are usually tabulated for various materials. Its units are  $\text{cm}^2/\text{g}$ .

Approximate analytic expressions can be derived for each of the three major attenuation mechanisms, as functions of  $Z$  and  $\rho$  and  $\mathcal{E}$ . Derivation of these expressions is beyond the scope of this course.

Over the diagnostic range of energies, the photoelectric component has a very strong  $Z$  dependence and dominates at low energies, whereas Compton scattering is  $Z$  independent and dominates at higher energies.

Fig. 3.7. X-ray attenuation coefficients for muscle, fat, and bone, as a function of photon energy.

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Summary

The attenuation coefficient  $\mu$  is related to material elemental composition, especially  $Z$ , as well as material density.

**Tradeoffs**

Lower energy, better contrast, but source less efficient (so fewer photons hence more noise for same scan time), and worse dose.

Lots of work on finding optimal contrast-to-noise ratio vs energy and other system design parameters.

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