

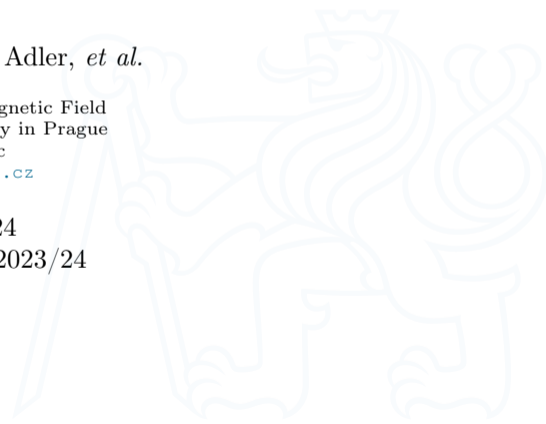
Lecture 3: Indexing, Relational and Logical Operators

B0B17MTB, BE0B17MTB – MATLAB

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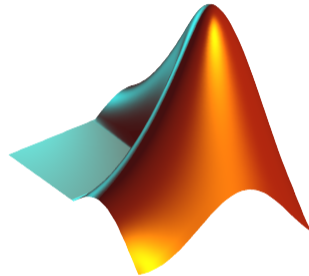
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1. Indexing
2. Relational Operators
3. Logical Operators
4. Exercises





Indexing in MATLAB

- ▶ Now we know all the stuff necessary to deal with indexing in MATLAB.
- ▶ Mastering **indexing is crucial** for efficient work with MATLAB.
- ▶ Up to now, we have been working with entire matrices, quite often we need, however, to access individual elements of arrays.
- ▶ Two ways of accessing matrices/vectors are distinguished.
 - ▶ Access using round brackets “()”.
 - ▶ Matrix indexing: refers to position of elements in a matrix.
 - ▶ Access using square brackets “[]”.
 - ▶ Matrix concatenation: refers to element’s order in a matrix.



Indexing in MATLAB I.

- ▶ Let's consider following triplet of matrices.
 - ▶ Execute individual commands and find out their meaning.
 - ▶ Start from inner part of the commands.
 - ▶ Note the meaning of the pointer end.

$$\mathbf{N}_1 = \begin{bmatrix} -5 \\ 0 \\ 5 \end{bmatrix}$$

$$\mathbf{N}_2 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 2 & 3 & 5 & 7 & 11 \end{bmatrix}$$

$$\mathbf{N}_3 = \begin{bmatrix} 11 & 12 & 13 & 14 \\ 22 & 24 & 26 & 28 \\ 33 & 36 & 39 & 42 \\ 44 & 48 & 52 & 56 \end{bmatrix}$$

```
N1 = (-5:5:5)'; N2 = [1:5;2:2:10;primes(11)]; N3 = (1:4)'+*(11:14);
```

```
N1(1:3)
N1([1 2 3])
N1(3:-1:1)
N1([1 3])
N1([1 3].')
N1([1 3]).'
N1([1; 3])
N1([1 3],1)
```

```
N2(1, 3)
N2(3, 1)
N2(1, end)
N2(end, end)
N2(1, :)
N2(1, :).'
```

```
N2(:, 2)
N2(:, 3:end)
```

```
N3(2:3, [1 1 1]) % like repmat
N3(2:3, ones(1,3))
N3(2:3, ones(3,1))
N3([N2(2,1:2)/2 4], [2 3])
N3([1 end], [1:4 1:2:end])
N3(:, :, 2) = magic(4)
N3([1 3], 3:4, 3) = ...
[1/2 -1/2; pi*ones(1, 2)]
```



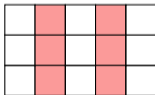
Indexing in MATLAB II.

- ▶ Remember the meaning of end and the application of colon operator “:”.
- ▶ Try to:

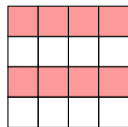
- ▶ Flip the elements of the vector \mathbf{N}_1 without use of `flipplr/flipud` functions.



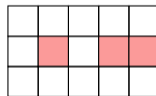
- ▶ Select only the even columns of \mathbf{N}_2 .



- ▶ Select only the odd rows of \mathbf{N}_3 .



- ▶ Select 2nd, 4th and 5th column of 2nd row of \mathbf{N}_2 .



- ▶ Create matrix \mathbf{A} of size 4×3 containing numbers 1 to 12 (row-wise, from left to right).





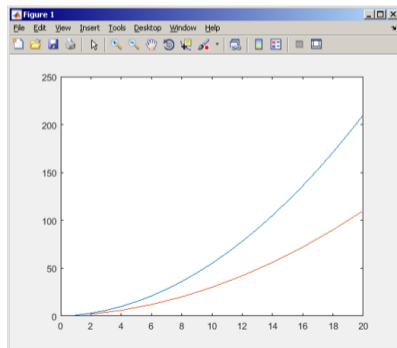
Indexing in MATLAB III.

- ▶ Calculate cumulative sum \mathbf{S} of a vector \mathbf{x} consisting of integers from 1 to 20.
- ▶ Search MATLAB help to find the appropriate function (*cumulative sum*).

$$\mathbf{x} = (1 \quad 2 \quad \dots \quad 20)$$

$$\mathbf{S} = (1 \quad 1 + 2 \quad \dots \quad 1 + 2 + \dots + 20)$$

- ▶ Calculate cumulative sum \mathbf{L} of even element of the vector \mathbf{x} .
- ▶ What is the value of the last element of vector \mathbf{L} ?





Indexing in MATLAB IV.

- Which one of the following returns corner elements of a matrix **A** (10×10)?

```
A([1, 1], [end, end])  
A({[1, 1], [1, end], [end, 1], [end, end]})  
A([1, end], [1, end])  
A(1:end, 1:end)
```



Deleting Elements of a Matrix

- ▶ Empty matrix is a crucial concept in deleting elements of a matrix.

```
T = [];
```

- ▶ We want to:

- ▶ Remove 2nd row of a matrix **A**.

```
A(2, :) = []
```

- ▶ Remove 3rd column of a matrix **A**.

```
A(:, 3) = []
```

- ▶ Remove 1st, 2nd and 5th column of a matrix **A**.

```
A(:, [1 2 5]) = []
```




Adding and Replacing Elements of a Matrix

- ▶ We want to replace:

- ▶ 3rd column of a matrix **A** (of size $M \times N$) by a vector **x** (length M).

```
A(:, 3) = x
```

- ▶ 2nd, 4th and 5th row of a matrix **A** by three rows of a matrix **B** (number of columns of both **A** and **B** is the same).

```
A([2 4 5], :) = B(1:3, :)
```

- ▶ We want to swap

- ▶ 2nd row of matrix **A** and 5th column of matrix **B** (number of columns of **A** is the same as number of rows of **B**).

```
A(2, :) = B(:, 5)
```

- ▶ Remember that always the size of matrices have to match!



Deleting, Adding and Replacing Matrices

- ▶ Which of the following deletes the first and the last column of matrix **A** (6×6)?
 - ▶ Create your own matrix and give it a try.
- ```
A[1, end] = 0
A(:, 1, end) = []
A(:, [1:end]) = []
A(:, [1 end]) = []
```
- ▶ Replace 2nd, 3rd and 5th row of matrix **A** by first row of matrix **B**.
    - ▶ Assume the number of columns of matrices **A** and **B** is the same.
    - ▶ Consider the case where **B** has more columns than **A**.
    - ▶ What happens if **B** has less columns than **A**?



# Matrix Creation, Element Replacement

- ▶ Create following 3D array:

$$\mathbf{M}(:, :, 1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{M}(:, :, 2) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{M}(:, :, 3) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}.$$

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 1 | 2 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 3 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 5 |

- ▶ Replace elements in the first two rows and columns of the first sheet of the array (*i.e.*, the matrix  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  with NaN elements).



# Linear Indexing I.

- ▶ Elements of an array of arbitrary number of dimensions and arbitrary size can be referred using simple index.
  - ▶ Indexing takes place along the main dimension (column-wise) then along the secondary dimension (row-wise) etc.

A = magic(3)

A =

|   |   |   |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

A(1:end)  
A(:)

|   |   |   |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

|   |
|---|
| 8 |
| 3 |
| 4 |
| 1 |
| 5 |
| 9 |
| 6 |
| 7 |
| 2 |

A([1 5])

|   |   |   |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

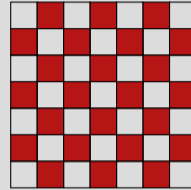
A([1 5], :)

Index in position 1  
exceeds array bounds  
(must not exceed 3).

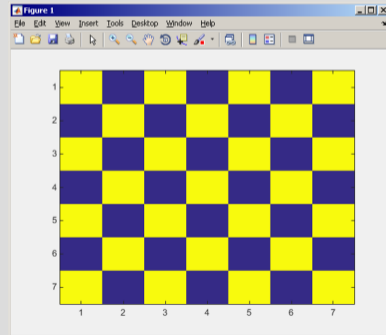


# Linear Indexing II.

- ▶ Consider following matrix:  $M = \text{ones}(7)$ .
  - ▶ We set all the red-highlighted elements to zero:



```
M(2:2:end) = 0;
imagesc(M);
```





# Matrix Indexing Using Own Values

- ▶ Create matrix **A**

```
N = 4;
A = magic(N);
```

- ▶ First think about what will be the result of the following operation and only then carry it out

```
B = A(A);
```

- ▶ Does the result correspond to what you expected?
  - ▶ Can you explain why the result looks the way it looks?
  - ▶ Notice the interesting mathematical properties of the matrices **A** and **B**.
  - ▶ Are you able to estimate the evolution?  $C = B(B)$
- ▶ Try similar process for  $N = 3$  or  $N = 5$ .



# Linear Indexing III. - ind2sub, sub2ind

- ▶ `ind2sub` recalculates linear index to subscript corresponding to size and dimensions of the matrix
  - ▶ Applicable to an array of arbitrary size and dimension.

```
ind = 3:6;
[rw, col] = ind2sub([3, 3], ind)
% rw = [3 1 2 3]
% col = [1 2 2 2]
```

- ▶ `sub2ind` recalculates subscripts to linear index.
  - ▶ Applicable to an array of arbitrary size and dimension.

```
ind2 = sub2ind([3, 3], rw, col)
% ind2 = [3 4 5 6]
```

|   |   |   |
|---|---|---|
| 1 | 4 | 7 |
| 2 | 5 | 8 |
| 3 | 6 | 9 |

→

|     |     |     |
|-----|-----|-----|
| 1,1 | 1,2 | 1,3 |
| 2,1 | 2,2 | 2,3 |
| 3,1 | 3,2 | 3,3 |

|     |     |     |
|-----|-----|-----|
| 1,1 | 1,2 | 1,3 |
| 2,1 | 2,2 | 2,3 |
| 3,1 | 3,2 | 3,3 |

→

|   |   |   |
|---|---|---|
| 1 | 4 | 7 |
| 2 | 5 | 8 |
| 3 | 6 | 9 |



## Linear Indexing IV.

- ▶ For a two-dimensional array, find a formula to calculate linear index from position given by `row` (`row`) and `col` (`column`).
  - ▶ Check with a matrix **A** of size  $4 \times 4$ , where
    - ▶ `row = [2, 4, 1, 2]`,
    - ▶ `col = [1, 2, 2, 8]`,
  - ▶ and therefore
    - ▶ `ind = [2, 8, 5, 14]`.

```
A = zeros(4);
A(:) = (1:16)
```





# Linear Indexing V.

- ▶ Consider following matrix:

```
A = magic(4);
```

- ▶ Use linear indexing so that only the element with highest value in each row of **A** was left (all other values set to 0); call the new matrix **B**.



# Relational Operators I.

- ▶ To find out, to compare, **whether “something” is greater than, less than, equal to, etc.**
- ▶ The result of the comparison is always either
  - ▶ positive (`true`), logical one “1”,
  - ▶ negative (`false`), logical zero “0”.
- ▶ All relation operators are vector-wise.
  - ▶ It is also possible to compare vector vs. vector, matrix vs. matrix, ...
- ▶ Often in combination with logical operators (*see later*)
  - ▶ Multiple relational operators can be applied to complex expressions.

|                    |                          |
|--------------------|--------------------------|
| <code>&gt;</code>  | greater than             |
| <code>&gt;=</code> | greater than or equal to |
| <code>&lt;</code>  | less than                |
| <code>&lt;=</code> | less than or equal to    |
| <code>==</code>    | equal to                 |
| <code>~=</code>    | not equal to             |



## Relational Operators II.

- ▶ Having the vector  $\mathbf{G} = \left( \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi \right)$ , find elements of  $\mathbf{G}$  that are
  - ▶ greater than  $\pi$ ,
  - ▶ less than or equal to  $\pi$ ,
  - ▶ not equal to  $\pi$ .
  - ▶ Try similar operations for  $\mathbf{H} = \mathbf{G}^T$ .
- ▶ Try to use relational operators in case of matrices and scalars as well.
  - ▶ Find out whether  $\mathbf{V} \geq \mathbf{U}$ :
    - ▶  $\mathbf{V} = \begin{pmatrix} -\pi & \pi & 1 & 0 \end{pmatrix}$ ,
    - ▶  $\mathbf{U} = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$ .



## Relational Operators III.

- ▶ Find out the results of following relations.
  - ▶ Try to interpret the results.

```
2 < 1 ~= 1 % ???
```

```
r = 1/2;
0 < r < 1 % ???
```

```
(1 > A) <= true
```



# Logical Operators I.

- ▶ To find out, **whether particular condition is fulfilled.**
- ▶ The result is always either
  - ▶ positive (`true`), logical one “1”,
  - ▶ negative (`false`), logical zero “0”.
- ▶ `all`, `any` is used to convert logical array into a scalar.
- ▶ MATLAB interprets any numerical value except 0 as `true`.
- ▶ All logical operators are vector-wise.
  - ▶ It is also possible to compare vector vs. vector, matrix vs. matrix, ...
- ▶ Function `is*` extends possibilities of logical expressions.
  - ▶ We will see later

|                    |                  |
|--------------------|------------------|
| <code>&amp;</code> | <code>and</code> |
| <code> </code>     | <code>or</code>  |
| <code>~</code>     | <code>not</code> |
|                    | <code>xor</code> |
|                    | <code>all</code> |
|                    | <code>any</code> |



## Logical Operators II.

- ▶ Assume a vector of 10 random numbers ranging from -10 to 10.

```
a = 20*rand(10, 1) - 10
```

- ▶ Following command returns `true` for elements fulfilling the condition.

```
a < -5 % relation operator
```

- ▶ Following command returns values of those elements fulfilling the condition (logical indexing).

```
a(a < -5)
```

- ▶ Following command puts value of -5 to the position of elements fulfilling the condition.

```
a(a < -5) = -5
```

- ▶ Following command sets value of the elements in the range from -5 to 5 equal to zero (opposite to thresholding).

```
a(a > -5 & a < 5) = 0
```

- ▶ Thresholding function (values below -5 set equal to -5, values above 5 set equal to 5).

```
a(a < -5 | a > 5) = ...
sign(a(a < -5 | a > 5))*5
```



## Logical Operators III.

- ▶ Determine which of the elements of the vector  $\mathbf{A} = \left[ \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi \right]$  fulfill following condition.
  - ▶ Which elements are equal to  $\pi$  or are equal to  $2\pi$ .
    - ▶ Pay attention to the type of the results (=logical values true/false).
  - ▶ Which elements are greater than  $\frac{\pi}{2}$  and at the same time are not equal to  $2\pi$ .
- ▶ Group elements from the previous condition with vector  $\mathbf{A}$ .



# Logical Operators IV.

- ▶ Create a row vector in the interval from 1 to 20 with step of 3.
  - ▶ Create the vector filled with elements from the previous vector that are:
    - ▶ greater than 10
    - ▶ and at the same time
    - ▶ less than 16.
  - ▶ Use logical operators.





## Logical Operators V.

- ▶ Create matrix **M** (`M = magic(3)`) and answer following questions using functions `all` and `any`.

- ▶ In which of the columns all elements are greater than 2?

$$\text{any} \left( \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \right) = [ 1 \quad 1 \quad 1 ],$$

- ▶ In which of the rows there is at least one element greater than or equal to 8?

$$\text{all} \left( \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \right) = [ 0 \quad 1 \quad 0 ],$$

- ▶ Does the matrix **M** contain only positive numbers?

$$\text{any}(\text{all} \left( \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \right)) = \text{any}([ 0 \quad 1 \quad 0 ]) = 1$$



## Logical Operators VI.

- ▶ In the case we need to compare scalar values only then “short-circuited” evaluation can be used.
- ▶ Evaluation keeps on going until the point where it makes no sense to continue
  - ▶ *e.g.*, when evaluating

```
clear;
a = true;
b = false;
a && b && c && d
```

- ▶ However:

```
clear;
a = true;
b = true;
a && b && c && d
```

- ▶ There are no problems with undefined variables *c* and *d*, because the execution is terminated before evaluating those variables.
- ▶ This is terminated with error ...  
*Unrecognized function or variable 'c'.*



## Logical Operators VII.

- ▶ Find out the result of the following operation and interpret it.

```
~(~[1 2 0 -2 0])
```

- ▶ Test whether variable  $b$  is not equal to zero and then test whether at the same time  $a/b > 3$ .
  - ▶ Following operation tests whether both conditions are fulfilled while avoiding division by zero.

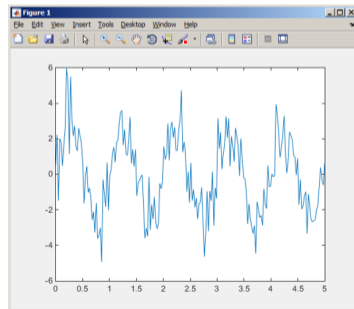
# Exercises



## Exercise I.

- ▶ Consider signal:  $s(t) = \sqrt{2\pi} \sin(2\omega_0 t) + n(\mu, \sigma)$ ,  $\omega_0 = \pi$ , where the mean and standard deviation of normal distribution  $n$  are:  $\mu = 0$  (mu = 0),  $\sigma = 1$  (sigma = 1).
  - ▶ Create time dependence of the signal spanning over  $N = 5$  periods of the signal using  $V = 40$  samples per period.
  - ▶ One period is  $T = 1$  such that  $t \in [kT, (k + N)T]$ ,  $k \in \mathbb{Z}^0$  (choose  $k$  equal for instance to 0).
  - ▶ The function  $n(\mu, \sigma)$  has the following MATLAB syntax:

```
n = mu + sigma*randn(1, N*V); % noise
```



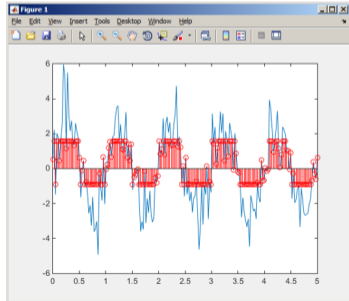


## Exercise II.

- ▶ Apply threshold function to generated signal from the previous exercise to limit its maximum and minimum value:
  - ▶ The result is vector `sp_t`.
  - ▶ Use function `min` and `max` with two input parameters (see MATLAB help for details).
  - ▶ Use the following code to check your output:

```
close all;
plot(t, s_t); hold on;
stem(t, sp_t, 'r');
```

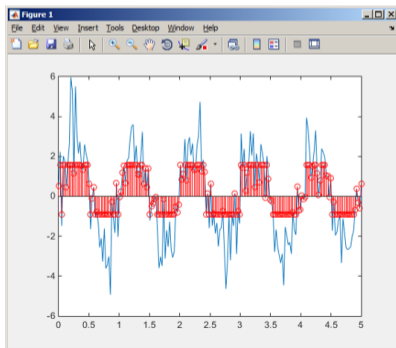
$$s_p(t) = \begin{cases} s_{\min} \Leftrightarrow s(t) < s_{\min} \\ s_{\max} \Leftrightarrow s(t) > s_{\max} \\ s(t) \dots \text{otherwise} \end{cases} \quad \begin{aligned} s_{\min} &= -\frac{9}{\pi} \\ s_{\max} &= \frac{2}{\pi} \end{aligned}$$





## Exercise III.

- ▶ Recall the signal from Exercise I.
  - ▶ Try again to limit the signal by values  $s_{\min}$  and  $s_{\max}$ .
  - ▶ Use relational operators ( $>/<$ ) and logical indexing ( $s(a>b) = c$ ) instead of functions `min` and `max`.
    - ▶ Solve the task item-by-item.



```
N = 5; V = 40;
t = linspace(0, N, N*V);
s_t = randn(1, N*V) +
sqrt(2*pi)*sin(2*pi*t);
```



## Exercise IV.a

- ▶ Create a script to calculate compound interest<sup>1</sup>.
  - ▶ The problem can be described as

$$P = \frac{rA \left(1 + \frac{r}{n}\right)^{nk}}{n \left( \left(1 + \frac{r}{n}\right)^{nk} - 1 \right)},$$

where  $P$  is regular repayment of debt  $A$ , paid  $n$ -times per year in the course of  $k$  years with interest rate  $r$  (decimal number).

- ▶ Create a new script and save it.
- ▶ At the beginning delete variables and clear Command window.
- ▶ Implement the formula first, then proceed with inputs (`input`) and outputs (`disp`).
- ▶ Try to vectorize the code, *e.g.*, for various values of  $n$ ,  $r$  or  $k$ .
- ▶ Check your results (for  $A = 1000$ ,  $n = 12$ ,  $k = 15$ ,  $r = 0.1$  is  $P = 10.7461$ ).

---

<sup>1</sup>Interest from the prior period is added to principal.





## Exercise IV.b



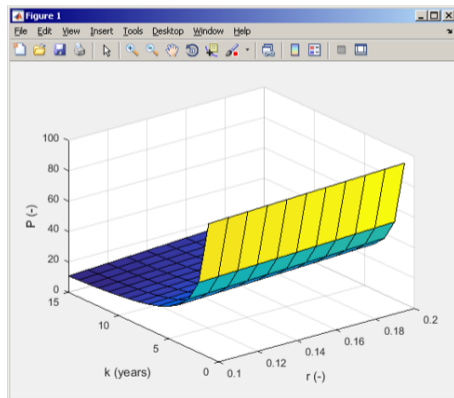
- ▶ Try to vectorize the code, both for  $r$  and  $k$ .
- ▶ Use scripts for future work with MATLAB.
  - ▶ Bear in mind, however, that parts of the code can be debugged using command line.

$$P = \frac{rA \left(1 + \frac{r}{n}\right)^{nk}}{n \left( \left(1 + \frac{r}{n}\right)^{nk} - 1 \right)}$$



## Exercise IV.c

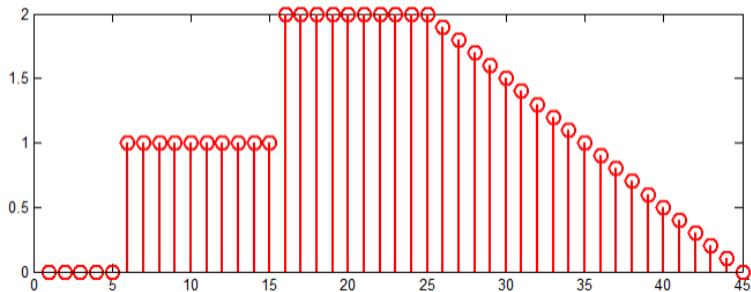
- ▶ Vectorized code for both  $r$  and  $k$ .
  - ▶ The compatible size array feature used.
  - ▶ `surf` created 3D surface plot.





## Exercise V.a

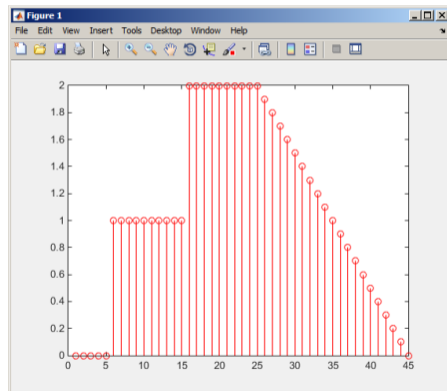
- ▶ Generate vector containing following sequence.
  - ▶ Note the  $x$ -axis (interval, number of samples).
  - ▶ Split the problem into several parts to be solved separately.
  - ▶ Several ways how to solve the problem.
  - ▶ Use `stem(x)` instead of `plot(x)` for plotting.
- ▶ Try to generate the same signal beginning with zero ...





## Exercise V.b

- ▶ Generate vector containing following sequence.





## Exercise VI.

- ▶ Consider following matrix  $\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \end{bmatrix}$ .
- ▶ Write an expression testing whether all elements of  $\mathbf{A}$  are positive and at the same time all elements of the first row are integers.



## Exercise VII.a

- ▶ Reflection coefficient  $S_{11}$  of a one-port device of impedance  $Z$  is given by:

$$S_{11} = 10 \log_{10} \left( \left| \frac{Z - Z_0}{Z + Z_0} \right|^2 \right),$$

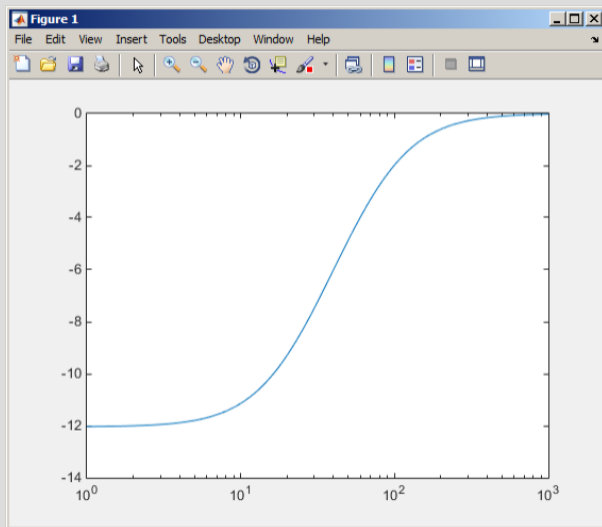
where  $Z_0 = 50\Omega$  and  $Z = R + jX$ .

- ▶ Calculate and depict the dependence of  $S_{11}$  for  $R = 30\Omega$  and  $X$  on the interval  $[1, 1000]$  with 100 evenly spaced points in logarithmic scale.
- ▶ Use the code below and correct errors in the code. Correct solution generates plot depicted on the next slide.

```
500 = Z0; % reference impedance
R == 30; % real part of the impedance
X = Logspace(0, 3, 1e2); % reactance vector
clear;
Z = i*(R + 1i*X); % impedance
S11 = 10*log(abs(Z-Z0)./(Z+Z0))^2); % reflection coeff. in dB
semilogx(S11, X) % plotting using log. x-axis
```



## Exercise VII.b



# Questions?

B0B17MTB, BE0B17MTB – MATLAB  
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