Elements of Geometry for Computer Vision and Computer Graphics

Translation of Euclid's Elements by Adelardus Bathensis (1080–1152)

Lecture 7: The Projective Plane

CZECH INSTITUTE OF INFORMATICS ROBOTICS AND CYBERNETICS CTU IN PRAGUE

Torsten Sattler

Comparing Geometrical and Algebraic Models

Point position	Projection	
	Geometrical model in aff. space	Algebraic model in aff. space
$X \notin \sigma$	one point of π	$\eta \neq 0, \vec{x}_\beta = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}, (\vec{x}_\beta \neq \vec{0})$
$C \neq X \in \sigma$	no point	$\eta \neq 0, \vec{x}_\beta = \begin{bmatrix} u \\ v \\ 0 \end{bmatrix}, \vec{x}_\beta \neq \vec{0}$
$X = C$	all points of π	$\eta \neq 0, \vec{x}_\beta = \vec{0}$

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1. We can always assume $\eta \neq 0$.
2. The "projection" of C is represented by the zero vector while the projections of all other points are represented by non-zero vectors.
3. The algebraic projection model can represent projections of all points in the affine space.
4. The geometrical projection model is less capable than the algebraic projection model since it can't model the projection of points in σ different from C.

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Quiz

cw.fel.cvut.cz/b212/courses/gvg/start → BRUTE → GVG Geometry of Computer Vision and Graphics

Q: Which points project to point(s) in the image plane using the geometric model of perspective projections in affine space?

1. A point X in the plane σ parallel to the image plane π.
2. The camera center C.
3. A point X in the plane π.

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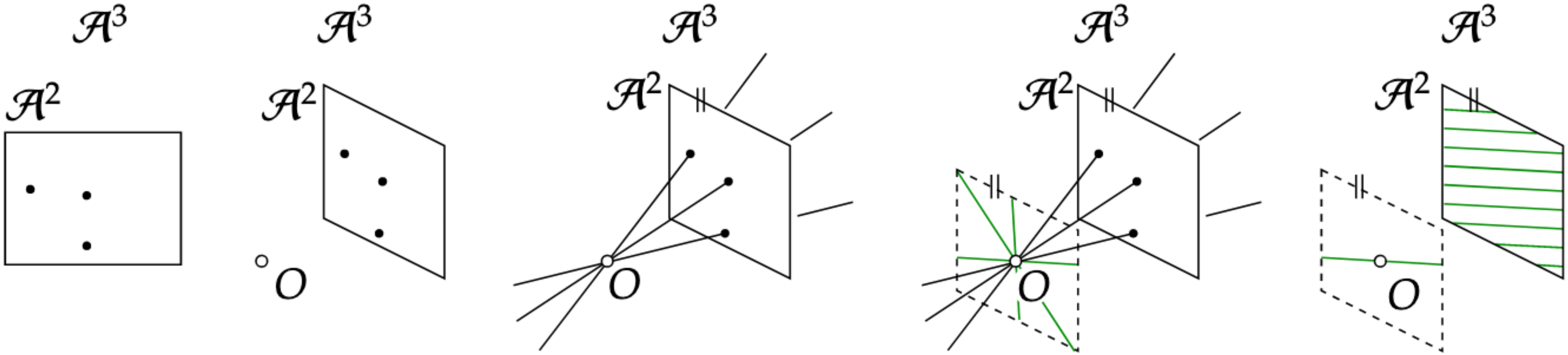
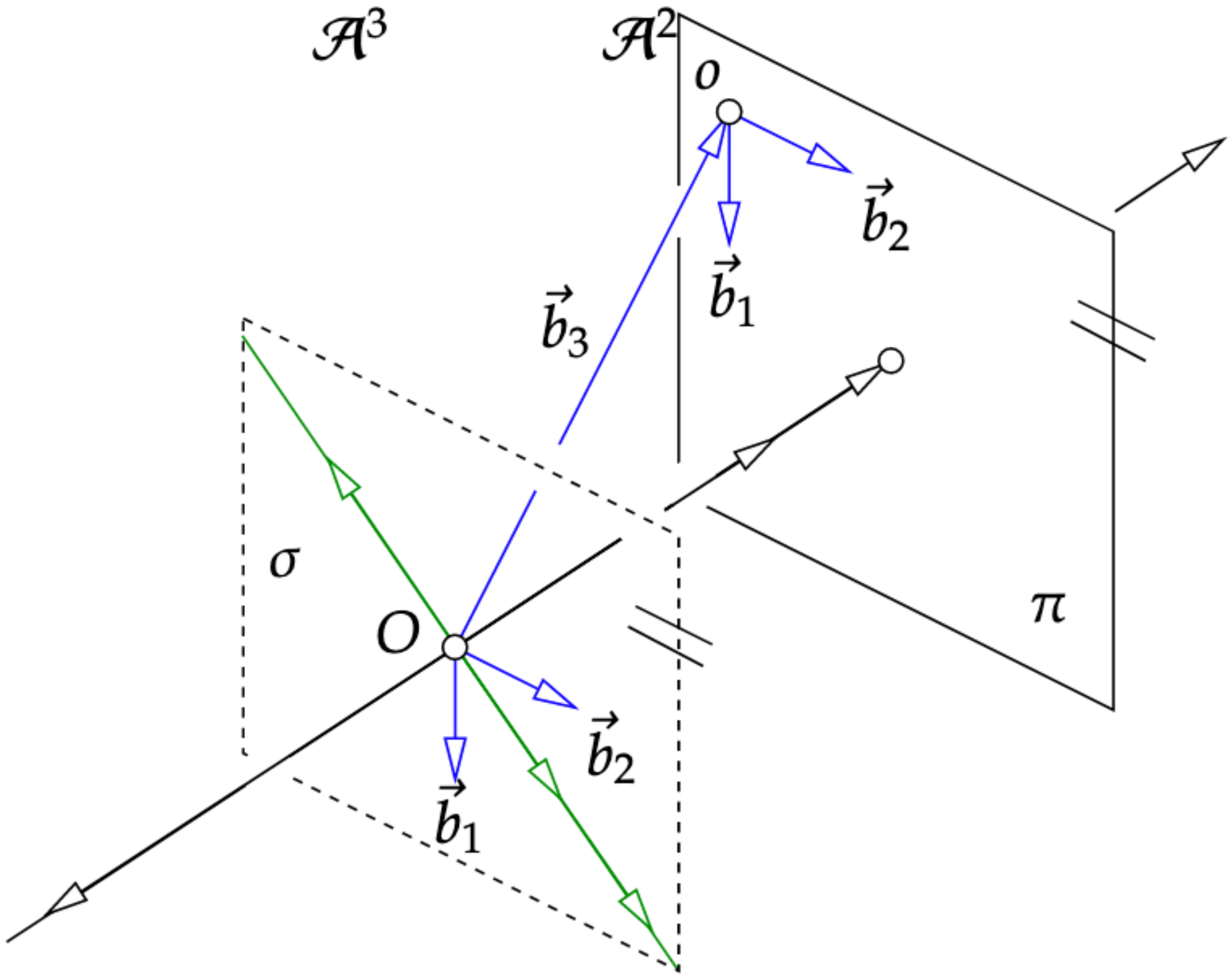
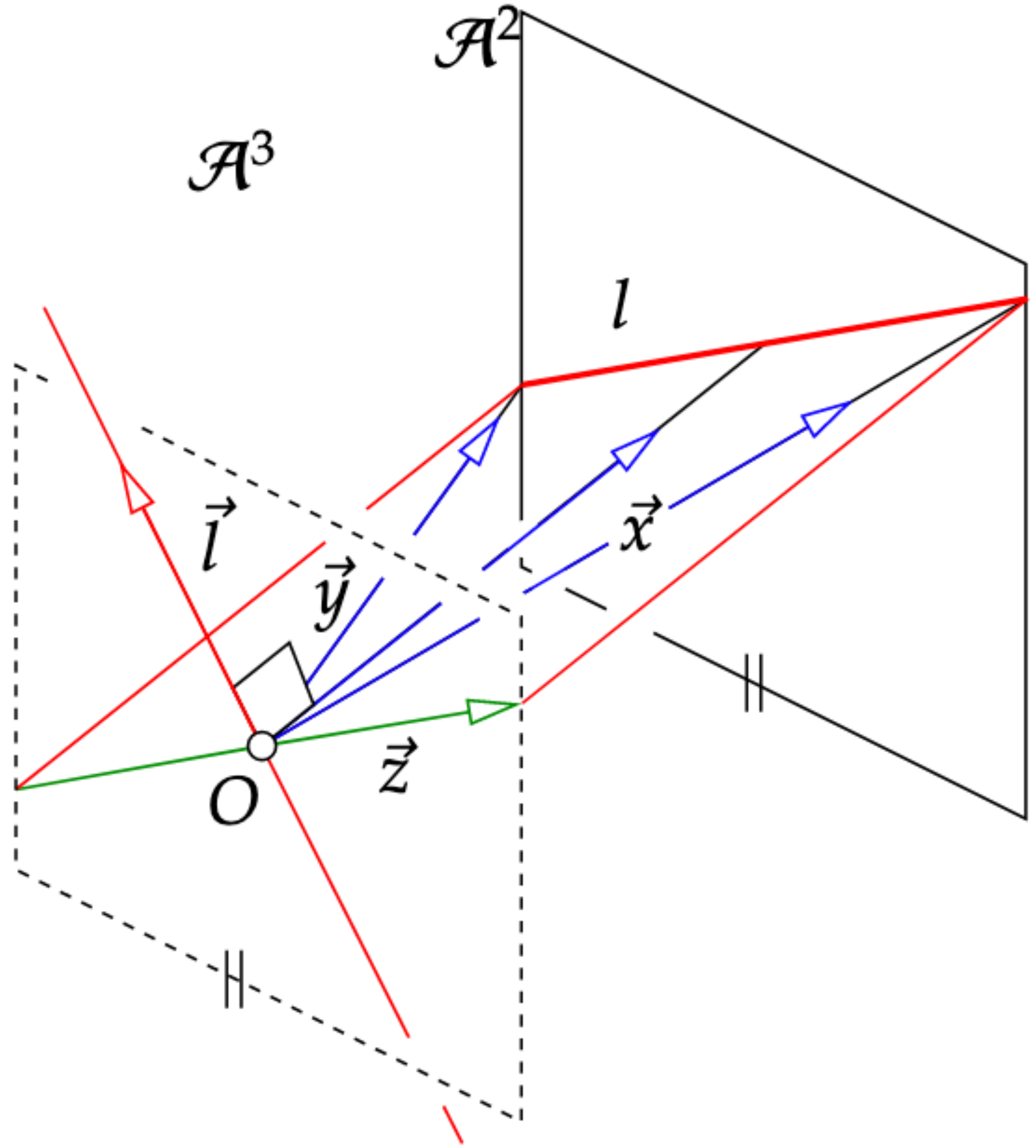
The Real Projective Plane - Geometrical Model



The Real Projective Plane - Algebraic Model



Lines of the Real Projective Plane



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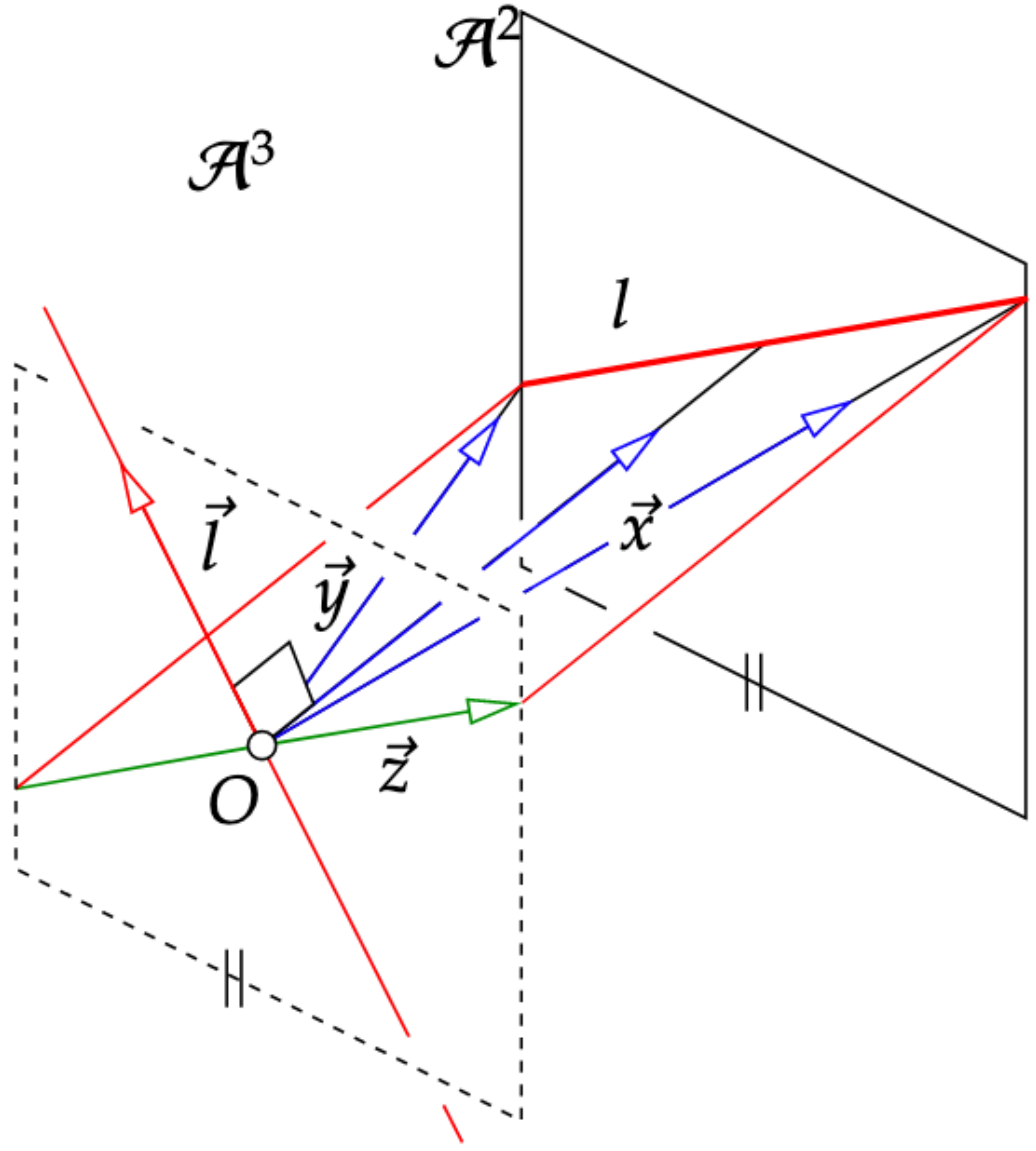

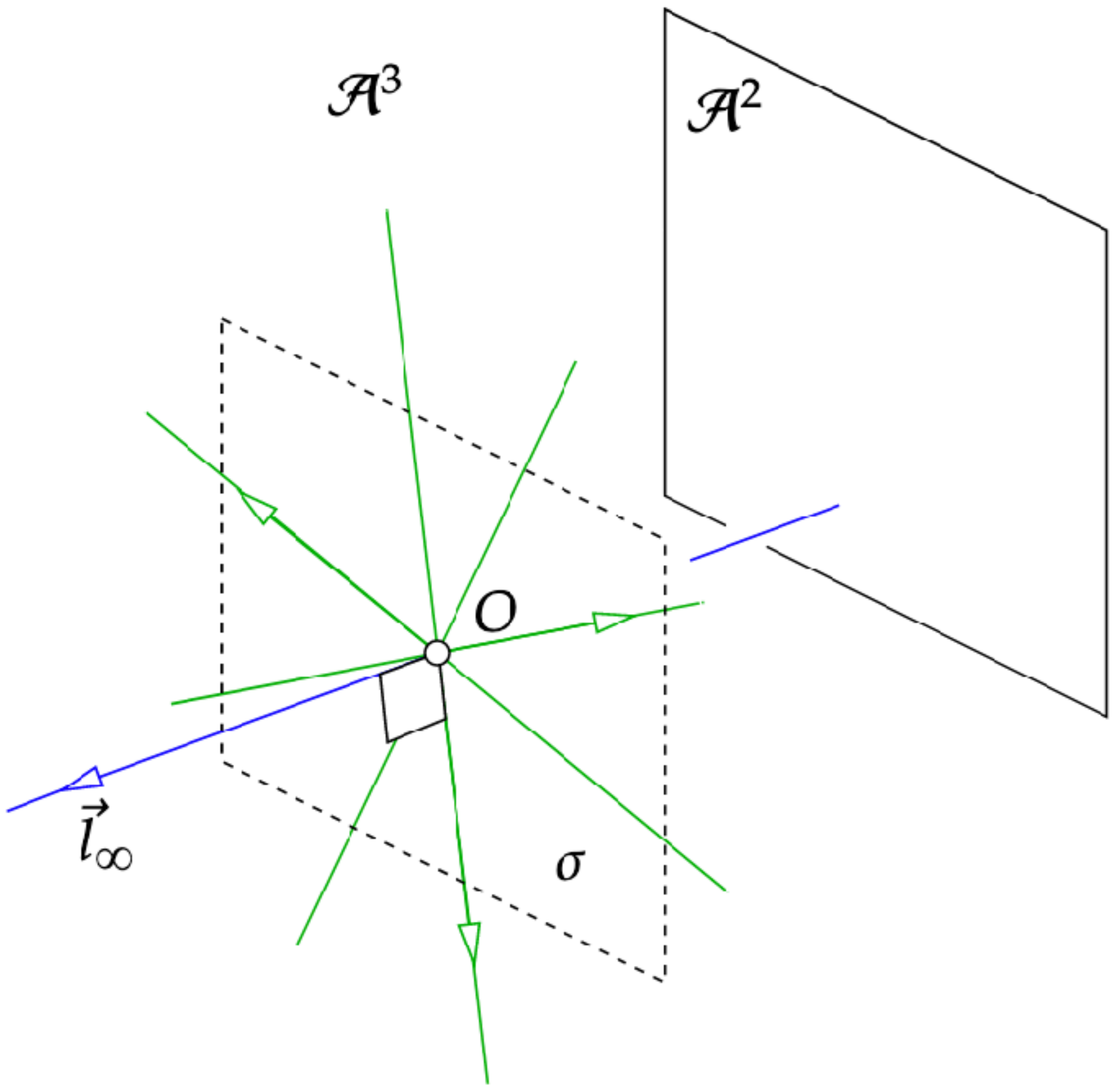

ideal line

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Quiz

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Q: Mark ideal points (points at infinity).

1. $[0\ 0\ 0]^T$
2. $[1\ 0\ 0]^T$
3. $[0\ 1\ 0]^T$
4. $[0\ 0\ 1]^T$

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Q: Mark homogenous coordinates representing the point [0 1]^T in the affine plane.

1. $[1\ 0\ 1]^T$
2. $[0\ 101\ 1]^T$
3. $[0\ 101\ 101]^T$
4. $[0\ 1\ 0.5]^T$

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