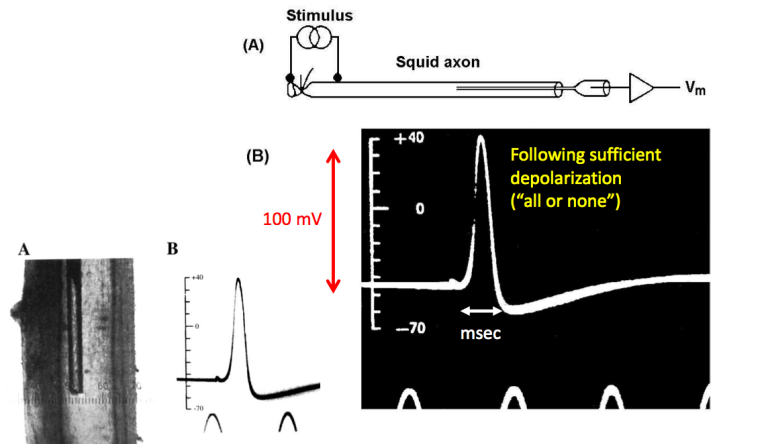


# Neuroinformatics, Prague

March 27, 2024

Hodkin and Huxley models

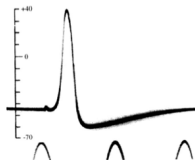
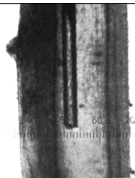
# First direct (intracellular) recorded action-potential (spike) - 1939!!



# Very nice theory



Sir Alan Lloyd  
Hodgkin



Sir Andrew Fielding  
Huxley

$$I = C_m \dot{V} + g_{Na} h m^3 (V - V_{Na}) + g_K n^4 (V - V_K) + G_L (V - V_L) \quad (1)$$

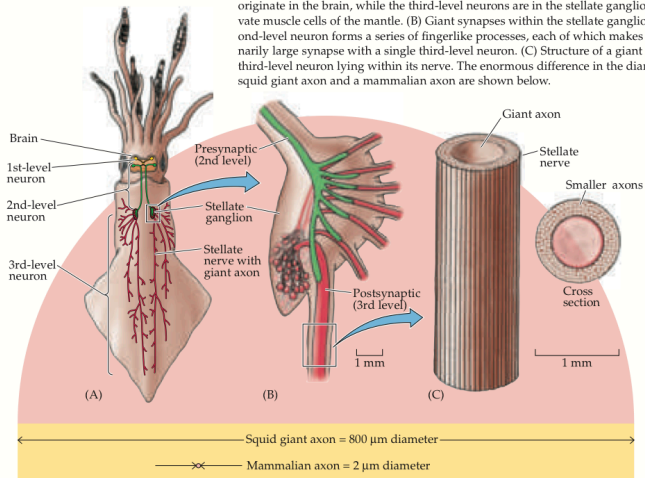
$$\frac{d}{dt} m = \alpha_m (V) (1 - m) - \beta_m (V) m \quad (2)$$

$$\frac{d}{dt} n = \alpha_n (V) (1 - n) - \beta_n (V) n \quad (3)$$

$$\frac{d}{dt} h = \alpha_h (V) (1 - h) - \beta_h (V) h \quad (4)$$

# Giant Nerve Cells of Squid

(A) Diagram of a squid, showing the location of its giant nerve cells. Different colors indicate the neuronal components of the escape circuitry. The first- and second-level neurons originate in the brain, while the third-level neurons are in the stellate ganglion and innervate muscle cells of the mantle. (B) Giant synapses within the stellate ganglion. The second-level neuron forms a series of fingerlike processes, each of which makes an extraordinarily large synapse with a single third-level neuron. (C) Structure of a giant axon of a third-level neuron lying within its nerve. The enormous difference in the diameters of a squid giant axon and a mammalian axon are shown below.



# Hodkin and Huxley experiment NOBEL 1963



**Sir Alan Lloyd  
Hodgkin**

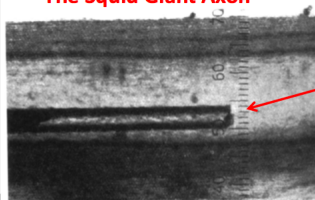


**The Squid Giant Axon**



**Sir Andrew Fielding  
Huxley**

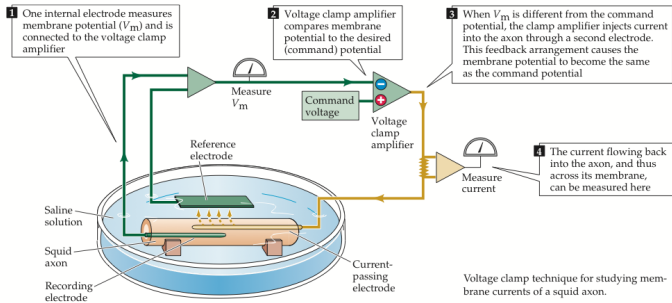
~ 0.5 mm

A red double-headed vertical arrow indicating the length of the axon shown in the adjacent image.

Axial electrode

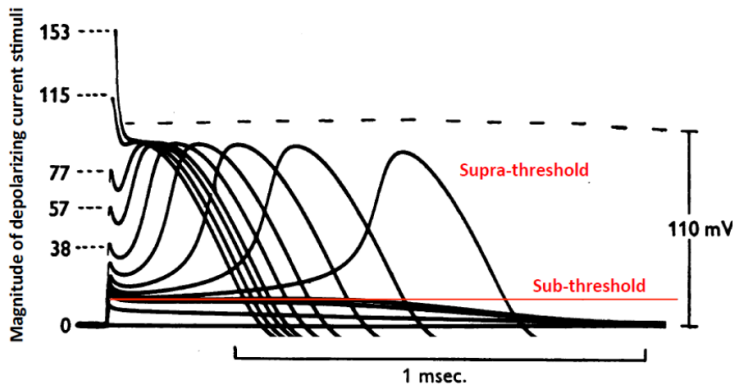
A red arrow pointing from the text 'Axial electrode' to the dark cylindrical structure in the micrograph.

# Voltage Clamp Method



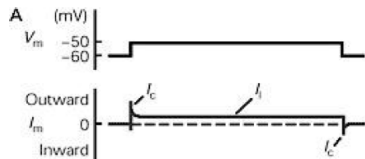
## All or None

### The “all or none” nature of the spike

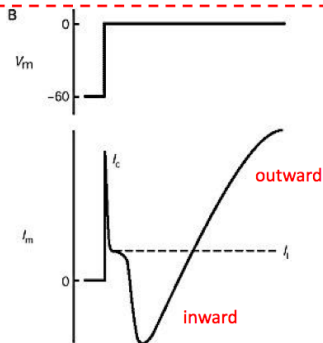


*Hodgkin, Huxley and Katz, 1952*

## Membrane current in response to voltage clamp (VC)



For **subthreshold** depolarizing voltage clamp, the recorded membrane current is the current that flows via the leak (passive) conductance + a small capacitive current (at start and end of the VC)

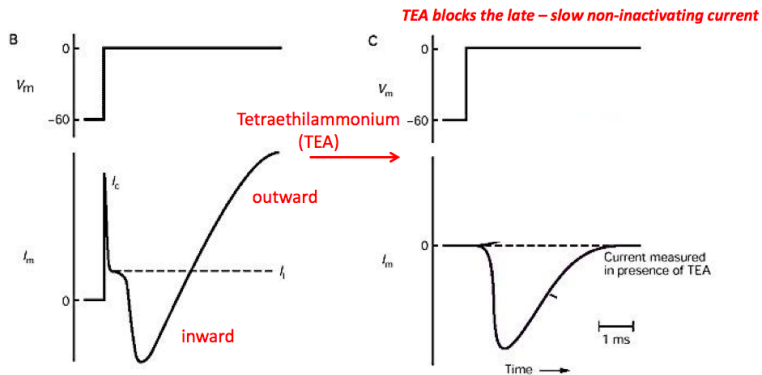


For **suprathreshold** depolarizing voltage clamp, the recorded membrane current (after the fast capacitive current) flows **first inwards** (into the axon) and later **outward** (from inside to the outside)

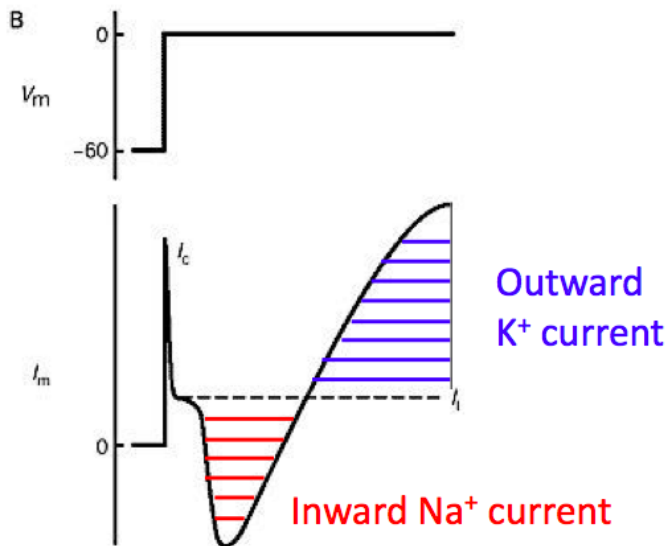


# Separating voltage-dependent active (excitable) currents Using pharmacological agents

2 different currents flow via the membrane during the spike

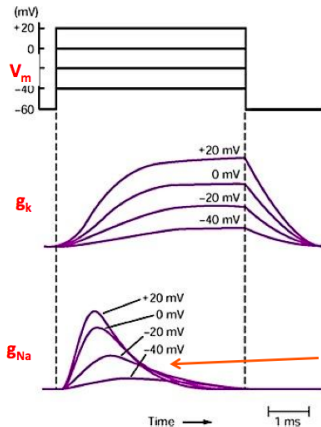


Changing ion concentration at bath with giant axon showed that early current is carried by  $\text{Na}^+$  ions and late one by  $\text{K}^+$  ions



# Ion currents ( $K^+$ and $Na^+$ ) for various depolarizing voltage clamp (and extracting respective ion conductances)

$$I_K = g_K (V_m - E_K); \quad I_{Na} = g_{Na} (V_m - E_{Na})$$

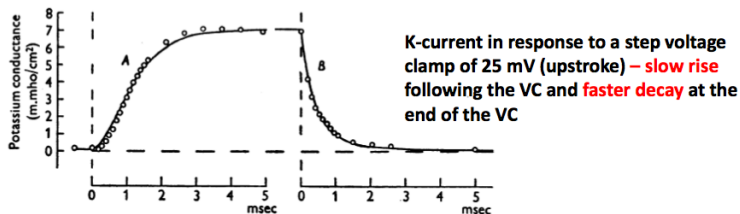


1. The slow (K) current (conductance) does not inactivate during VC

2. The K conductance rises slower than it decays at end of VC

3. The fast (early) Na conductance inactivates during VC

## Fitting an equation for the K current (K-conductance) during/following VC



Mathematically – the rising phase of K-current can be described as a power of 4 (namely as  $(1 - \exp(-t))$ <sup>4</sup> and the decay as  $\exp(-4t)$ )

$$g_K = \bar{g}_K n^4$$

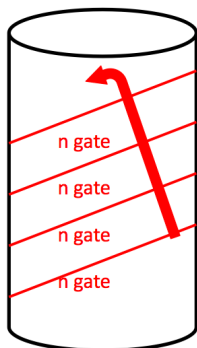
**n represents the proportion of K-ion channels in the open state**

*“These equations may be given a physical basis if we assume that potassium ions can only cross the membrane when four similar particles occupy a certain region of the membrane...”* Hodgkin AL, Huxley AF. 1952 J Physiol (Lond) 117:500–544

13 אפריל

# Graphical interpretation of H&H model for the K channel

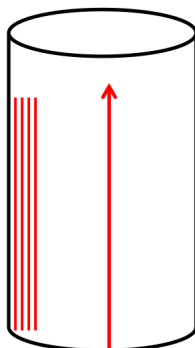
Closed K channel (by 4 n gates)



$K^+$   
INSIDE

4 n gates open with  
depolarization

Open K channel (by 4 n gates)



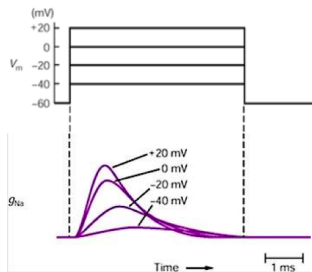
$K^+$   
INSIDE

The activation function,  $n$ , and the rate functions  $\alpha_n$  and  $\beta_n$

$$g_{\mathbf{K}} = \bar{g}_{\mathbf{K}} n^4,$$
$$\frac{dn}{dt} = \alpha_n (1 - n) - \beta_n n,$$

where  $\bar{g}_{\mathbf{K}}$  is a constant with the dimensions of conductance/cm<sup>2</sup>,  $\alpha_n$  and  $\beta_n$  are rate constants which vary with voltage but not with time and have dimensions of [time]<sup>-1</sup>,  $n$  is a dimensionless variable which can vary between 0 and 1.

Similar procedure is used to extract the activation ( $m$ ) and inactivation ( $h$ ) parameters for the Na current

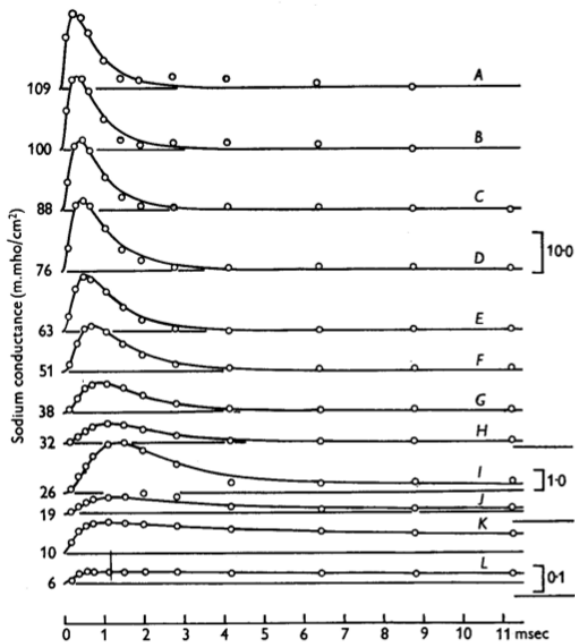


$$g_{Na} = m^3 h \bar{g}_{Na},$$

$$\frac{dm}{dt} = \alpha_m (1 - m) - \beta_m m,$$

$$\frac{dh}{dt} = \alpha_h (1 - h) - \beta_h h,$$

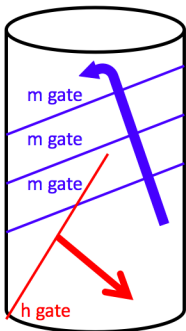
# Fitting Na current for different VC depolarizing values





# Graphical interpretation of H&H model for the Na channel

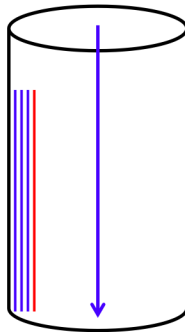
Na channel (by 3 activated m gates and 1 inactivated h gate)



3 (fast) m (activated) gates open with depolarization

1 (slow) h (inactivated) gate closes with depolarization

Open Na channel  
Na outside



# Overlay of the action potential (voltage) and underlying Na and K conductances

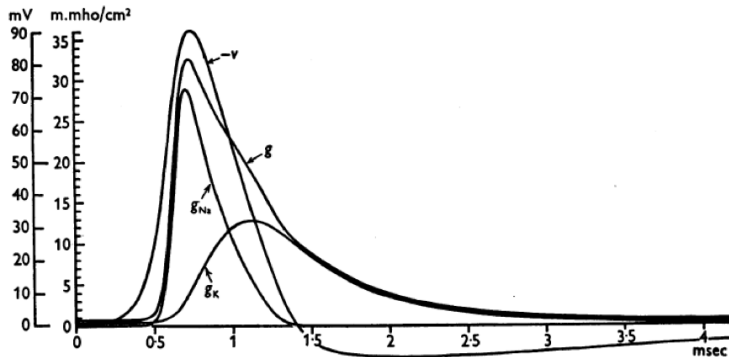


Fig. 17. Numerical solution of eqn. (31) showing components of membrane conductance ( $g$ ) during propagated action potential ( $-V$ ). Details of the analysis are as in Fig. 15.

# Hodgkin–Huxley model

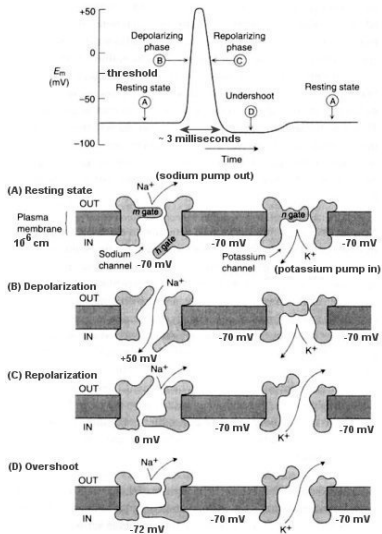
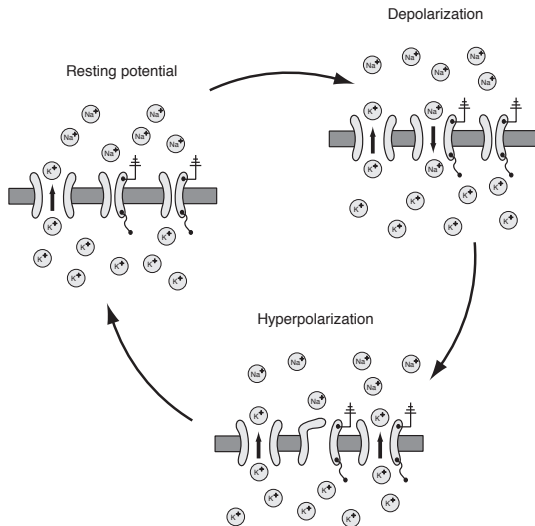


Figure: Typical form of an action potential; redrawn from an oscilloscope picture from Hodgkin and Huxley (1939).

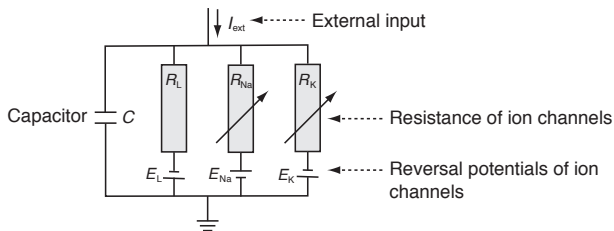
# The minimal mechanisms



## HH stucture

- ▶  $I_{ion} = g_{ion}(V - E_{ion})$
- ▶ voltage and time dependent variables  $n(V, t), m(V, t), h(V, t)$

$$\hat{g}_K(V, t) = g_K n^4$$
$$\hat{g}_{Na}(V, t) = g_{Na} m^3 h$$



# Hodgkin–Huxley equations and simulation

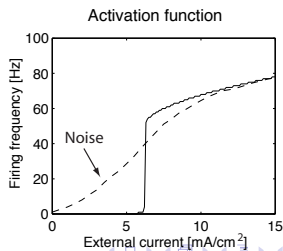
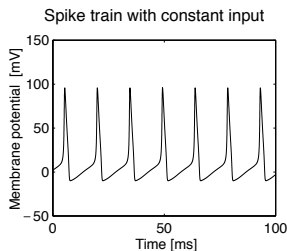
$$C \frac{dV}{dt} = -g_K n^4 (V - E_K) - g_{Na} m^3 h (V - E_{Na}) - g_L (V - E_L) + I_{ext}(t)$$

$$\tau_n(V) \frac{dn}{dt} = -[n - n_0(V)]$$

$$\tau_m(V) \frac{dm}{dt} = -[m - m_0(V)]$$

$$\tau_h(V) \frac{dh}{dt} = -[h - h_0(V)]$$

$$\frac{dx}{dt} = -\frac{1}{\tau_x(V)} [x - x_0(V)] \rightarrow x(t + \Delta t) = \left(1 - \frac{\Delta t}{\tau_x}\right) x(t) + \frac{\Delta t}{\tau_x} x_0$$



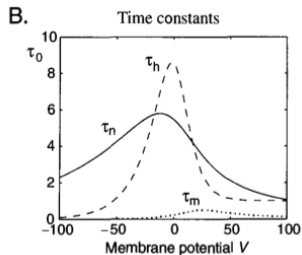
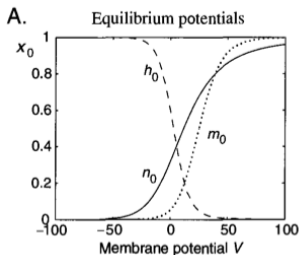
## Ion channels resistance

$$x(0) = \frac{\alpha}{\alpha + \beta}, t_x = \alpha\beta, x \in \{n, m, h\}$$

$$\alpha_n = \frac{10 - V}{100(e^{\frac{10-V}{10}} - 1)}, \beta_n = 0.125e^{-\frac{V}{80}}$$

$$\alpha_m = \frac{25 - V}{10(e^{\frac{25-V}{10}} - 1)}, \beta_m = 4e^{-\frac{V}{18}}$$

$$\alpha_h = 0.07e^{\frac{V}{20}}, \beta_h = \frac{1}{e^{\frac{30-V}{10}} + 1}$$



# Matlab implementation

```
%% Integration of Hodgkin--Huxley equations with Euler method
clear; figure;%clf;
%% Setting parameters
% Maximal conductances (in units of mS/cm^2); 1=K, 2=Na, 3=R
g(1)=36; g(2)=120; g(3)=0.3;
% Battery voltage ( in mV); 1=n, 2=m, 3=h
E(1)=-12; E(2)=115; E(3)=10.613;
% Initialization of some variables
I_ext=0; V=-10; x=zeros(1,3); x(3)=1; t_rec=0;
% Time step for integration
dt=0.01;
%% Integration with Euler method
for t=-30:dt:500
    if t==10; I_ext=6; end % turns external current on at t=10
    if t==400; I_ext=0; end % turns external current off at t=400
    % alpha functions used by Hodgkin-and Huxley
    Alpha(1)=(10-V)/(100*(exp((10-V)/10)-1));
    Alpha(2)=(25-V)/(10*(exp((25-V)/10)-1));
    Alpha(3)=0.07*exp(-V/20);
    % beta functions used by Hodgkin-and Huxley
    Beta(1)=0.125*exp(-V/80);
    Beta(2)=4*exp(-V/18);
    Beta(3)=1/(exp((30-V)/10)+1);
    % tau_x and x_0 (x=1,2,3) are defined with alpha and beta
    tau_x=1./(Alpha+Beta);
    x_0=Alpha.*tau_x;
    % leaky integration with Euler method
    x=(1-dt./tau_x).*x+dt./tau_x.*x_0; % x is m,n,h
    % calculate actual conductances g with given n, m, h
    gnmh(1)=g(1)*x(1)^4;
    gnmh(2)=g(2)*x(2)^3*x(3);
    gnmh(3)=g(3);
    % Ohm's law
    I=gnmh.*(V-E);
    % update voltage of membrane
    V=V+dt*(I_ext-sum(I));
    % record some variables for plotting after equilibration
    if t>=0;
        t_rec=t_rec+1;
        x_plot(t_rec)=t;
        y_plot(t_rec)=V;
    end
end
```



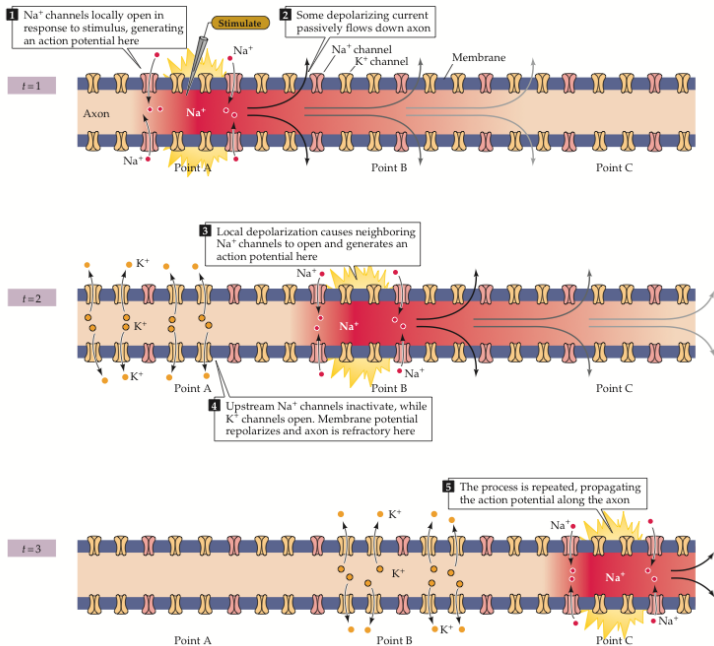
## Refractory period

- ▶ waiting for inactivation of sodium channels about 1 ms
- ▶ absolute refractory period limiting firing rate to 1000Hz
- ▶ hyperpolarizing activity further limits the neuron's rate
- ▶ relative refractory period
- ▶ brainstem neurons 600Hz, cortical neurons 3Hz

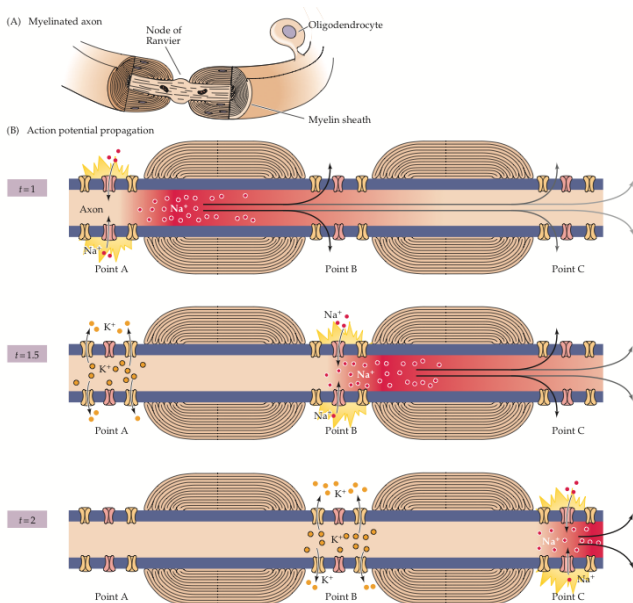
# Propagation of action potentials

- ▶ action potentials=spikes travel about 10 m/s.
- ▶ non-loss signal transfer - SLOW
- ▶ myelin = FAST lossy signal transfer in axon
- ▶ Ranvier nodes = AP regeneration
- ▶ myelination happens after second year of age
- ▶ Alzheimer deased - DESmyelination!

# NON-LOSS transfer

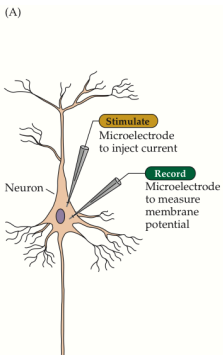


# LOSSY transfer

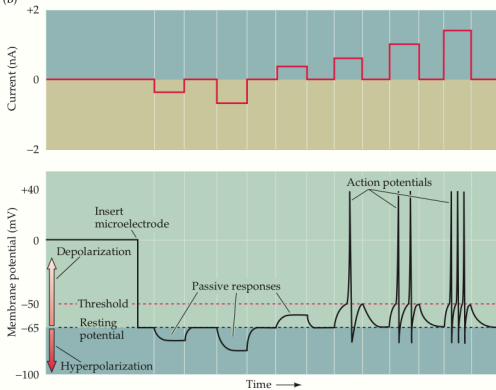


# Stimulation of neuron

(A)



(B)



## HH - simplification: Hugh Wilson model for neocortical neurons

- ▶  $h = 1 - n$
- ▶  $\tau_m \approx m_0(V)$
- ▶  $h = 1$  no inactivation of the fast  $Na^+$  channel combining leakage and Na channel, only for cortical neurons
- ▶  $R$  describes recovery of membrane potential
- ▶ 2 differential equations

$$C \frac{dV}{dt} = -g_K R (V - E_K) - g_{Na}(V)(V - E_{Na}) + I_{ext}(t)$$

$$\tau_R \frac{dR}{dt} = -[R - R_0(V)]$$

## Wilson model

- ▶ more realistic mammalian neocortical neurons
- ▶ two more channels types → more diverse firing
- ▶ cation  $C_a^{2+}$  described by gating variable  $T$
- ▶ slow hyperpolarizing current  $Ca^{2+}$ -mediated  $K^+$  described by gating variable  $H$

$$C \frac{dV}{dt} = -g_{Na}(V - E_{Na}) - g_K R(V - E_K) - g_T(V - E_T) - g_H H(V - E_H)$$

$$\tau_R \frac{dR}{dt} = -[R - R_0(V)]$$

$$\tau_T \frac{dT}{dt} = -[T - T_0(V)]$$

$$\tau_H \frac{dH}{dt} = -[H - 3T(V)]$$

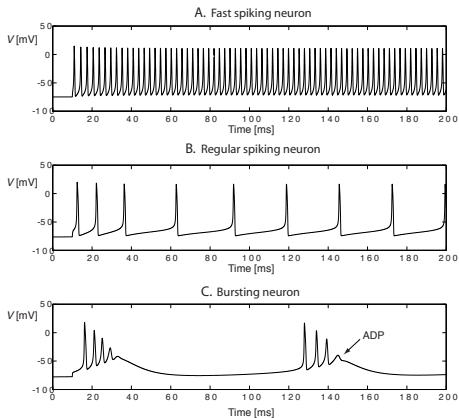
$$g_{Na}(V) = 17.8 + 0.476V + 33.8V^2$$

$$R_0(V) = 1.24 + 3.7V + 3.2V^2$$

$$T_0(V) = 4.205 + 11.6V + 8V^2$$

## Wilson model:results

- ▶ RS: regular spiking neuron
- ▶ FS: fast spiking neuron
- ▶ CS: continuously spiking neuron
- ▶ IB: bursting neuron





# Matlab implementation

```
%% Integration of Wilson model with the Euler method
clear; clf;
%% Parameters of the model: 1=K,R 2=Ca,T 3=KCa,H 4=Na
g(1)=26; g(2)=2.25; g(3)=9.5; g(4)=1;
E(1)=-.95; E(2)=1.20; E(3)=E(1); E(4)=-.50;

%% Initial values
dt=.01; I_ext=0; V=-1; x=zeros(1,4);
tau(1)=dt./4.2; tau(2)=dt./14; tau(3)=dt./45; tau(4)=1;

%% Integration
t_rec=0;

for t=-100:dt:200
    switch t;
        case 0; I_ext=1;
            end

    x0(1)=1.24 + 3.7*V + 3.2*V^2;
    x0(2)=4.205 + 11.6*V + 8 *V^2;
    x0(3)=3*x(2);
    x0(4)=17.8 + 47.6*V +33.8*V^2;

    x=x-tau.*(x-x0); %rem x(4)=x0(4) because tau(4)=1
    I=g.*x.*(V-E);
    V=V+dt*(I_ext-sum(I));

    if t>=0;
        t_rec=t_rec+1;
        x_plot(t_rec)=t;
        y_plot(t_rec)=V;
    end
end % time loop

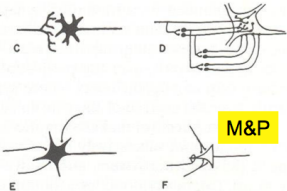
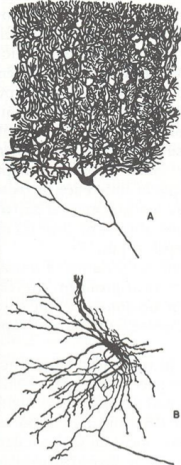
%% Plotting results
plot(x_plot,100*y_plot); xlabel('Time'); ylabel('Membrane potential');
```

# Physiology versus Neurons Models

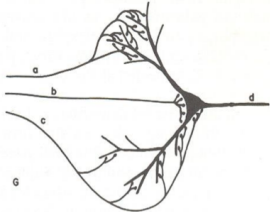
**Rall (1964)**

**Histological Vs. Schmectic Neurons**

Histological Neurons



Schematic Neurons



# Physiology versus Neurons Models

Understand experimental synaptic potentials recorded at the soma



1. **Most of the input current flows into the dendrites** (not directly to soma)
2. **Dendrites are non-isopotential electrical devices**
  - (i) **voltage attenuates from synapse to soma;**
  - (ii) **it takes time (delay) for the PSP to reach the soma;**
  - (iii) **somatic EPSP/IPSP shape is expected to change with synaptic location**

# Rall Cable Theory for Dendrites

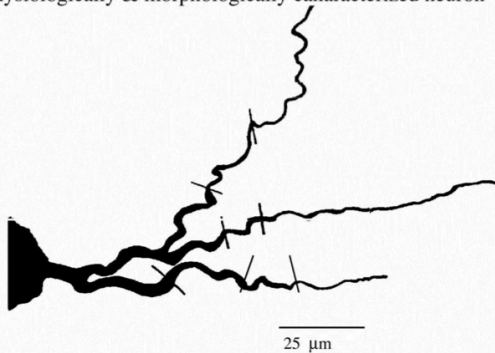
**Understanding (mathematically) the impact of  
(remote) dendritic synapses (the input)  
on the soma/axon (output) region**



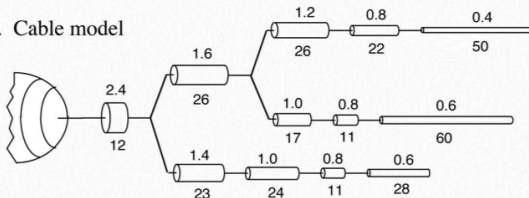
*Wilfrid Rall*

# Cylindric model

A. Physiologically & morphologically characterized neuron

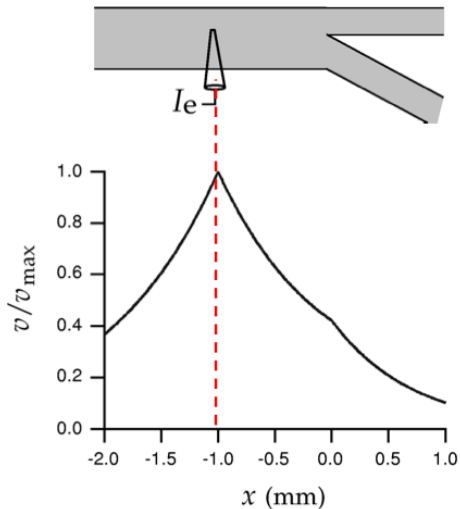


B. Cable model

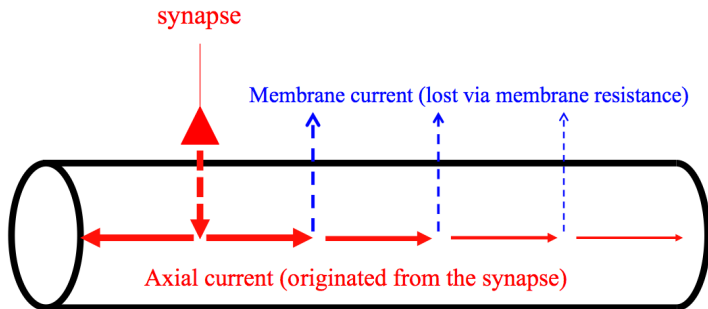


## Voltage attenuation

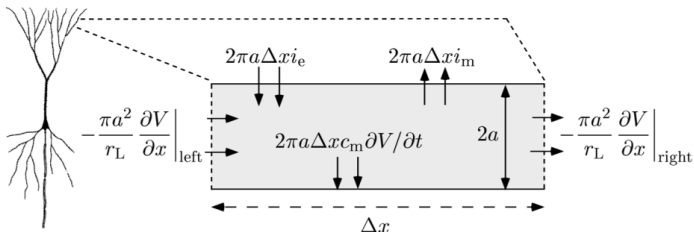
**Synaptic potentials attenuate from the synapse origin towards other regions of the dendrites**



## Axial and membrane current



# Passive cable equations



$$\frac{r_m}{r_i} \frac{\partial^2 V(x,t)}{\partial x^2} - r_m c_m \frac{\partial V(x,t)}{\partial t} - V(x,t) = 0$$

$$\frac{\partial^2 V}{\partial X^2} = \frac{\partial V}{\partial T} + V(X,T)$$

$$X = x/\lambda$$

$$T = t/\tau_m$$

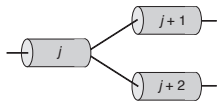


# Compartmental models

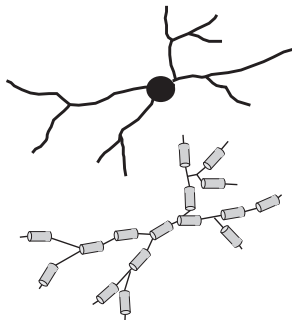
A. Chain of compartments



B. Branching compartments



C. Compartmental reconstruction



# Cable theory

- ▶ discretization - compartments like branching  $j, j + 1, j + 1$

$$\lambda^2 \frac{\partial V_m(x, t)}{\partial x^2} - \tau_m \frac{\partial V_m(x, t)}{\partial t} - V_m(x, t) + V_0 = R_m I_{inj}(x, t)$$

$$\lambda = \sqrt{\frac{dR_m}{2R_i}}$$

$$\tau_m = R_m C_m$$

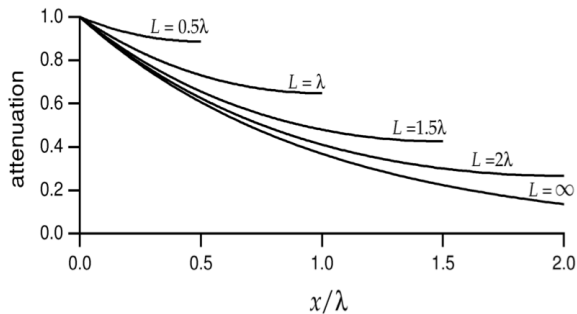
$$V_m = V_0 e^{-\frac{x}{\lambda}}$$

$$\frac{\partial V_m(x, t)}{\partial x^2} \leftarrow \frac{V_{j+1} - 2V_j(t) + V_{j-1}(t)}{(x_{j-1} - x_j)^2}$$

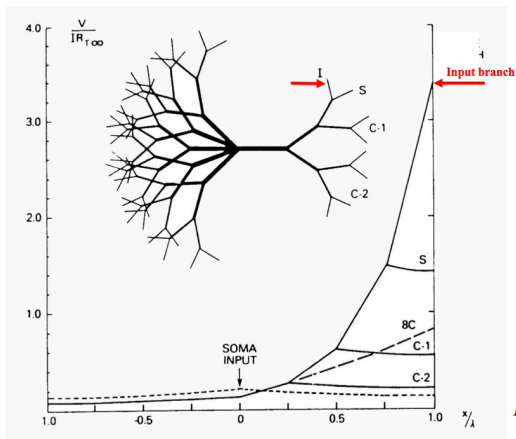
## Steady state condition

(“Sealed-end” boundary)  $dV/dX = 0$ ;  $x=L$

$$\frac{\partial^2 V}{\partial X^2} = \cancel{\frac{\partial V}{\partial T}} + V(X, T)$$



# Simulating voltage attenuation



*Rall and Rinzel, 1973*

# Simulators

The screenshot displays the NEURON software interface with several windows open:

- NEURON Main Menu:** File Edit Build Tools Graph Vector Window
- ModelView[1]:** File menu open. Model statistics: 79 sections; 150 segments. \* 1 real cells, 0 artificial cells, 0 NetCon objects, 0 LinearMechanism objects. \* Density Mechanisms, \* 1 point processes (0 can receive events) of 1 type.
- Graph[0]:** Plot of `soma.v(.5)` showing a membrane potential spike from -65 mV to approximately +10 mV over time.
- PointProcessManager:** SelectPointProcess window showing `IClamp[0]` at `soma(0.5)` with a corresponding diagram of a neuron.
- RunControl:** Control panel for simulation parameters:
  - Init (mV): -65
  - Init & Run / Stop buttons
  - Continue til (ms): 5
  - Continue for (ms): 1
  - Single Step / t (ms): 5 / tstop (ms): 5 / dt (ms): 0.025
  - Points plotted/ms: 40
  - Scrn update intvl (s): 0.05
  - Real Time (s): 0.07
- NEURON Demonstrati...:** Configuration for `Pyramidal: HH soma, passive dendrites`.
  - Patch: HH
  - Stylized
  - Pyramidal
  - Release
  - Synchronizing net (artificial cells)
  - LinearCircuit: Dynamic Clamp
  - Stochastic Single Channels: HH
  - No model
- Temperature:** celsius (degC): 15
- VariableTimeStep:** Use variable dt, Absolute Tolerance: 0.001, Atol Scale Tool, Details

## Further Readings

- Mark F. Bear, Barry W. Connors, and Michael A. Paradiso (2006), **Neuroscience: exploring the brain**, Lippincott Williams & Wilkins , 3rd edition.
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- Gordon M. Shepherd (1994), **Neurobiology**, Oxford University Press, 3rd edition.
- Christof Koch (1999), **Biophysics of computation; information processing in single neurons**, Oxford University Press
- Christof Koch and Idan Segev (eds.) (1998), **Methods in neural modelling**, MIT Press, 2nd edition.
- C. T. Tuckwell (1988), **Introduction to theoretical neurobiology**, Cambridge University Press.
- Hugh R. Wilson (1999) **Spikes, decisions and actions: dynamical foundations of neuroscience**, Oxford University Press. See also his paper in J. Theor. Biol. 200: 375–88, 1999.