Reinforcement learning

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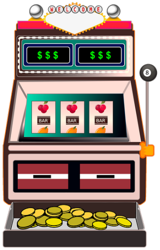
Notes

(Multi-armed) Bandits



Think about not one but 10 arms you may choose to pull.

Notes

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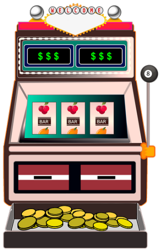
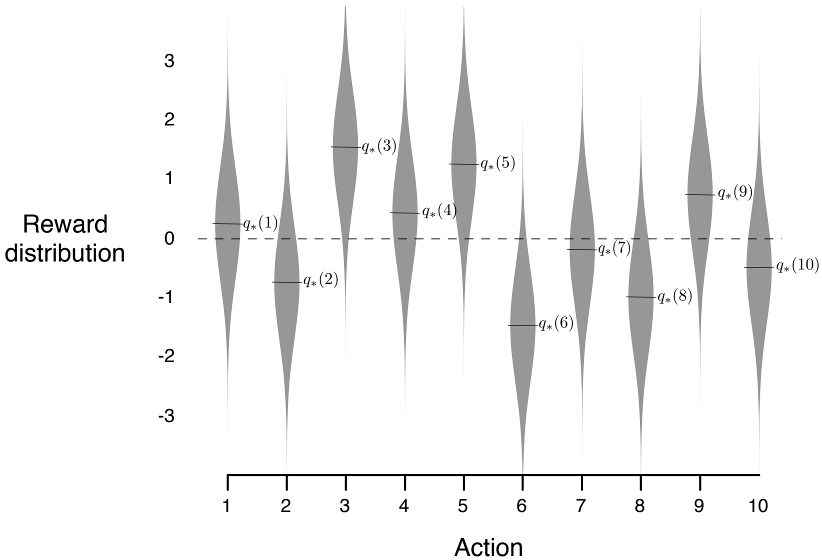


Think about not one but 10 arms you may choose to pull.

$p(s'|s,a)$ and $r(s,a,s')$ not known!

Notes

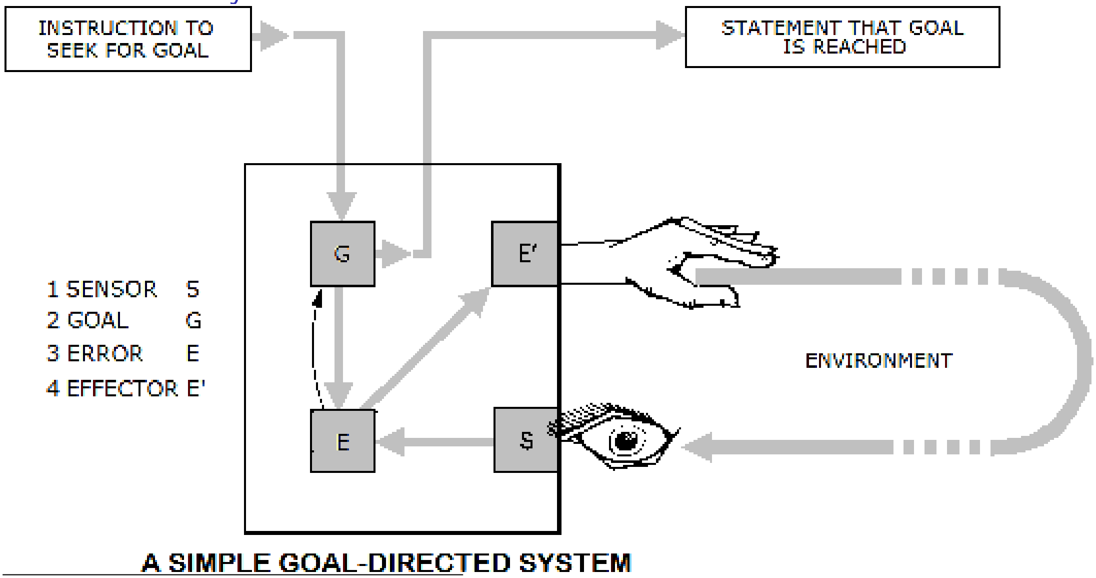
10 armed bandit, what arm to pull?



Notes

- 10 different arms
- action pulling $k-$th arm
- value of the action, i.e. $q(a)$ is stochastic (Gaussian around $q^*(a)$)
- Playing (pulling) many times, what is the policy?

Goal-directed system

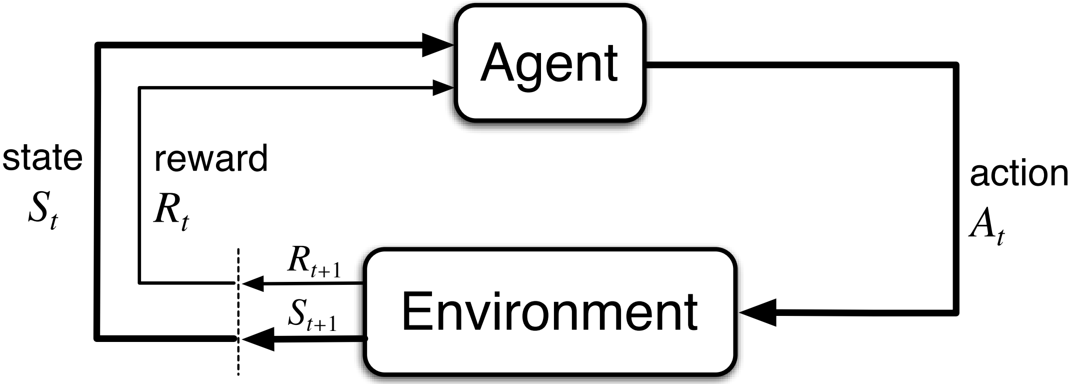


[1]

[1] Figure from http://www.cybsoc.org/gcyb.htm

Notes

Reinforcement Learning - performing actions, learning from rewards



[2]

- Feedback in form of Rewards
- Learn to act so as to maximize expected rewards.

[2] Scheme from [4]

Notes

Examples, robot learning, Atari games, ...

Video: Learning safe policies[3]

[3] M. Pecka, V. Salansky, K. Zimmermann, T. Svoboda. Autonomous flipper control with safety constraints. In Intelligent Robots and Systems (IROS), 2016, https://youtu.be/_oUMbBtoRcs

Notes

Policy search is a more advanced topic, only touched by this course. Later in master programme. Reinforement learning beating humans in playing Atari games: https://deepmind.google/discover/blog/agent57-outperforming-the-human-atari-benchmark/

From off-line (MDPs) to on-line (RL)

Markov decision process – MDPs. Off-line search, we know:

- A set of states $s \in \mathcal{S}$ (map)
- A set of actions per state. $a \in \mathcal{A}$
- A transition model $T(s, a, s')$ or $p(s'|s, a)$ (robot)
- A reward function $r(s, a, s')$ (map, robot)

Looking for the optimal policy $\pi(s)$. We can plan/search before the robot enters the environment.

Notes

For MDPs, we know p, r for all possible states and actions.

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On-line problem:

- Transition model p and reward function r not known.
- Agent/robot must act and learn from experience.

Notes

For MDPs, we know p, r for all possible states and actions.

(Transition) Model-based learning

The main idea: Do something and:

- Learn an approximate model from experiences.
- Solve as if the model was correct.

Notes

- Where to start?
- When does it end?
- How long does it take?
- When to stop (the learning phase)?

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- Solve as if the model was correct.

Learning MDP model:

- In s try a, observe s', count (s, a, s').
- Normalize to get and estimate of $p(s' \mid s, a)$.
- Discover (by observation) each $r(s, a, s')$ when experienced.

Notes

- Where to start?
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- In s try a, observe s', count (s, a, s').
- Normalize to get and estimate of $p(s' \mid s, a)$.
- Discover (by observation) each $r(s, a, s')$ when experienced.

Solve the learned MDP.

Notes

- Where to start?
- When does it end?
- How long does it take?
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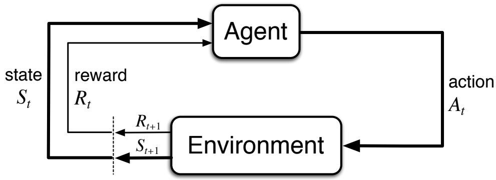
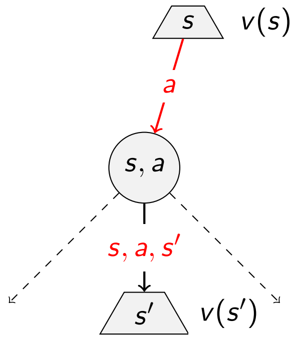
Reward function $r(s, a, s')$

- $r(s, a, s')$ - reward for taking a in s and landing in s'.
- In Grid world, we assumed $r(s, a, s')$ to be the same everywhere.
- In the real world, it is different (going up, down, ...)





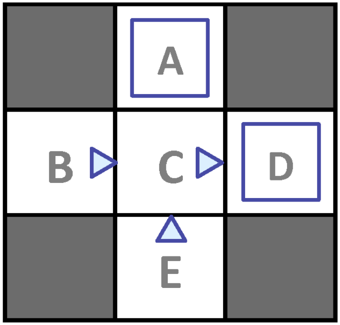
In ai-gym `env.step(action)` returns s', $r(s, \texttt{action}, s')$.

Notes

In ai-gym `env.step(action)` returns s', $r(s, \texttt{action}, s')$, ... It is defined by the environment (robot simulator, system, ...) not by the (algorithms)

Model-based learning: Grid example

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)[4]

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

[4] Figure from [1]

Notes

Learned Model

$$\hat{T}(s, a, s')$$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
...

$$\hat{R}(s, a, s')$$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
...

Learning transition model

$\hat{p}(\mathrm{D} \mid \mathrm{C}, \mathrm{east}) = ?$

Episode 1	Episode 2
B, east, C, -1	B, east, C, -1
C, east, D, -1	C, east, D, -1
D, exit, x, +10	D, exit, x, +10

Episode 3	Episode 4
E, north, C, -1	E, north, C, -1
C, east, D, -1	C, east, A, -1
D, exit, x, +10	A, exit, x, -10

Notes

(C, east) combination performed 4 times, 3 times landed in D, once in A. Hence, $\hat{p}(\mathrm{D} \mid \mathrm{C}, \mathrm{east}) = 0.75$.

Learning reward function

$\hat{r}(\mathrm{C}, \mathrm{east}, \mathrm{D}) = ?$

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B, east, C, -1
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Episode 2

B, east, C, -1
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D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Notes

Whenever $(\mathrm{C}, \mathrm{east}, \mathrm{D})$ performed, received reward was -1. Hence, $\hat{r}(\mathrm{C}, \mathrm{east}, \mathrm{D}) = -1$.

Model based vs model-free: Expected age $\mathsf{E}[A]$

Random variable age A.

$$\mathsf{E}[A] = \sum_a P(A = a)a$$

We do not know $P(A = a)$. Instead, we collect N samples $[a_1, a_2, \ldots a_N]$.

Notes

Just to avoid confusion. There are many more samples than possible ages (positive integer). Think about $N \gg 100$.

- Model based – eventually, we learn the correct model.
- Model free – no need for weighting; this is achieved through the frequencies of different ages within the samples (most frequent and hence most probable ages simply come up many times).

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Model based

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$\mathrm{E}[A] \approx \sum_a \hat{P}(a)a$$

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Model based

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$\mathsf{E}[A] \approx \sum_a \hat{P}(a)a$$

Model free

$$\mathsf{E}[A] \approx \frac{1}{N}\sum_i a_i$$

Notes

Just to avoid confusion. There are many more samples than possible ages (positive integer). Think about $N \gg 100$.

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Model-free learning

Notes

Passive learning (evaluating given policy)

- **Input:** a fixed policy $\pi(s)$
- We want to know how good it is.
- r, p not known.
- Execute policy ...
- and learn on the way.
- **Goal:** learn the state values $v^\pi(s)$

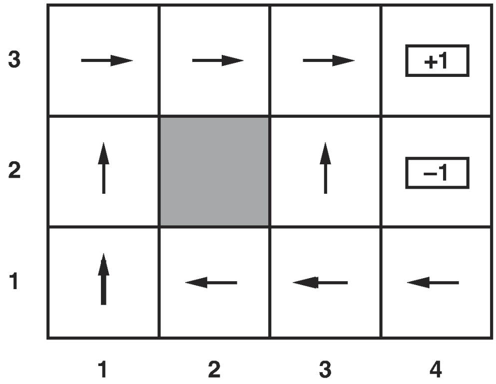


Image from [2]

Notes

Executing policies - training, then learning from the observations. We want to do the policy evaluation but the necessary model is not known.

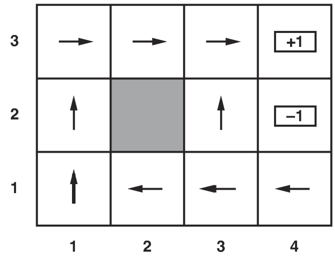
The word passive means we just follow a prescribed policy $\pi(s)$.

Direct evaluation from episodes

Value of s for π – expected sum of discounted rewards – expected return

$$v^{\pi}(S_t) = \mathrm{E}\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}\right]$$

$$v^{\pi}(S_t) = \mathrm{E}\left[G_t\right]$$



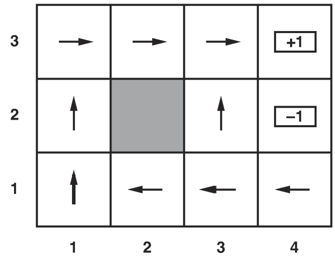
Notes

- Act according to the policy.
- When visiting a state, remember what the sum of discounted rewards (returns) turned out to be.
- Compute average of the returns.
- Each trial episode provides a sample of v^{π}.

What is $v^{\pi}(3,2)$ after these episodes?

Direct evaluation from episodes

Value of s for π – expected sum of discounted rewards – expected return



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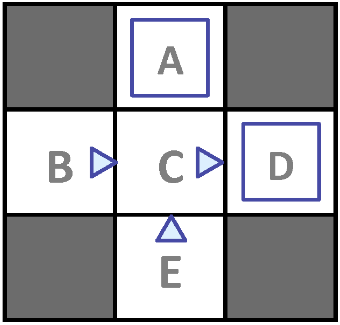
What is $v(3,2)$ after these episodes?

Notes

- Not visited during the first episode.
- Visited once in the second, gathered return $G = -0.04 - 0.04 + 1 = 0.92$.
- Visited once in the third, return $G = -0.04 - 1 = -1.04$.
- Value, average return is $(0.92 - 1.04)/2 = -0.06$.

Direct evaluation: Grid example

Input Policy π



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E, north, C, -1
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Episode 4

E, north, C, -1
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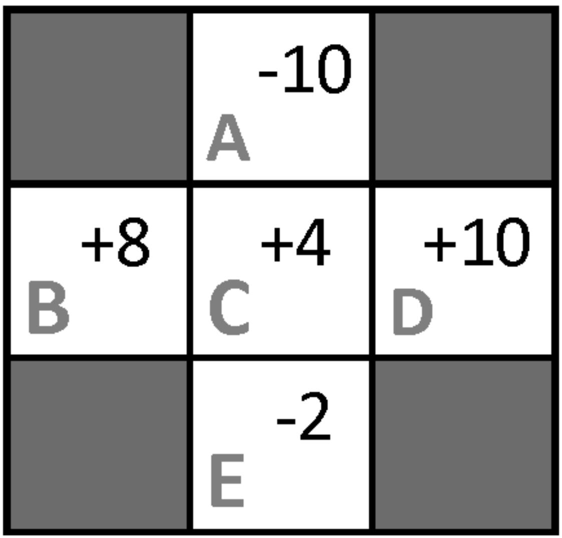
Notes



Direct evaluation: Grid example, $\gamma = 1$

What is $v(\mathrm{C})$ after the 4 episodes?

Episode 1	Episode 2	Episode 3	Episode 4
B, east, C, -1	B, east, C, -1	E, north, C, -1	E, north, C, -1
C, east, D, -1	C, east, D, -1	C, east, D, -1	C, east, A, -1
D, exit, x, +10	D, exit, x, +10	D, exit, x, +10	A, exit, x, -10

Notes

- Episode 1, $G = -1 + 10 = 9$
- Episode 2, $G = -1 + 10 = 9$
- Episode 3, $G = -1 + 10 = 9$
- Episode 4, $G = -1 - 10 = -11$
- Average return $v(\mathrm{C}) = (9 + 9 + 9 - 11)/4 = 4$

For first-visit variant, B is correct. For every-visit variant, D is correct.
N can be lower than M (state does not have to be attended in every episode). For every-visit variant, N can be higher than M (a state can be visited several times in one episode).

Direct evaluation: Grid example, $\gamma = 1$

What is $v(\mathrm{C})$ after the 4 episodes?

Let M be the number of recorded episodes.
Let N be the number of samples used to compute the averages.
What is the relation of M and N?

A $N = M$

B $N \leq M$

C $N \geq M$

D N has no relation to M

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

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Episode 4

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Direct evaluation algorithm (every-visit version)

$(1,1)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (2,3)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (4,3)_{\textbf{+1}}$
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Input: a policy π to be evaluated
Initialize:
  $V(s) \in \mathbb{R}$, arbitrarily, for all $s \in \mathcal{S}$
  $Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$
Loop forever (for each episode):
  Generate an episode following π: $S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T$
  $G \leftarrow 0$
  Loop backwards for each step of episode, $t = T-1, T-2, \ldots, 0$:
    $G \leftarrow R_{t+1} + \gamma G$
    Append G to $Returns(S_t)$
    $V(S_t) \leftarrow \text{average}(Returns(S_t))$

Notes

The algorithm can be easily expanded to $Q(S_t, A_t)$. Instead of visiting S_t we consider visiting of a pair S_t, A_t.

Direct evaluation algorithm (first-visit version)

$$(1,1)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (2,3)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (4,3)_{\textbf{+1}}$$
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Initialize:
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Loop forever (for each episode):
- Generate an episode following π: $S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T$
- $G \leftarrow 0$
- Loop backwards for each step of episode, $t = T-1, T-2, \ldots, 0$:
  - $G \leftarrow R_{t+1} + \gamma G$
  - If S_t does not appear in $S_0, S_1, \ldots, S_{t-1}$: *// Use the return for the first visit only*
    - Append G to $Returns(S_t)$
    - $V(S_t) \leftarrow$ average($Returns(S_t)$)

Notes

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Direct evaluation: analysis

The good:

- Simple, easy to understand and implement.
- Does not need p, r and eventually it computes the true v^{π}.

Notes

In second trial, we visit $(3, 2)$ for the first time. We already know that the successor $(3, 3)$ has probably a high value but the method does not use until the end of the trial episode.

Before updating $V(s)$ we have to wait until the training episode ends.

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The bad:

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- Each state value learned in isolation.
- State values are not independent
- $v^\pi(s) = \sum_{s'} p\big(s' \mid s, \pi(s)\big)\big[r(s, \pi(s), s') + \gamma\, v^\pi(s')\big]$

Notes

In second trial, we visit $(3,2)$ for the first time. We already know that the successor $(3,3)$ has probably a high value but the method does not use until the end of the trial episode.

Before updating $V(s)$ we have to wait until the training episode ends.

(on-line) Policy evaluation?

In MDP, we did:

- Initialize the values: $V_0^\pi(s) = 0$
- In each iteration, replace V with a one-step-look-ahead:
  $V_{k+1}^\pi(s) \leftarrow \sum_{s'} p\big(s' \mid s, \pi(s)\big)\big[r(s, \pi(s), s') + \gamma\, V_k^\pi(s')\big]$

Notes

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Problem: both $p\big(s' \mid s, \pi(s)\big)$ and $r(s,\pi(s),s')$ unknown!

Notes

Use samples for evaluating policy?

MDP (p, r known) : Update V estimate by a weighted average:

$V^{\pi}_{k+1}(s) \leftarrow \sum_{s'} p\big(s' \mid s, \pi(s)\big)\big[r(s, \pi(s), s') + \gamma\, V^{\pi}_{k}(s')\big]$

Notes

It looks promising. Unfortunately, we cannot do it that way. After an action, the robot is in a next state and cannot go back to the very same state where it was before. Energy was consumed and some actions may be irreversible; think about falling into a hole. We have to utilize the s, a, s' experience anytime when performed/visited.

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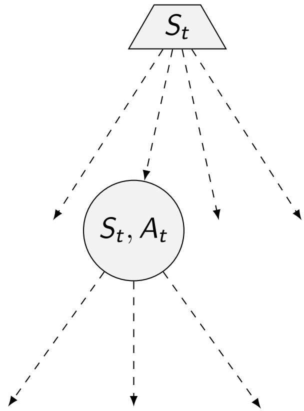
What about stop, try, try, ..., and average?
Trials at time t. $\pi(S_t) \rightarrow A_t$, repeat A_t.



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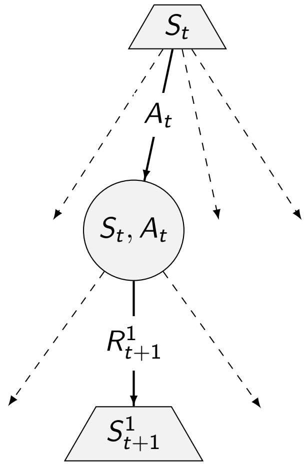
Trials at time t. $\pi(S_t) \rightarrow A_t$, repeat A_t.

$$\text{trial}^1 \;=\; R^1_{t+1} + \gamma\, V(S^1_{t+1})$$



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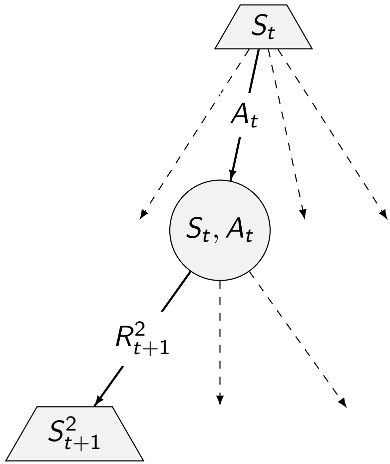
$$\begin{aligned}\text{trial}^1 &= R^1_{t+1} + \gamma\, V(S^1_{t+1})\\ \text{trial}^2 &= R^2_{t+1} + \gamma\, V(S^2_{t+1})\end{aligned}$$



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\vdots &= \vdots \\
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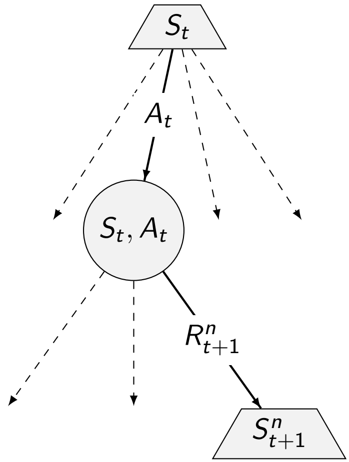

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\vdots &= \vdots \\
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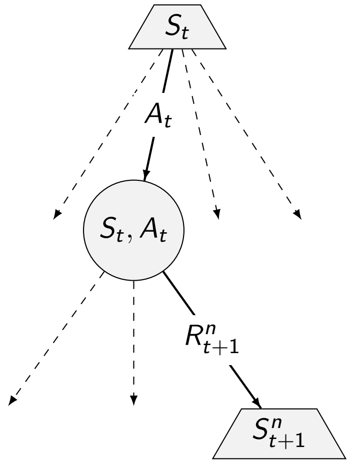
\end{aligned}
$$

$$V(S_t) \leftarrow \frac{1}{n} \sum_i \text{trial}^i$$



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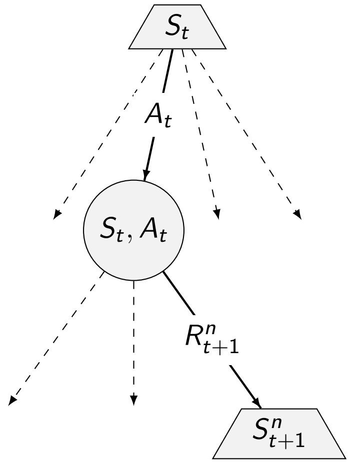

Problem: We cannot re-set to S_t easily.

Notes

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Temporal-difference value learning

$$(1,1)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (1,2)_{\textbf{-.04}} \rightsquigarrow (1,3)_{\textbf{-.04}} \rightsquigarrow (2,3)_{\textbf{-.04}} \rightsquigarrow (3,3)_{\textbf{-.04}} \rightsquigarrow (4,3)_{\textbf{+1}}$$
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$\gamma = 1$

Notes

Trial episode: acting, observing, until it stops (in a terminal state or by a limit).
We visit $S(1,3)$ twice during the first episode. Its value estimate is the average of two returns.
Note the main difference. In *Direct evaluation*, we had to wait until the end of the episode, compute G_t for each t on the way, and then we update $V(S_t)$. We can do it α incrementally

$$V(S_t) \leftarrow V(S_t) + \alpha\Big(G_t - V(S_t)\Big)$$

In *TD learning*, we update as we go.

Temporal-difference value learning

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From first trial (episode): $V(2,3) =$, $V(1,3) =$, ...

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From first trial (episode): $V(2,3) = 0.92$, $V(1,3) = 0.84, \ldots$

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In second episode, going from $S_t = (1,3)$ to $S_{t+1} = (2,3)$ with reward $R_{t+1} = -0.04$, hence:

$$V(1,3) = R_{t+1} + V(2,3) = -0.04 + 0.92 = 0.88$$

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- First estimate 0.84 is a bit lower than 0.88. $V(S_t)$ is different than $R_{t+1} + \gamma V(S_{t+1})$

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- Update ($\alpha \times$ difference): $V(S_t) \leftarrow V(S_t) + \alpha\Big(\big[R_{t+1} + \gamma V(S_{t+1})\big] - V(S_t)\Big)$
- α is the learning rate.

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- α is the learning rate.
- $V(S_t) \leftarrow (1-\alpha)V(S_t) + \alpha\,(\text{new sample})$

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Exponential moving average

$$\overline{x}_n = (1-\alpha)\overline{x}_{n-1} + \alpha x_n$$

What does it remember about the past? Try to derive:

$$\overline{x}_n = f(\alpha, x_n, x_{n-1}, x_{n-2}, x_{n-3}, \dots)$$

Notes

Recursively insetring we end up with

$$\overline{x}_n = \alpha \left[x_n + (1-\alpha)x_{n-1} + (1-\alpha)^2 x_{n-2} + \cdots \right]$$

We already know the sum of geometric series for $r < 1$

$$1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r}$$

Putting $r = 1 - \alpha$, we see that

$$\frac{1}{\alpha} = 1 + (1-\alpha) + (1-\alpha)^2 + \cdots$$

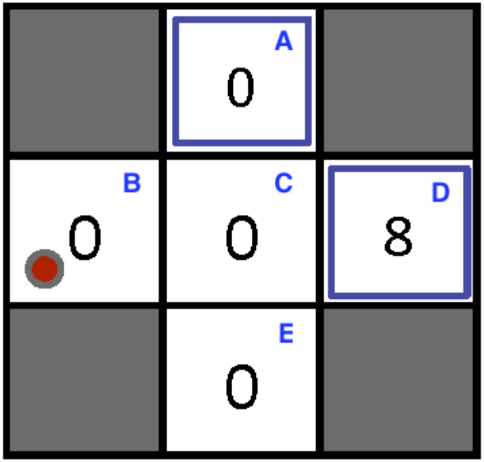
And hence:

$$\overline{x}_n = \frac{x_n + (1-\alpha)x_{n-1} + (1-\alpha)^2 x_{n-2} + \cdots}{1 + (1-\alpha) + (1-\alpha)^2 + (1-\alpha)^3 + \cdots}$$

a weighted average that exponentially forgets about the past.

Example: TD Value learning

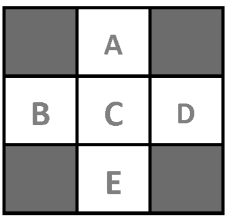
$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$



- Values represent initial $V(s)$
- Assume: $\gamma = 1, \alpha = 0.5, \pi(s) = \rightarrow$

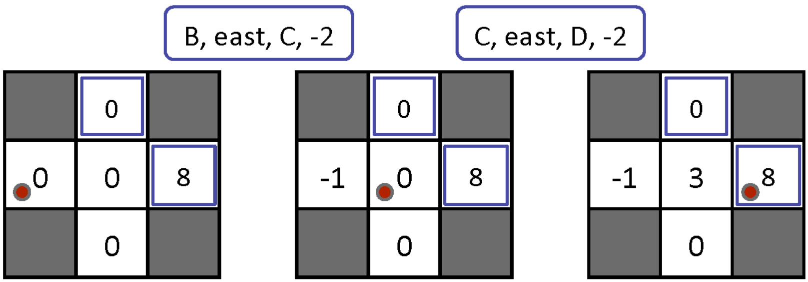
Notes

States



Assume: $\gamma = 1$, $\alpha = 1/2$

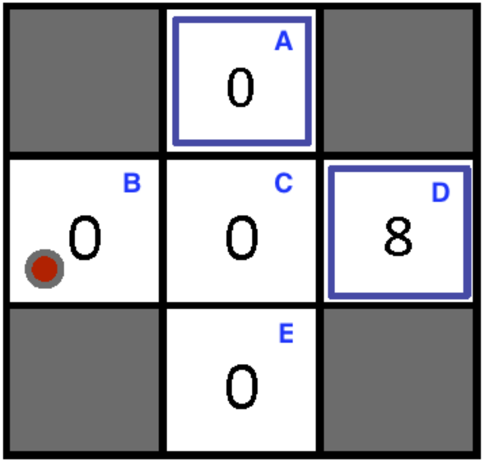
Observed Transitions



$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

Example: TD Value learning

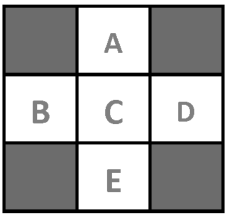
$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$



- Values represent initial $V(s)$
- Assume: $\gamma = 1, \alpha = 0.5, \pi(s) = \rightarrow$
- $(B, \rightarrow, C), -2, \Rightarrow V(B)$?

Notes

States



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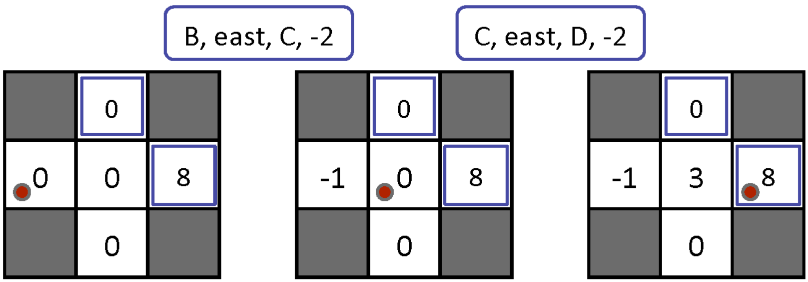

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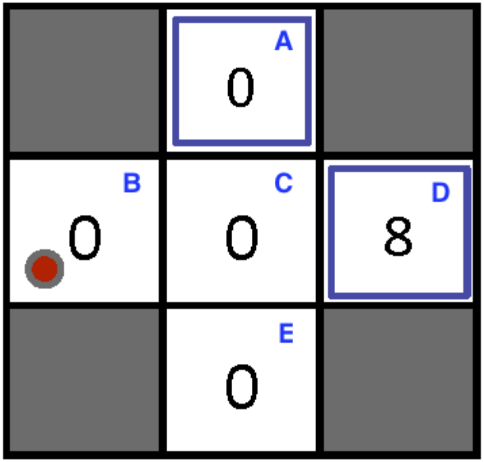

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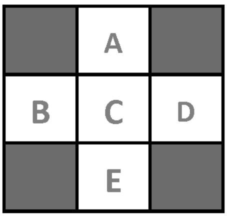
Notes

States



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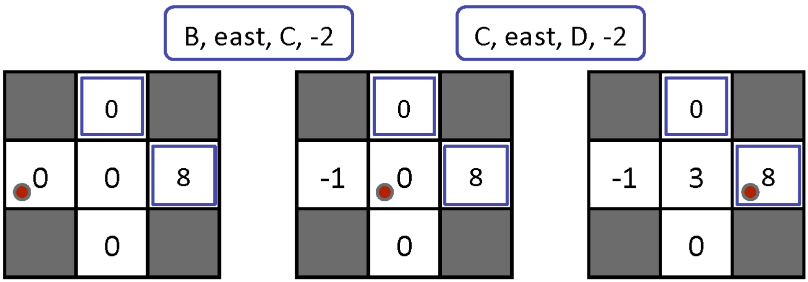

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Temporal difference value learning: algorithm

Input: the policy π to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(\mathit{terminal}) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    $A \leftarrow$ action given by π for S
    Take action A, observe R, S'
    $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
    $S \leftarrow S'$
  until S is terminal

Notes

What is wrong with the temporal difference Value learning?

The Good: Model-free value learning by mimicking Bellman updates.

Notes

Learn Q-values, not V-values, and make the action selection model-free too!

What is wrong with the temporal difference Value learning?

The Good: Model-free value learning by mimicking Bellman updates.

The Bad: How to turn values into a (new) policy?

- $\pi(s) = \arg\max_a \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma V(s') \right]$

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What is wrong with the temporal difference Value learning?

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- $\pi(s) = \arg\max_a \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma V(s') \right]$
- $\pi(s) = \arg\max_a Q(s, a)$

Notes

Learn Q-values, not V-values, and make the action selection model-free too!

Q-learning

Notes

So far we walked as prescribed by a $\pi(s)$ because we did not know how to act better.

Reminder: V, Q-value iteration for MDPs

Value/Utility iteration (depth limited evaluation):

- Start: $V_0(s) = 0$
- In each step update V by looking one step ahead:
  $V_{k+1}(s) \leftarrow \max_a \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma V_k(s') \right]$

Q values more useful (think about updating π)

- Start: $Q_0(s, a) = 0$
- In each step update Q by looking one step ahead:
  $Q_{k+1}(s, a) \leftarrow \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$

Notes

Draw the (s)-(s,a)-(s')-(s',a') tree. It will be also handy when discussing exploration vs. exploitation – where to drive next.

Q-learning (one episode of)

MDP update: $Q_{k+1}(s,a) \leftarrow \sum_{s'} p(s' \mid s,a) \left[r(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$

Notes

There are alternatives how to compute the trial value. SARSA method takes $Q(S_{t+1}, A_{t+1})$ directly, not the max. More next week.

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Learn Q values as the robot/agent goes (temporal difference)

- Drive the robot and fetch rewards (s, a, s', R)

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- Drive the robot and fetch rewards (s, a, s', R)
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- A new trial/sample estimate at time t

  $\text{trial} = R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$

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  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(\text{trial} - Q(S_t, A_t))$

  or (the same)

  $Q(S_t, A_t) \leftarrow (1-\alpha) Q(S_t, A_t) + \alpha\, \text{trial}$

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In each step Q approximates the optimal q^* function.

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Q-learning: algorithm (repeating episodes, until terminal or exhausted)

step size $0 < \alpha \leq 1$
initialize $Q(s, a)$ for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$
repeat episodes:
    initialize S
    for for each step of episode: **do**
        choose A from $\mathcal{A}(S)$
        take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
        $S \leftarrow S'$
    until S is terminal
until Time is up, ...

Notes

From Q-learning to Q-learning agent

- Drive the robot and fetch rewards. (s, a, s', R)
- We know old estimates $Q(s, a)$ (and $Q(s', a')$), if not, initialize.
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Notes

Q-function for a discrete, finite problem? But what about continous space or discrete but a very large one?
Use the (s)-(s,a)-(s')-(s',a') tree to discuss the next-action selection.

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Technicalities for the Q-learning agent

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Technicalities for the Q-learning agent

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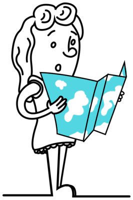
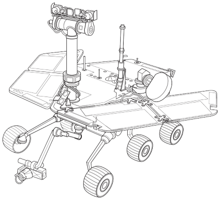
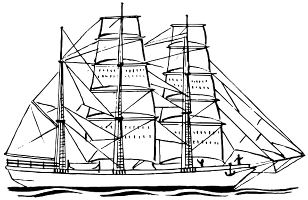
Technicalities for the Q-learning agent

- How to represent the Q-function?
- What is the value for terminal? $Q(s, \text{Exit})$ or $Q(s, \text{None})$
- How to drive? Where to drive next? Does it change over the course?

Notes

Q-function for a discrete, finite problem? But what about continous space or discrete but a very large one?
Use the (s)-(s,a)-(s')-(s',a') tree to discuss the next-action selection.

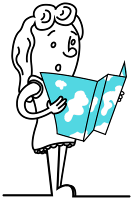
Exploration vs. Exploitation

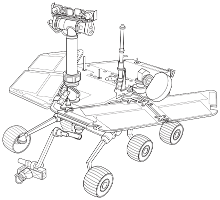
  

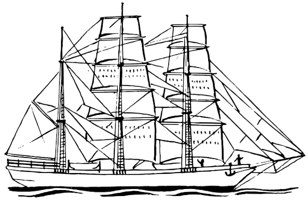
- Drive the known road or try a new one?

Notes

Exploration vs. Exploitation







- Drive the known road or try a new one?
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Notes

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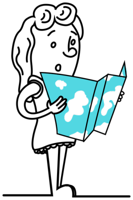

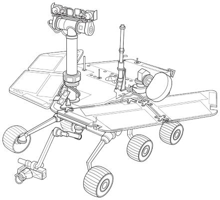

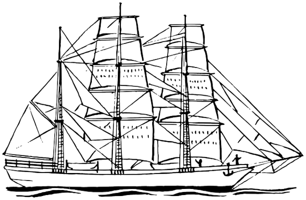

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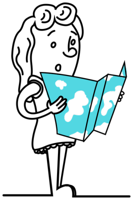
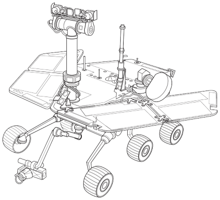
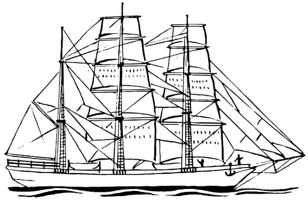
- Use the SW (operating system) I know or try a new one?

Notes

Exploration vs. Exploitation

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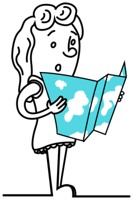

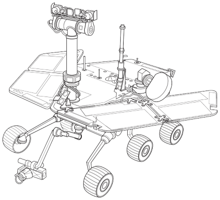

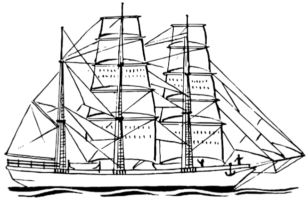

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Notes

How to explore?

Random (ϵ-greedy):

Notes

- We can think about lowering ϵ as the learning progresses.
- Favor unexplored states - be optimistic - exploration functions - $f(u, n) = u + k/n$, where u is the value estimated, and n is the visit count, and k is the training/simulation episode.

How to explore?

Random (ϵ-greedy):

- Flip a coin every step.

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- ϵ same everywhere?

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What we have learned

- Agent/robot may learn by acting an getting rewards
- Model based vs. model-free methods
- Direct learning vs. temporal-difference learning
- From learning state values to Q-learning

Notes

References I

Further reading: Chapter 21 of [2] (chapter 23 of [3]). More detailed discussion in [4], chapters 5 and 6.

[1] Dan Klein and Pieter Abbeel.
UC Berkeley CS188 Intro to AI – course materials.
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Used with permission of Pieter Abbeel.

[2] Stuart Russell and Peter Norvig.
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[3] Stuart Russell and Peter Norvig.
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Notes

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[4] Richard S. Sutton and Andrew G. Barto.
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Notes