

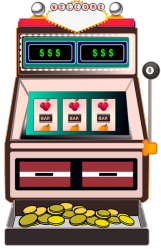
Reinforcement learning

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(Multi-armed) Bandits

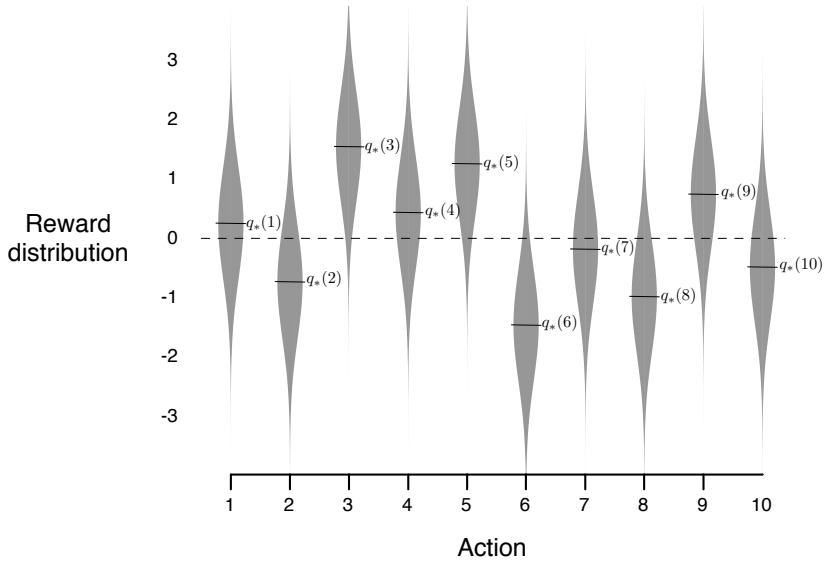


Think about not one but 10 arms you may choose to pull.

$p(s'|s, a)$ and $r(s, a, s')$ not known!

Notes

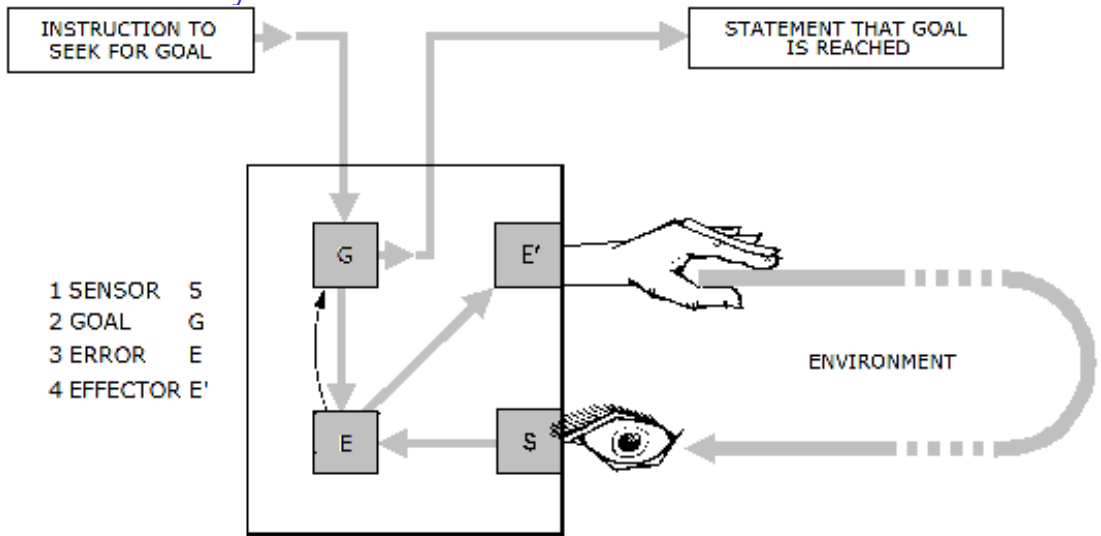
10 armed bandit, what arm to pull?



Notes

- 10 different arms
- action pulling k -th arm
- value of the action, i.e. $q(a)$ is stochastic (Gaussian around $q^*(a)$)
- Playing (pulling) many times, what is the policy?

Goal-directed system



A SIMPLE GOAL-DIRECTED SYSTEM

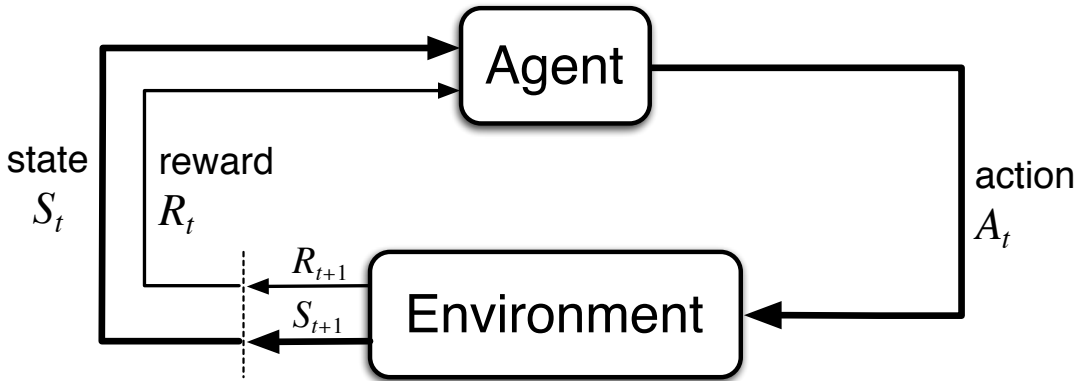
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¹Figure from <http://www.cybsoc.org/gcyb.htm>

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Notes

Reinforcement Learning - performing actions, learning from rewards



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- ▶ Feedback in form of Rewards
- ▶ Learn to act so as to maximize expected rewards.

²Scheme from [4]

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Autonomous Flipper Control with Safety Constraints

Martin Pecka, Vojtěch Šalanský,
Karel Zimmermann, Tomáš Svoboda

experiments utilizing
Constrained Relative Entropy Policy Search

Video: Learning safe policies³

³M. Pecka, V. Salansky, K. Zimmermann, T. Svoboda. Autonomous flipper control with safety constraints. In Intelligent Robots and Systems (IROS), 2016, https://youtu.be/_oUMbBtoRcs

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Notes

Policy search is a more advanced topic, only touched by this course. Later in master programme. Reinforcement learning beating humans in playing Atari games: <https://deepmind.google/discover/blog/agent57-outperforming-the-human-atari-benchmark/>

From off-line (MDPs) to on-line (RL)

Markov decision process – MDPs. Off-line search, we know:

- ▶ A set of states $s \in \mathcal{S}$ (map)
- ▶ A set of actions per state. $a \in \mathcal{A}$
- ▶ A transition model $T(s, a, s')$ or $p(s'|s, a)$ (robot)
- ▶ A reward function $r(s, a, s')$ (map, robot)

Looking for the optimal policy $\pi(s)$. We can plan/search before the robot enters the environment.

On-line problem:

- ▶ Transition model p and reward function r not known.
- ▶ Agent/robot must act and learn from experience.

Notes

For MDPs, we know p, r for all possible states and actions.

(Transition) Model-based learning

The main idea: Do something and:

- ▶ Learn an approximate model from experiences.
- ▶ Solve as if the model was correct.

Learning MDP model:

- ▶ In s try a , observe s' , count (s, a, s') .
- ▶ Normalize to get an estimate of $p(s' | s, a)$.
- ▶ Discover (by observation) each $r(s, a, s')$ when experienced.

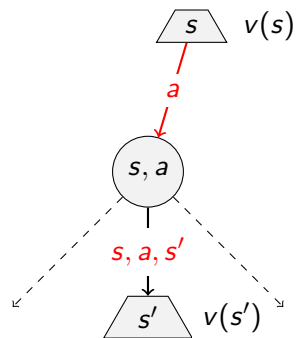
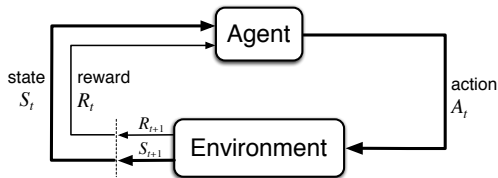
Solve the learned MDP.

Notes

- Where to start?
- When does it end?
- How long does it take?
- When to stop (the learning phase)?

Reward function $r(s, a, s')$

- ▶ $r(s, a, s')$ - reward for taking a in s and landing in s' .
- ▶ In Grid world, we assumed $r(s, a, s')$ to be the same everywhere.
- ▶ In the real world, it is different (going up, down, ...)



In ai-gym env.step(action) returns $s', r(s, \text{action}, s')$.

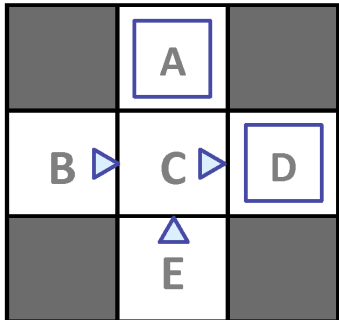
Notes

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In ai-gym env.step(action) returns $s', r(s, \text{action}, s'), \dots$. It is defined by the environment (robot simulator, system, ...) not by the (algorithms)

Model-based learning: Grid example

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

4

⁴Figure from [1]

Notes

Learned Model

$$\hat{T}(s, a, s')$$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
...

$$\hat{R}(s, a, s')$$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
...

Learning transition model

$$\hat{p}(D \mid C, \text{east}) = ?$$

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Notes

(C, east) combination performed 4 times, 3 times landed in D, once in A. Hence, $\hat{p}(D \mid C, \text{east}) = 0.75$.

Learning reward function

$\hat{r}(C, \text{east}, D) = ?$

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Notes

Whenever (C, east, D) performed, received reward was -1 . Hence, $\hat{r}(C, \text{east}, D) = -1$.

Model based vs model-free: Expected age $E[A]$

Random variable age A .

$$E[A] = \sum_a P(A = a)a$$

We do not know $P(A = a)$. Instead, we collect N samples $[a_1, a_2, \dots, a_N]$.

Model based

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a)a$$

Model free

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

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Notes

Just to avoid confusion. There are many more samples than possible ages (positive integer). Think about $N \gg 100$.

- Model based – eventually, we learn the correct model.
- Model free – no need for weighting; this is achieved through the frequencies of different ages within the samples (most frequent and hence most probable ages simply come up many times).

Model-free learning

Passive learning (evaluating given policy)

- ▶ **Input:** a fixed policy $\pi(s)$
- ▶ We want to know how good it is.
- ▶ r, p not known.
- ▶ Execute policy ...
- ▶ and learn on the way.
- ▶ **Goal:** learn the state values $v^\pi(s)$

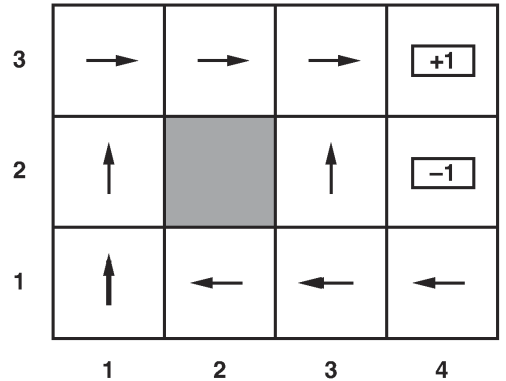


Image from [2]

Notes

Executing policies - training, then learning from the observations. We want to do the policy evaluation but the necessary model is not known.

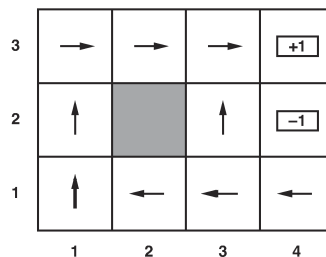
The word passive means we just follow a prescribed policy $\pi(s)$.

Direct evaluation from episodes

Value of s for π – expected sum of discounted rewards – expected return

$$v^\pi(S_t) = E \left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \right]$$

$$v^\pi(S_t) = E[G_t]$$



$(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) \xrightarrow{+1}$
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 $(1, 1) \xrightarrow{-0.04} (2, 1) \xrightarrow{-0.04} (3, 1) \xrightarrow{-0.04} (3, 2) \xrightarrow{-0.04} (4, 2) \xrightarrow{-1}$

Notes

- Act according to the policy.
- When visiting a state, remember what the sum of discounted rewards (returns) turned out to be.
- Compute average of the returns.
- Each trial episode provides a sample of v^π .

What is $v^\pi(3, 2)$ after these episodes?

Direct evaluation from episodes, $v^\pi(S_t) = E[G_t]$, $\gamma = 1$

$(1, 1)_{-0.04} \rightsquigarrow (1, 2)_{-0.04} \rightsquigarrow (1, 3)_{-0.04} \rightsquigarrow (1, 2)_{-0.04} \rightsquigarrow (1, 3)_{-0.04} \rightsquigarrow (2, 3)_{-0.04} \rightsquigarrow (3, 3)_{-0.04} \rightsquigarrow (4, 3)_{+1}$
 $(1, 1)_{-0.04} \rightsquigarrow (1, 2)_{-0.04} \rightsquigarrow (1, 3)_{-0.04} \rightsquigarrow (2, 3)_{-0.04} \rightsquigarrow (3, 3)_{-0.04} \rightsquigarrow (3, 2)_{-0.04} \rightsquigarrow (3, 3)_{-0.04} \rightsquigarrow (4, 3)_{+1}$
 $(1, 1)_{-0.04} \rightsquigarrow (2, 1)_{-0.04} \rightsquigarrow (3, 1)_{-0.04} \rightsquigarrow (3, 2)_{-0.04} \rightsquigarrow (4, 2)_{-1}$.

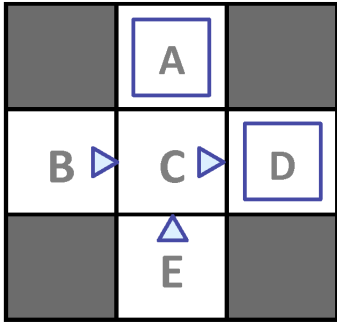
What is $v(3, 2)$ after these episodes?

Notes

- Not visited during the first episode.
- Visited once in the second, gathered return $G = -0.04 - 0.04 + 1 = 0.92$.
- Visited once in the third, return $G = -0.04 - 1 = -1.04$.
- Value, average return is $(0.92 - 1.04)/2 = -0.06$.

Direct evaluation: Grid example

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
 C, east, D, -1
 D, exit, x, +10

Episode 2

B, east, C, -1
 C, east, D, -1
 D, exit, x, +10

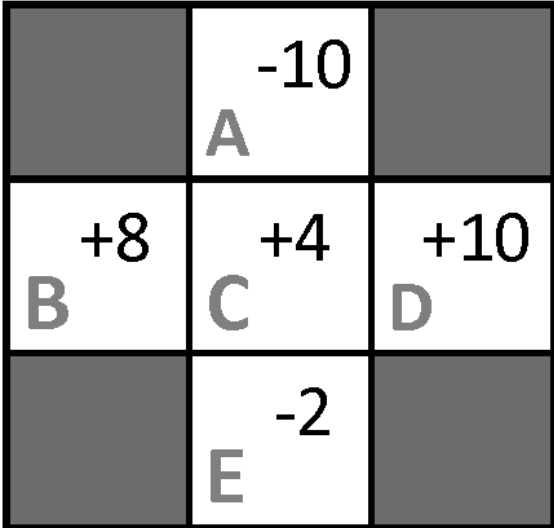
Episode 3

E, north, C, -1
 C, east, D, -1
 D, exit, x, +10

Episode 4

E, north, C, -1
 C, east, A, -1
 A, exit, x, -10

Notes



Direct evaluation: Grid example, $\gamma = 1$

What is $v(C)$ after the 4 episodes?

Let M be the number of recorded episodes.

Let N be the number of samples used to compute the averages.

What is the relation of M and N ?

- A** $N = M$
- B** $N \leq M$
- C** $N \geq M$
- D** N has no relation to M

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Notes

- Episode 1, $G = -1 + 10 = 9$
- Episode 2, $G = -1 + 10 = 9$
- Episode 3, $G = -1 + 10 = 9$
- Episode 4, $G = -1 - 10 = -11$
- Average return $v(C) = (9 + 9 + 9 - 11)/4 = 4$

For first-visit variant, B is correct. For every-visit variant, D is correct.

N can be lower than M (state does not have to be attended in every episode). For every-visit variant, N can be higher than M (a state can be visited several times in one episode).

Direct evaluation algorithm (every-visit/first-visit version)

$(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) +1$
 $(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (3, 2) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) +1$
 $(1, 1) \xrightarrow{-0.04} (2, 1) \xrightarrow{-0.04} (3, 1) \xrightarrow{-0.04} (3, 2) \xrightarrow{-0.04} (4, 2) -1 .$

Input: a policy π to be evaluated

Initialize:

$V(s) \in \mathbb{R}$, arbitrarily, for all $s \in \mathcal{S}$

$Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$

Loop forever (for each episode):

Generate an episode following π : $S_0, A_0, R_1, S_1, A_1, R_2, \dots, S_{T-1}, A_{T-1}, R_T$

$G \leftarrow 0$

Loop backwards for each step of episode, $t = T - 1, T - 2, \dots, 0$:

$G \leftarrow R_{t+1} + \gamma G$

Append G to $Returns(S_t)$

$V(S_t) \leftarrow \text{average}(Returns(S_t))$

If S_t does not appear in S_0, S_1, \dots, S_{t-1} : // Use the return for the first visit only

Append G to $Returns(S_t)$

$V(S_t) \leftarrow \text{average}(Returns(S_t))$

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Notes

The algorithm can be easily expanded to $Q(S_t, A_t)$. Instead of visiting S_t we consider visiting of a pair S_t, A_t .

Direct evaluation: analysis

The good:

- ▶ Simple, easy to understand and implement.
- ▶ Does not need p, r and eventually it computes the true v^π .

The bad:

$(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) +1$
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- ▶ Each state value learned in isolation.
- ▶ State values are not independent
- ▶ $v^\pi(s) = \sum_{s'} p(s' | s, \pi(s)) [r(s, \pi(s), s') + \gamma v^\pi(s')]$

Notes

In second trial, we visit $(3, 2)$ for the first time. We already know that the successor $(3, 3)$ has probably a high value but the method does not use until the end of the trial episode.

Before updating $V(s)$ we have to wait until the training episode ends.

(on-line) Policy evaluation?

In MDP, we did:

- ▶ Initialize the values: $V_0^\pi(s) = 0$
- ▶ In each iteration, replace V with a one-step-look-ahead:
$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} p(s' | s, \pi(s)) [r(s, \pi(s), s') + \gamma V_k^\pi(s')]$$

Problem: both $p(s' | s, \pi(s))$ and $r(s, \pi(s), s')$ unknown!

Use samples for evaluating policy?

MDP (p, r known) : Update V estimate by a weighted average:

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} p(s' | s, \pi(s)) [r(s, \pi(s), s') + \gamma V_k^\pi(s')]$$

What about stop, try, try, ..., and average?

Trials at time t . $\pi(S_t) \rightarrow A_t$, repeat A_t .

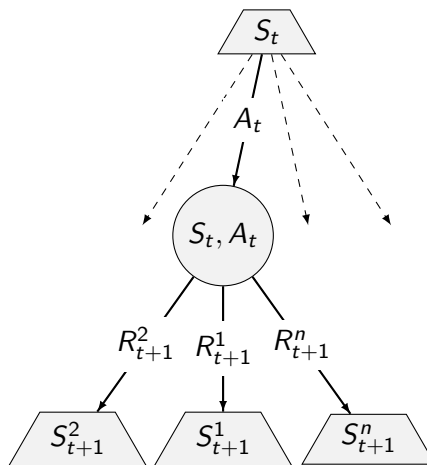
$$\text{trial}^1 = R_{t+1}^1 + \gamma V(S_{t+1}^1)$$

$$\text{trial}^2 = R_{t+1}^2 + \gamma V(S_{t+1}^2)$$

$$\vdots = \vdots$$

$$\text{trial}^n = R_{t+1}^n + \gamma V(S_{t+1}^n)$$

$$V(S_t) \leftarrow \frac{1}{n} \sum_i \text{trial}^i$$



Problem: We cannot re-set to S_t easily.

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Notes

It looks promising. Unfortunately, we cannot do it that way. After an action, the robot is in a next state and cannot go back to the very same state where it was before. Energy was consumed and some actions may be irreversible; think about falling into a hole. We have to utilize the s, a, s' experience anytime when performed/visited.

Temporal-difference value learning

$(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) \xrightarrow{+1}$
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$\gamma = 1$

From first trial (episode): $V(2, 3) = 0.92$, $V(1, 3) = 0.84, \dots$

In second episode, going from $S_t = (1, 3)$ to $S_{t+1} = (2, 3)$ with reward $R_{t+1} = -0.04$, hence:

$$V(1, 3) = R_{t+1} + V(2, 3) = -0.04 + 0.92 = 0.88$$

- ▶ First estimate 0.84 is a bit lower than 0.88. $V(S_t)$ is different than $R_{t+1} + \gamma V(S_{t+1})$
- ▶ Update ($\alpha \times$ difference): $V(S_t) \leftarrow V(S_t) + \alpha \left([R_{t+1} + \gamma V(S_{t+1})] - V(S_t) \right)$
- ▶ α is the learning rate.
- ▶ $V(S_t) \leftarrow (1 - \alpha)V(S_t) + \alpha(\text{new sample})$

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Notes

Trial episode: acting, observing, until it stops (in a terminal state or by a limit).

We visit $S(1, 3)$ twice during the first episode. Its value estimate is the average of two returns.

Note the main difference. In *Direct evaluation*, we had to wait until the end of the episode, compute G_t for each t on the way, and then we update $V(S_t)$. We can do it α incrementally

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t - V(S_t) \right)$$

In *TD learning*, we update as we go.

Exponential moving average

$$\bar{x}_n = (1 - \alpha)\bar{x}_{n-1} + \alpha x_n$$

What does it remember about the past? Try to derive:

$$\bar{x}_n = f(\alpha, x_n, x_{n-1}, x_{n-2}, x_{n-3}, \dots)$$

Notes

Recursively inserting we end up with

$$\bar{x}_n = \alpha \left[x_n + (1 - \alpha)x_{n-1} + (1 - \alpha)^2 x_{n-2} + \dots \right]$$

We already know the sum of geometric series for $r < 1$

$$1 + r + r^2 + r^3 + \dots = \frac{1}{1 - r}$$

Putting $r = 1 - \alpha$, we see that

$$\frac{1}{\alpha} = 1 + (1 - \alpha) + (1 - \alpha)^2 + \dots$$

And hence:

$$\bar{x}_n = \frac{x_n + (1 - \alpha)x_{n-1} + (1 - \alpha)^2 x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + (1 - \alpha)^3 + \dots}$$

a weighted average that exponentially forgets about the past.

Example: TD Value learning

$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$

	A 0	
B 0	C 0	D 8
	E 0	

- ▶ Values represent initial $V(s)$
- ▶ Assume: $\gamma = 1, \alpha = 0.5, \pi(s) = \rightarrow$
- ▶ $(B, \rightarrow, C), -2, \Rightarrow V(B)?$
- ▶ $(C, \rightarrow, D), -2, \Rightarrow V(C)?$

Notes

States

	A	
B	C	D
	E	

Assume: $\gamma = 1, \alpha = 1/2$

Observed Transitions

B, east, C, -2

C, east, D, -2

	0	
0	0	8
	0	

	0	
-1	0	8
	0	

	0	
-1	3	8
	0	

$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

Temporal difference value learning: algorithm

Input: the policy π to be evaluated

Algorithm parameter: step size $\alpha \in (0, 1]$

Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(\text{terminal}) = 0$

Loop for each episode:

 Initialize S

 Loop for each step of episode:

$A \leftarrow$ action given by π for S

 Take action A , observe R, S'

$V(S) \leftarrow V(S) + \alpha [R + \gamma V(S') - V(S)]$

$S \leftarrow S'$

 until S is terminal

What is wrong with the temporal difference Value learning?

The Good: Model-free value learning by mimicking Bellman updates.

The Bad: How to turn values into a (new) policy?

$$\blacktriangleright \pi(s) = \arg \max_a \sum_{s'} p(s' | s, a) [r(s, a, s') + \gamma V(s')]$$

$$\blacktriangleright \pi(s) = \arg \max_a Q(s, a)$$

Notes

Learn Q-values, not V-values, and make the action selection model-free too!

Q-learning

Notes

So far we walked as prescribed by a $\pi(s)$ because we did not know how to act better.

Reminder: V , Q -value iteration for MDPs

Value/Utility iteration (depth limited evaluation):

- ▶ Start: $V_0(s) = 0$
- ▶ In each step update V by looking one step ahead:
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} p(s' | s, a) [r(s, a, s') + \gamma V_k(s')]$$

Q values more useful (think about updating π)

- ▶ Start: $Q_0(s, a) = 0$
- ▶ In each step update Q by looking one step ahead:
$$Q_{k+1}(s, a) \leftarrow \sum_{s'} p(s' | s, a) \left[r(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

Notes

Draw the (s) - (s,a) - (s') - (s',a') tree. It will be also handy when discussing exploration vs. exploitation – where to drive next.

Q-learning (one episode of)

$$\text{MDP update: } Q_{k+1}(s, a) \leftarrow \sum_{s'} p(s' | s, a) \left[r(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

Learn Q values as the robot/agent goes (temporal difference)

- ▶ Drive the robot and fetch rewards (s, a, s', R)
- ▶ We know old estimates $Q(s, a)$ (and $Q(s', a')$), if not, initialize.

- ▶ A new trial/sample estimate at time t

$$\text{trial} = R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$$

- ▶ α update
 $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(\text{trial} - Q(S_t, A_t))$
or (the same)
 $Q(S_t, A_t) \leftarrow (1 - \alpha)Q(S_t, A_t) + \alpha \text{ trial}$

In each step Q approximates the optimal q^* function.

Notes

There are alternatives how to compute the trial value. SARSA method takes $Q(S_{t+1}, A_{t+1})$ directly, not the max. More next week.

Q-learning: algorithm (repeating episodes, until terminal or exhausted)

step size $0 < \alpha \leq 1$

initialize $Q(s, a)$ for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$

repeat episodes:

 initialize S

for for each step of episode: **do**

 choose A from $\mathcal{A}(S)$

 take action A , observe R, S'

$Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_a Q(S', a) - Q(S, A)]$

$S \leftarrow S'$

 until S is terminal

until Time is up, ...

From Q-learning to Q-learning agent

- ▶ Drive the robot and fetch rewards. (s, a, s', R)
- ▶ We know old estimates $Q(s, a)$ (and $Q(s', a')$), if not, initialize.
- ▶ A new trial/sample estimate: $\text{trial} = R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$
- ▶ α update: $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(\text{trial} - Q(S_t, A_t))$

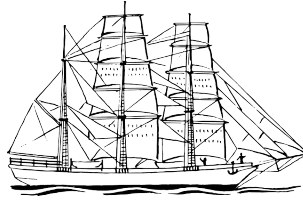
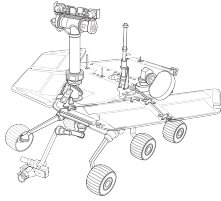
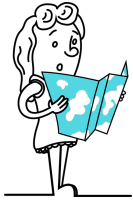
Technicalities for the Q-learning agent

- ▶ How to represent the Q-function?
- ▶ What is the value for terminal? $Q(s, \text{Exit})$ or $Q(s, \text{None})$
- ▶ How to drive? Where to drive next? Does it change over the course?

Notes

Q-function for a discrete, finite problem? But what about continuous space or discrete but a very large one?
Use the $(s)-(s,a)-(s')-(s',a')$ tree to discuss the next-action selection.

Exploration vs. Exploitation



- ▶ Drive the known road or try a new one?
- ▶ Go to the university menza or try a nearby restaurant?
- ▶ Use the SW (operating system) I know or try a new one?
- ▶ Go to bussiness or study a demanding program?
- ▶ ...

How to explore?

Random (ϵ -greedy):

- ▶ Flip a coin every step.
- ▶ With probability ϵ , act randomly.
- ▶ With probability $1 - \epsilon$, use the policy.

Problems with randomness?

- ▶ Keeps exploring forever.
- ▶ Should we keep ϵ fixed (over learning)?
- ▶ ϵ same everywhere?

Notes

- We can think about lowering ϵ as the learning progresses.
- Favor unexplored states - be optimistic - exploration functions - $f(u, n) = u + k/n$, where u is the value estimated, and n is the visit count, and k is the training/simulation episode.

What we have learned

- ▶ Agent/robot may learn by acting and getting rewards
- ▶ Model based vs. model-free methods
- ▶ Direct learning vs. temporal-difference learning
- ▶ From learning state values to Q-learning

References I

Further reading: Chapter 21 of [2] (chapter 23 of [3]). More detailed discussion in [4], chapters 5 and 6.

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