Where the hell am I and where is the stuff around me?

SLAM in SE(2) with (i) measurement models of 2D/3D marker detectors, UWB, GPS/GNSS, odometry, and (ii) differential drive motion model

Karel Zimmermann

Problem definition

Complete states: $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_t \in \mathcal{R}^n$

Actions: $\mathbf{u}_1, \ldots, \mathbf{u}_t \in \mathcal{R}^m$

Measurements: $\mathbf{z}_1, \ldots, \mathbf{z}_t \in \mathcal{R}^k$

Algorithm: $\mathbf{u}_{t+1} = \pi(\mathbf{z}_{1:t}, \mathbf{u}_{1:t})$

Rewards: $r_t = r(\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_t) \in \mathcal{R}$

Criterion: $J_\pi = \mathbb{E}_{\tau\sim\pi}\{\sum_{r_t\sim\tau} \gamma^t r_t\} \in \mathcal{R}$

Goal: $\pi^* = \arg\max_\pi J_\pi$

Algorithm: $\mathbf{z}_0, \mathbf{u}_1, \mathbf{z}_1, \ldots$ => estimate $p(\mathbf{x}_t | \mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ $\overset{\pi(\mathbf{x}_t)}{=>}$ decide following action $\mathbf{u}_{t+1}$

perception (local, SLAM, object detection) control (planning, RL, opt.control

Localisation problem definition

Previous lecture only 1D/2D translations (no rotations)

States: $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_t \in \mathcal{R}^n$ .... ~~6DOF~~ robot's poses (no map for now)

Actions: $\mathbf{u}_1, \ldots, \mathbf{u}_t \in \mathcal{R}^m$ .... generated by external source

Measurements: $\mathbf{z}_1, \ldots, \mathbf{z}_t \in \mathcal{R}^k$ .... comes from variety of sensors

MAP: $\mathbf{x}^* = \arg\max_{\mathbf{x}} p(\mathbf{x} \,|\, \mathbf{z}, \mathbf{u}) = \arg\max_{\mathbf{x}_0 \ldots \mathbf{x}_t} p(\mathbf{x}_0 \ldots \mathbf{x}_t \,|\, \mathbf{z}_1 \ldots \mathbf{z}_t, \mathbf{u}_1 \ldots \mathbf{u}_t)$

Unknown

1. Construct p(x|z)
2. Optimize poses

Localisation problem definition

Today only 2D translations + 1D rotation

States: $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_t \in \mathcal{R}^n$ .... ~~6DOF~~ robot's poses (no map for now)

Actions: $\mathbf{u}_1, \ldots, \mathbf{u}_t \in \mathcal{R}^m$ .... generated by external source

Measurements: $\mathbf{z}_1, \ldots, \mathbf{z}_t \in \mathcal{R}^k$ .... comes from variety of sensors

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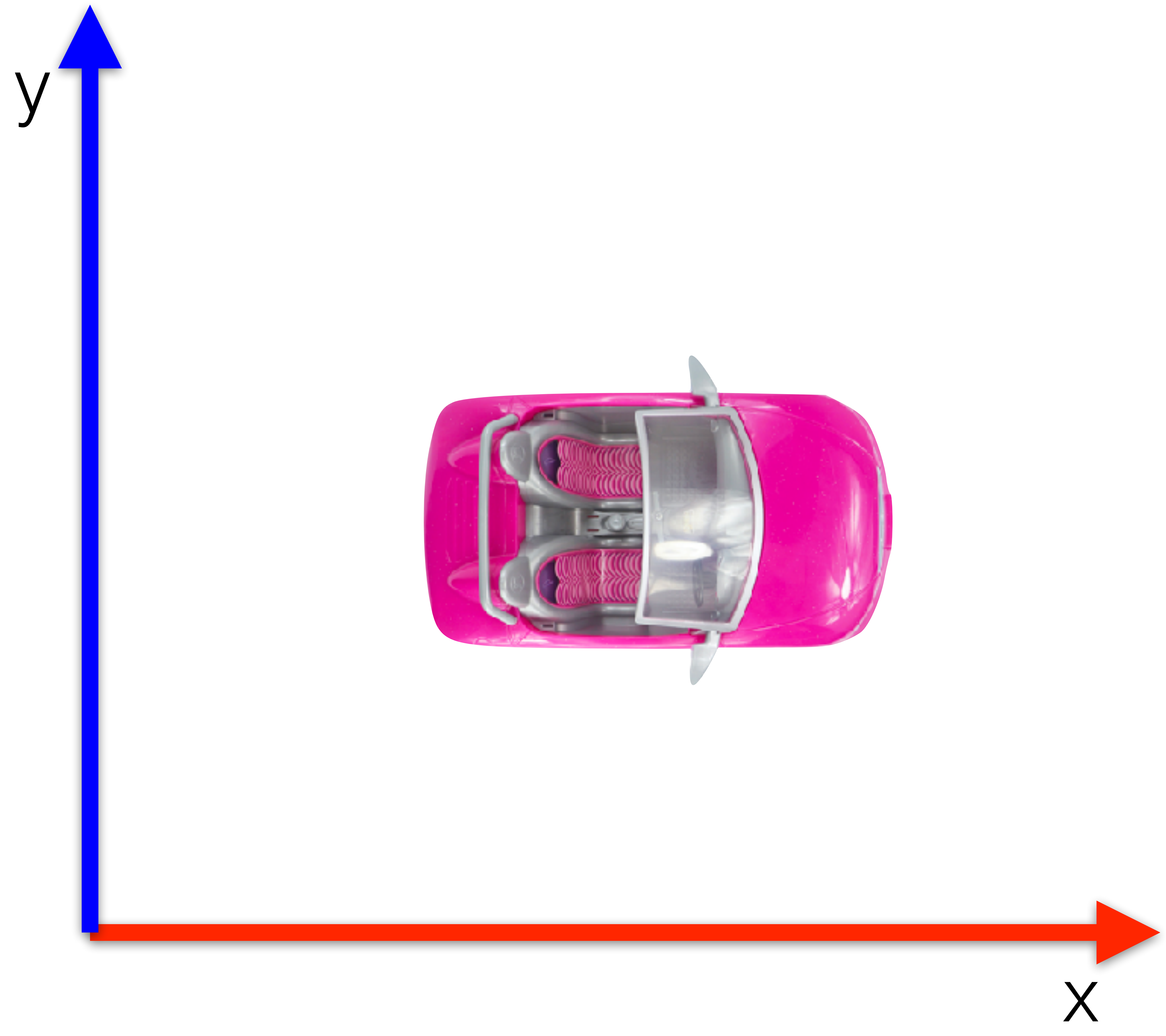
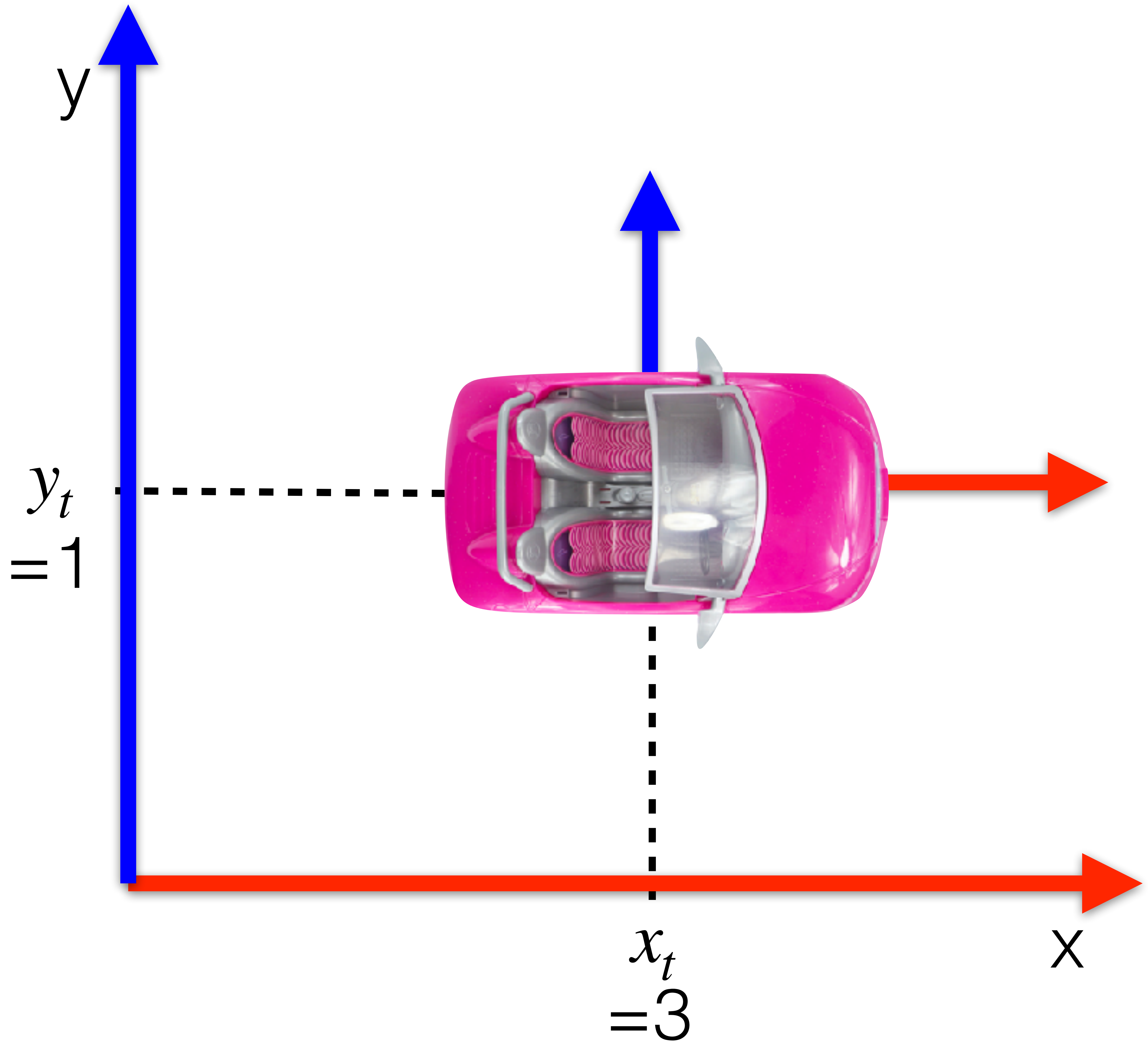
MAP: $\mathbf{x}^* = \arg\max_{\mathbf{x}} p(\mathbf{x} \,|\, \mathbf{z}, \mathbf{u}) = \arg\max_{\mathbf{x}_0 \ldots \mathbf{x}_t} p(\mathbf{x}_0 \ldots \mathbf{x}_t \,|\, \mathbf{z}_1 \ldots \mathbf{z}_t, \mathbf{u}_1 \ldots \mathbf{u}_t)$

1. Construct p(x|z)
2. Optimize poses

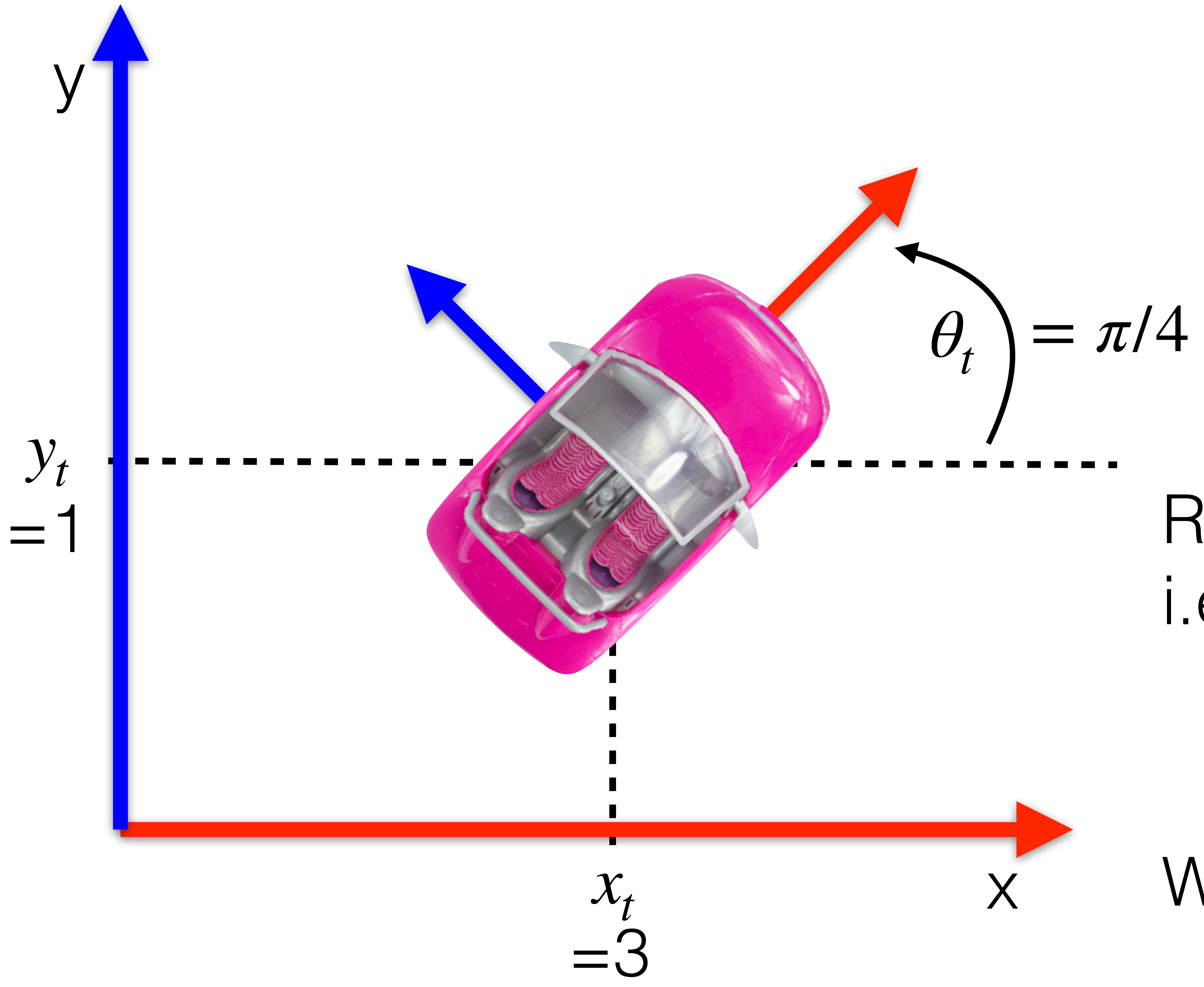


World coordinate frame (wcf)



Robot coordinate frame (rcf)
i.e. pose of robot in wcf

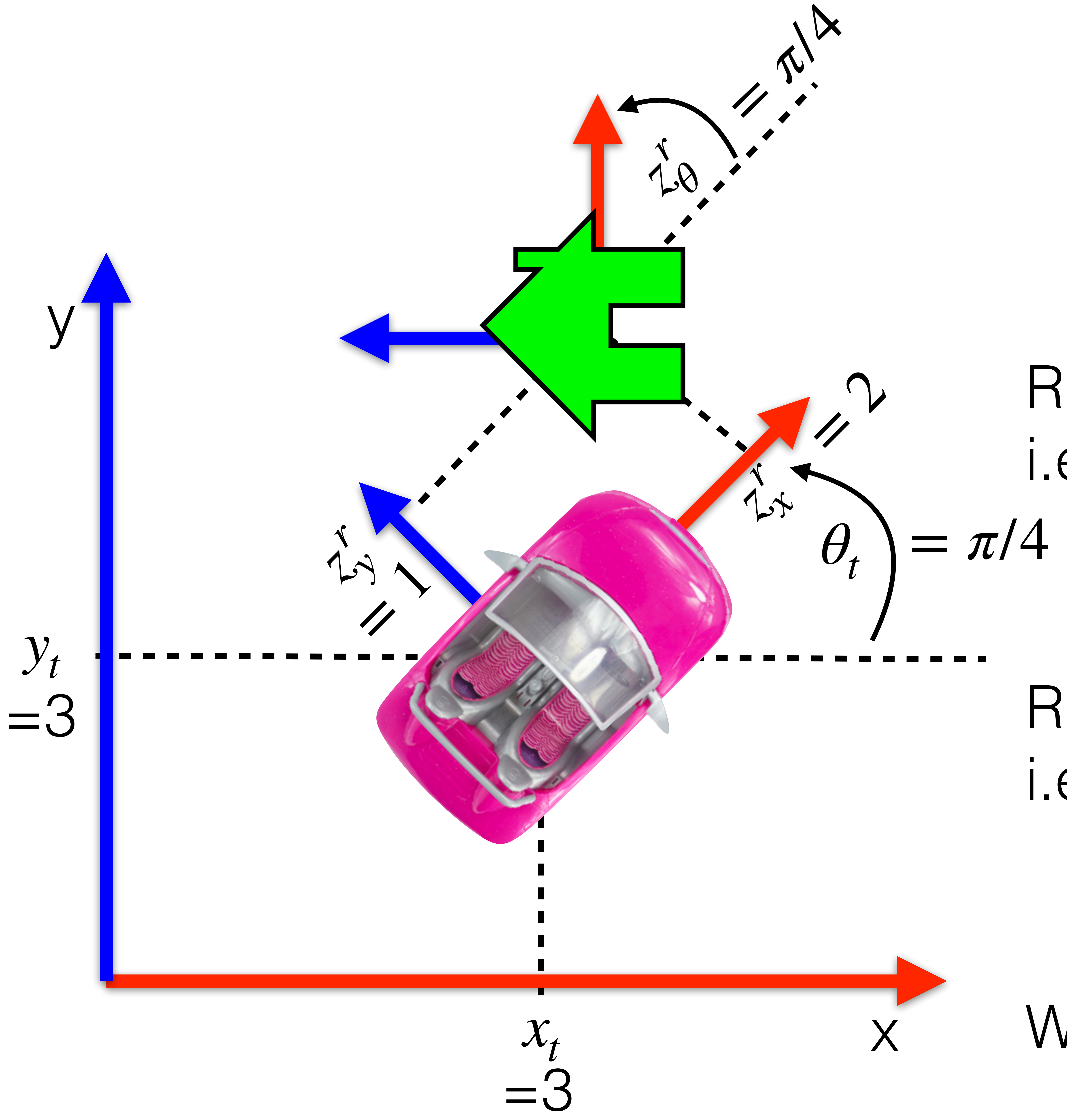
World coordinate frame (wcf)



Robot coordinate frame (rcf)
i.e. pose of robot in wcf

$$\mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}$$

World coordinate frame (wcf)



Robot sees (measures) house in rcf
i.e. pose of house in rcf

$$\mathbf{z} = \begin{bmatrix} z_x^r \\ z_y^r \\ z_\theta^r \end{bmatrix}$$

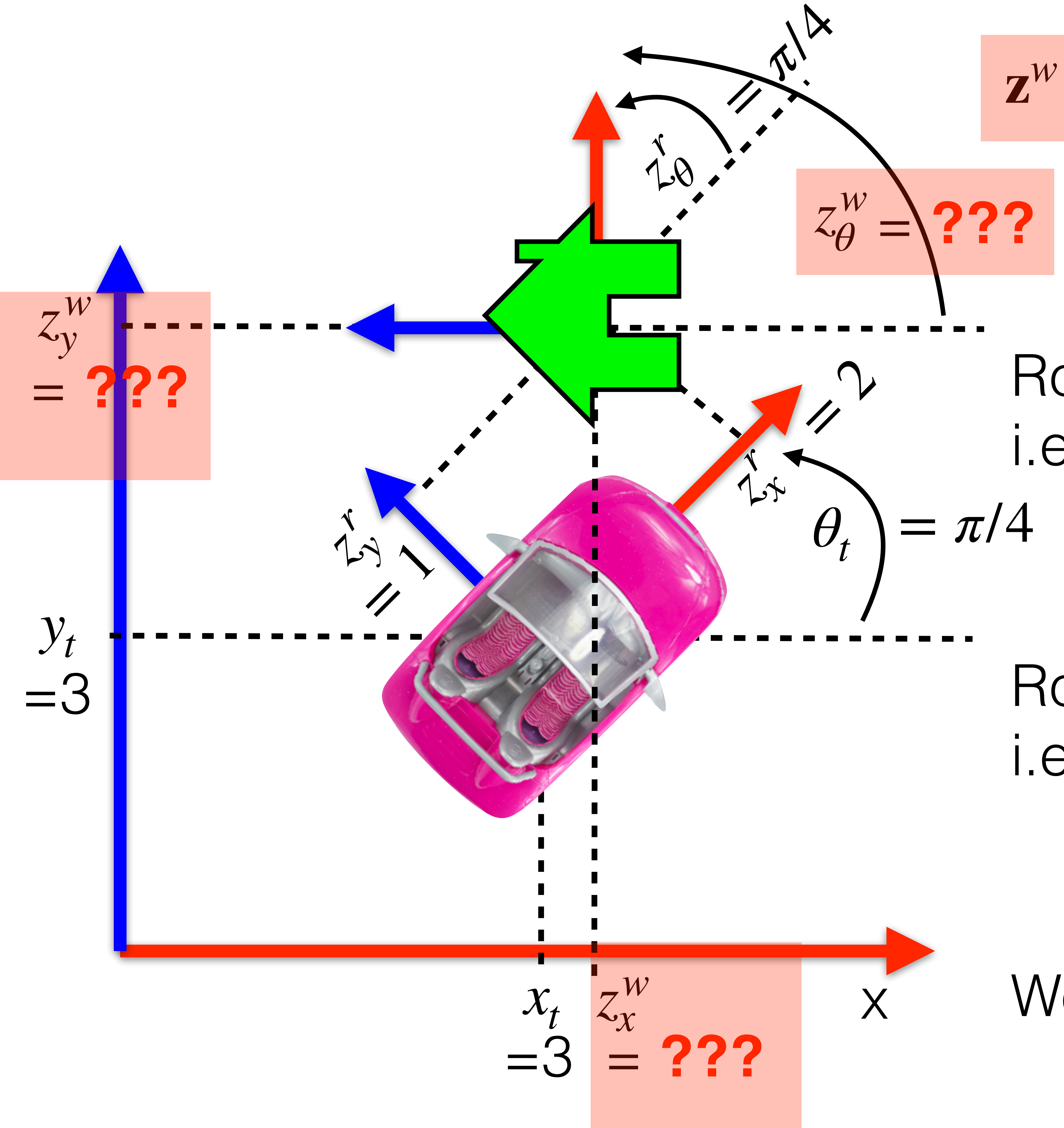
Robot coordinate frame (rcf)
i.e. pose of robot in wcf

$$\mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}$$

World coordinate frame (wcf)

Pose of house in wcf:

$$\mathbf{z}^w = \begin{bmatrix} z_x^w \\ z_y^w \\ z_\theta^w \end{bmatrix} = \begin{bmatrix} \cos\theta_t & -\sin\theta_t & 0 \\ \sin\theta_t & \cos\theta_t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_x^r \\ z_y^r \\ z_\theta^r \end{bmatrix} + \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}$$



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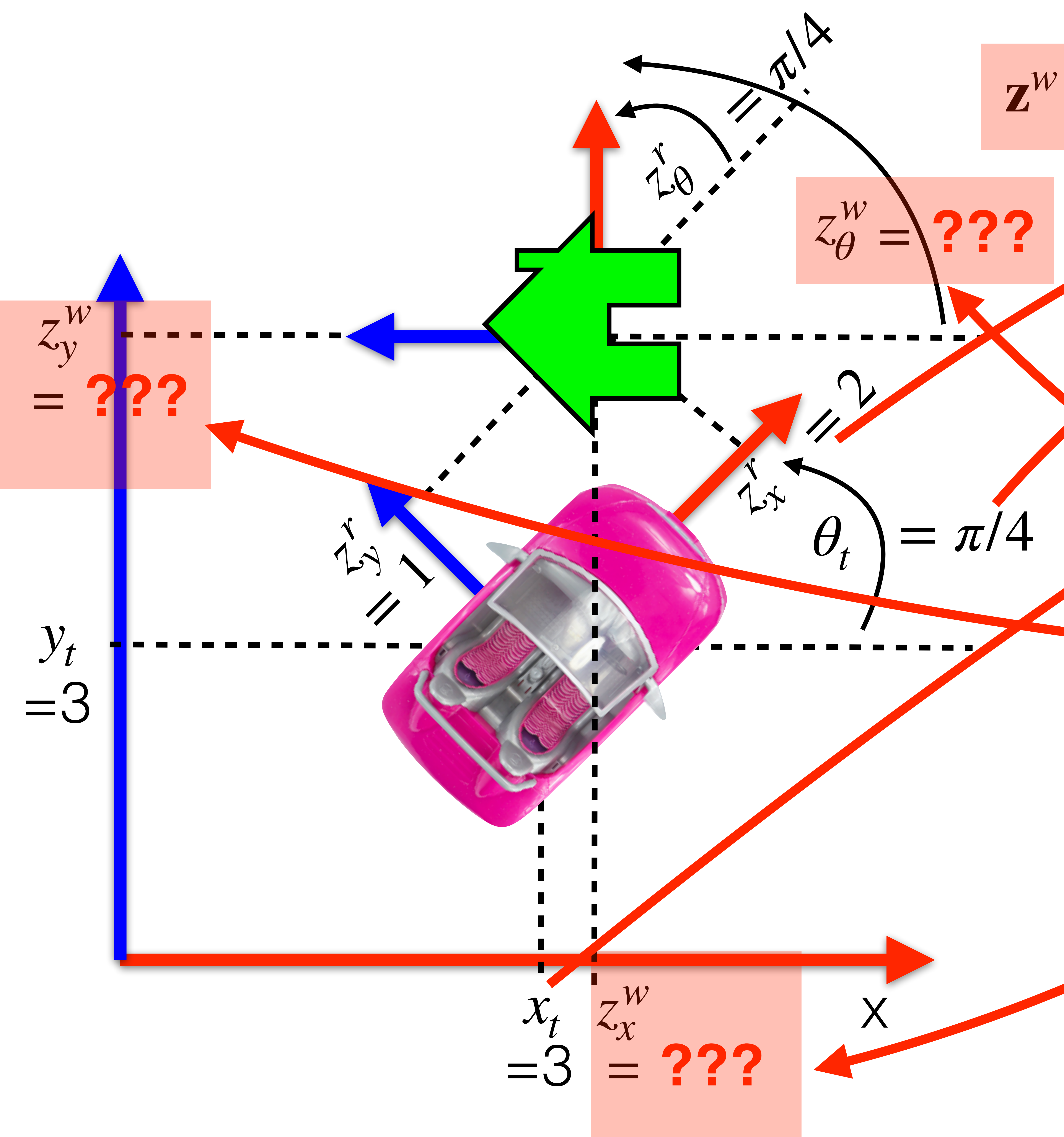
Pose of house in wcf:

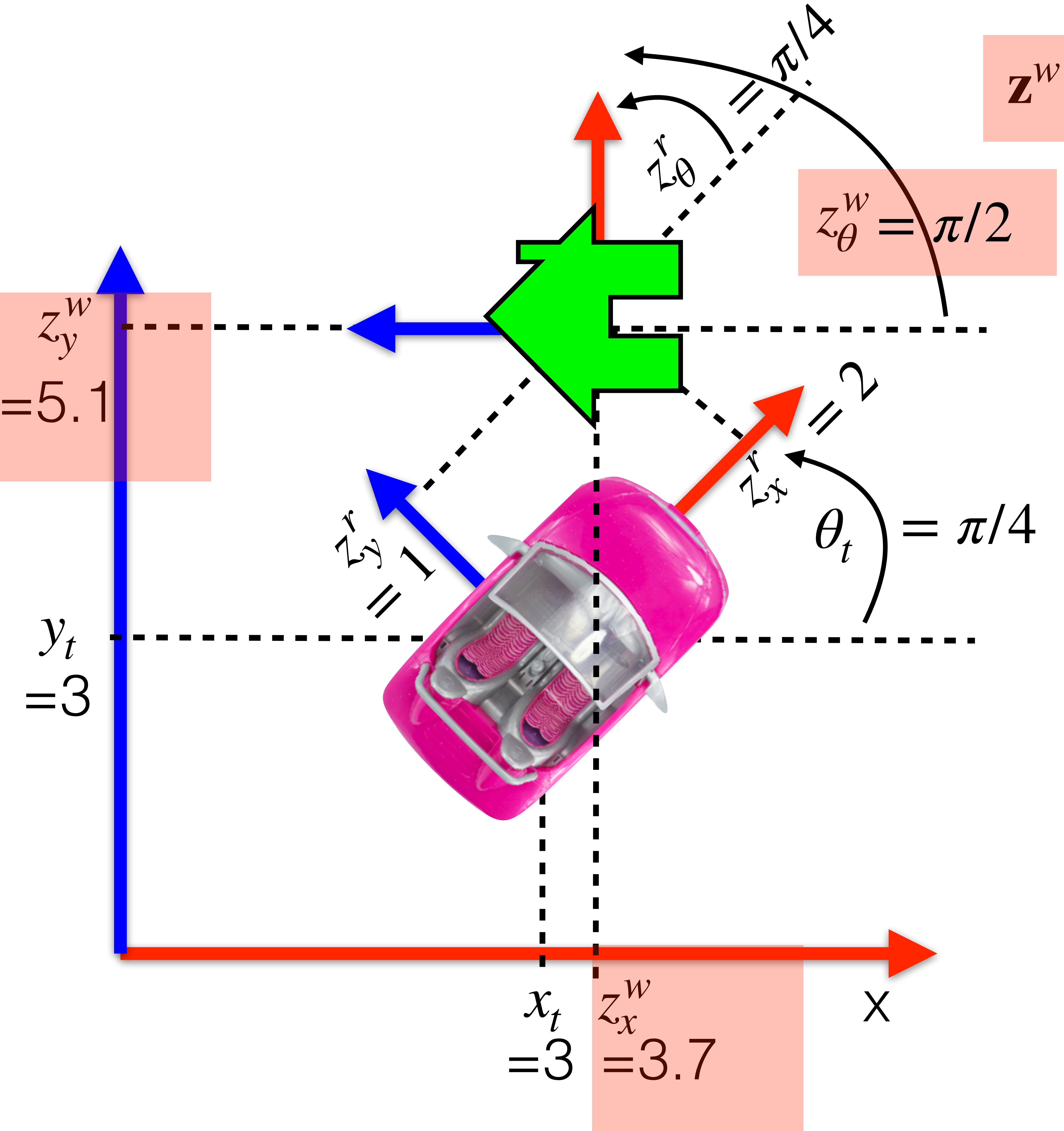


$$\mathbf{z}^w = \begin{bmatrix} z_x^w \\ z_y^w \\ z_\theta^w \end{bmatrix} = \begin{bmatrix} \cos\theta_t & -\sin\theta_t & 0 \\ \sin\theta_t & \cos\theta_t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_x^r \\ z_y^r \\ z_\theta^r \end{bmatrix} + \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}$$

$$= \begin{bmatrix} \cos\pi/4 & -\sin\pi/4 & 0 \\ \sin\pi/4 & \cos\pi/4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ \pi/4 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ \pi/4 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2}+3 \\ 3/\sqrt{2}+3 \\ \pi/2 \end{bmatrix} \approx \begin{bmatrix} 3.7 \\ 5.1 \\ \pi/2 \end{bmatrix}$$

Pose of house in wcf:



$$\mathbf{z}^w = \begin{bmatrix} z_x^w \\ z_y^w \\ z_\theta^w \end{bmatrix} = \begin{bmatrix} \cos\theta_t & -\sin\theta_t & 0 \\ \sin\theta_t & \cos\theta_t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_x^r \\ z_y^r \\ z_\theta^r \end{bmatrix} + \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}$$

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$$= \begin{bmatrix} 1/\sqrt{2} + 3 \\ 3/\sqrt{2} + 3 \\ \pi/2 \end{bmatrix} \approx \begin{bmatrix} 3.7 \\ 5.1 \\ \pi/2 \end{bmatrix}$$

Pose of the house transformed from rcf to wcf:

$$\mathbf{z}^w = \begin{bmatrix} z_x^w \\ z_y^w \\ z_\theta^w \end{bmatrix} = \begin{bmatrix} \cos\theta_t & -\sin\theta_t & 0 \\ \sin\theta_t & \cos\theta_t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_x^r \\ z_y^r \\ z_\theta^r \end{bmatrix} + \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} R(\theta_t) & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{z}^r + \mathbf{x}_t \; = T(\mathbf{z}^r, \mathbf{x}_t) \; = \mathrm{r2w}(\mathbf{z}^r, \mathbf{x}_t)$$

Pose of the house transformed from wcf to rcf:

$$\mathbf{z}^r = \begin{bmatrix} R(\theta_t)^\top & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix} (\mathbf{z}^w - \mathbf{x}_t) = T^{-1}(\mathbf{z}^w, \mathbf{x}_t) \; = \mathrm{w2r}(\mathbf{z}^w, \mathbf{x}_t)$$

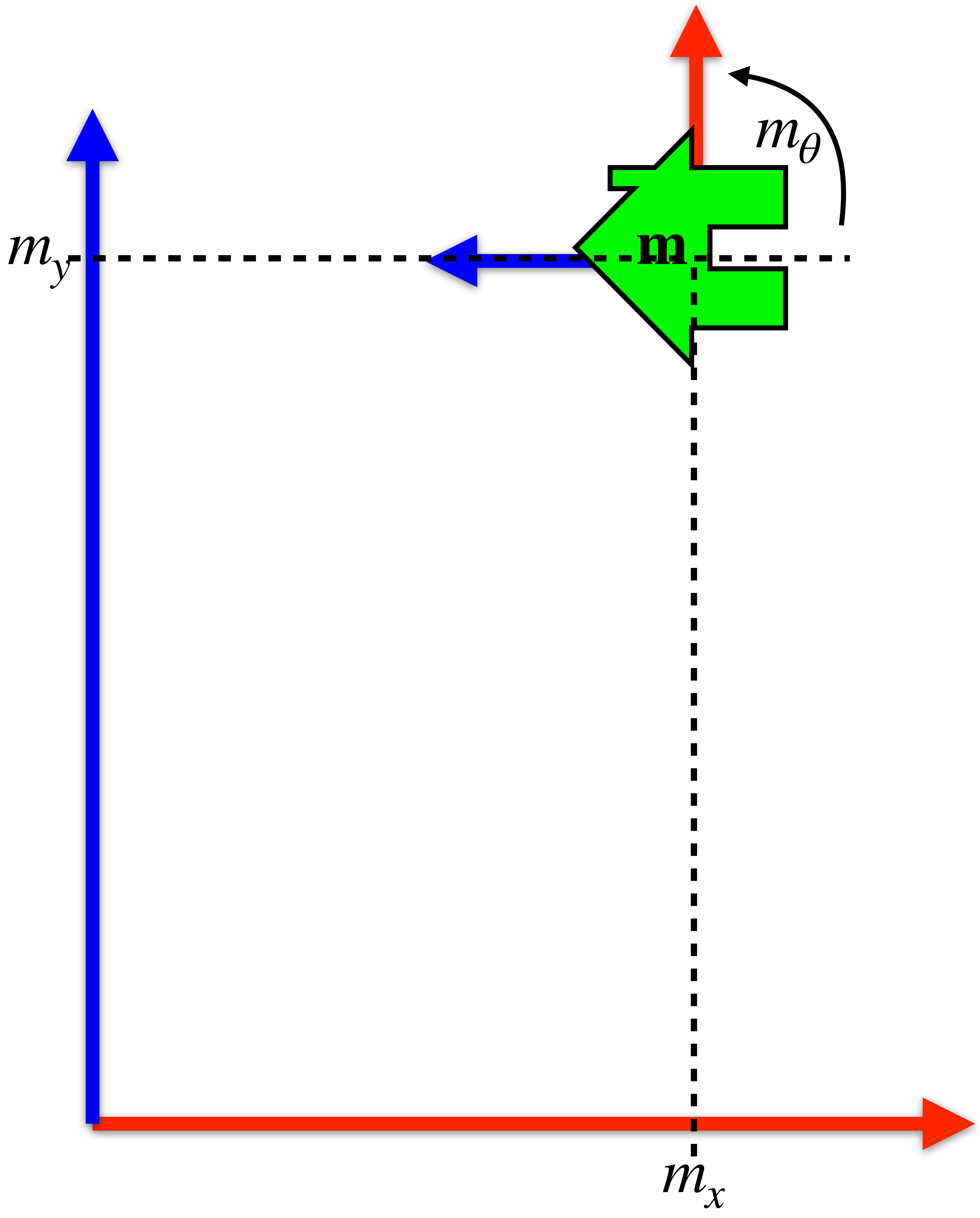
```
def r2w(z_r, x):
    z_w = torch.zeros(3)
    R = torch.vstack((torch.hstack((torch.cos(x[2]), -torch.sin(x[2]))), …
    z_w[0:2] = R @ z_r[0:2] + x[0:2]
    z_w[2] = z_r[2] + x[2]
    return z_w

def w2r(z_w, x):
    z_r = torch.zeros(3)
    R = torch.vstack((torch.hstack((torch.cos(x[2]), -torch.sin(x[2]))), …
    z_r[0:2] = R.t() @ (z_w[0:2] - x[0:2])
    z_r[2] = z_w[2] - x[2]
    return z_r
```

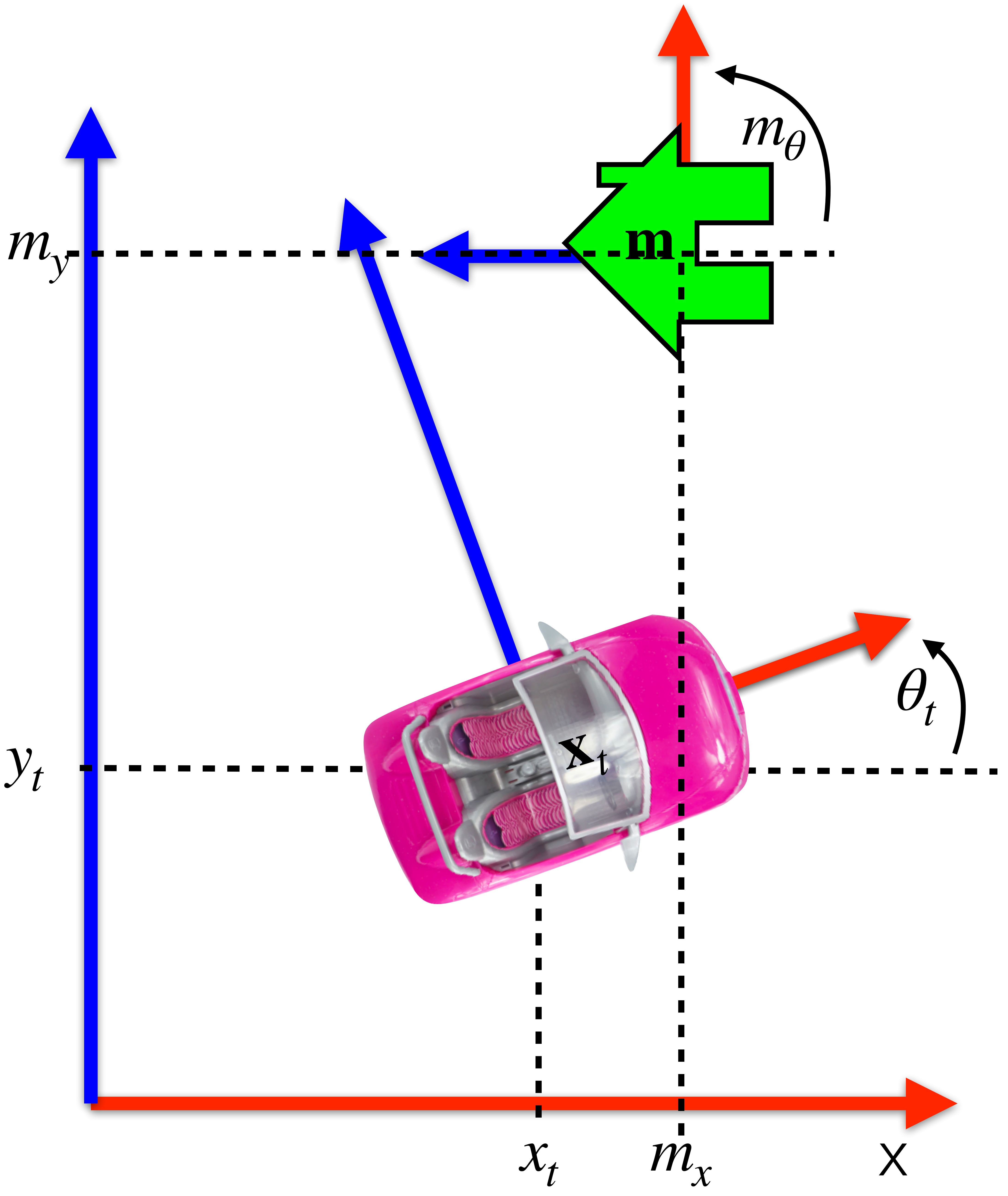
Localization of robot in wcf from known position of the house

Known marker pose in wcf

$$\mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_\theta \end{bmatrix}$$



Localization of robot in wcf from known position of the house



Known marker pose in wcf $\mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_\theta \end{bmatrix}$

Unknown robot pose in wcf $\mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}$

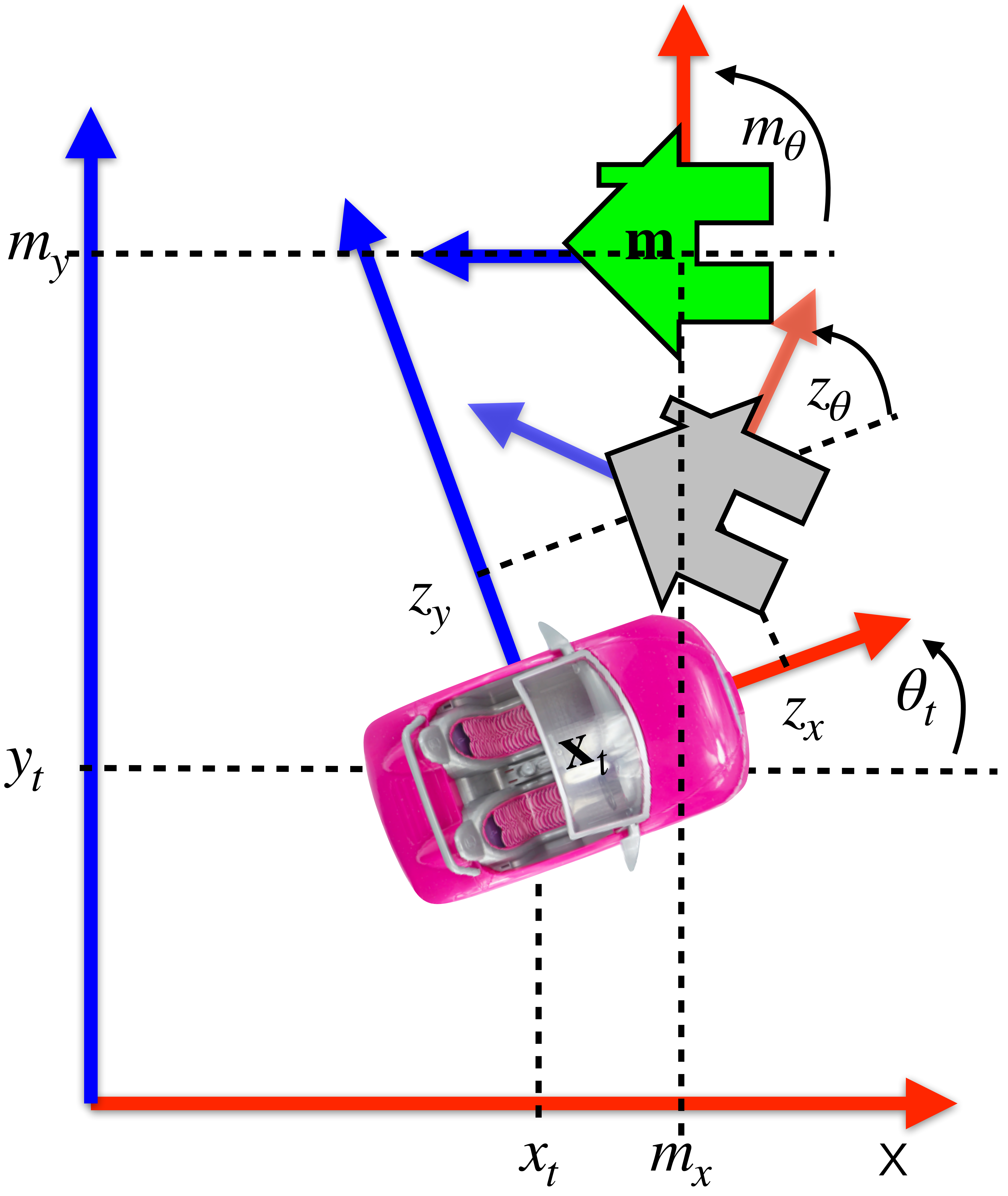
Robot is equipped by exteroceptive sensor:

RGBD camera

RGB camera

Lidar

Localization of robot in wcf from known position of the house



Known marker pose in wcf $\quad \mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_\theta \end{bmatrix}$

Unknown robot pose in wcf $\quad \mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}$

Is this pose $\mathbf{x}_t$ correct?

Robot measures the house in rcf $\quad \mathbf{z} = \begin{bmatrix} z_x^r \\ z_y^r \\ z_\theta^r \end{bmatrix}$

What is the correct pose of robot?

Robot is equipped by exteroceptive sensor:

RGBD camera

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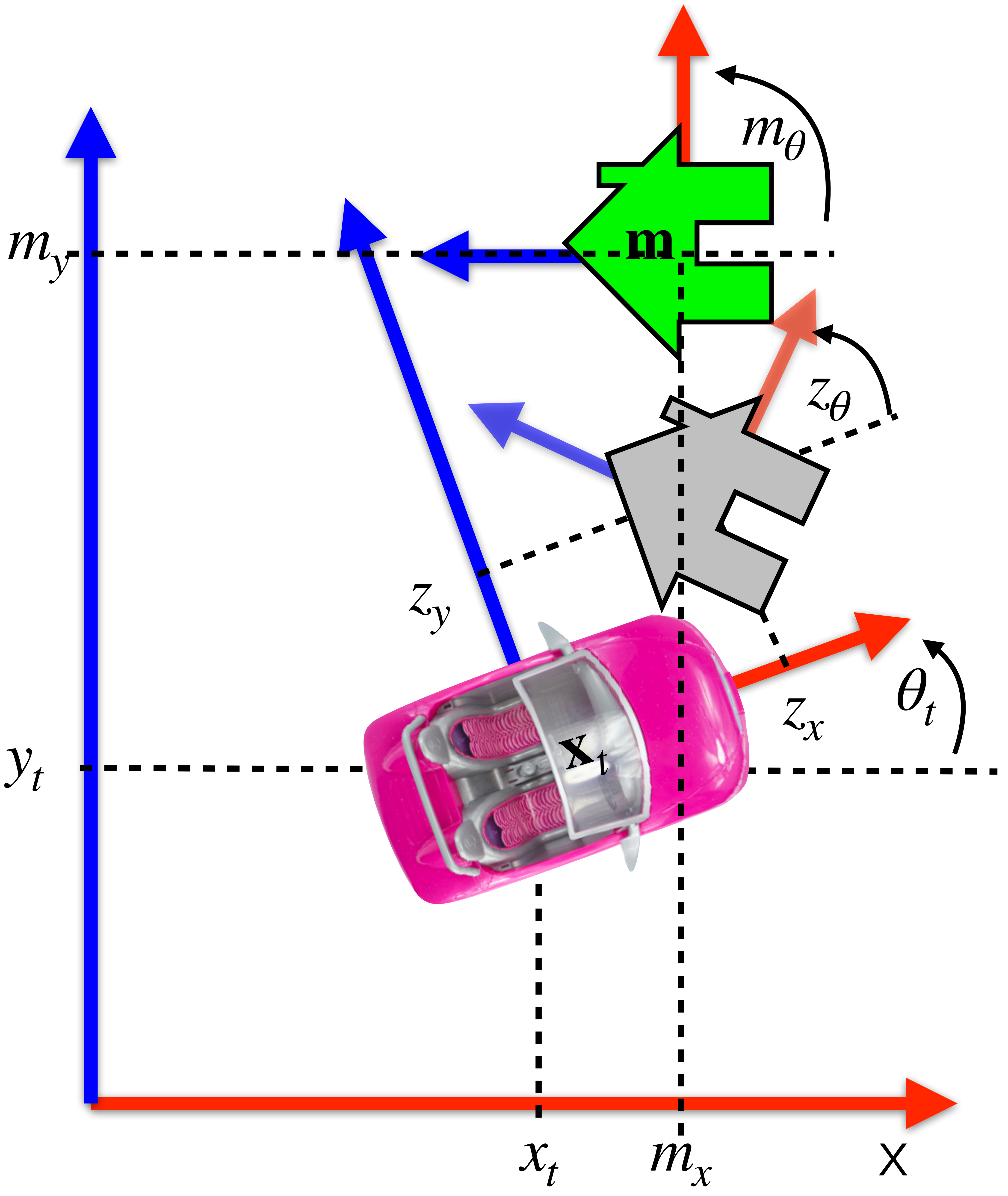
Localization of robot in wcf from known position of the house



Known marker pose in wcf $\quad \mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_\theta \end{bmatrix}$

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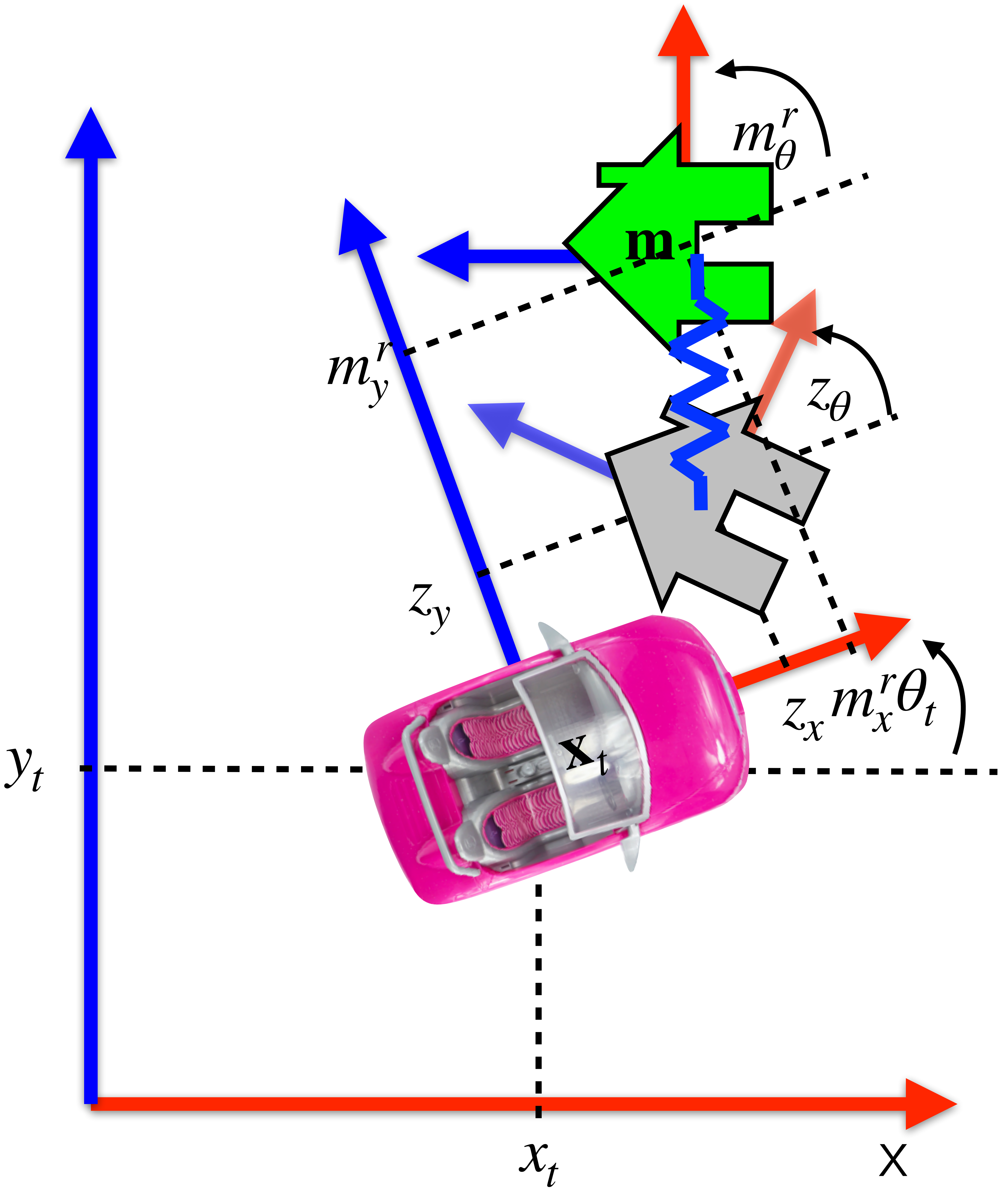
What is the correct pose of robot?

Correct pose is the one that aligns [grey house] with [green house] but they are in different coordinate frames!

Which coordinate frame should I use to measure their distance?

Localization of robot in wcf from known position of the house



Known marker pose in wcf $\quad \mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_\theta \end{bmatrix}$

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Is this pose $\mathbf{x}_t$ correct?

Robot measures the house in rcf $\quad \mathbf{z} = \begin{bmatrix} z_x^r \\ z_y^r \\ z_\theta^r \end{bmatrix}$

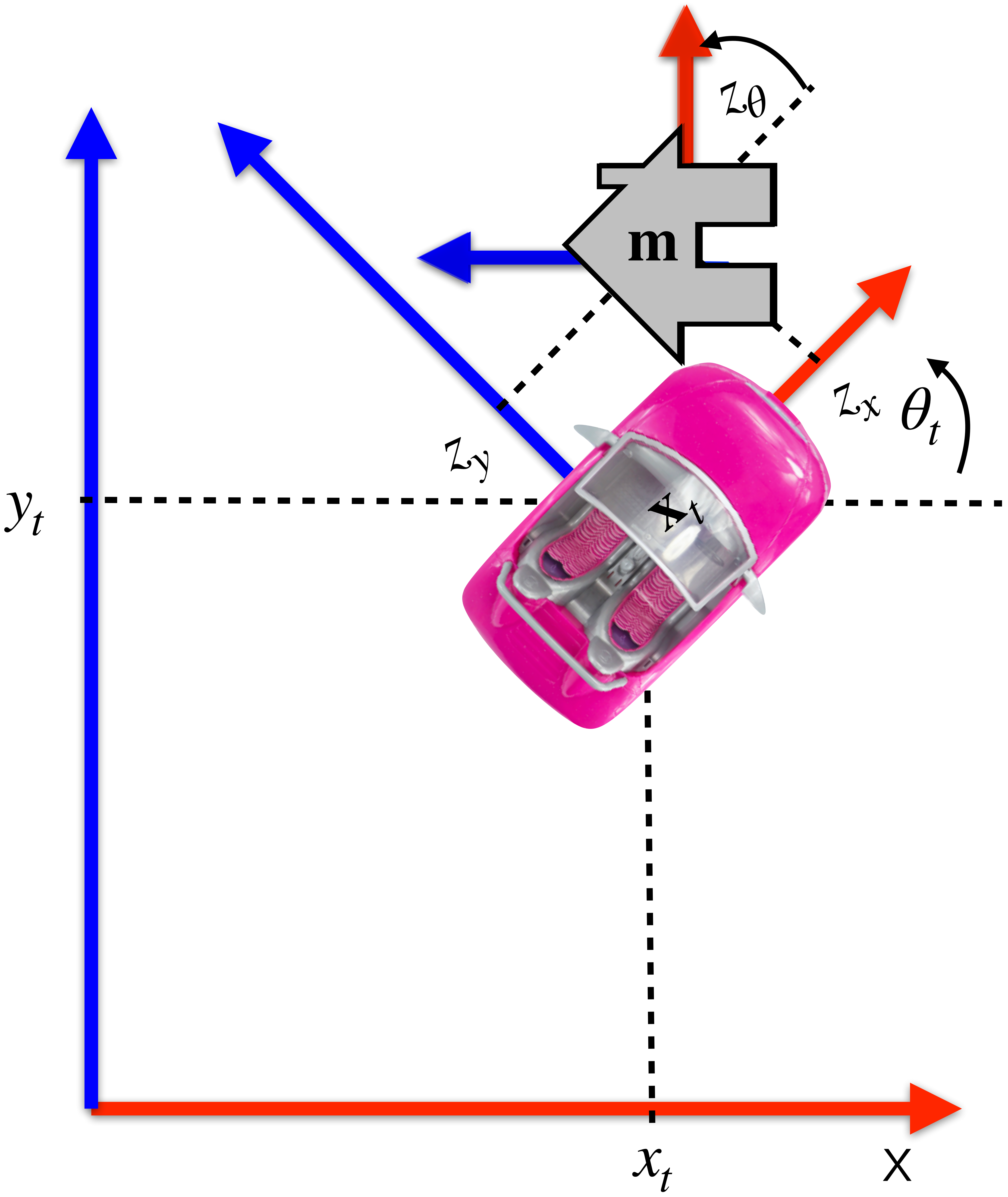
What is the correct pose of robot?

Correct pose is the one that aligns  with 
but they are in different coordinate frames!

Marker pose in rcf $\quad \mathbf{m}^r = \mathrm{w2r}(\mathbf{m}, \mathbf{x}_t)$

$$\mathbf{x}_t^\star = \arg\min_{\mathbf{x}_t} \|\mathbf{m}^r - \mathbf{z}\|^2 = \arg\min_{\mathbf{x}_t} \|\mathrm{w2r}(\mathbf{m}, \mathbf{x}_t) - \mathbf{z}\|^2$$

Localization of robot in wcf from known position of the house



Known marker pose in wcf $\quad \mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_\theta \end{bmatrix}$

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Is this pose $\mathbf{x}_t$ correct?

Robot measures the house in rcf $\quad \mathbf{z} = \begin{bmatrix} z^r_x \\ z^r_y \\ z^r_\theta \end{bmatrix}$

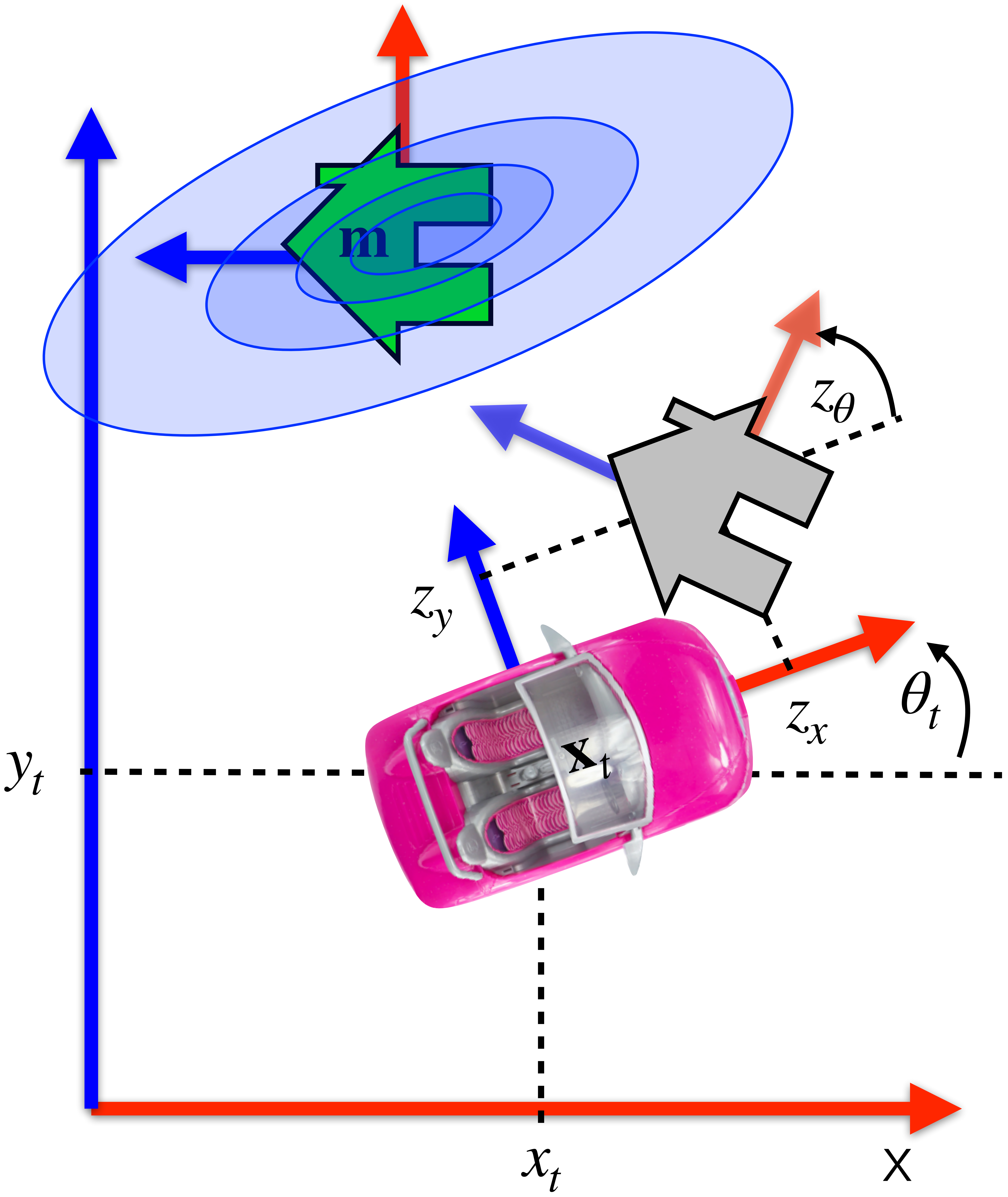
What is the correct pose of robot?

Correct pose is the one that aligns (grey house) with (green house) but they are in different coordinate frames!

Marker pose in rcf $\quad \mathbf{m}^r = \mathrm{w2r}(\mathbf{m}, \mathbf{x}_t)$

$$\mathbf{x}^\star_t = \arg\min_{\mathbf{x}_t} \|\mathbf{m}^r - \mathbf{z}\|^2 = \arg\min_{\mathbf{x}_t} \|\mathrm{w2r}(\mathbf{m}, \mathbf{x}_t) - \mathbf{z}\|^2$$
$$= \arg\min_{\mathbf{x}_t} \|\mathrm{r2w}(\mathbf{z}, \mathbf{x}_t) - \mathbf{m}\|^2$$

Localization of robot in wcf from known position of the house

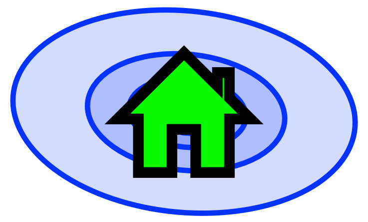


We completely ignored measurement inaccuracy

Given measurements $\mathbf{z}$ in rcf are normally distrib.

$$p(\mathbf{z}\,|\,\mathbf{x}_t) = \mathcal{N}(\mathbf{z};\ \underbrace{\mathrm{w2r}(\mathbf{m}, \mathbf{x}_t)}_{\mathbf{m}^r},\ \Sigma)$$

Dimensionality?

Correct pose maximize prob of  in 

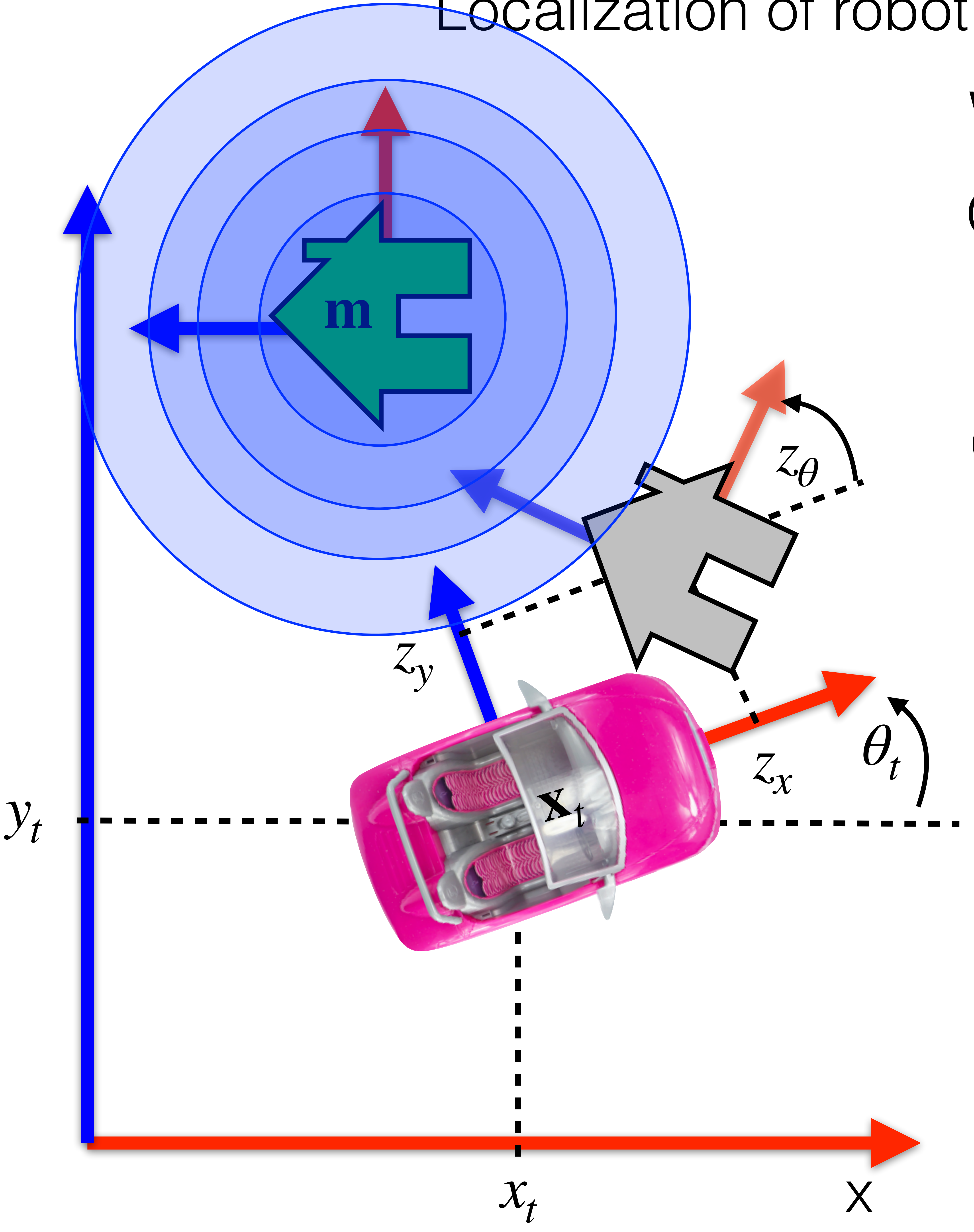
$$\mathbf{x}_t^\star = \arg\max_{\mathbf{x}_t} \mathcal{N}(\mathbf{z};\ \mathrm{w2r}(\mathbf{m}, \mathbf{x}_t),\ \Sigma)$$

$$= \arg\min_{\mathbf{x}_t} \|\mathrm{w2r}(\mathbf{m}, \mathbf{x}_t) - \mathbf{z}\|_\Sigma^2$$

$$\neq \arg\min_{\mathbf{x}_t} \|\mathrm{r2w}(\mathbf{z}, \mathbf{x}_t) - \mathbf{m}\|_\Sigma^2$$

… **why ???**

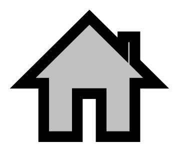
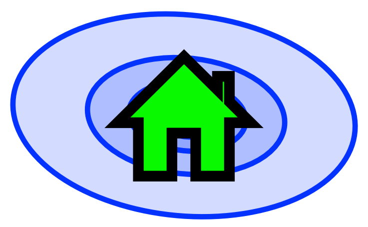
Localization of robot in wcf from known position of the house



We completely ignored measurement inaccuracy

Given $\mathbf{m}^r$ measurements will be normally distrib.

$$p(\mathbf{z}\,|\,\mathbf{x}_t) = \mathcal{N}(\mathbf{z};\ \underbrace{\mathrm{w2r}(\mathbf{m}, \mathbf{x}_t)}_{\mathbf{m}^r},\ \Sigma)$$

Correct pose maximize prob of  in 

$$\begin{aligned}\mathbf{x}_t^\star &= \arg\max_{\mathbf{x}_t} \mathcal{N}(\mathbf{z};\ \mathrm{w2r}(\mathbf{m}, \mathbf{x}_t),\ \Sigma)\\ &= \arg\min_{\mathbf{x}_t} \|\mathrm{w2r}(\mathbf{m}, \mathbf{x}_t) - \mathbf{z}\|_\Sigma^2\\ &\neq \arg\min_{\mathbf{x}_t} \|\mathrm{r2w}(\mathbf{z}, \mathbf{x}_t) - \mathbf{m}\|_\Sigma^2\end{aligned}$$

… **why ???**

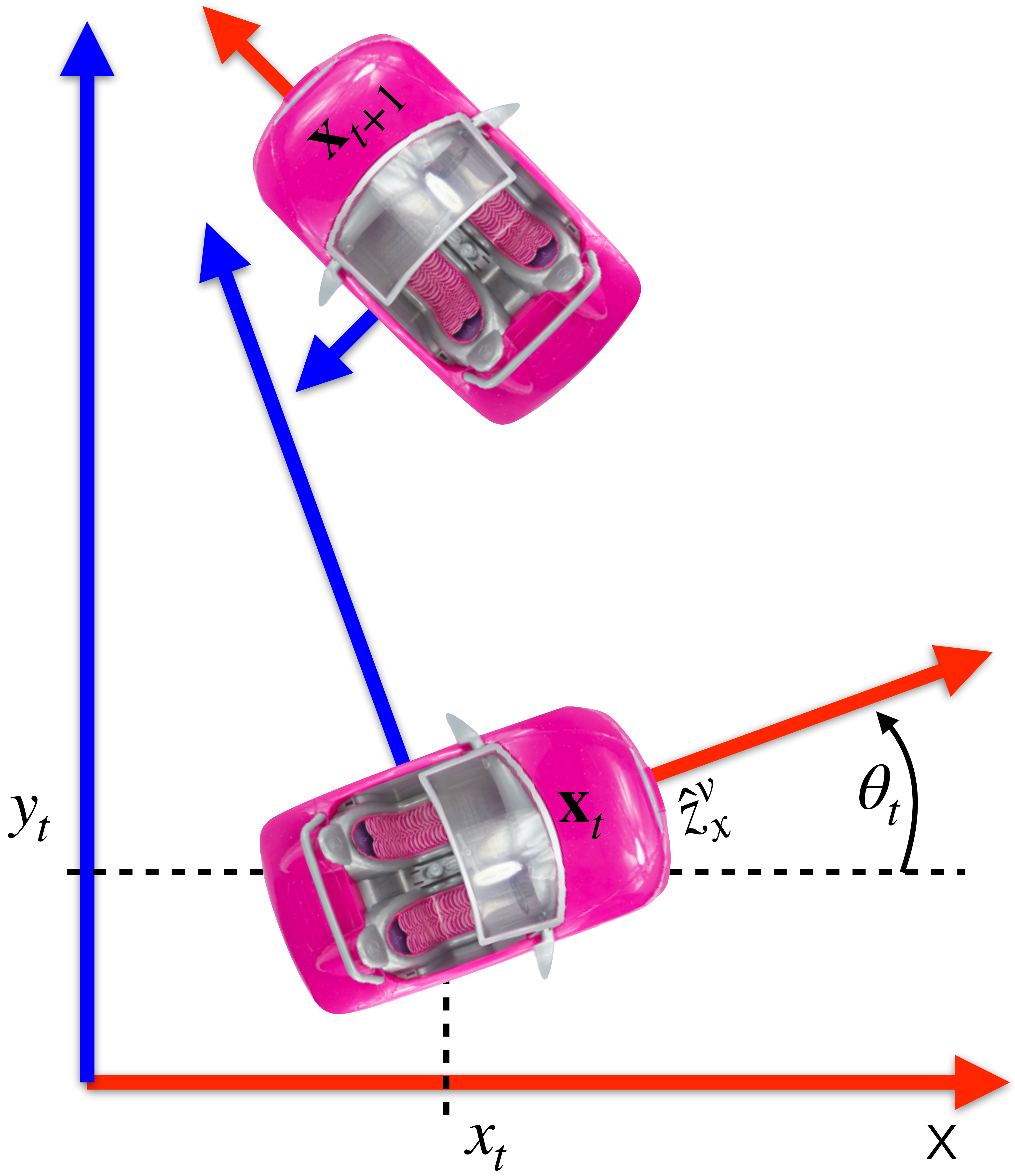
Especially if: $\Sigma = \frac{1}{c}\cdot \mathbf{I} = \begin{bmatrix} 1/c & 0 & 0\\ 0 & 1/c & 0 \\ 0 & 0 & 1/c\end{bmatrix}$

Then: $\mathbf{x}_t^\star = \arg\min_{\mathbf{x}_t} c\cdot \|\mathrm{w2r}(\mathbf{m}, \mathbf{x}_t) - \mathbf{z}\|^2$ rcf

$= \arg\min_{\mathbf{x}_t} c\cdot \|\mathrm{r2w}(\mathbf{z}, \mathbf{x}_t) - \mathbf{m}\|^2$ wcf

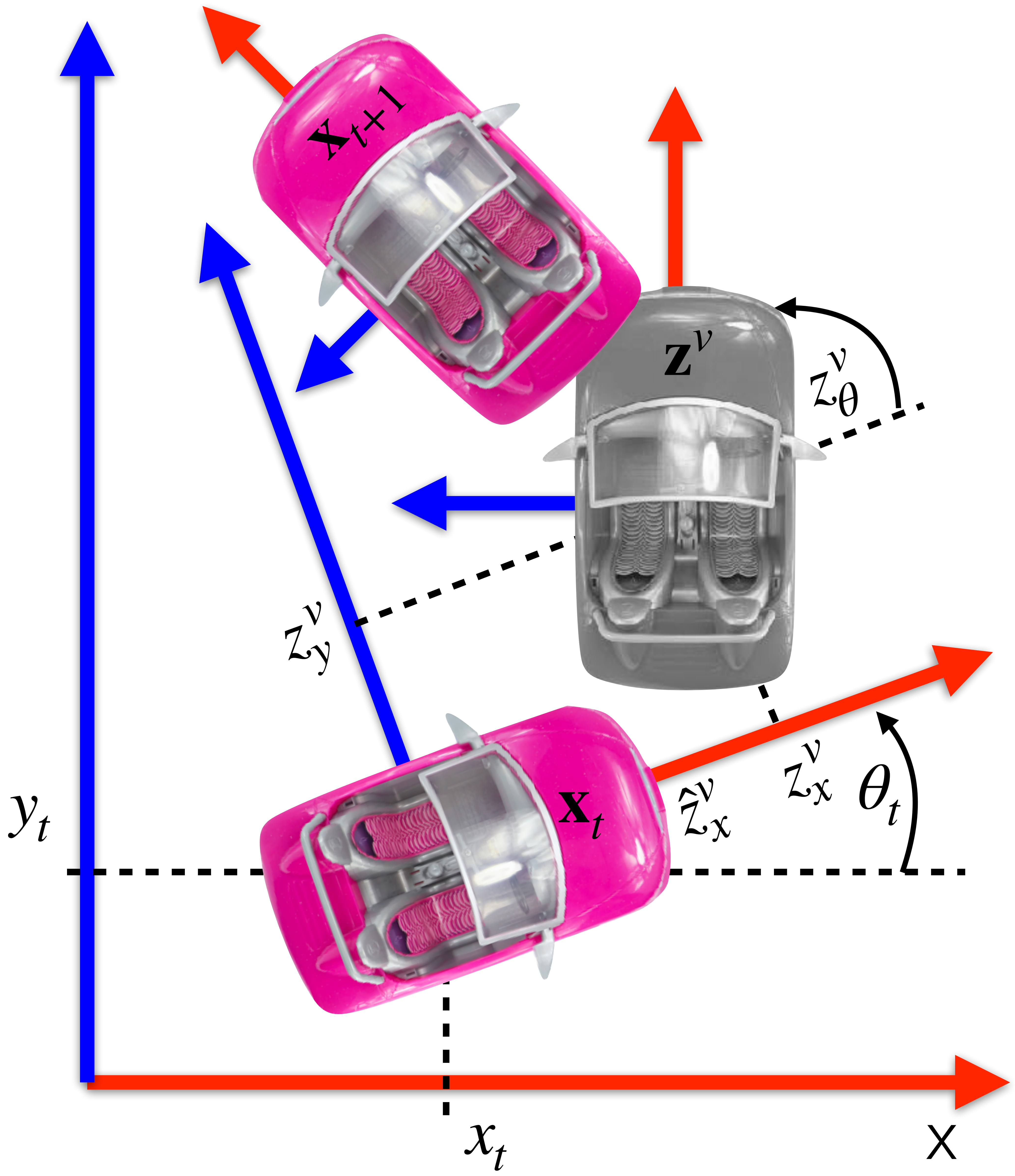
Localization of robot in wcf from known odometry

Odometry represented by linear+angular velocity



Robot poses in wcf: $\mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} \quad \mathbf{x}_{t+1} = \begin{bmatrix} x_{t+1} \\ y_{t+1} \\ \theta_{t+1} \end{bmatrix}$

Localization of robot in wcf from known odometry



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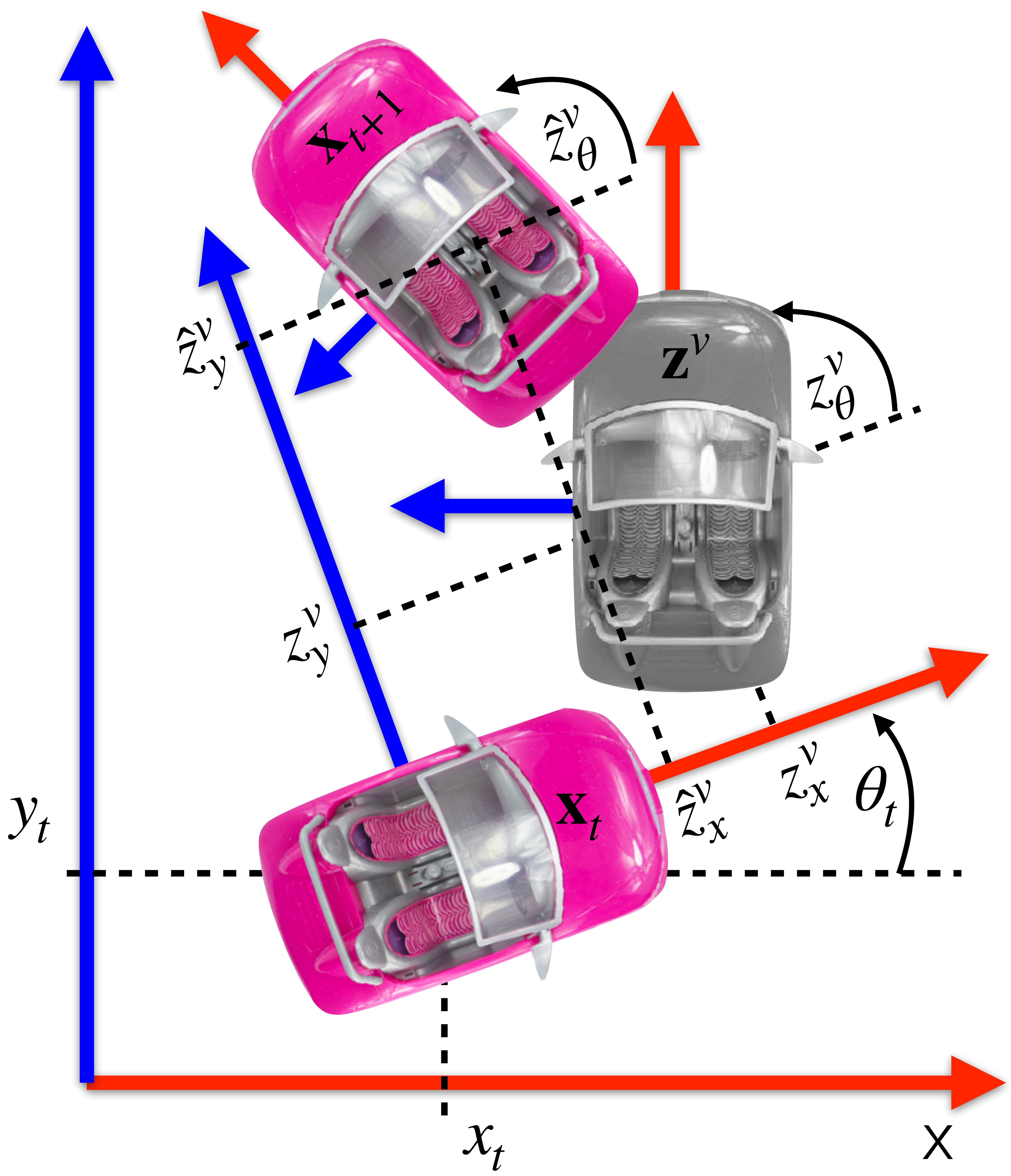
Robot measures velocity in $\mathbf{x}_t$-rcf: $\mathbf{z}^v = \begin{bmatrix} z_x^v \\ z_y^v \\ z_\theta^v \end{bmatrix}$

Localization of robot in wcf from known odometry



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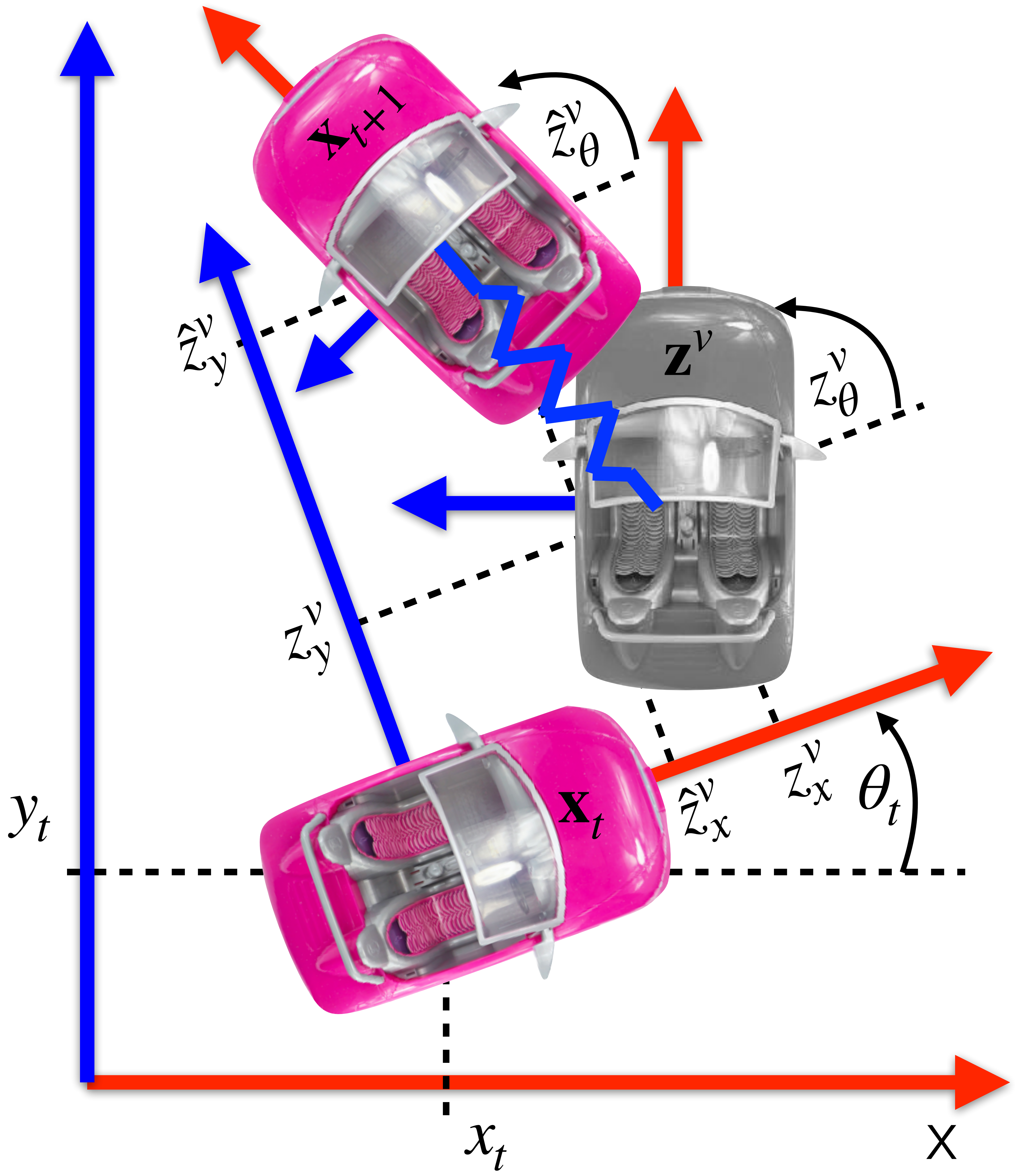
Robot measures velocity in $\mathbf{x}_t$-rcf: $\mathbf{z}^v = \begin{bmatrix} z^v_x \\ z^v_y \\ z^v_\theta \end{bmatrix}$

Next pose $\mathbf{x}_{t+1}$ in $\mathbf{x}_t$-rcf: $\hat{\mathbf{z}}^v = \mathrm{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t)$

What is the correct pose of robot?

Localization of robot in wcf from known odometry



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Next pose $\mathbf{x}_{t+1}$ in $\mathbf{x}_t$-rcf: $\hat{\mathbf{z}}^v = \text{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t)$

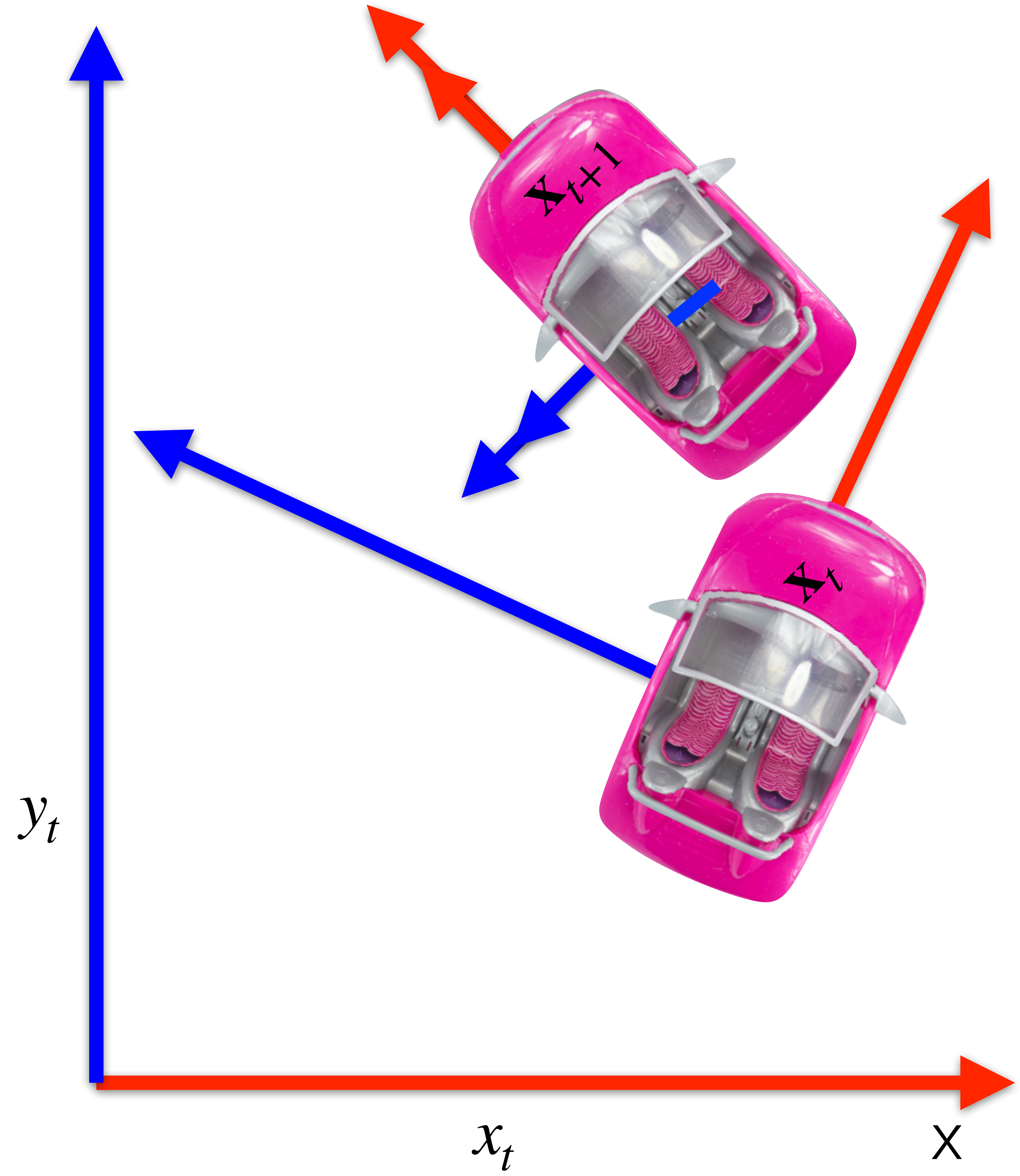
What is the correct pose of robot?

Find the correct poses

$$\mathbf{x}^\star_t, \mathbf{x}^\star_{t+1} = \arg\max_{\mathbf{x}_t, \mathbf{x}_{t+1}} \mathcal{N}(\mathbf{z}^v;\ \text{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t), \Sigma^v)$$

$$= \arg\min_{\mathbf{x}_t, \mathbf{x}_{t+1}} \|\text{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t) - \mathbf{z}^v\|^2_{\Sigma^v}$$

Localization of robot in wcf from known odometry



Odometry represented by linear+angular velocity

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Find the correct poses

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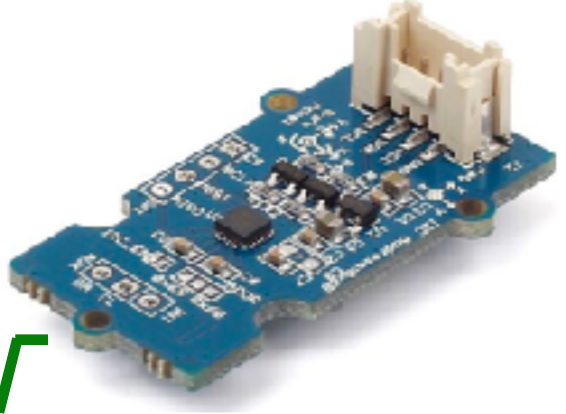
$$= \arg\min_{\mathbf{x}_t, \mathbf{x}_{t+1}} \|\text{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t) - \mathbf{z}^v\|_{\Sigma^v}^2$$

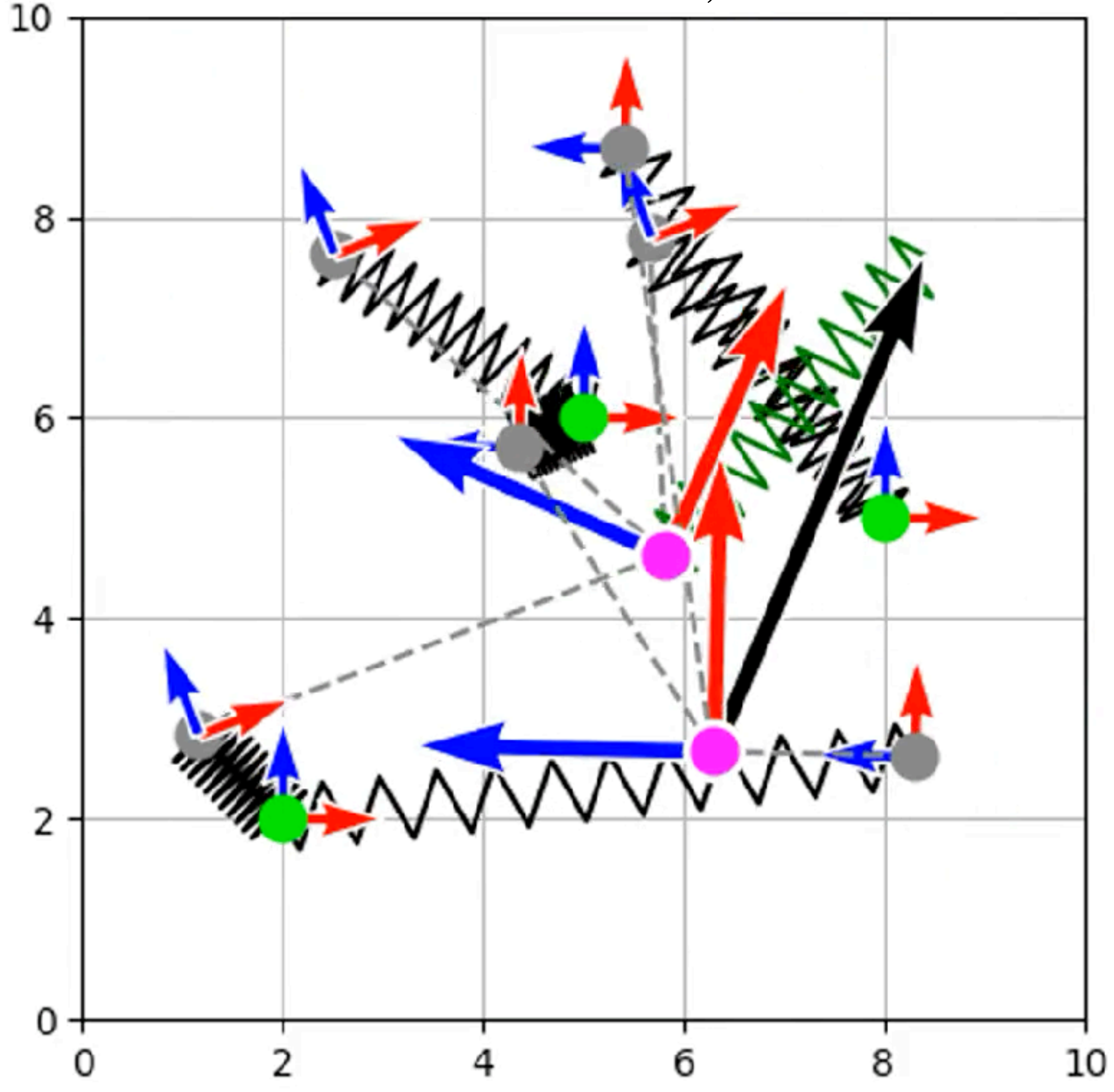
Localization

3D marker detector (RGBD camera) +



Odometry (IMU)

$$\mathbf{x}^\star = \arg\min_{\mathbf{x}_t} \sum_{i,t} \|\mathrm{w2r}(\mathbf{m}_i, \mathbf{x}_t) - \mathbf{z}_t^{\mathbf{m}_i}\|^2 + \|\mathrm{w2r}(\mathbf{x}_2, \mathbf{x}_1) - \mathbf{z}_{12}^{odom}\|^2$$



$\mathbf{x}_t$ … robot poses

$\mathbf{m}_i$ … known marker positions

$\mathbf{z}_t^{\mathbf{m}_i}$ … marker measurements

…… local coordinate frame

…… odometry

$\sum_{i,t} \|\mathrm{w2r}(\mathbf{m}_i, \mathbf{x}_t) - \mathbf{z}_t^{\mathbf{m}_i}\|^2$ … marker loss

$\|\mathrm{w2r}(\mathbf{x}_2, \mathbf{x}_1) - \mathbf{z}_{12}^{odom}\|^2$ … odom loss

Localization of robot in wcf from known marker pose, odometry and GPS

GPS | odometry | marker

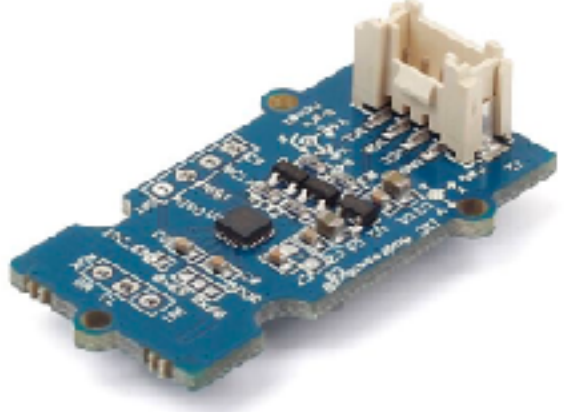
$$\mathbf{x}^\star = \arg\max_{\mathbf{x}_0,\dots\mathbf{x}_t} \prod_t p(\mathbf{z}_t^{gps} \,|\, \mathbf{x}_t) \cdot \prod_t p(\mathbf{z}_t^{v} \,|\, \mathbf{x}_t, \mathbf{x}_{t-1}) \cdot \prod_t p(\mathbf{z}_t^{m} \,|\, \mathbf{x}_t, \mathbf{m})$$

$$= \arg\max_{\mathbf{x}_0,\dots\mathbf{x}_t} \prod_t \mathcal{N}(\mathbf{z}^{gps};\; \mathbf{x}_t, \Sigma_t^{gps}) \cdot \prod_t \mathcal{N}(\mathbf{z}^{v};\; \mathrm{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t), \Sigma_t^{v}) \cdot \prod_t \mathcal{N}(\mathbf{z}^{m};\; \mathrm{w2r}(\mathbf{m}, \mathbf{x}_t), \Sigma_t^{m})$$

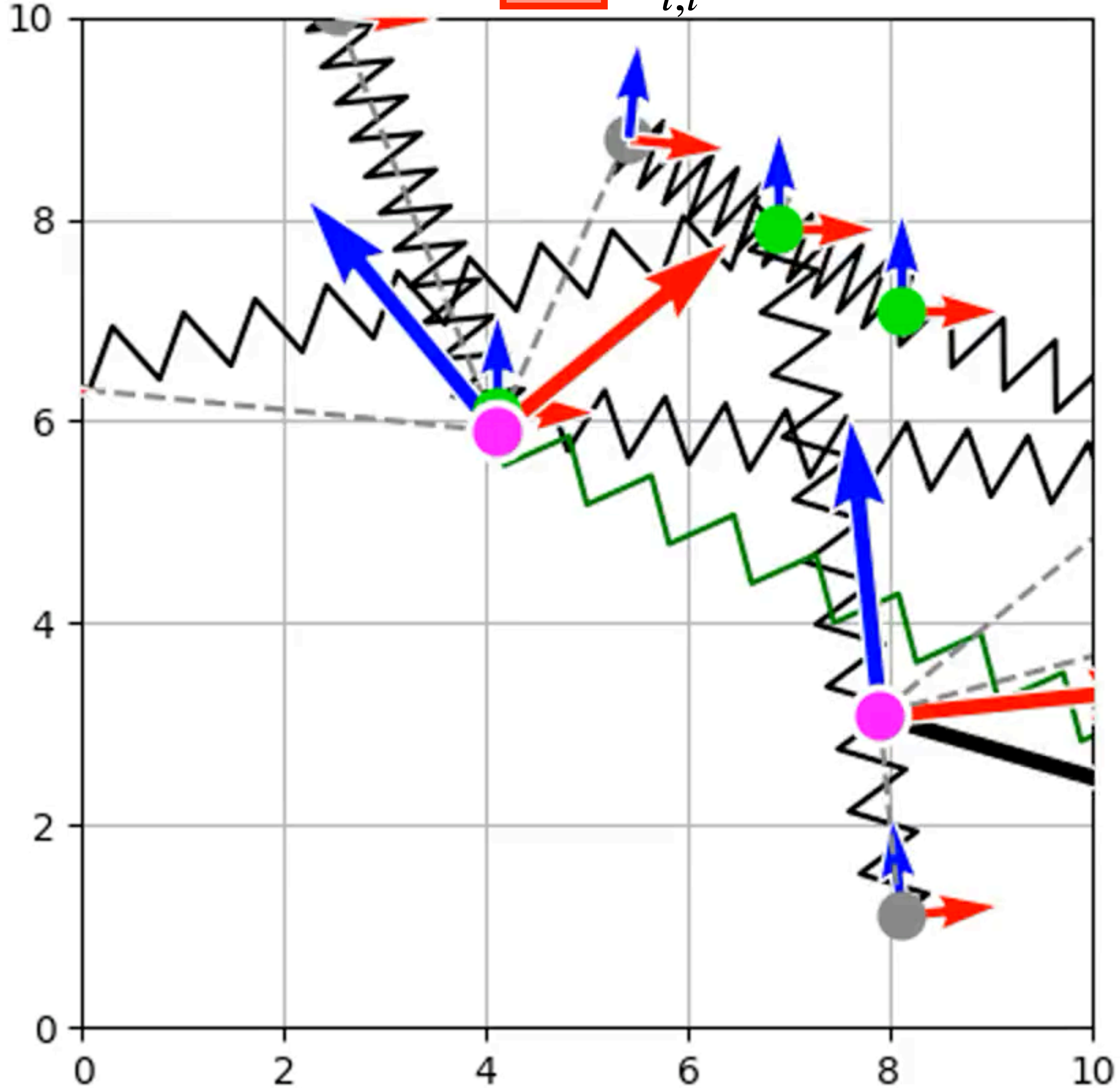
$$= \arg\min_{\substack{\mathbf{x}_0,\dots\mathbf{x}_T \\ \mathbf{m}^1\dots\mathbf{m}^J}} \sum_t \|\mathbf{x}_t - \mathbf{z}_t^{gps}\|^2_{\Sigma_t^{gps}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t) - \mathbf{z}_t^{v}\|^2_{\Sigma_t^{v}} + \sum_t \|\mathrm{w2r}(\mathbf{m}, \mathbf{x}_t) - \mathbf{z}^{m}\|^2_{\Sigma_t^{m}}$$

Localization => SLAM

Localization

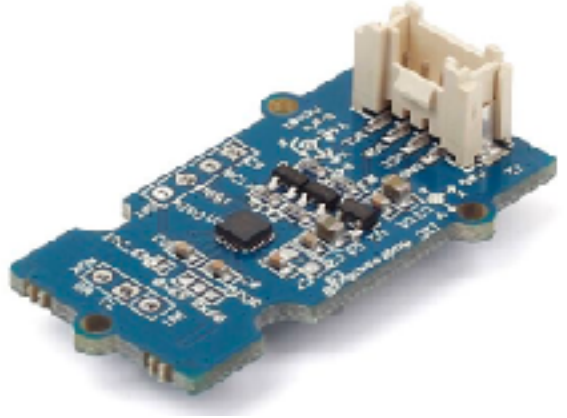
3D marker detector (RGBD camera) +  Odometry (IMU)

$$\mathbf{x}^\star = \arg\min_{\mathbf{m}_i, \mathbf{x}_t} \sum_{i,t} \|\mathrm{w2r}(\mathbf{m}_i, \mathbf{x}_t) - \mathbf{z}_t^{\mathbf{m}_i}\|^2 + \|\mathrm{w2r}(\mathbf{x}_2, \mathbf{x}_1) - \mathbf{z}_{12}^{odom}\|^2$$



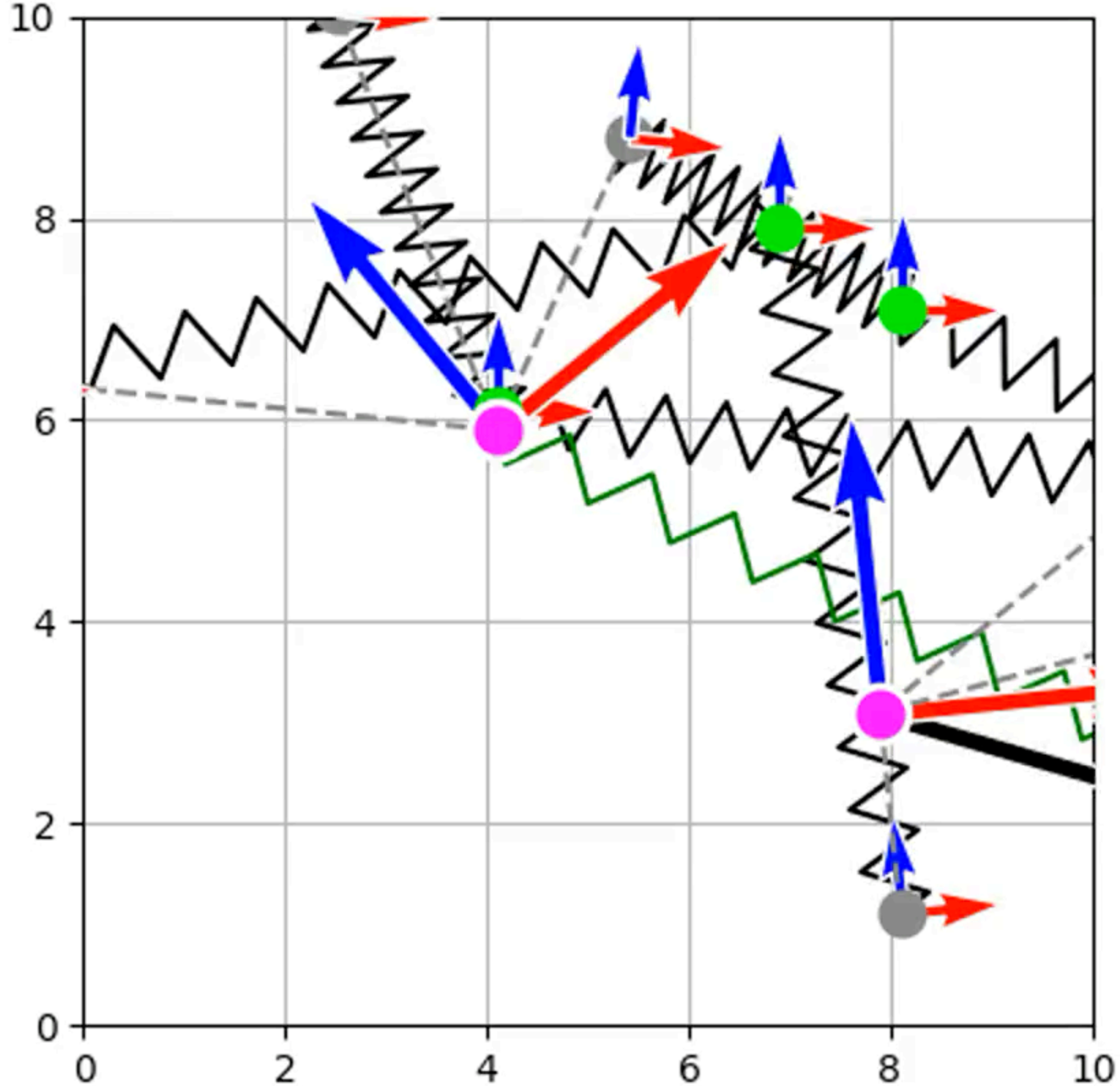
- $\mathbf{x}_t$ … robot poses
- $\mathbf{m}_i$ … known marker positions
- $\mathbf{z}_t^{\mathbf{m}_i}$ … marker measurements
- …… local coordinate frame
- …… odometry
- $\sum_{i,t} \|\mathrm{w2r}(\mathbf{m}_i, \mathbf{x}_t) - \mathbf{z}_t^{\mathbf{m}_i}\|^2$ … marker loss
- $\|\mathrm{w2r}(\mathbf{x}_2, \mathbf{x}_1) - \mathbf{z}_{12}^{odom}\|^2$ … odom loss

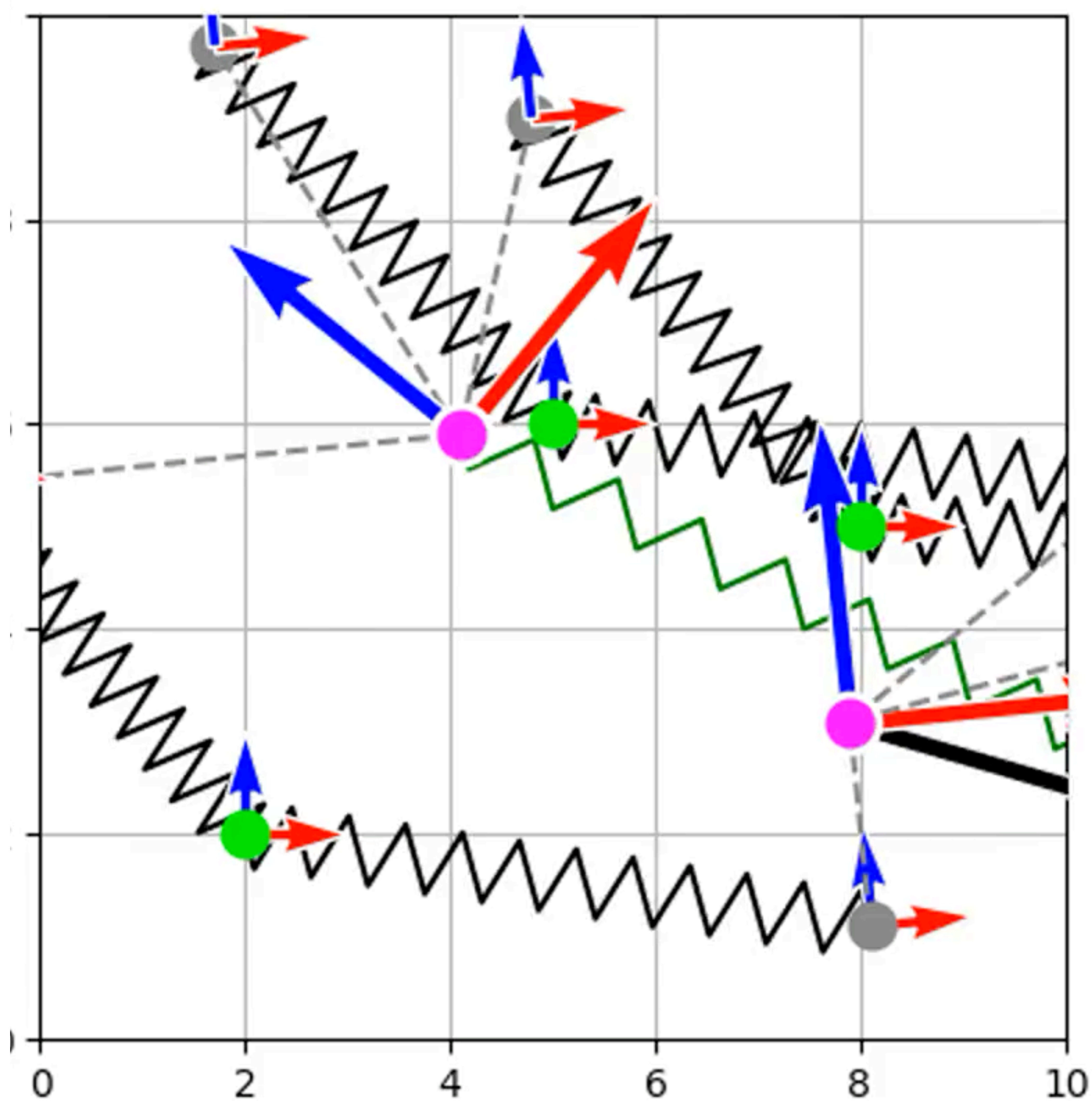
3D marker detector (RGBD camera) +



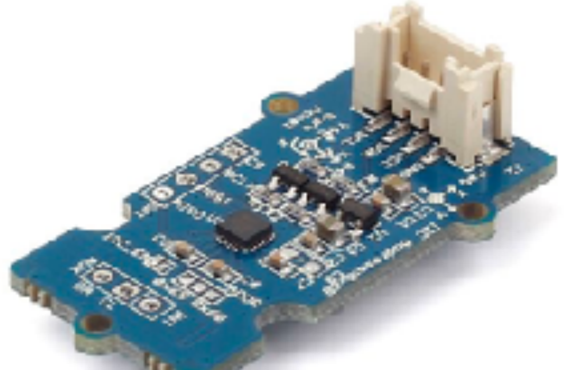
Odometry (IMU)

SLAM **Why is the result different?** Localization





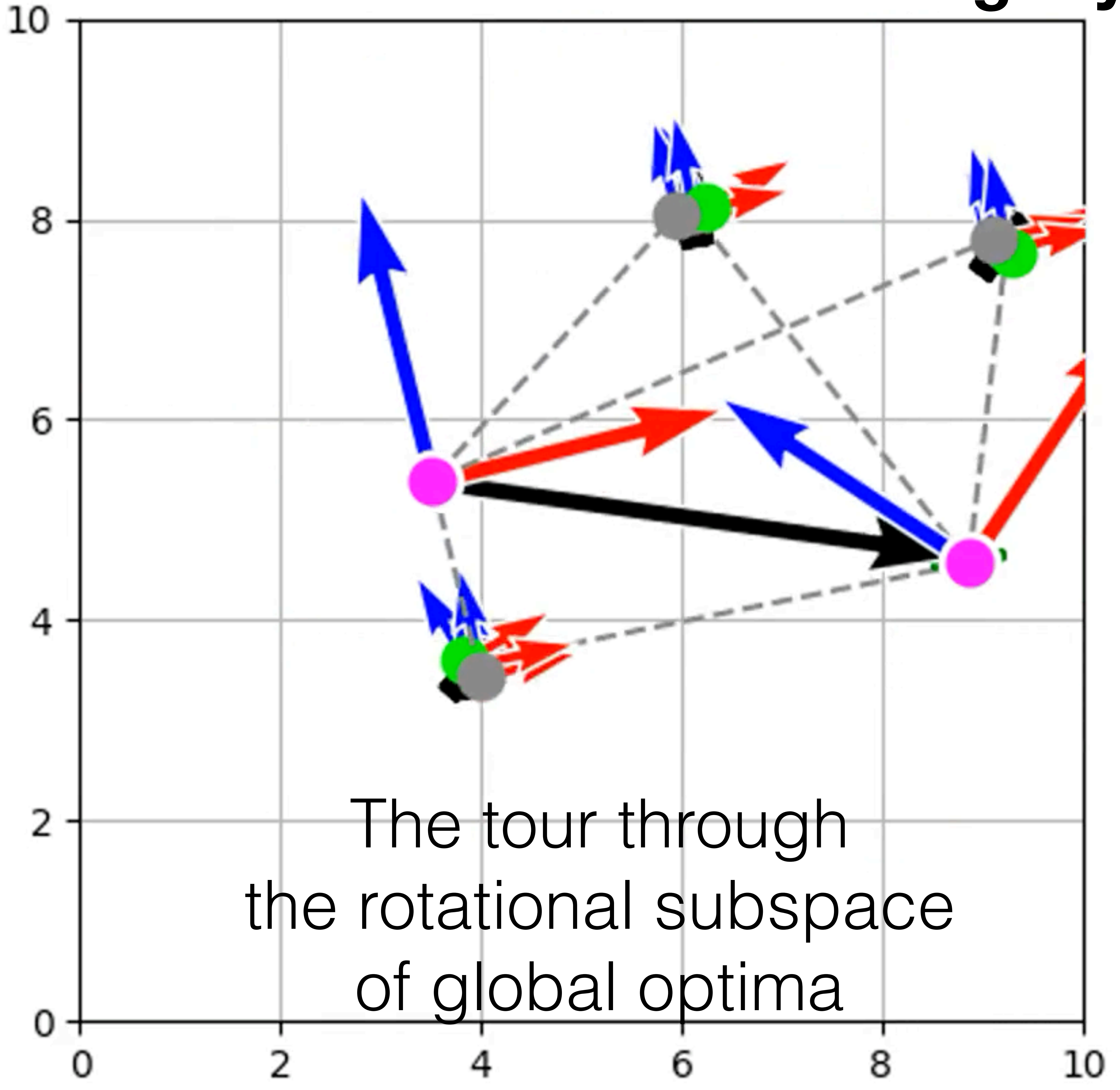
3D marker detector (RGBD camera) +



Odometry (IMU)

Nothing provides **absolute rotation+transl** => Global optimum is **3D subspace**

How can I resolve the ambiguity?

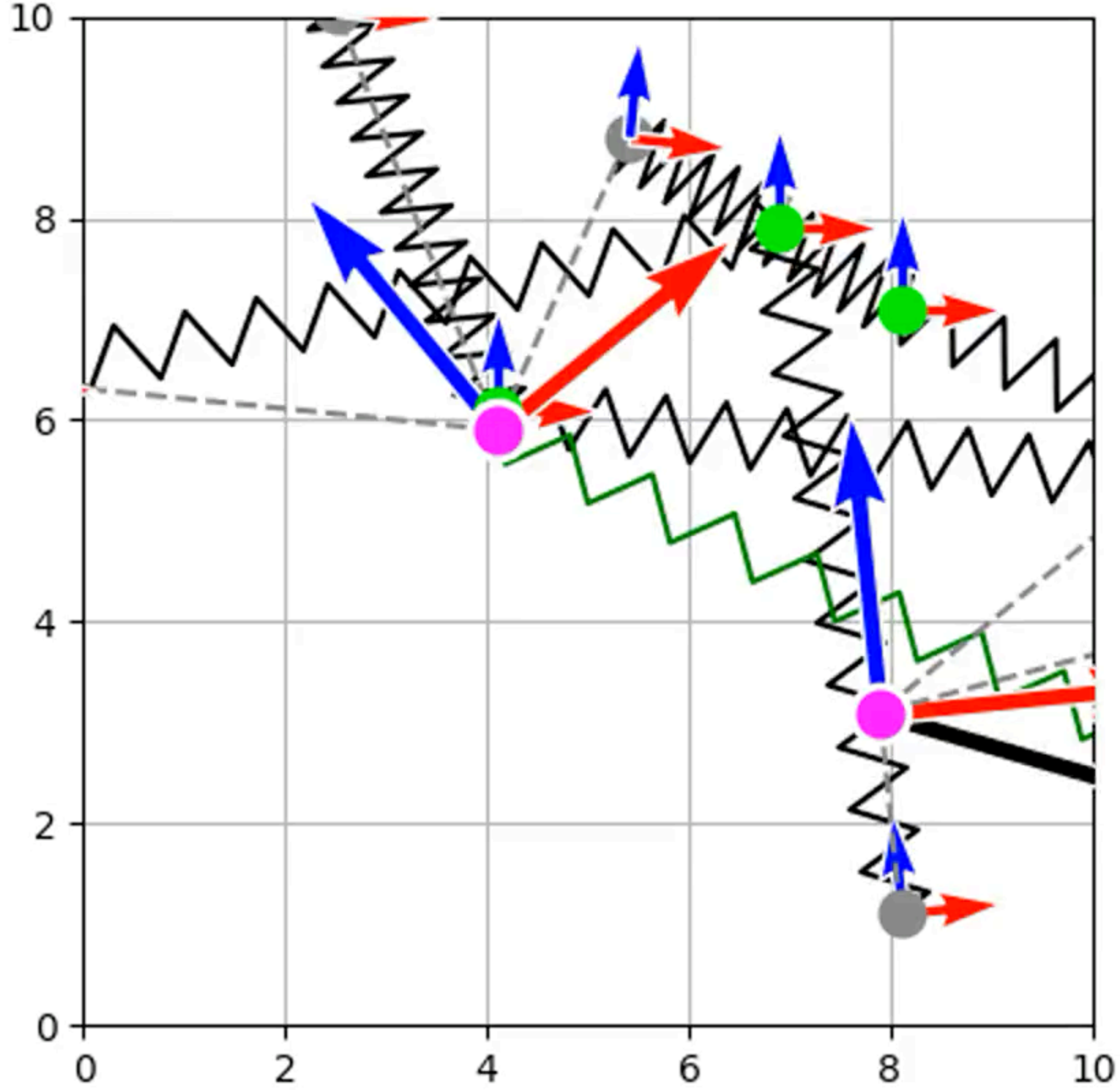


Removing the odometry does not matter

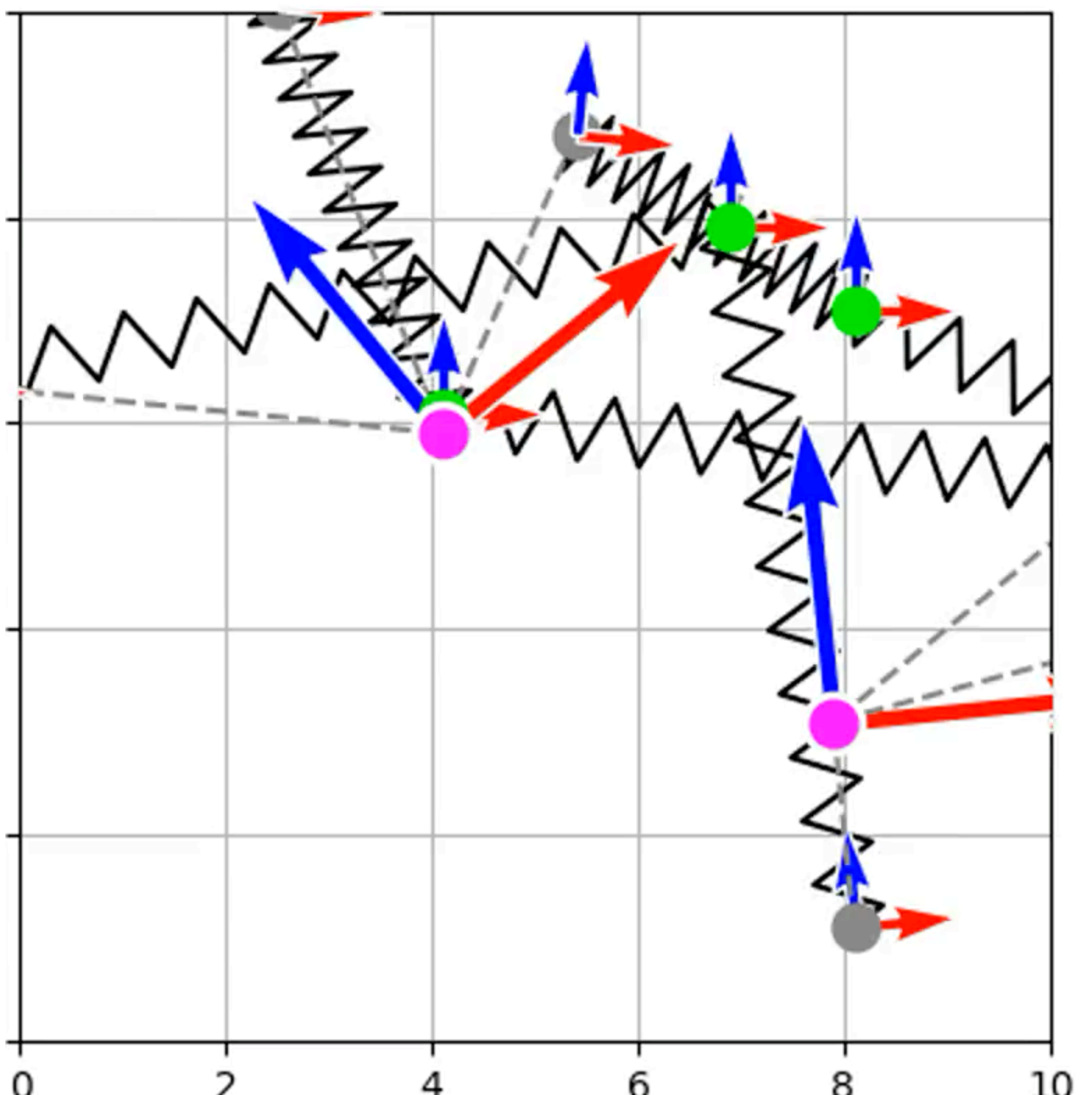
3D marker detector (RGBD camera) +

Odometry (IMU)

SLAM **with** odometry



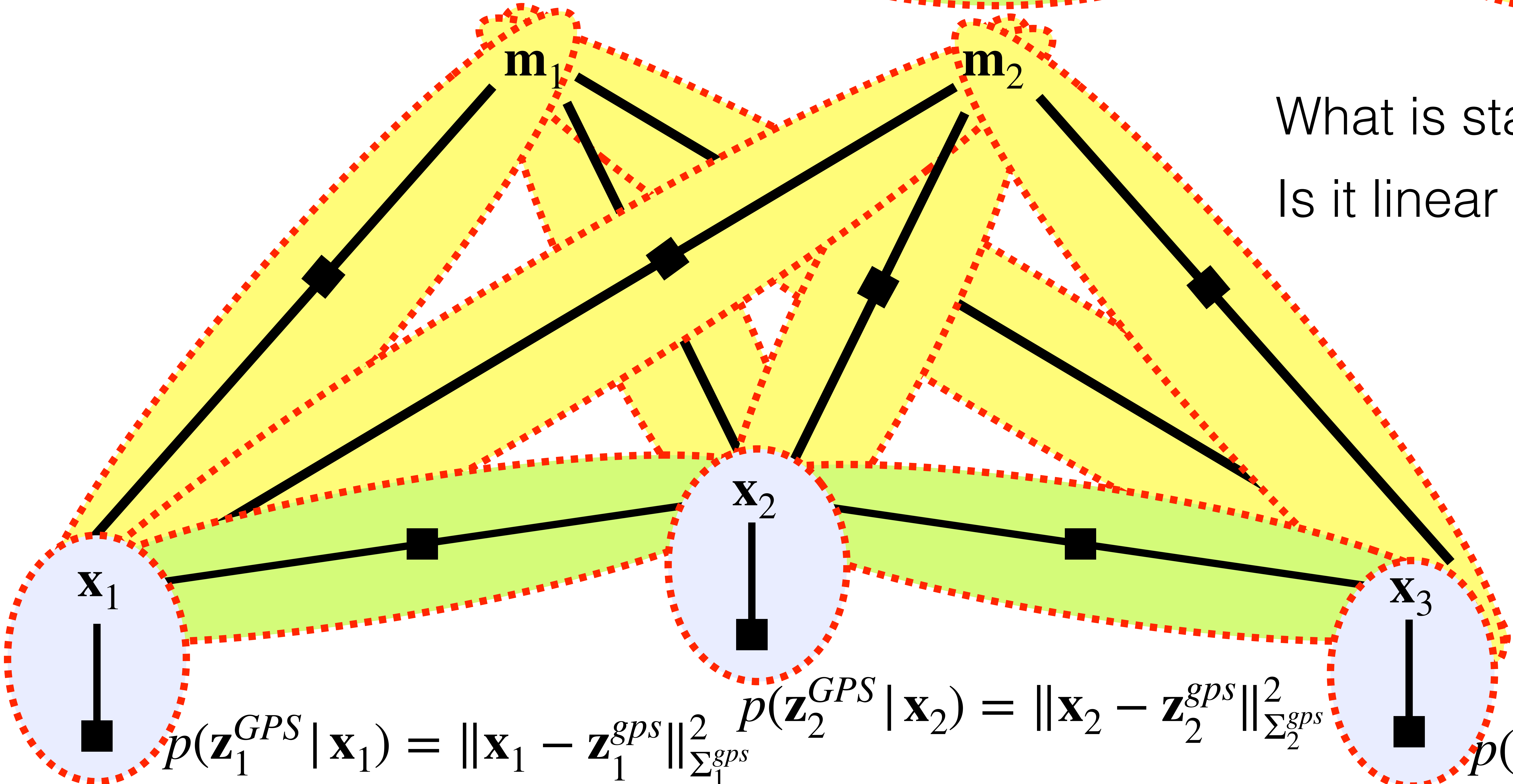
SLAM **without** odometry



Factorgraph

GPS — unary; odometry — pair-wise; 3D marker(s) — pair-wise

$$= \arg\min_{\substack{\mathbf{x}_0,\ldots\mathbf{x}_T \\ \mathbf{m}^1\ldots\mathbf{m}^J}} \sum_t \|\mathbf{x}_t - \mathbf{z}_t^{gps}\|^2_{\Sigma_t^{gps}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t) - \mathbf{z}_t^{v}\|^2_{\Sigma_t^{v}} + \sum_{t,j} \|\mathrm{w2r}(\mathbf{m}^j, \mathbf{x}_t) - \mathbf{z}\|^2_{\Sigma_t^{mj}}$$



What is state dimensionality?

Is it linear LS problem?

$p(\mathbf{z}_1^{GPS} \mid \mathbf{x}_1) = \|\mathbf{x}_1 - \mathbf{z}_1^{gps}\|^2_{\Sigma_1^{gps}}$

$p(\mathbf{z}_2^{GPS} \mid \mathbf{x}_2) = \|\mathbf{x}_2 - \mathbf{z}_2^{gps}\|^2_{\Sigma_2^{gps}}$

$p(\mathbf{z}_3^{GPS} \mid \mathbf{x}_3) = \|\mathbf{x}_3 - \mathbf{z}_3^{gp}$

Straightforward extensions

GPS — odometry — 3D marker(s)

$$= \arg\min_{\substack{\mathbf{x}_0,\ldots\mathbf{x}_T \\ \mathbf{m}^1\ldots\mathbf{m}^J}} \sum_t \|\mathbf{x}_t - \mathbf{z}_t^{gps}\|^2_{\Sigma_t^{gps}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t) - \mathbf{z}^v\|^2_{\Sigma_t^v} + \sum_{t,j} \|\mathrm{w2r}(\mathbf{m}^j, \mathbf{x}_t) - \mathbf{z}\|^2_{\Sigma_t^{m^j}}$$

priors — loop-closures

$$+ \sum_t \|\mathbf{x}_t - \mathbf{x}_t^{prior}\|^2_{\Sigma_t^{prior}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_0, \mathbf{x}_T)\|^2_{\Sigma_t^{lc}}$$

Straightforward extensions

GPS — odometry — 3D marker(s)

$$= \arg\min_{\substack{\mathbf{x}_0,\ldots\mathbf{x}_T \\ \mathbf{m}^1\ldots\mathbf{m}^J}} \sum_t \|\mathbf{x}_t - \mathbf{z}_t^{gps}\|^2_{\Sigma_t^{gps}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t) - \mathbf{z}^v\|^2_{\Sigma_t^v} + \sum_{t,j} \|\mathrm{w2r}(\mathbf{m}^j, \mathbf{x}_t) - \mathbf{z}\|^2_{\Sigma_t^{mj}}$$

priors — loop-closures

$$+ \sum_t \|\mathbf{x}_t - \mathbf{x}_t^{prior}\|^2_{\Sigma_t^{prior}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_0, \mathbf{x}_T)\|^2_{\Sigma_t^{lc}}$$

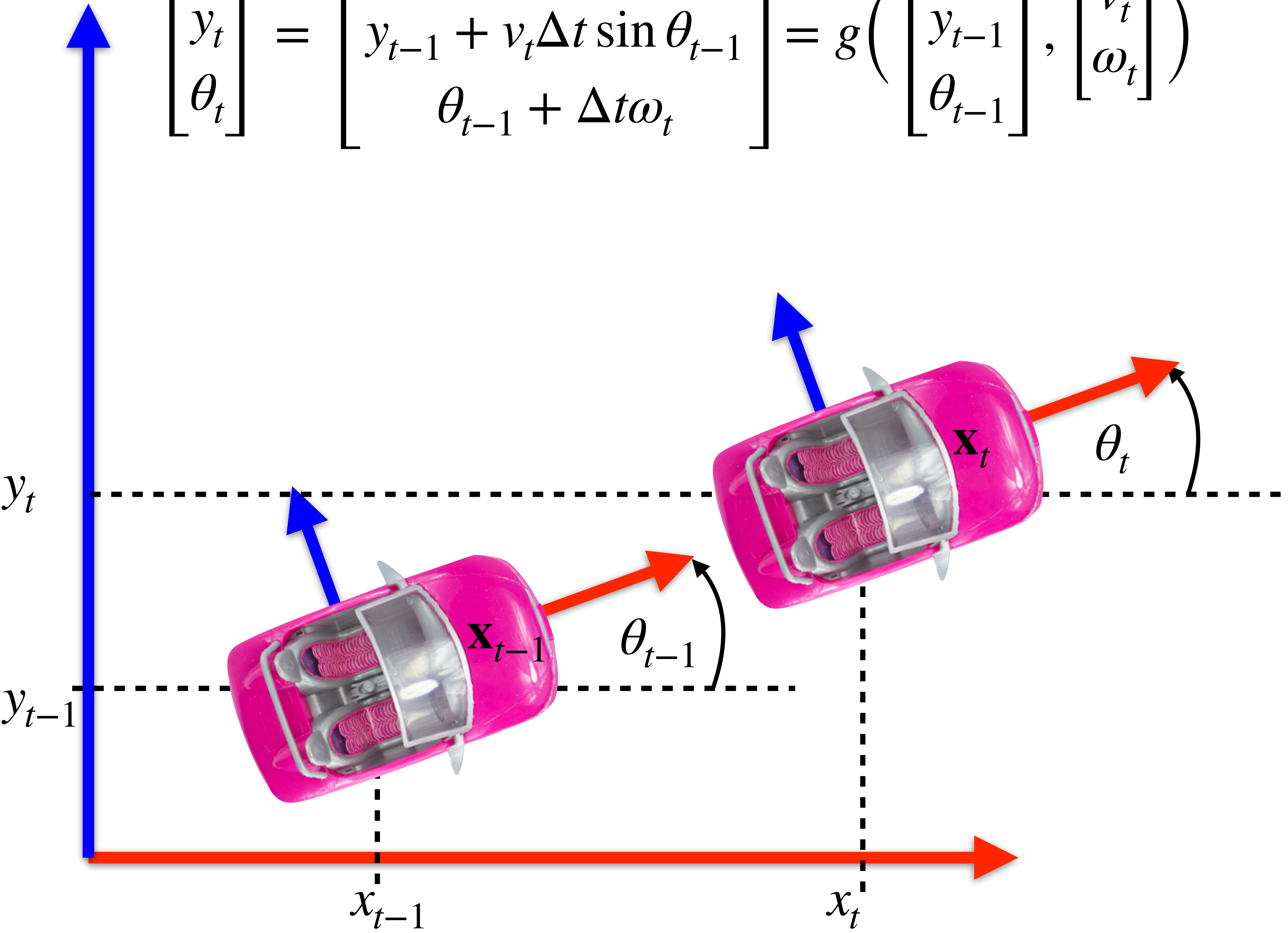
motion model — UWB — 2D marker(s)

\+ ??? + ??? + ???

e.g. camera detections

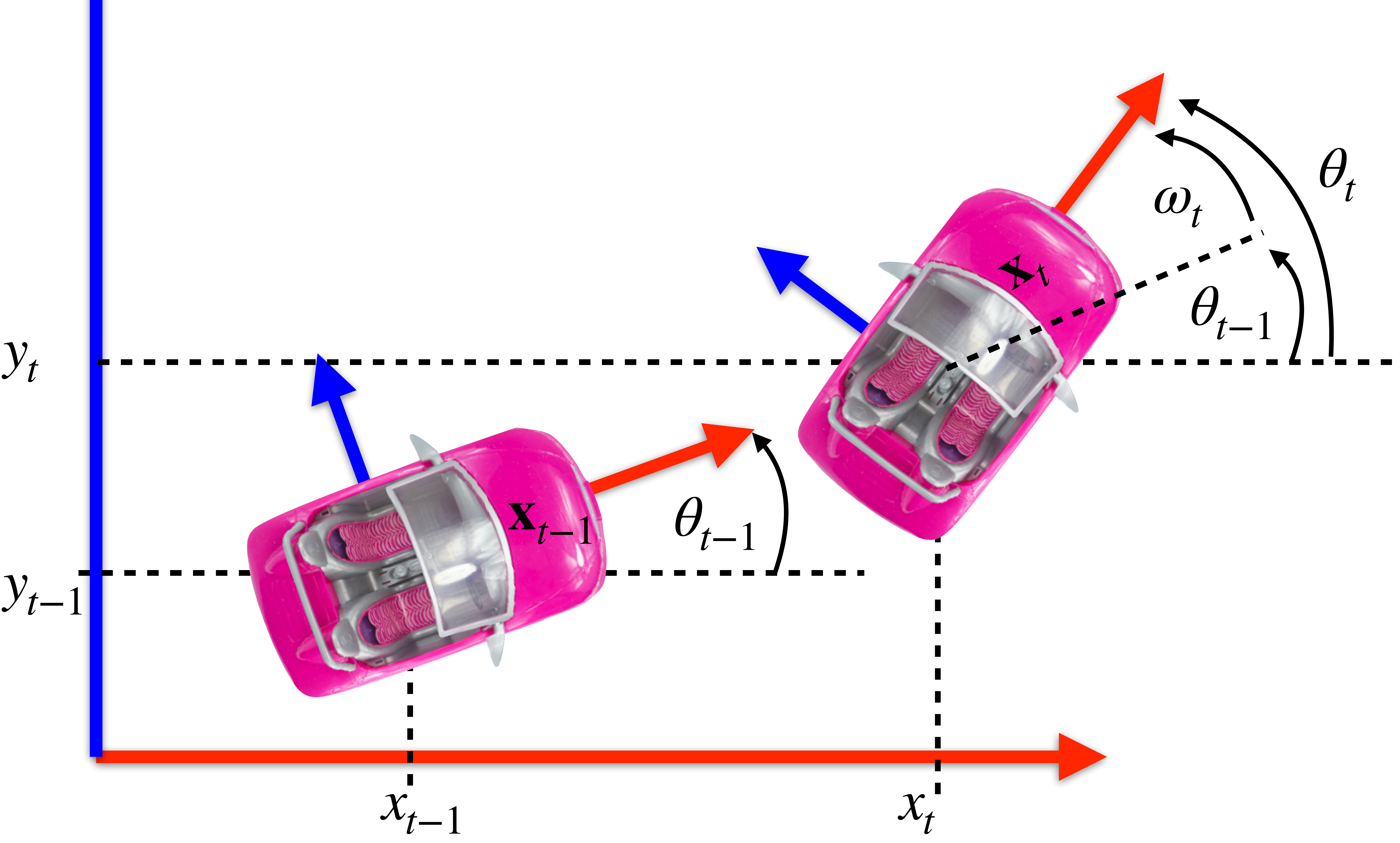
Differential drive model

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t \Delta t \cos \theta_{t-1} \\ y_{t-1} + v_t \Delta t \sin \theta_{t-1} \\ \theta_{t-1} + \Delta t \omega_t \end{bmatrix} = g\left(\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}, \begin{bmatrix} v_t \\ \omega_t \end{bmatrix} \right)$$



Differential drive model

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t \Delta t \cos \theta_{t-1} \\ y_{t-1} + v_t \Delta t \sin \theta_{t-1} \\ \theta_{t-1} + \Delta t \omega_t \end{bmatrix} = g\left(\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}, \begin{bmatrix} v_t \\ \omega_t \end{bmatrix} \right)$$



Example: GPS + motion model

Turtlebot transition probability:

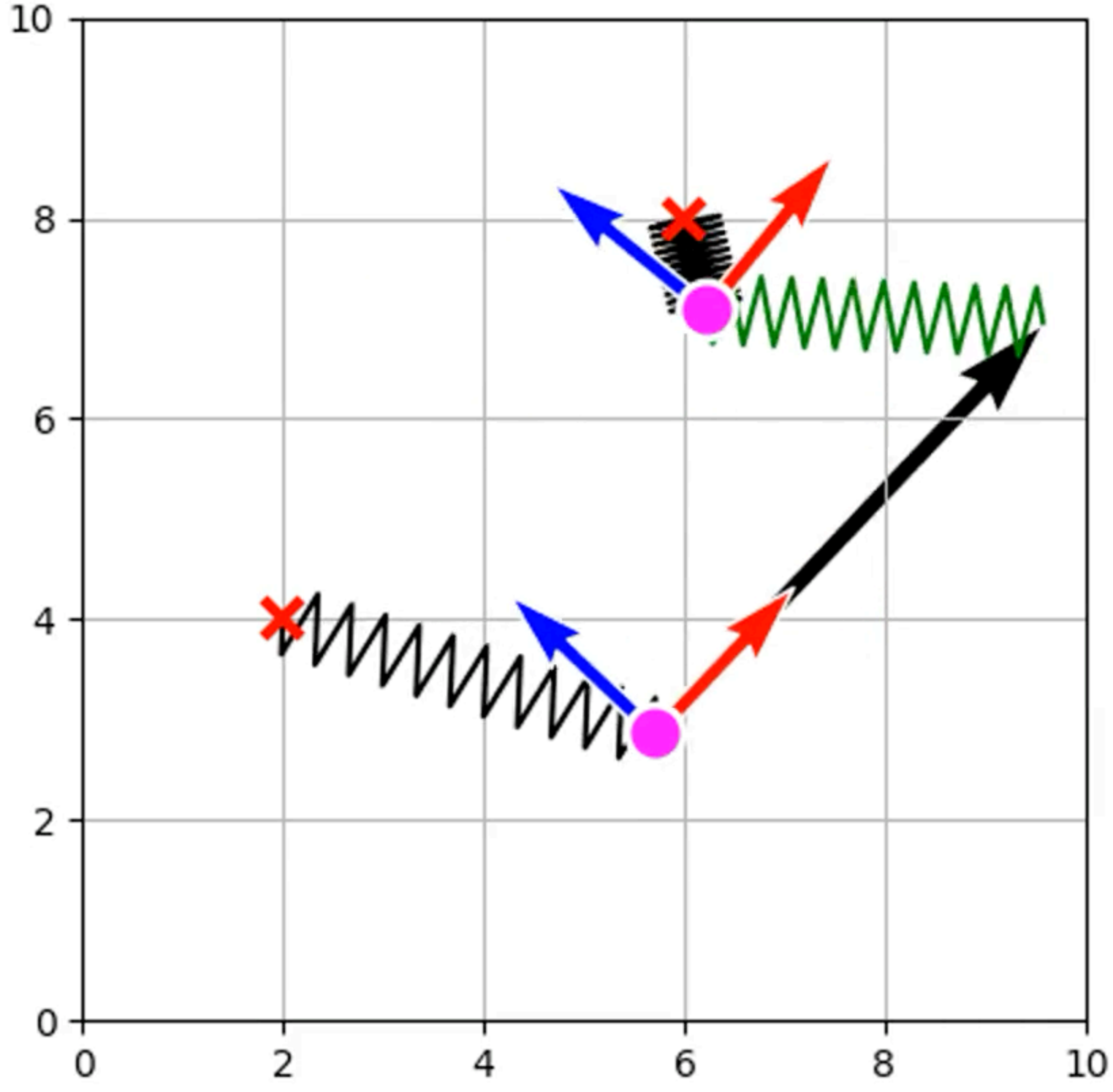
$$p\Big(\underbrace{\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}}_{\mathbf{x}_t} \Big| \underbrace{\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}}_{\mathbf{x}_{t-1}}, \underbrace{\begin{bmatrix} v_t \\ \omega_t \end{bmatrix}}_{\mathbf{u}_t}\Big) = \mathcal{N}\Big(\mathbf{x}_t; \underbrace{\begin{bmatrix} x_{t-1} + v_t \Delta t \cos\theta_{t-1} \\ y_{t-1} + v_t \Delta t \sin\theta_{t-1} \\ \theta_{t-1} + \omega_t \Delta t \end{bmatrix}}_{g(\mathbf{u}_t, \mathbf{x}_{t-1})}, \quad \mathbf{R}_t\Big)$$

GPS measurement probability:

$$p\Big(\underbrace{\begin{bmatrix} z_t^x \\ z_t^y \end{bmatrix}}_{\mathbf{z}_t^{\text{GPS}}} \Big| \underbrace{\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}}_{\mathbf{x}_t}\Big) = \mathcal{N}\Big(\mathbf{z}_t^{\text{GPS}}; \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}}_{h(\mathbf{x}_t)}, \quad Q_t^{\text{GPS}}\Big)$$

Example: GPS + motion model

$$= \arg\min_{\mathbf{x}_0,\ldots\mathbf{x}_T} \sum_t \|\mathbf{x}_t - \mathbf{z}_t^{gps}\|^2_{\Sigma_t^{gps}} \quad + \quad \sum_t \|g(\mathbf{x}_{t-1}, \mathbf{u}_t) - \mathbf{x}_t\|^2_{\Sigma_t^g}$$



$\mathbf{x}_t$ … robot poses

$\mathbf{z}_t^{gps}$ … gps measurements

…… local coordinate frame

$\mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_t)$ …… motion model

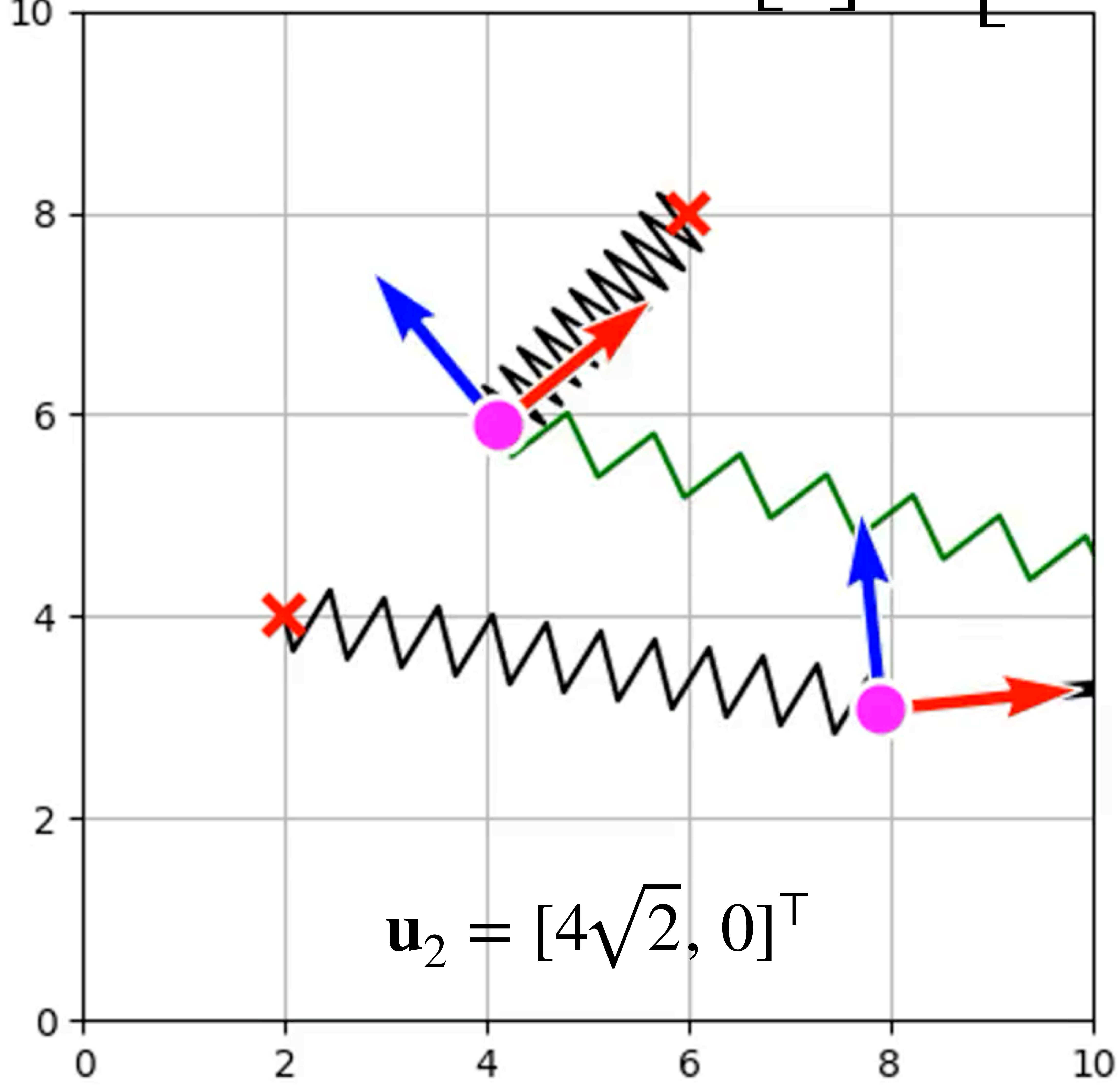
$\mathbf{u}_2 = [4\sqrt{2}, 0]^\top$

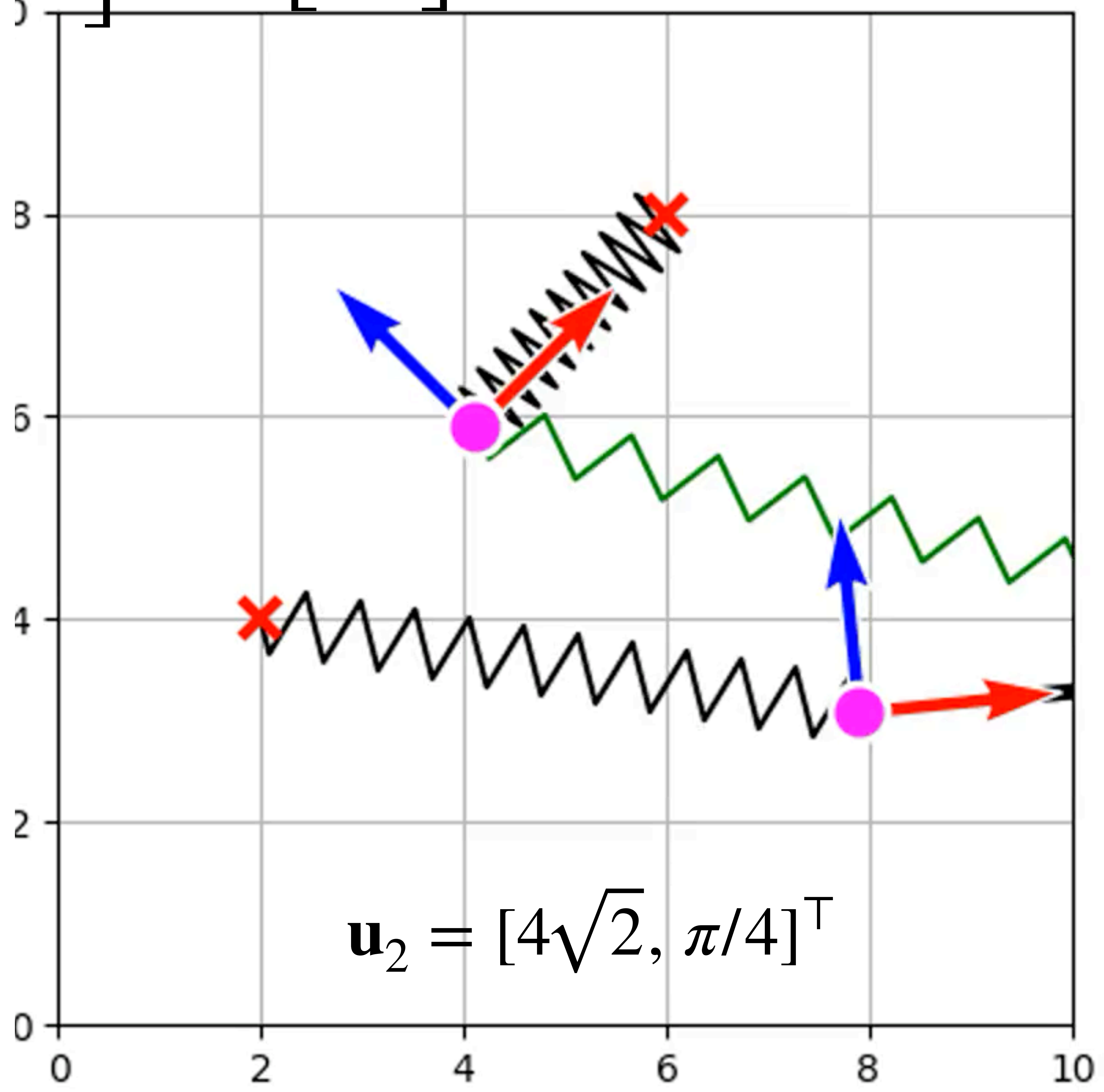
$\|\mathbf{x}_t - \mathbf{z}_t^{gps}\|^2_{\Sigma_t^{gps}}$ ………………. gps loss

$\|g(\mathbf{x}_{t-1}, \mathbf{u}_t) - \mathbf{x}_t\|^2_{\Sigma_t^g}$ …motion model loss

Example: GPS + motion model

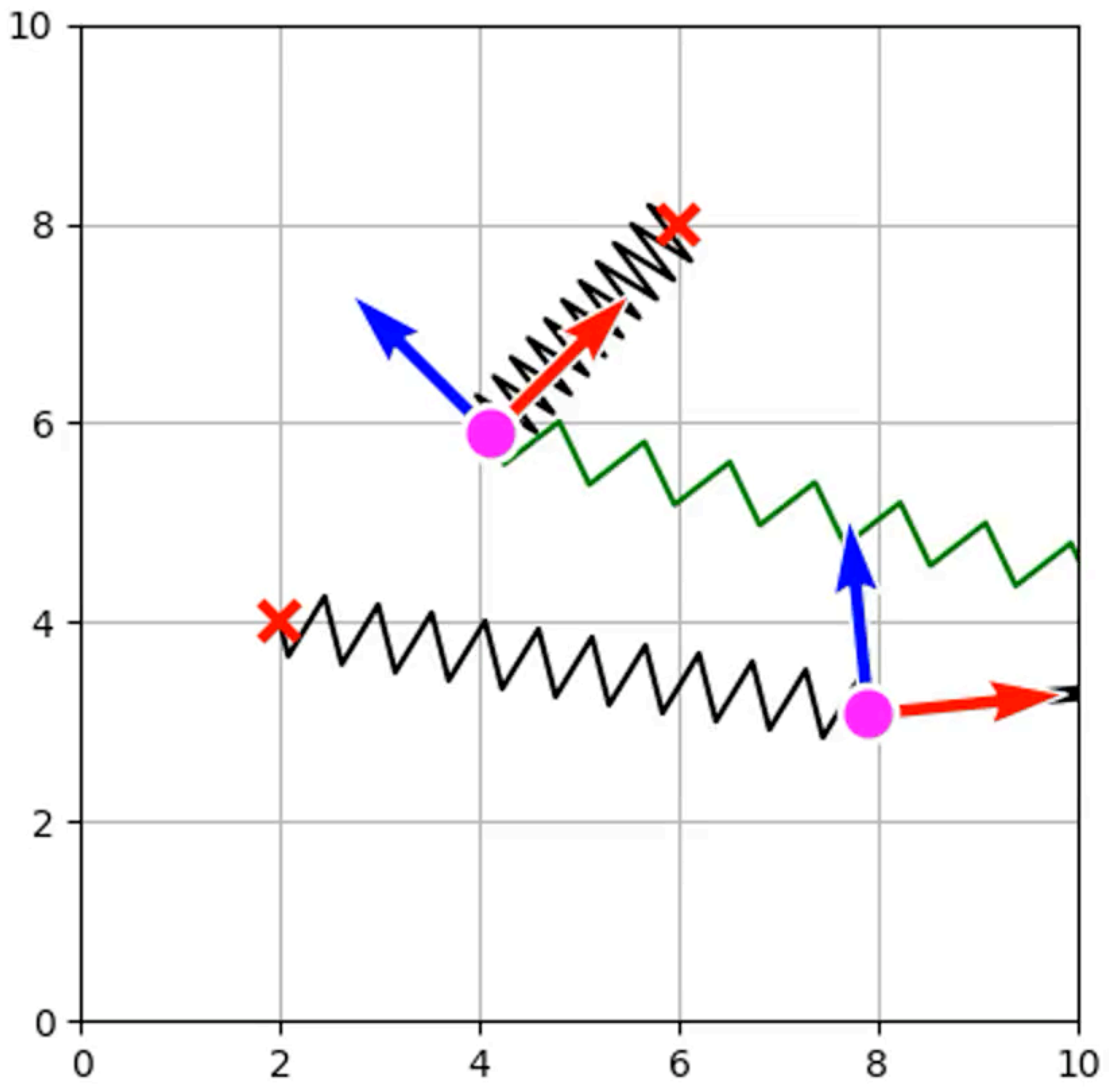
$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t \Delta t \cos \theta_{t-1} \\ y_{t-1} + v_t \Delta t \sin \theta_{t-1} \\ \theta_{t-1} + \omega_t \Delta t \end{bmatrix} = g\left(\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}, \begin{bmatrix} v_t \\ \omega_t \end{bmatrix} \right)$$





$\mathbf{u}_2 = [4\sqrt{2}, \pi/4]^\top$

Example: GPS + motion model



Move then turn

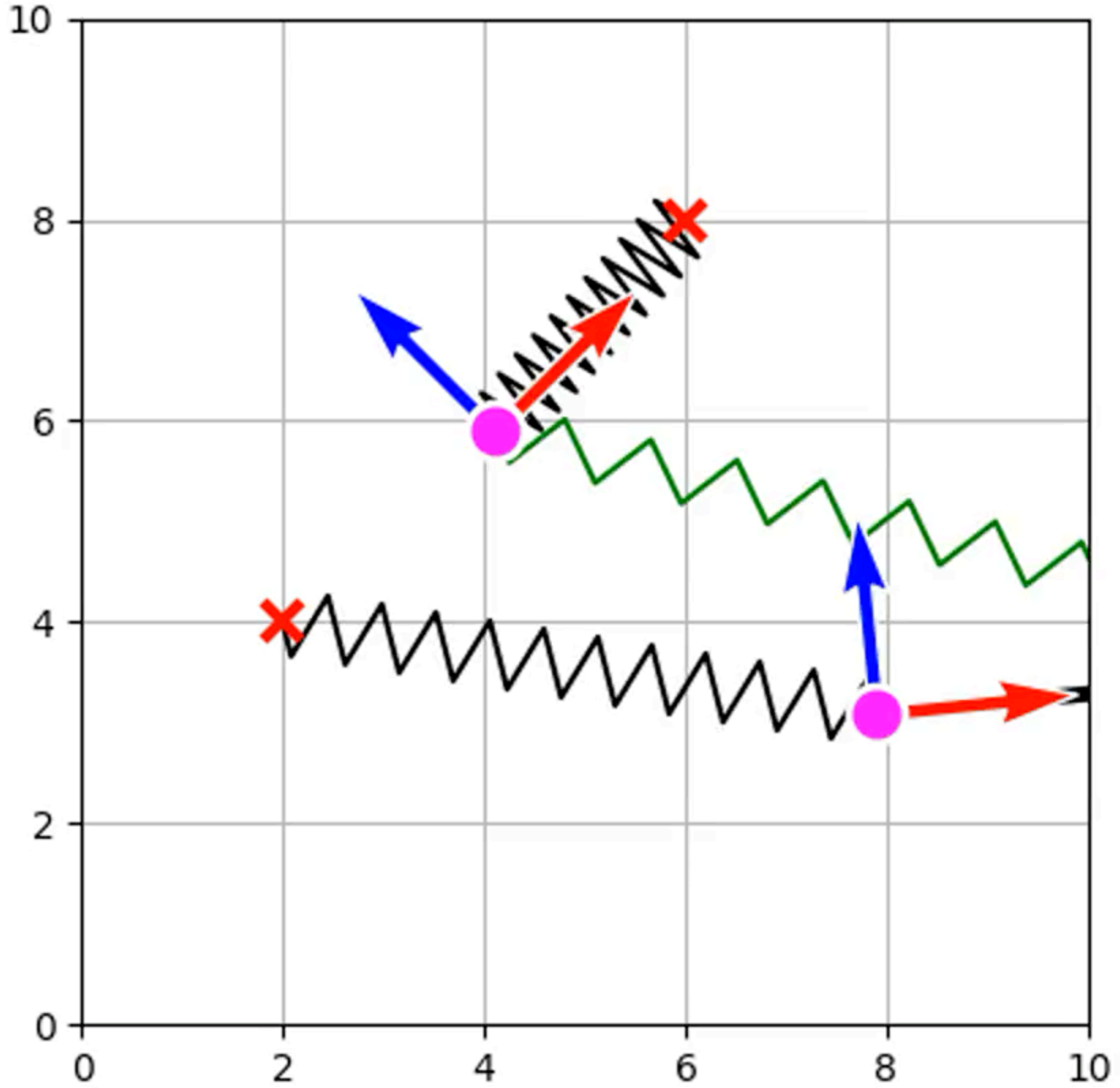
$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t \Delta t \cos \theta_{t-1} \\ y_{t-1} + v_t \Delta t \sin \theta_{t-1} \\ \theta_{t-1} + \omega_t \Delta t \end{bmatrix} = g\left(\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}, \begin{bmatrix} v_t \\ \omega_t \end{bmatrix} \right)$$

Turn then move

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t \Delta t \cos(\theta_{t-1} + \omega_t \Delta t) \\ y_{t-1} + v_t \Delta t \sin(\theta_{t-1} + \omega_t \Delta t) \\ \theta_{t-1} + \omega_t \Delta t \end{bmatrix} = g\left(\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}, \begin{bmatrix} v_t \\ \omega_t \end{bmatrix} \right)$$

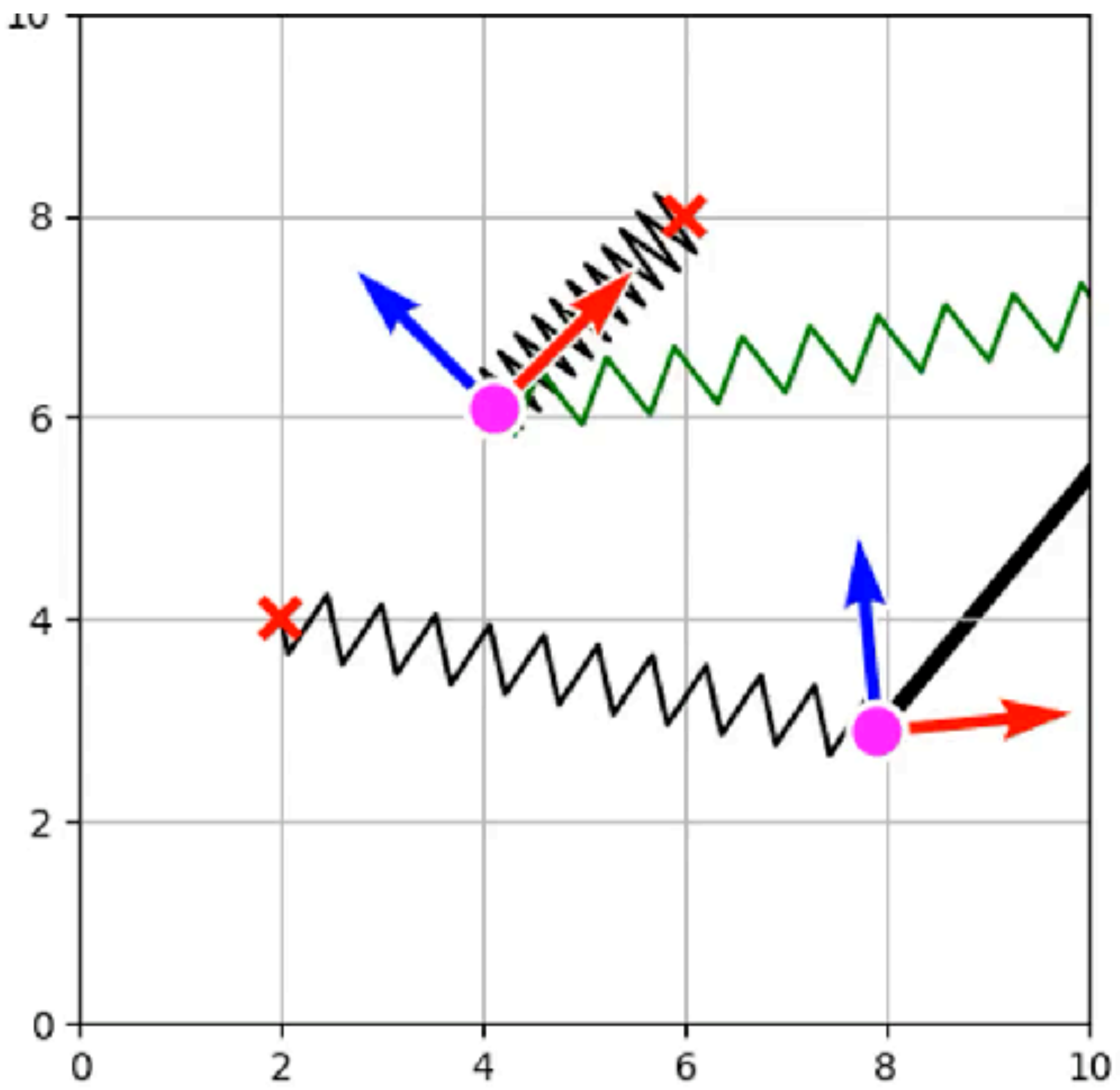
$\mathbf{u}_2 = [4\sqrt{2}, \pi/4]^\top$

Example: GPS + motion model



Move then turn

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t \Delta t \cos \theta_{t-1} \\ y_{t-1} + v_t \Delta t \sin \theta_{t-1} \\ \theta_{t-1} + \omega_t \Delta t \end{bmatrix} = g\left(\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}, \begin{bmatrix} v_t \\ \omega_t \end{bmatrix} \right)$$

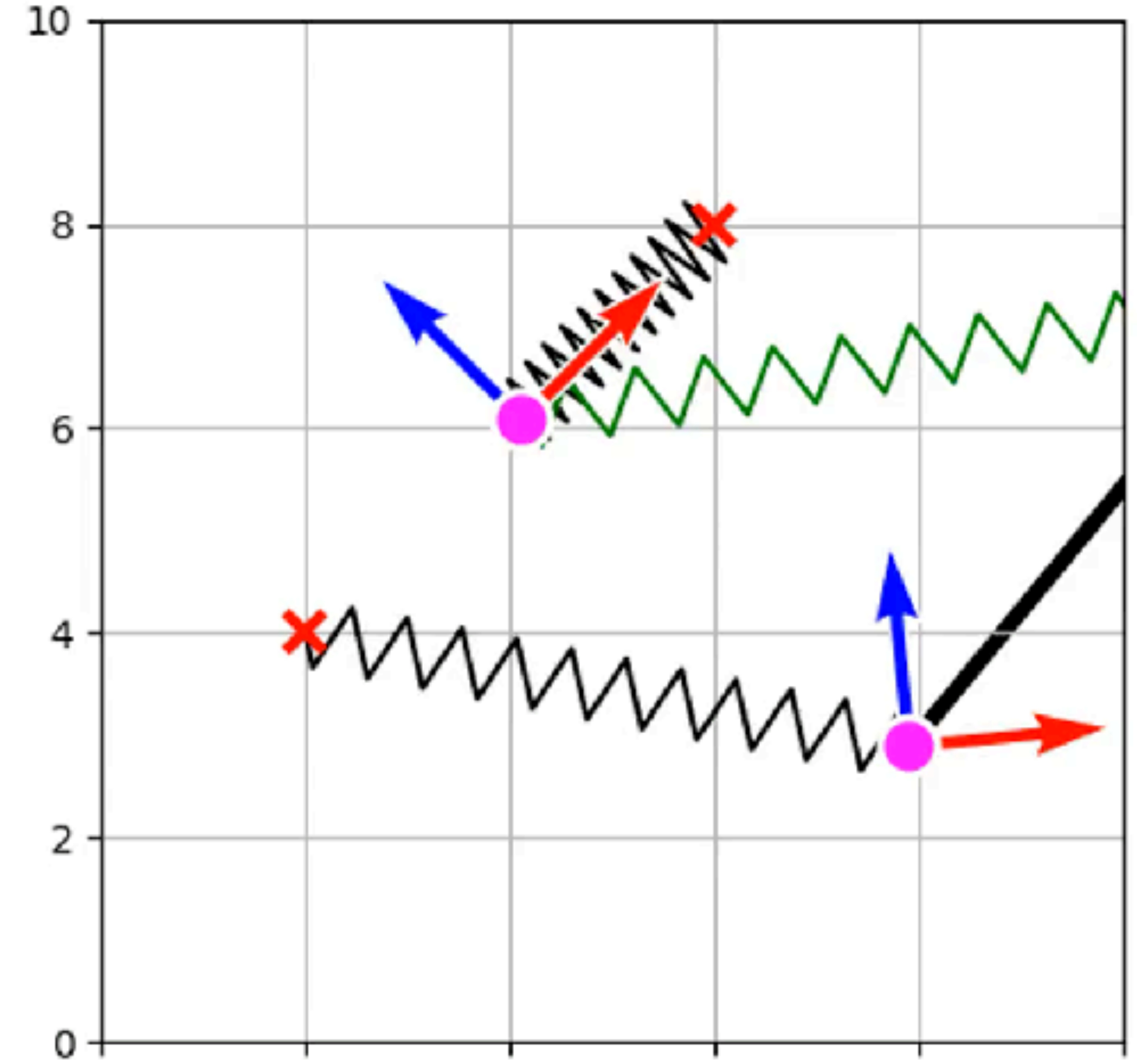


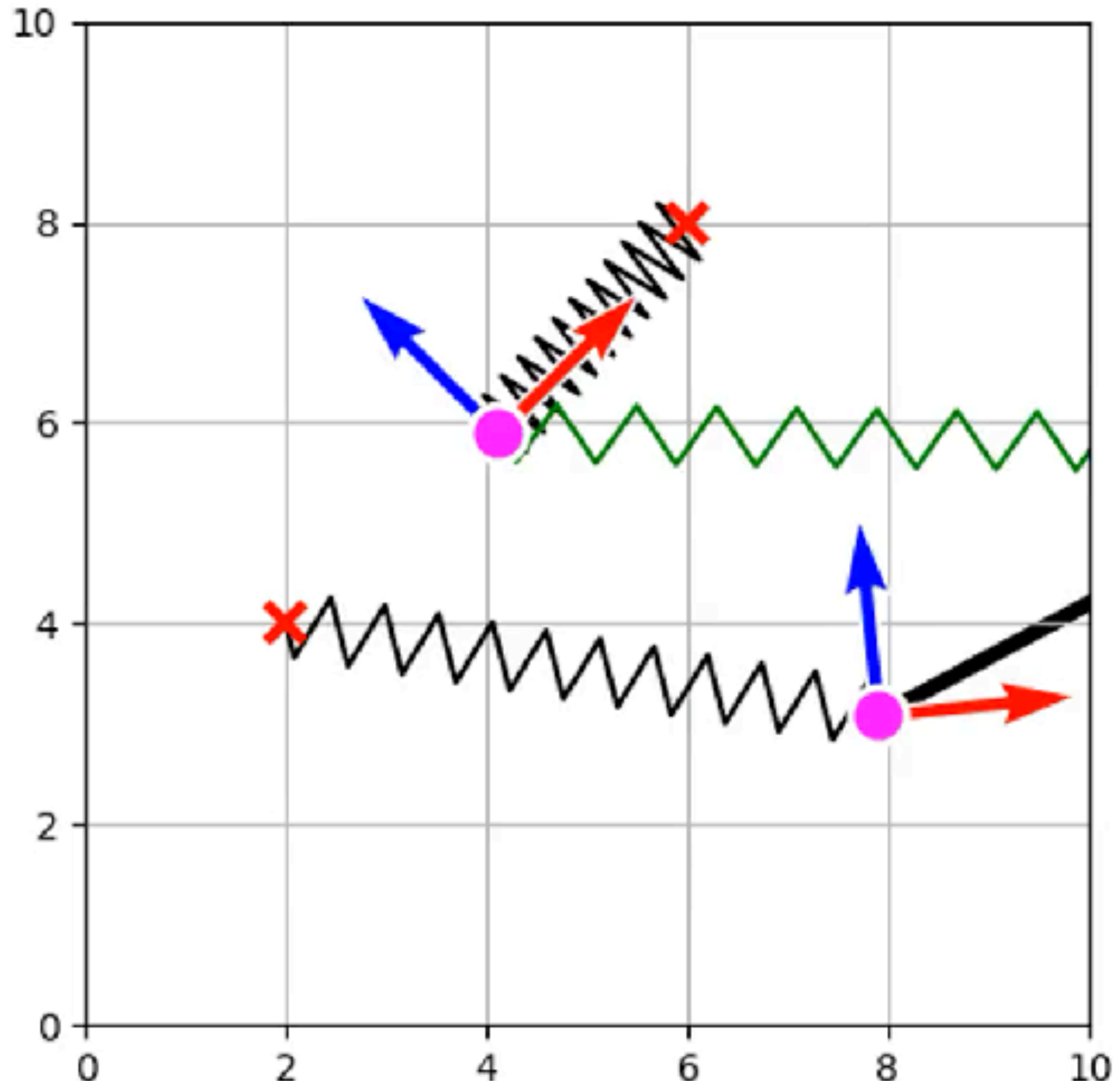
How do I get the best of both worlds?

Turn then move

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t \Delta t \cos(\theta_{t-1} + \omega_t \Delta t) \\ y_{t-1} + v_t \Delta t \sin(\theta_{t-1} + \omega_t \Delta t) \\ \theta_{t-1} + \omega_t \Delta t \end{bmatrix} = g\left(\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}, \begin{bmatrix} v_t \\ \omega_t \end{bmatrix} \right)$$

$$\mathbf{u}_2 = [4\sqrt{2}, \pi/4]^\top$$





Example: GPS + motion model

Turn then move

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t \Delta t \cos(\theta_{t-1} + \omega_t \Delta t) \\ y_{t-1} + v_t \Delta t \sin(\theta_{t-1} + \omega_t \Delta t) \\ \theta_{t-1} + \omega_t \Delta t \end{bmatrix} = g\left(\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}, \begin{bmatrix} v_t \\ \omega_t \end{bmatrix} \right)$$

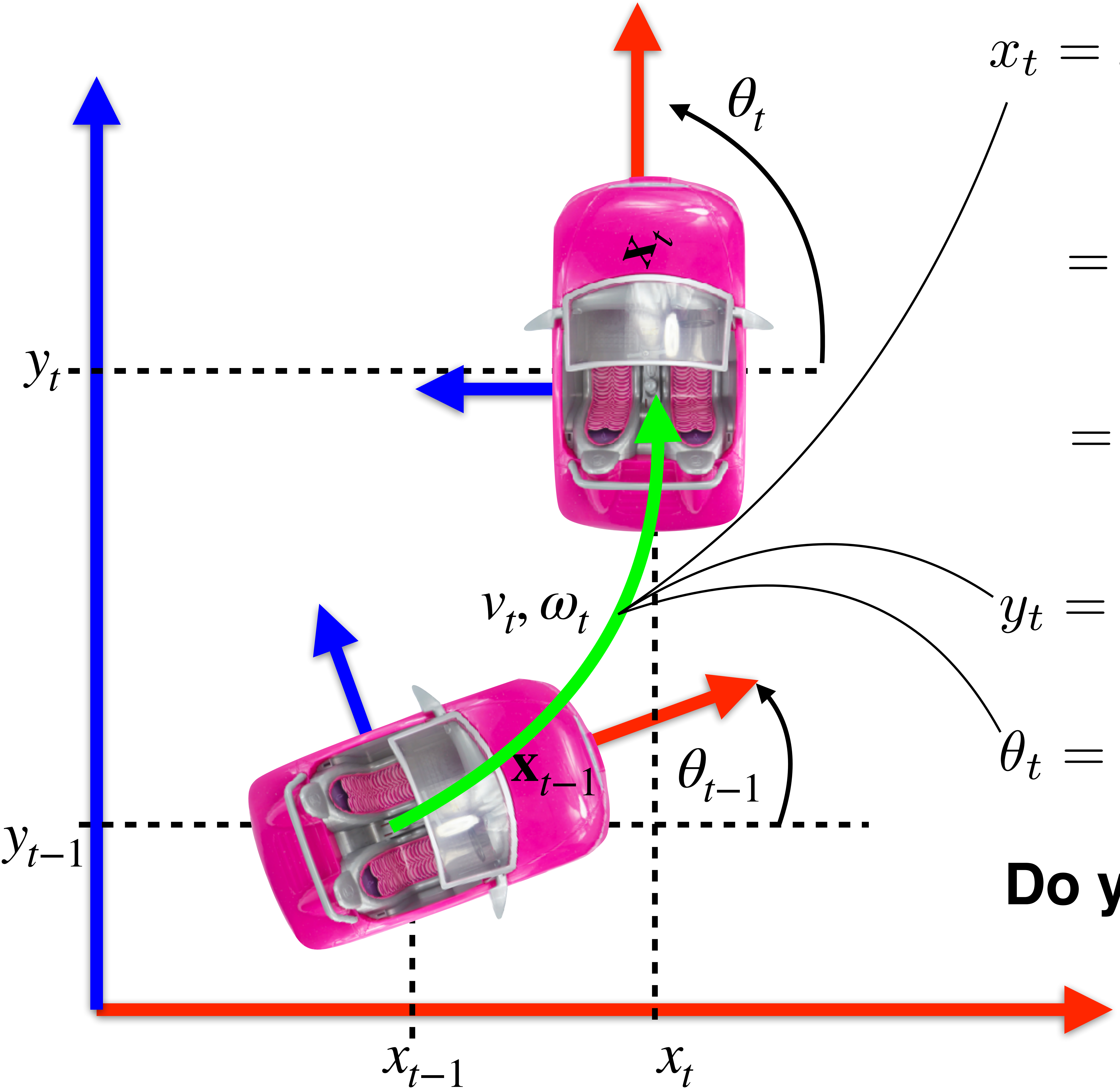
How do I get the best of both worlds?

Half-turn, then move, then half-turn

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t \Delta t \cos(\theta_{t-1} + \omega_t \Delta t/2) \\ y_{t-1} + v_t \Delta t \sin(\theta_{t-1} + \omega_t \Delta t/2) \\ \theta_{t-1} + \omega_t \Delta t \end{bmatrix} = g\left(\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}, \begin{bmatrix} v_t \\ \omega_t \end{bmatrix} \right)$$

Is there even better way to do it?

Differential drive model



$$x_t = x_{t-1} + \int_0^{\Delta t} \overbrace{v_t \cos(\underbrace{\theta_{t-1} + \omega_t t}_{\theta(t)})dt}^{\dot{x}_t}$$

$$= x_{t-1} + v_t \left[\frac{\sin(\theta_{t-1} + \omega_t t)}{\omega_t} \right]_0^{\Delta t}$$

$$= x_{t-1} + \frac{v_t}{\omega_t}\left(+ \sin(\theta_{t-1} + \omega_t \Delta t) - \sin(\theta_{t-1})\right)$$

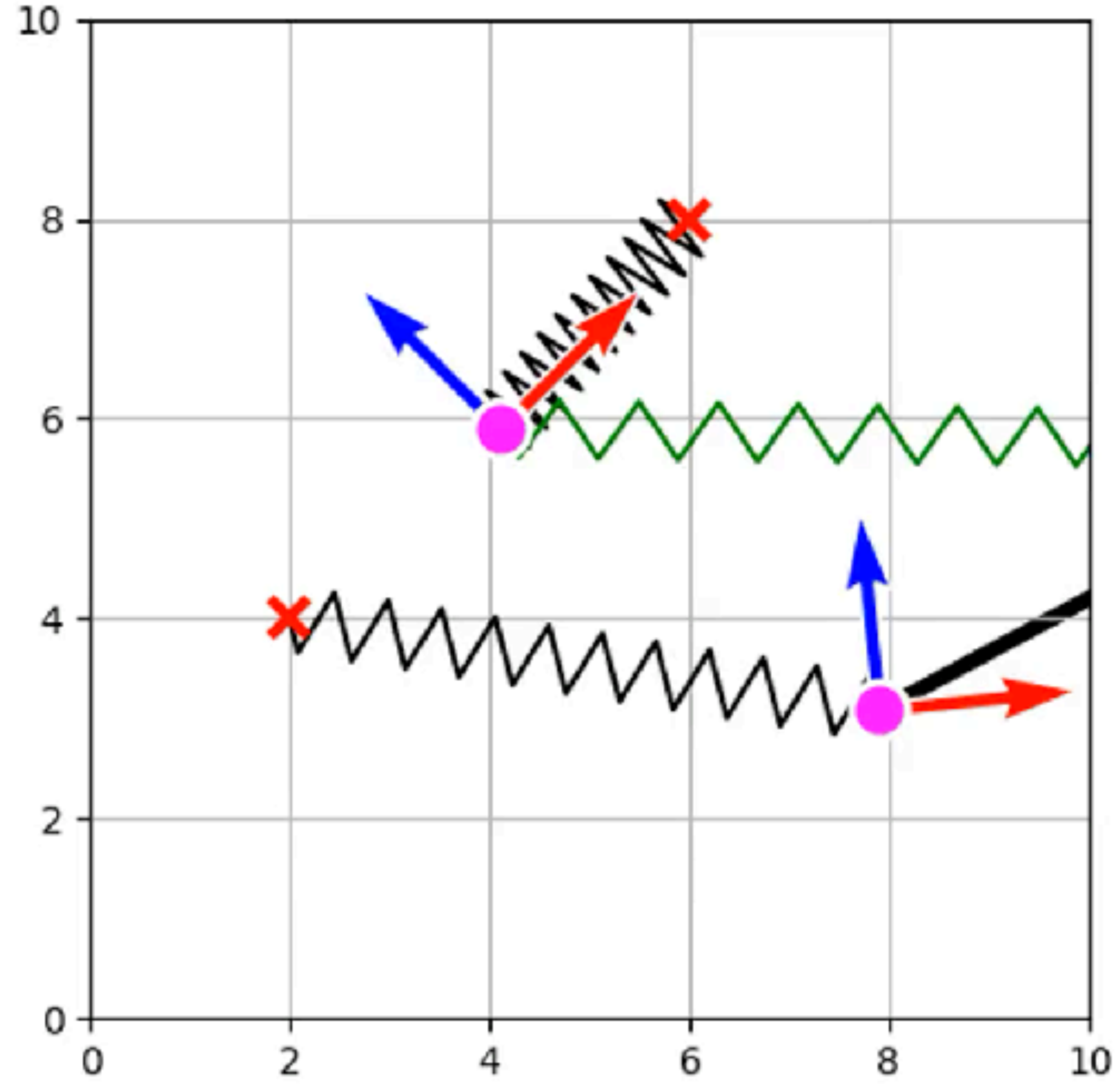
$$y_t = y_{t-1} + \frac{v_t}{\omega_t}\left(- \cos(\theta_{t-1} + \omega_t \Delta t) + \cos(\theta_{t-1})\right)$$

$$\theta_t = \theta_{t-1} + \omega \Delta t$$

Do you see any disadvantage of the model?

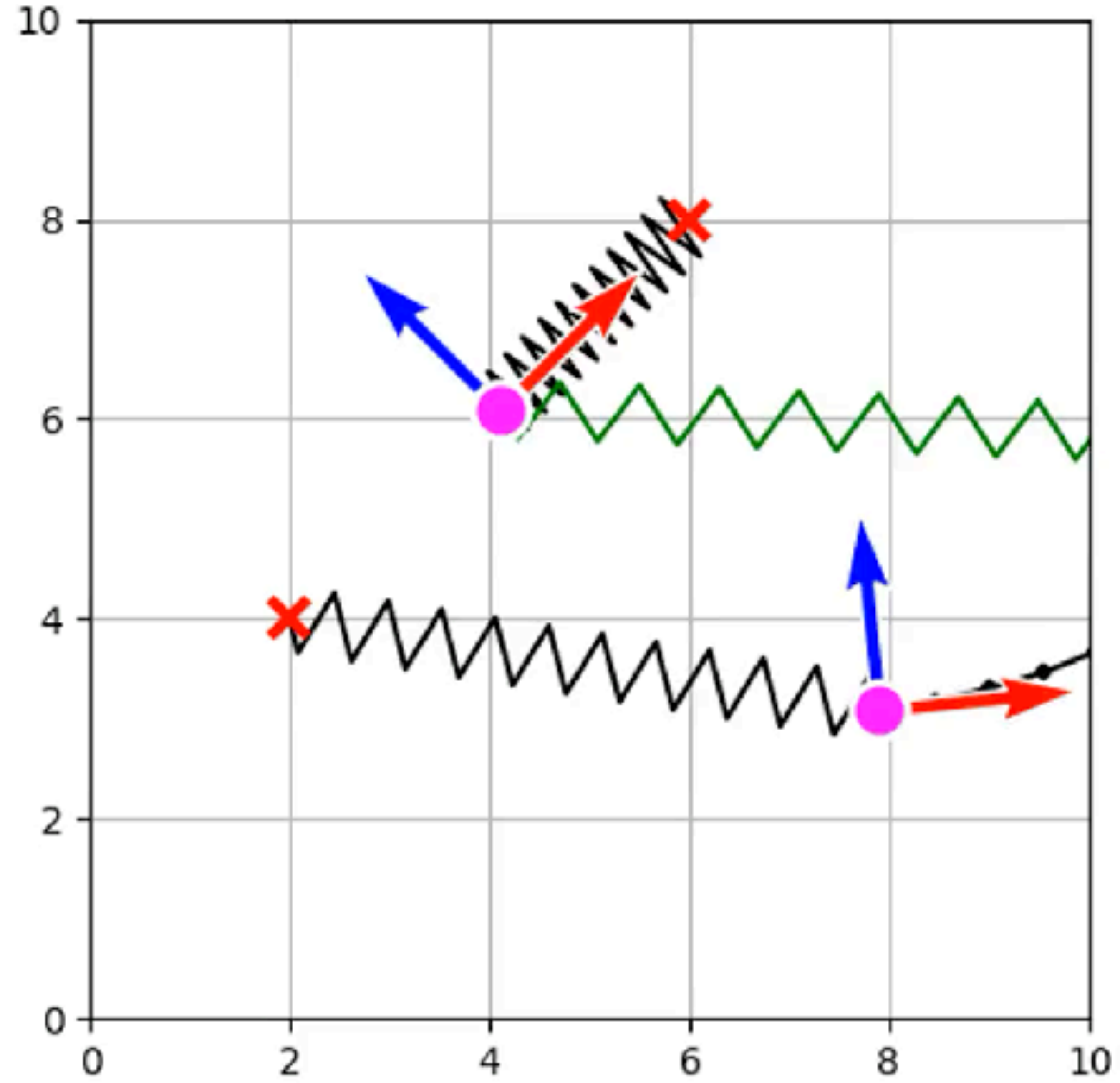
$\mathbf{u}_2 = [4\sqrt{2}, \pi/4]^\top$

Example: GPS + motion model



Half-turn, then move, then half-turn

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t \Delta t \cos(\theta_{t-1} + \omega_t \Delta t/2) \\ y_{t-1} + v_t \Delta t \sin(\theta_{t-1} + \omega_t \Delta t/2) \\ \theta_{t-1} + \omega_t \Delta t \end{bmatrix} = g\left(\begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix}, \begin{bmatrix} v_t \\ \omega_t \end{bmatrix} \right)$$

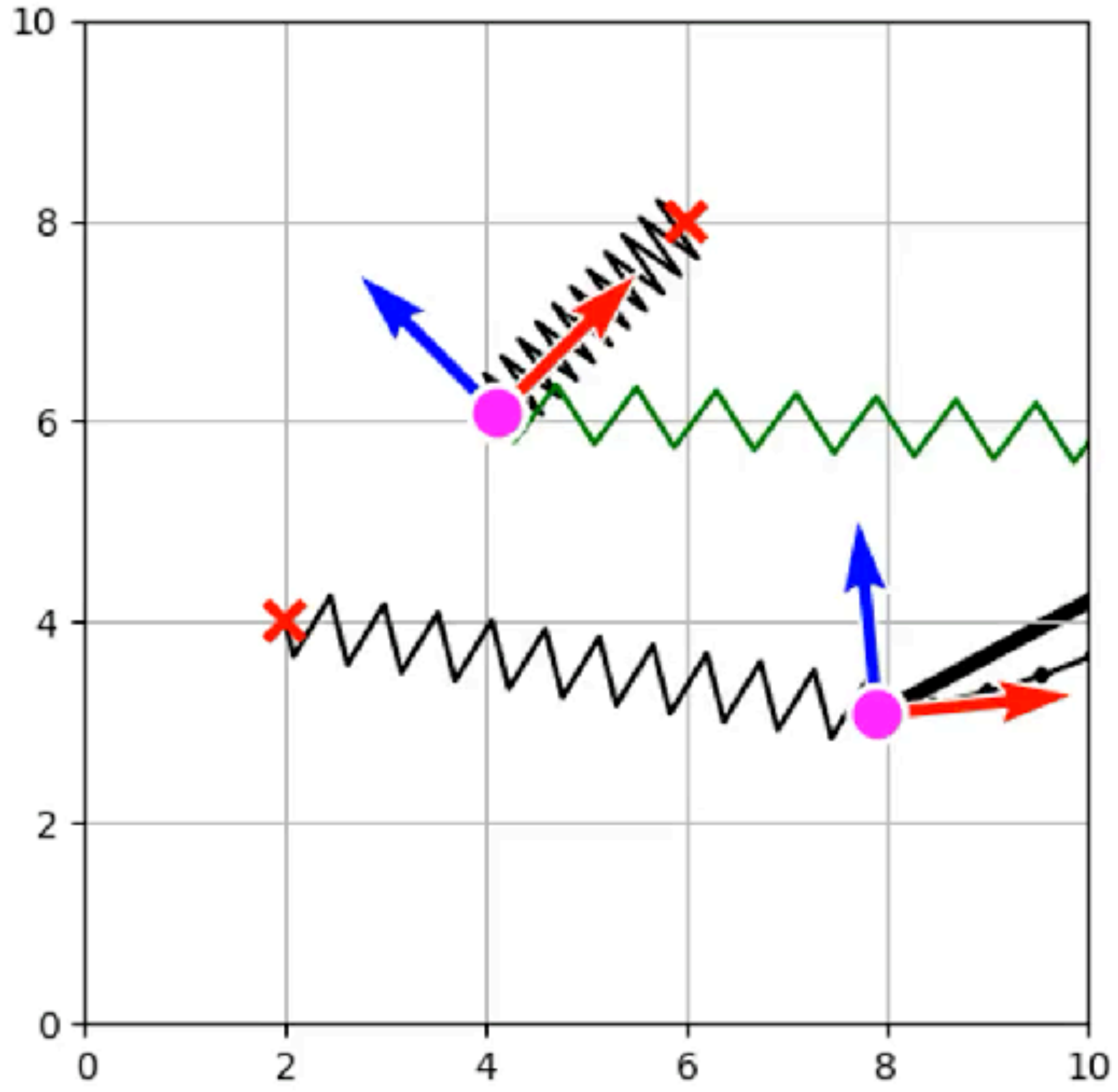


Analytical integration

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + v_t/\omega_t(\sin(\theta_{t-1} + \omega_t \Delta t) - \sin\theta_{t-1}) \\ y_{t-1} + v_t/\omega_t(-\cos(\theta_{t-1} + \omega_t \Delta t) + \cos\theta_{t-1}) \\ \theta_{t-1} + \omega_t \Delta t \end{bmatrix}$$

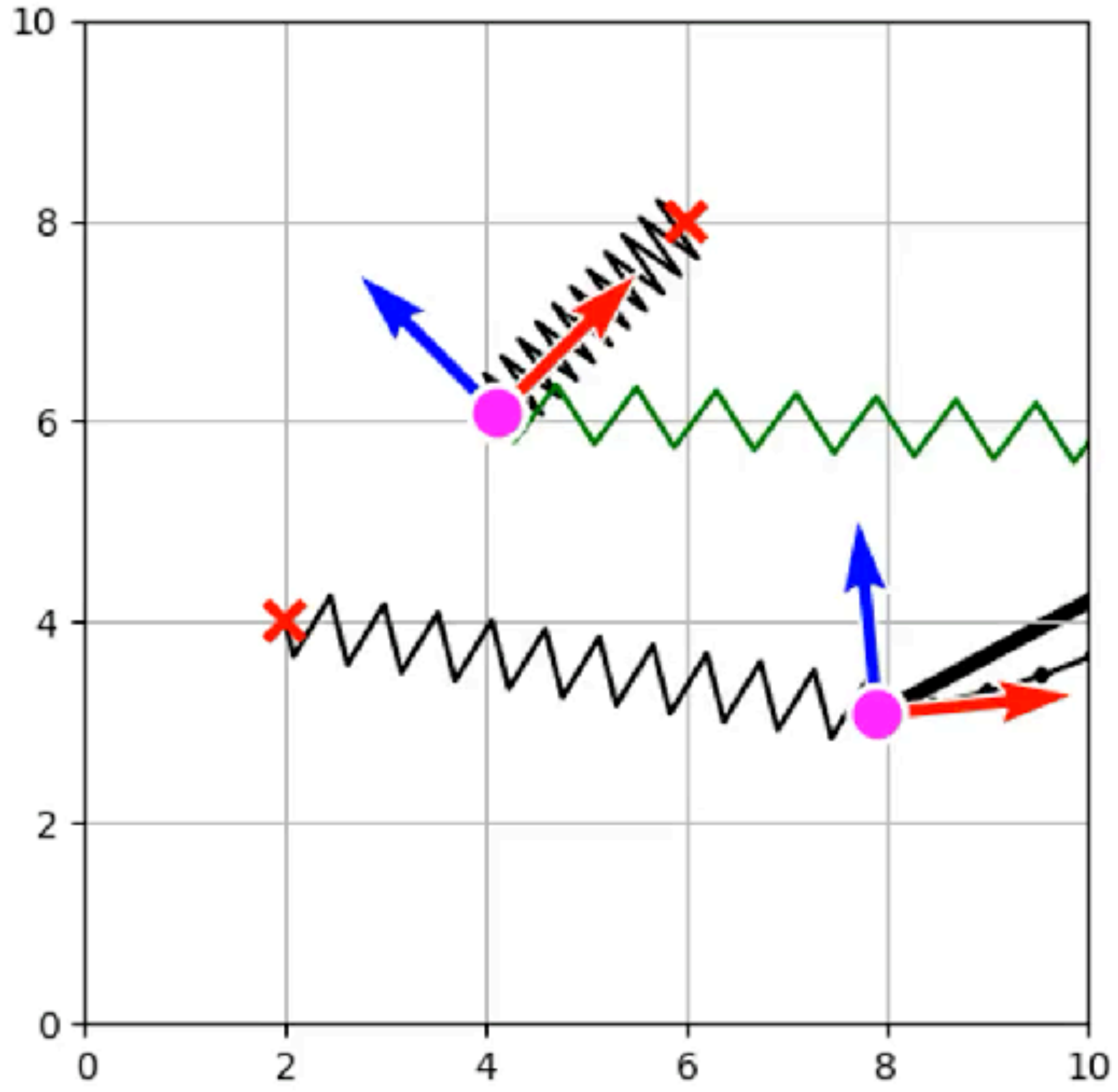
The difference is quite small for small angular velocity

$$\mathbf{u}_2 = [4\sqrt{2}, \pi/4]^\top$$

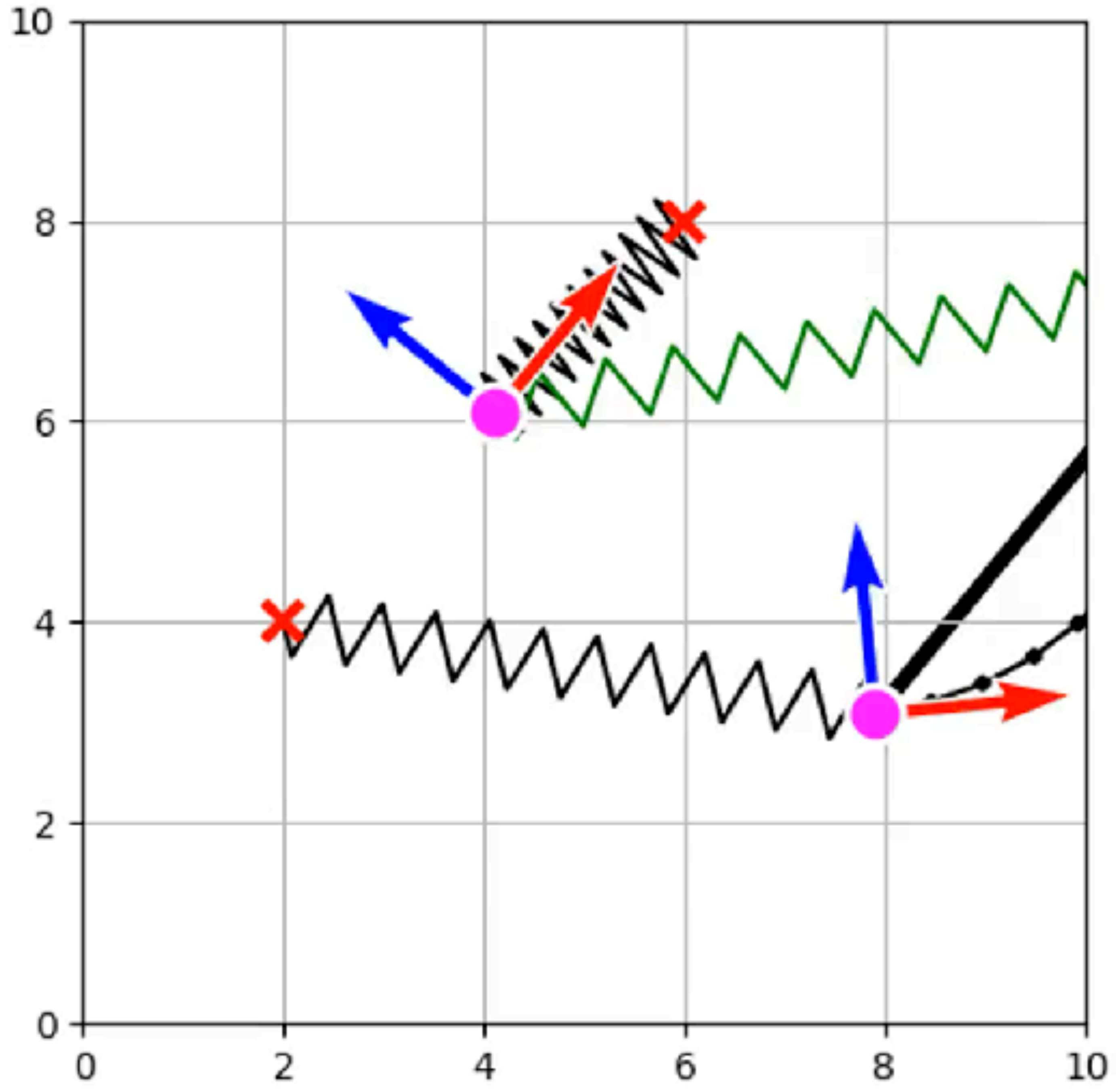


The difference is quite small for small angular velocity

$$\mathbf{u}_2 = [4\sqrt{2}, \pi/4]^\top$$

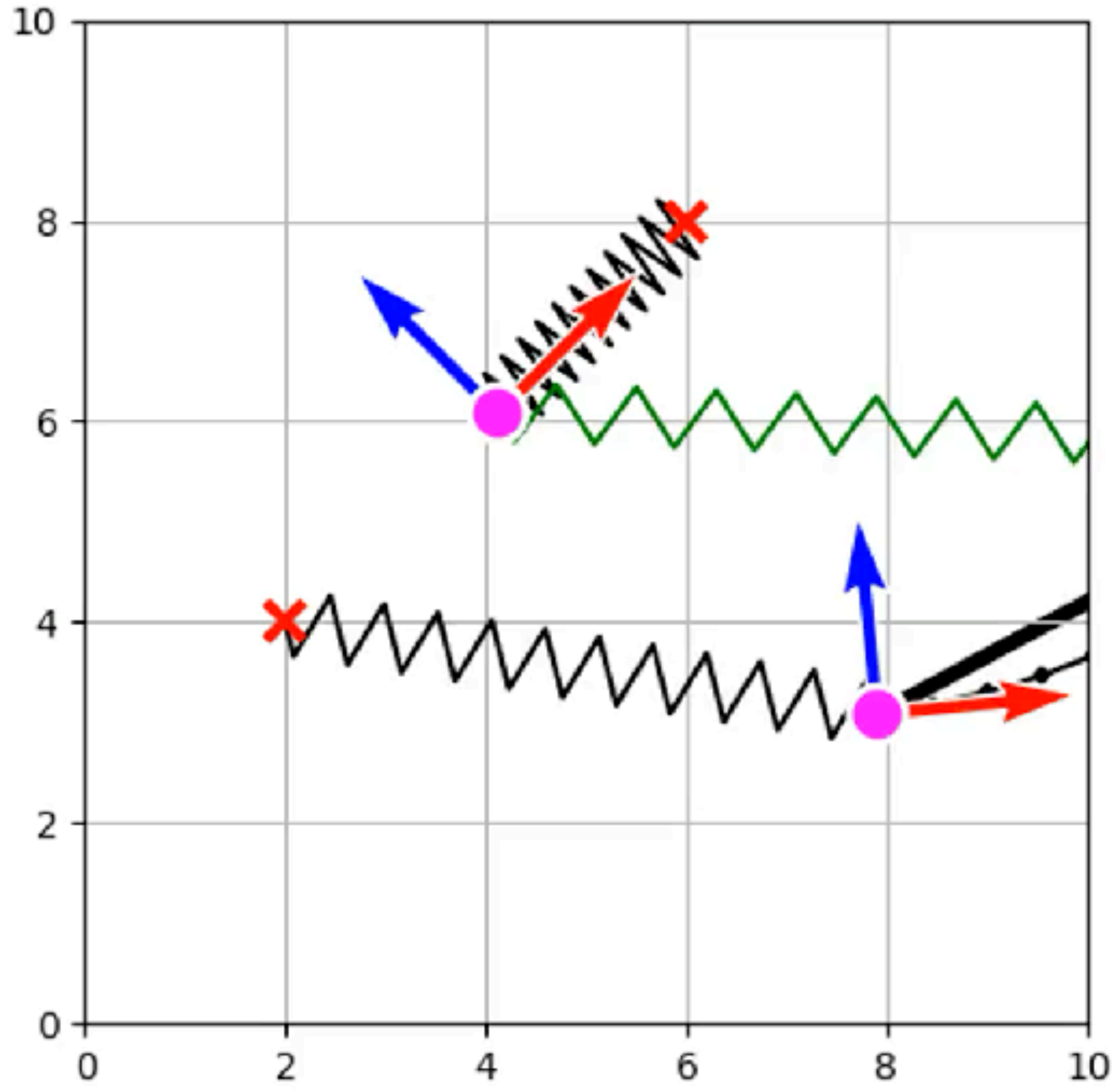


$$\mathbf{u}_2 = [4\sqrt{2}, \pi/2]^\top$$

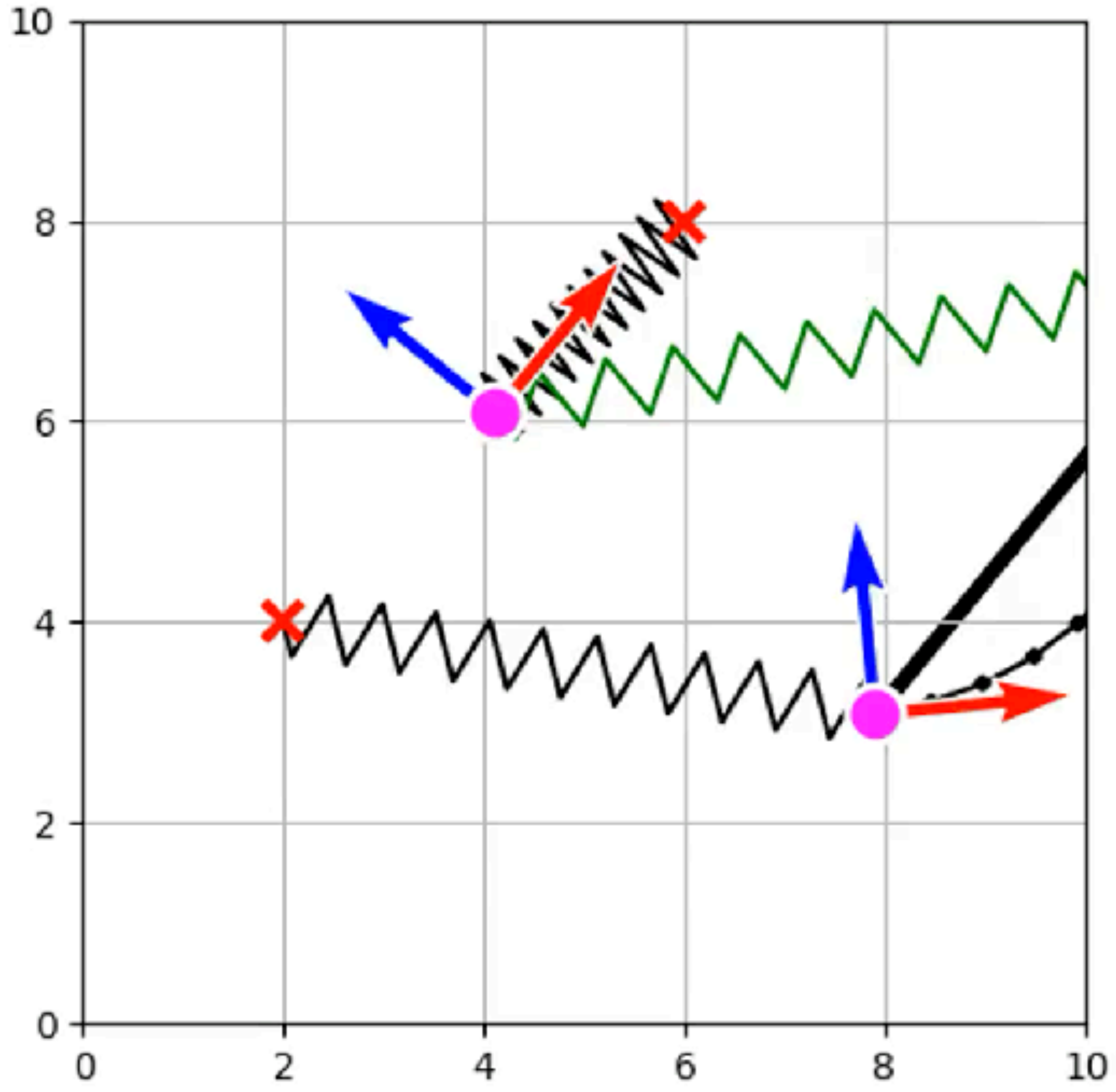


The difference is quite small for small angular velocity

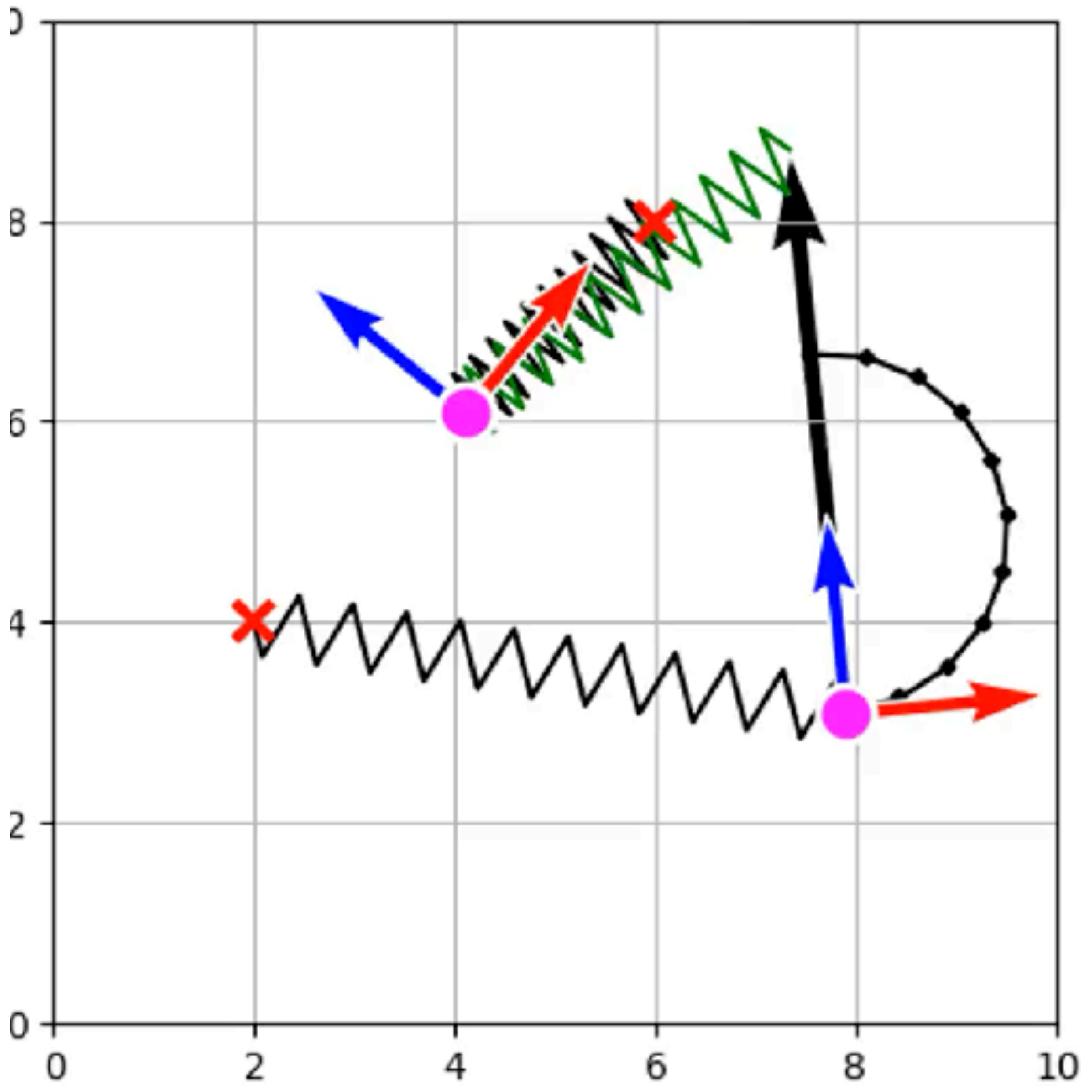
$\mathbf{u}_2 = [4\sqrt{2}, \pi/4]^\top$



$\mathbf{u}_2 = [4\sqrt{2}, \pi/2]^\top$



$\mathbf{u}_2 = [4\sqrt{2}, \pi]^\top$



Where do I get control command?

Standard control ROS topic `/cmd_vel`

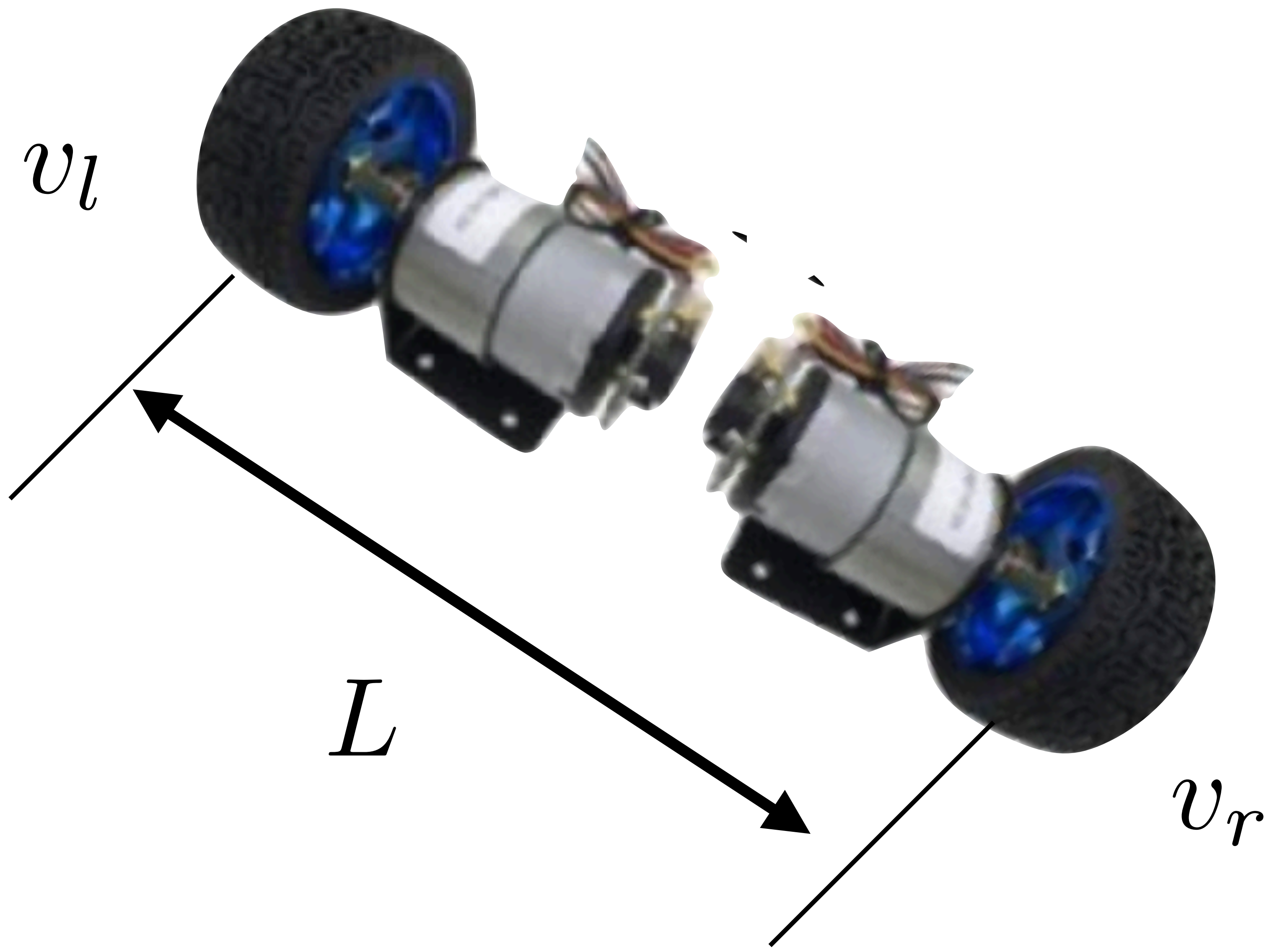
```
/cmd_vel= '{linear:  {x: 1.0, y: 0.0, z: 0.0}, angular: {x: 0.0,y: 0.0,z: 1.0}}'
```

Differential drive platforms:

$$\mathbf{u}_t = [\ \text{Linear velocity } v,\quad \text{Angular velocity } \omega\ ]$$

Control commands often replaced by wheel velocities

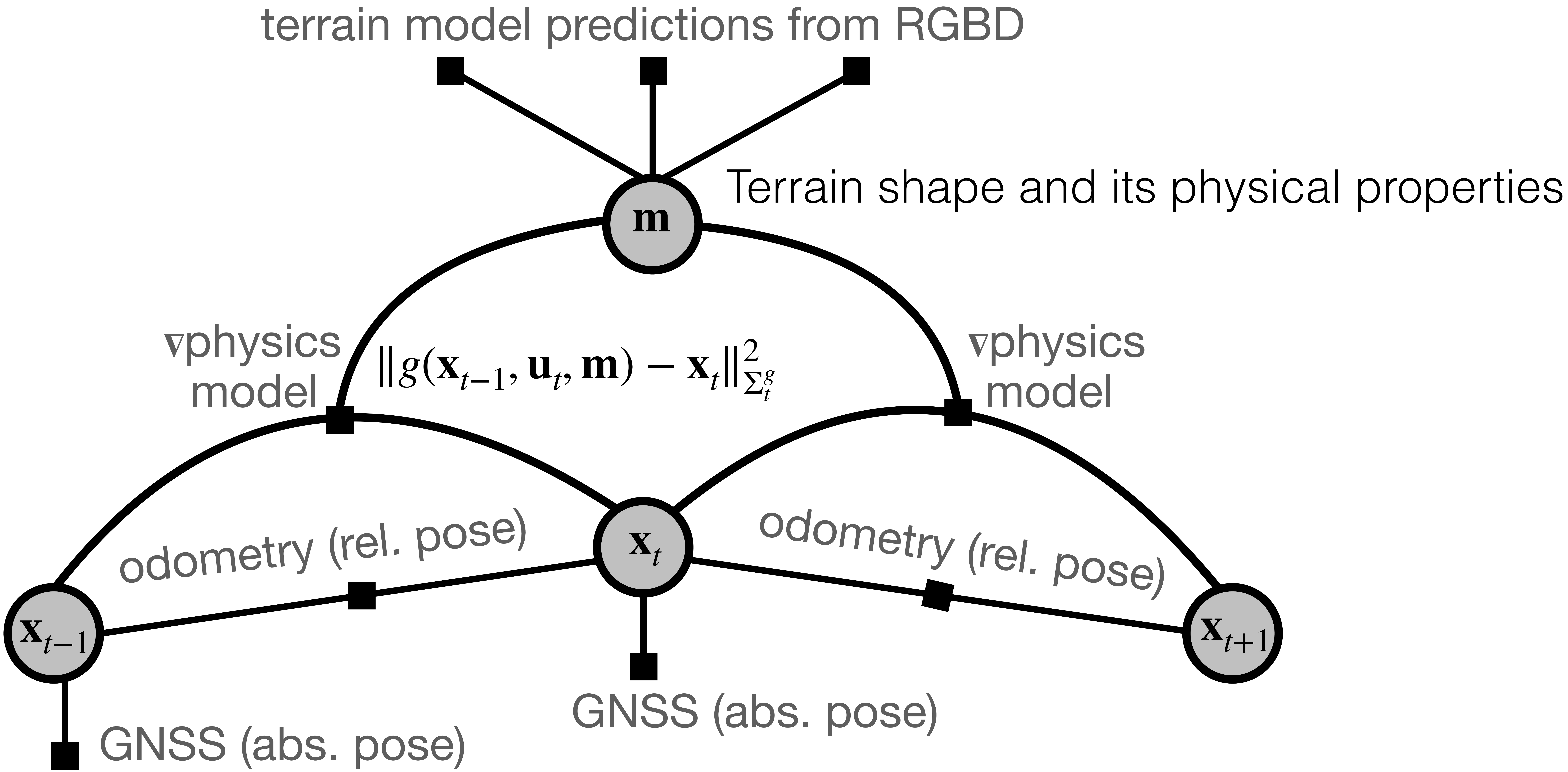
$$v = \frac{v_l + v_r}{2} \qquad \omega = \frac{v_l - v_r}{L}$$



What if I bounce into obstacle?

Are there any more sophisticated models that take the terrain in account?

SLAM - our approach

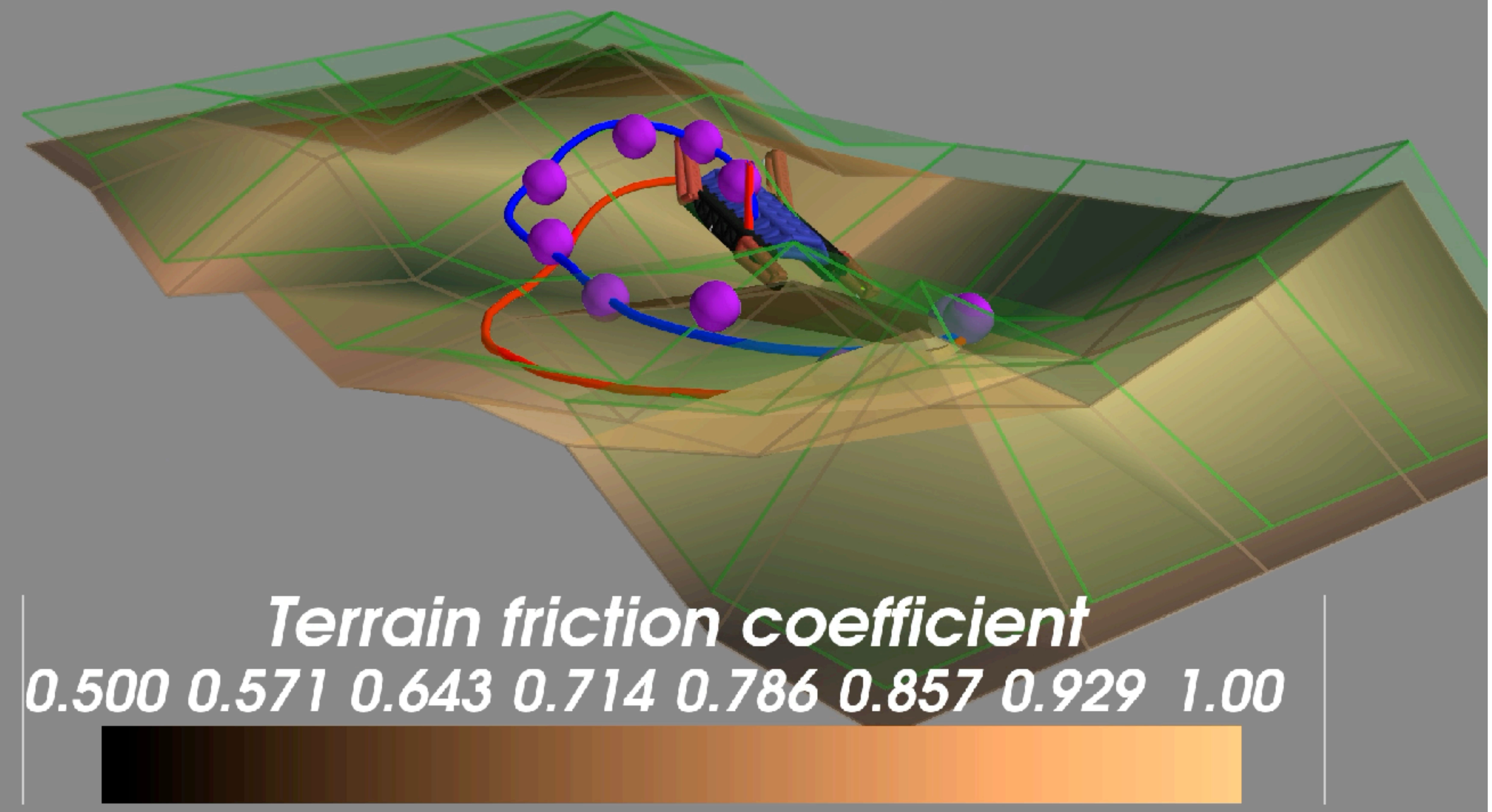


Input: noisy gps + imu + control

Output: trajectory + map (field of robot-terrain inter. forces)

loss = 26.205

GraphSLAM requires differentiable ODE solver



Google's BRAX - differentiable physics engine

https://github.com/google/brax

Brax simulates these environments at millions of physics steps per second on TPU

NVIDIA WARP - differentiable physics engine

https://developer.nvidia.com/warp-python

Cloth simulation

Particle-based simulation

Straightforward extensions

GPS, odometry, 3D marker(s)

$$= \arg \min_{\substack{\mathbf{x}_0,\ldots\mathbf{x}_T \\ \mathbf{m}^1\ldots\mathbf{m}^J}} \sum_t \|\mathbf{x}_t - \mathbf{z}_t^{gps}\|^2_{\Sigma_t^{gps}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t) - \mathbf{z}^v\|^2_{\Sigma_t^v} + \sum_{t,j} \|\mathrm{w2r}(\mathbf{m}^j, \mathbf{x}_t) - \mathbf{z}\|^2_{\Sigma_t^{m^j}}$$

priors, loop-closures

$$+ \sum_t \|\mathbf{x}_t - \mathbf{x}_t^{prior}\|^2_{\Sigma_t^{prior}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_0, \mathbf{x}_T)\|^2_{\Sigma_t^{lc}}$$

motion model, UWB, 2D marker(s)

$$+ \sum_t \|g(\mathbf{x}_{t-1}, \mathbf{u}_t) - \mathbf{x}_t\|^2_{\Sigma_t^g} + ??? + ???$$

e.g. camera detections

Straightforward extensions

GPS | odometry | 3D marker(s)

$$= \arg\min_{\substack{\mathbf{x}_0,\ldots\mathbf{x}_T \\ \mathbf{m}^1\ldots\mathbf{m}^J}} \sum_t \|\mathbf{x}_t - \mathbf{z}_t^{gps}\|^2_{\Sigma_t^{gps}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t) - \mathbf{z}^v\|^2_{\Sigma_t^v} + \sum_{t,j} \|\mathrm{w2r}(\mathbf{m}^j, \mathbf{x}_t) - \mathbf{z}\|^2_{\Sigma_t^{m^j}}$$

priors | loop-closures

$$+ \sum_t \|\mathbf{x}_t - \mathbf{x}_t^{prior}\|^2_{\Sigma_t^{prior}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_0, \mathbf{x}_T)\|^2_{\Sigma_t^{lc}}$$

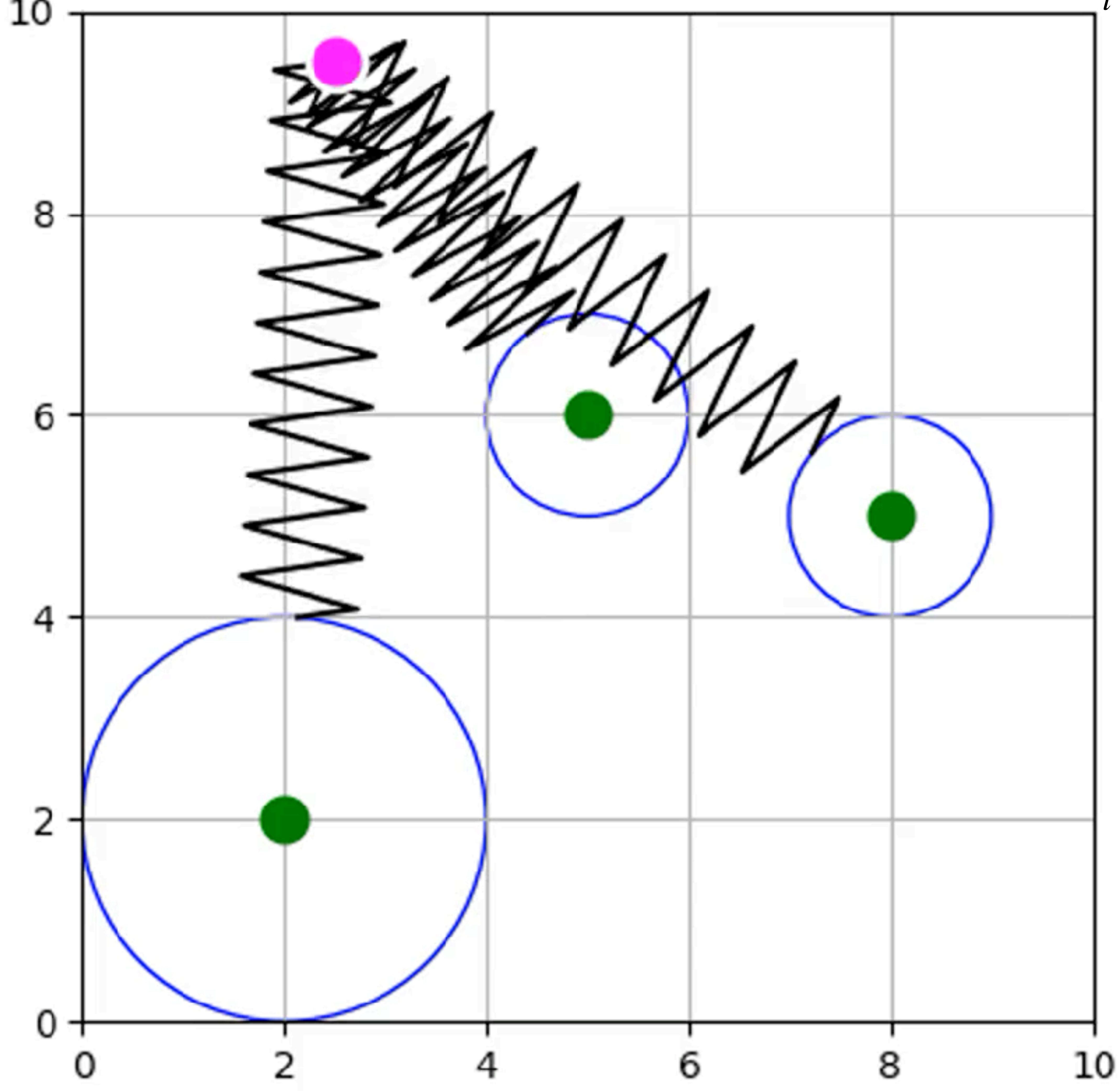
motion model | UWB | 2D marker(s)

$$+ \sum_t \|g(\mathbf{x}_{t-1}, \mathbf{u}_t) - \mathbf{x}_t\|^2_{\Sigma_t^g} + ??? + ???$$

e.g. camera detections

UWB

$$\mathbf{x}^{\star} = \arg\min_{\mathbf{x}_t} \sum_i \| \|\mathbf{x}_t - \mathbf{m}_i\| - \mathbf{z}_t^{UWB_i} \|^2_{\Sigma_t^{UWB_i}}$$



$\mathbf{x}_t$ … robot poses

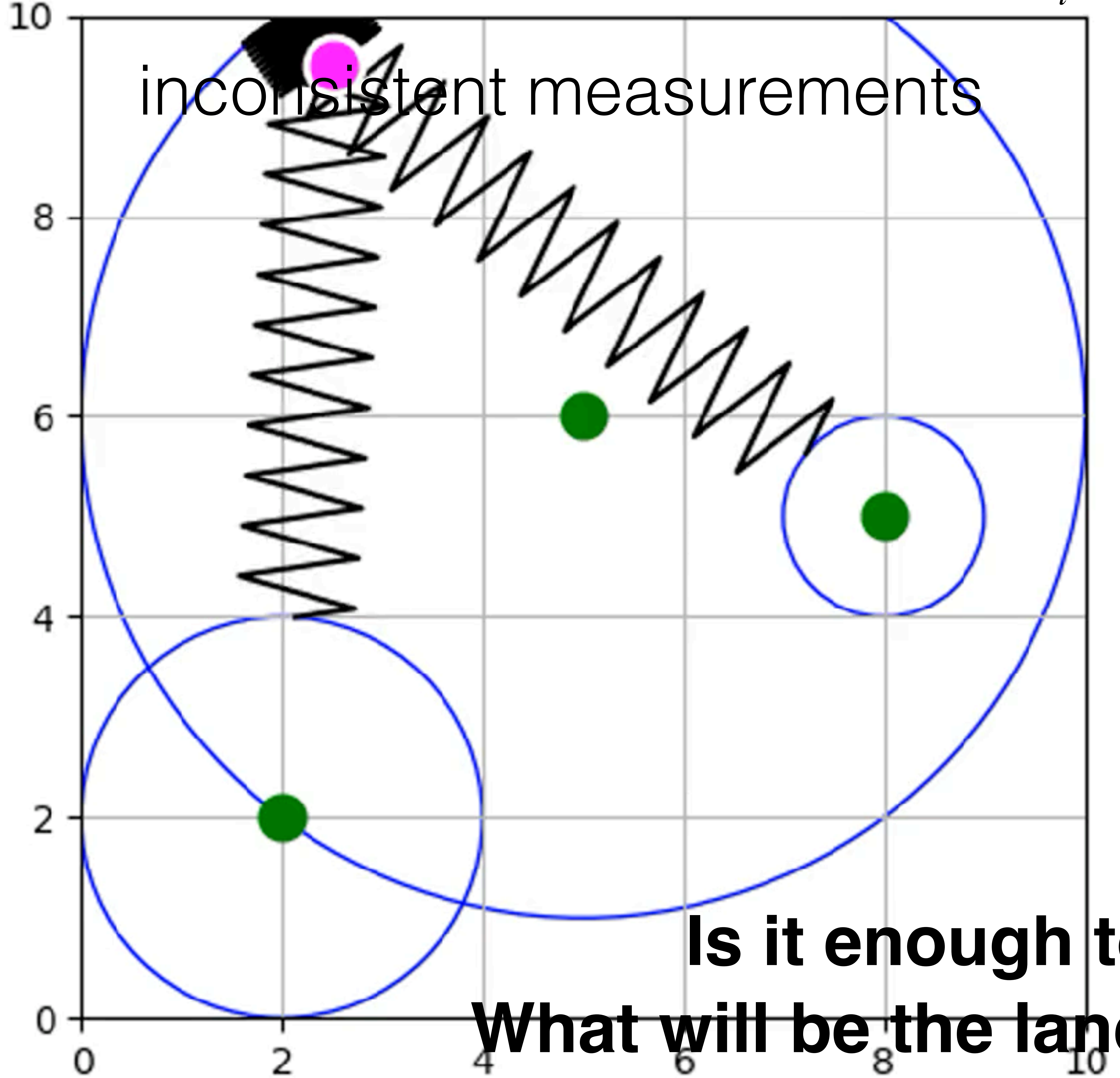
$\mathbf{m}_i$ … known marker positions

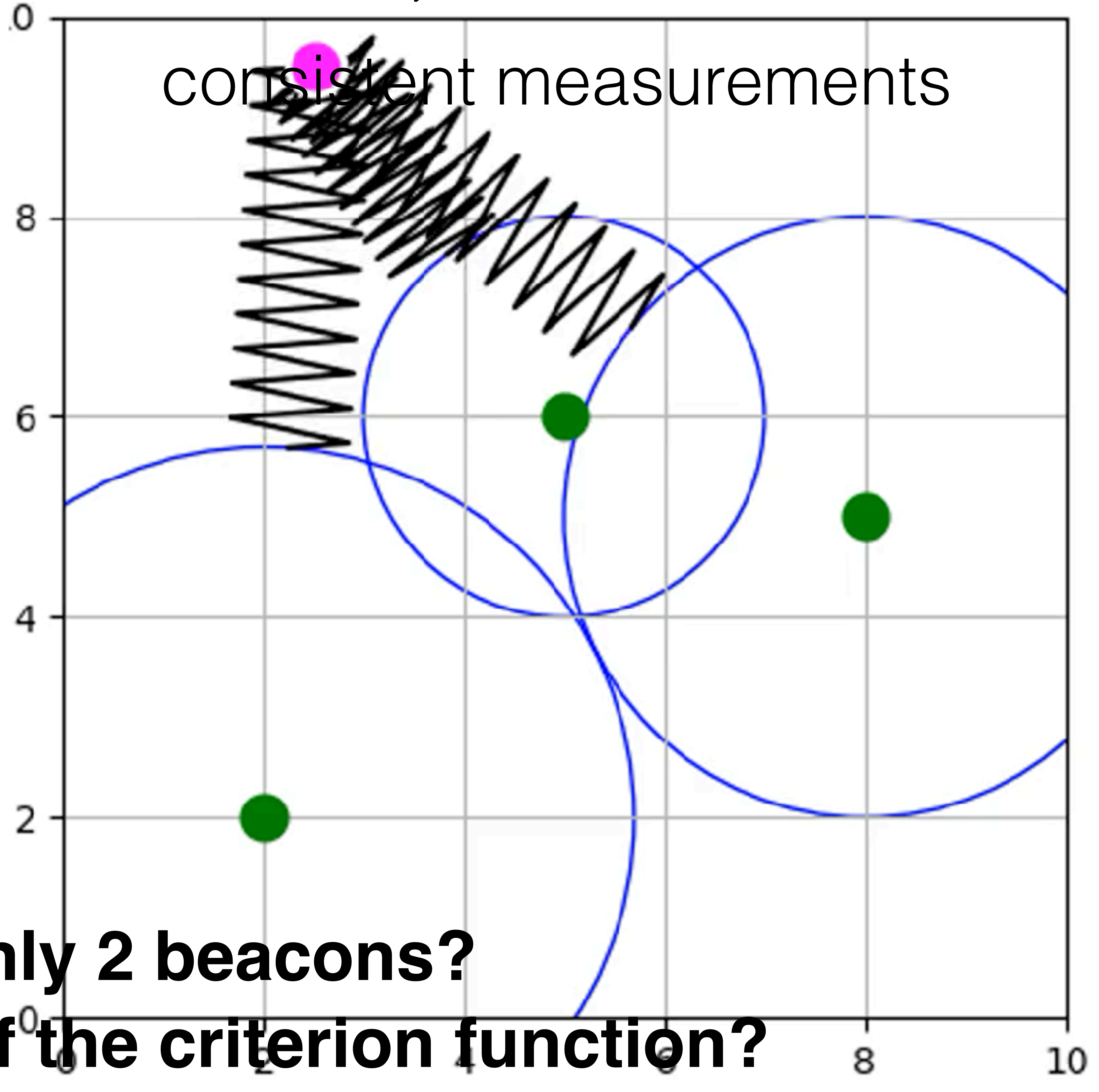
$\mathbf{z}_t^{UWB}$ … UWB measurements (distance)

… UWB loss

UWB

$$\mathbf{x}^\star = \arg\min_{\mathbf{x}_t} \sum_i \| \|\mathbf{x}_t - \mathbf{m}_i\| - \mathbf{z}_t^{UWB_i} \|^2_{\Sigma_t^{UWB_i}}$$

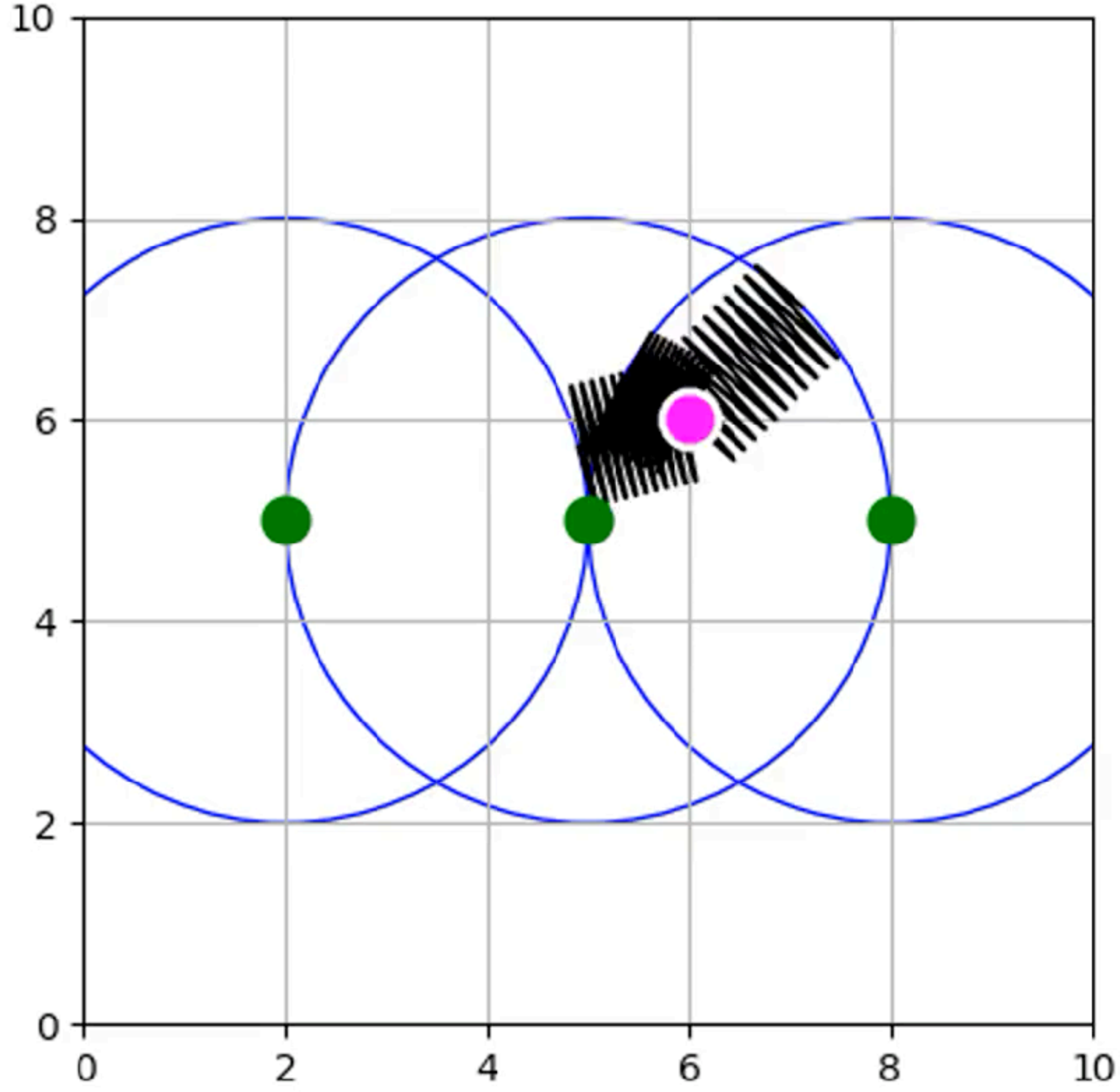




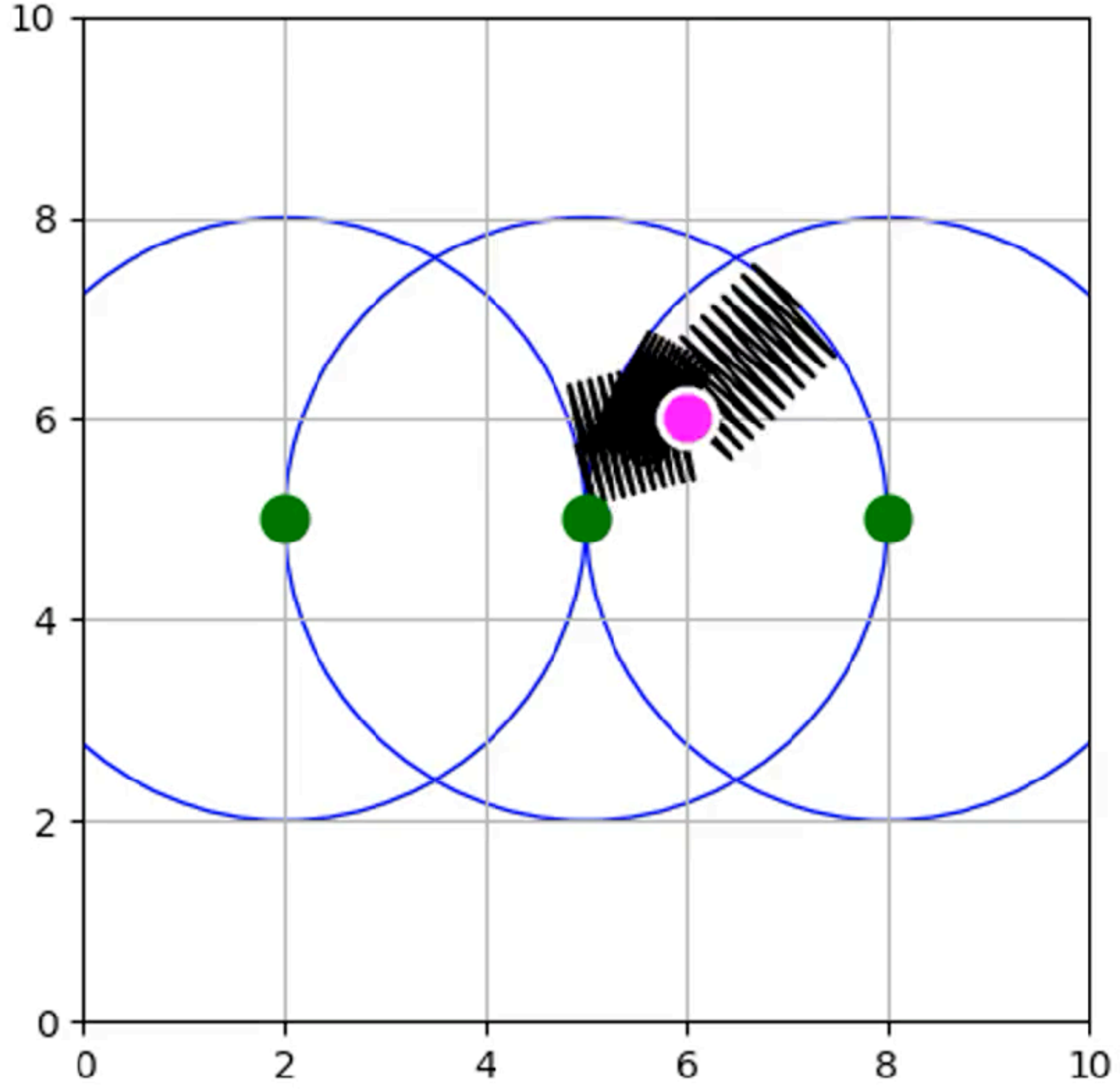
Is it enough to have only 2 beacons?

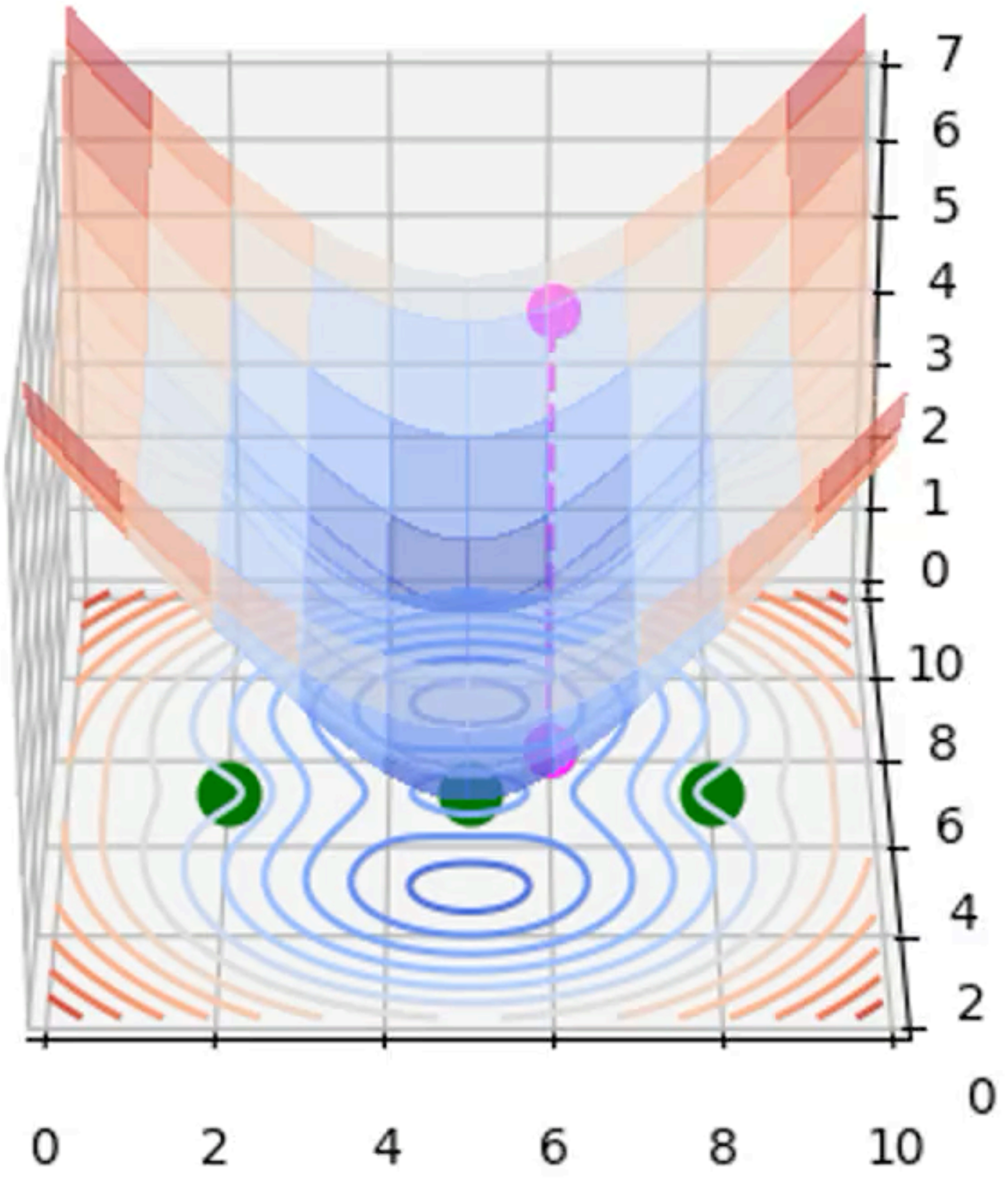
What will be the landscape of the criterion function?

What will be the landscape of the criterion function?



What will be the landscape of the criterion function?





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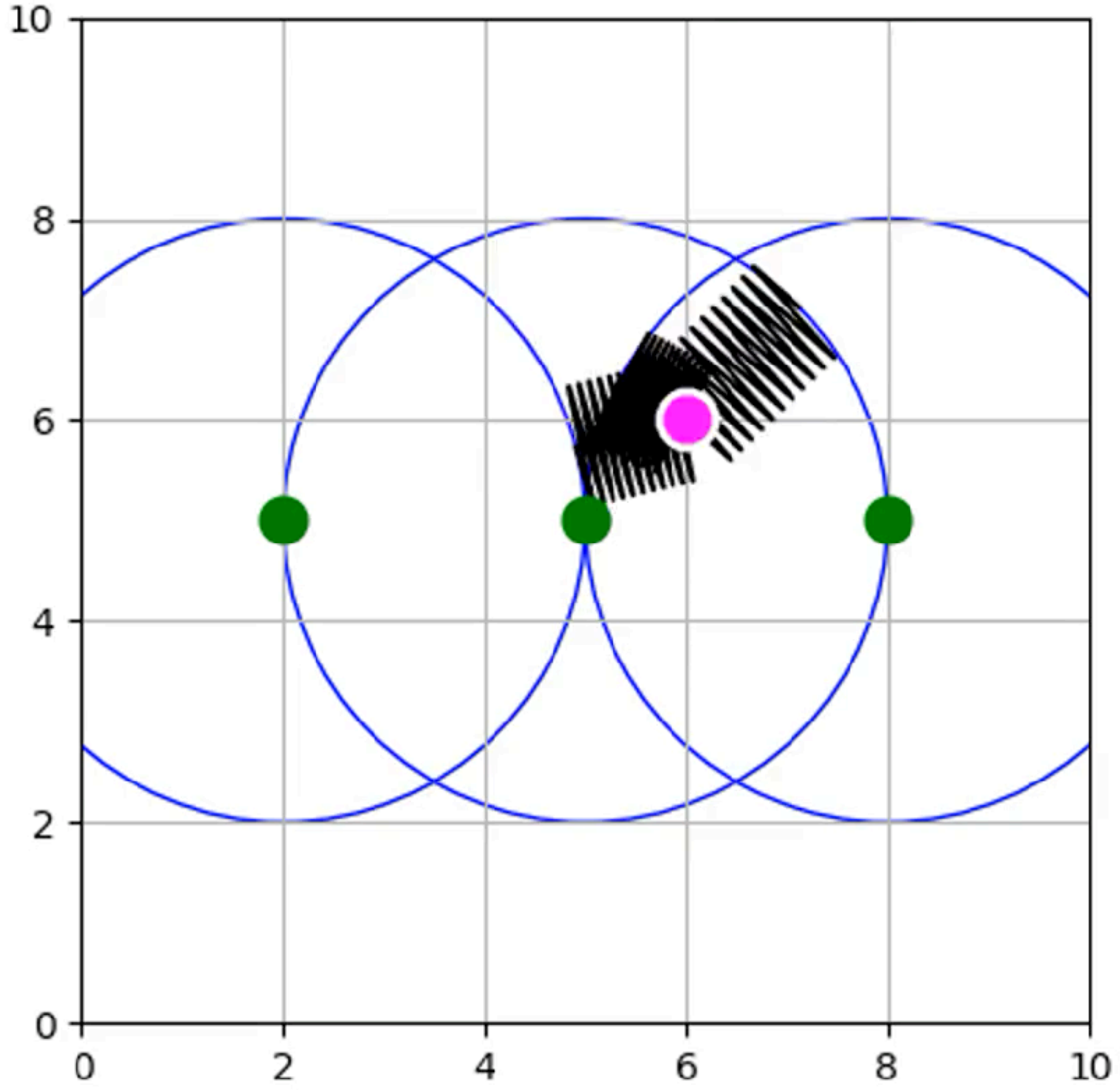
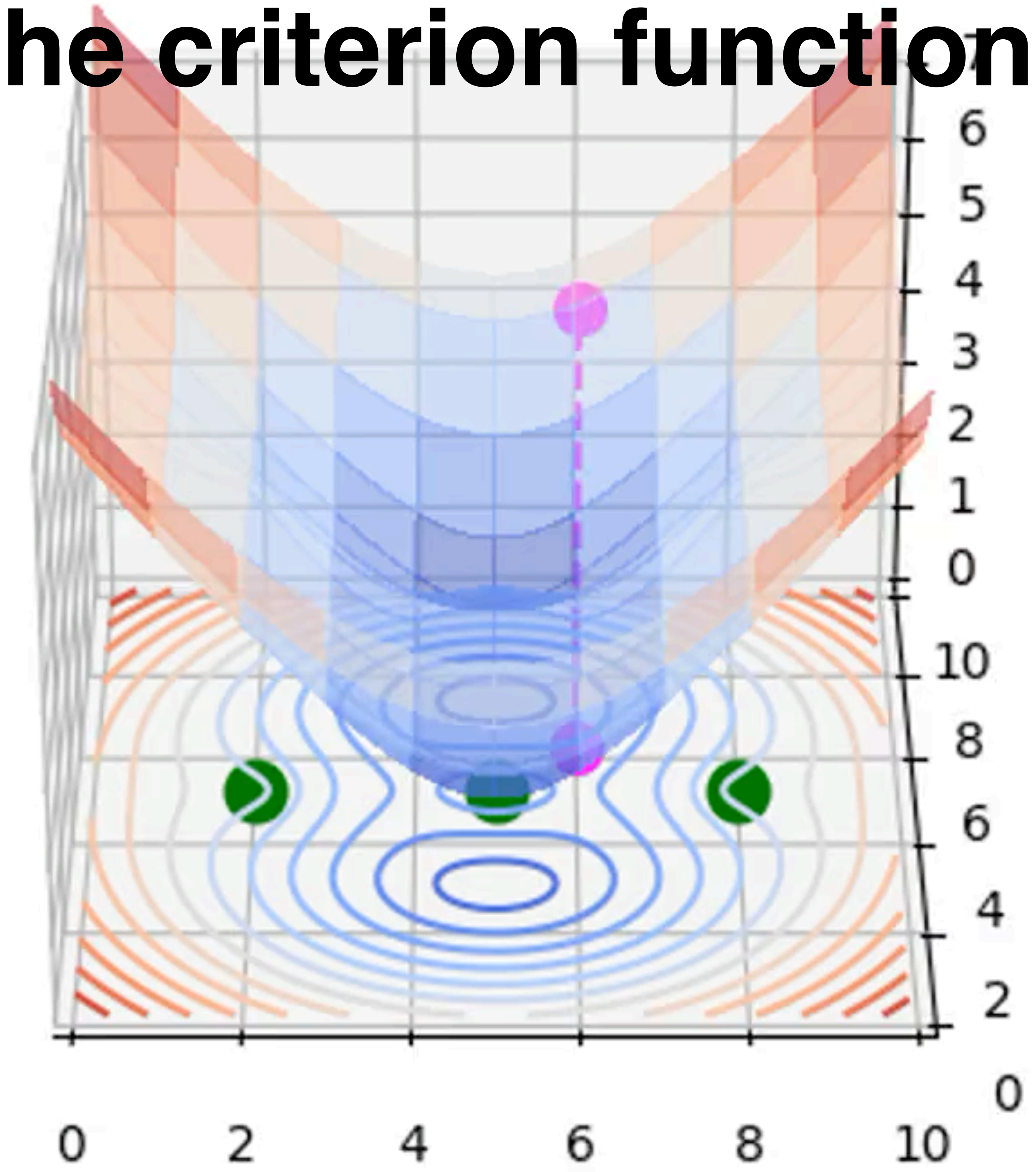
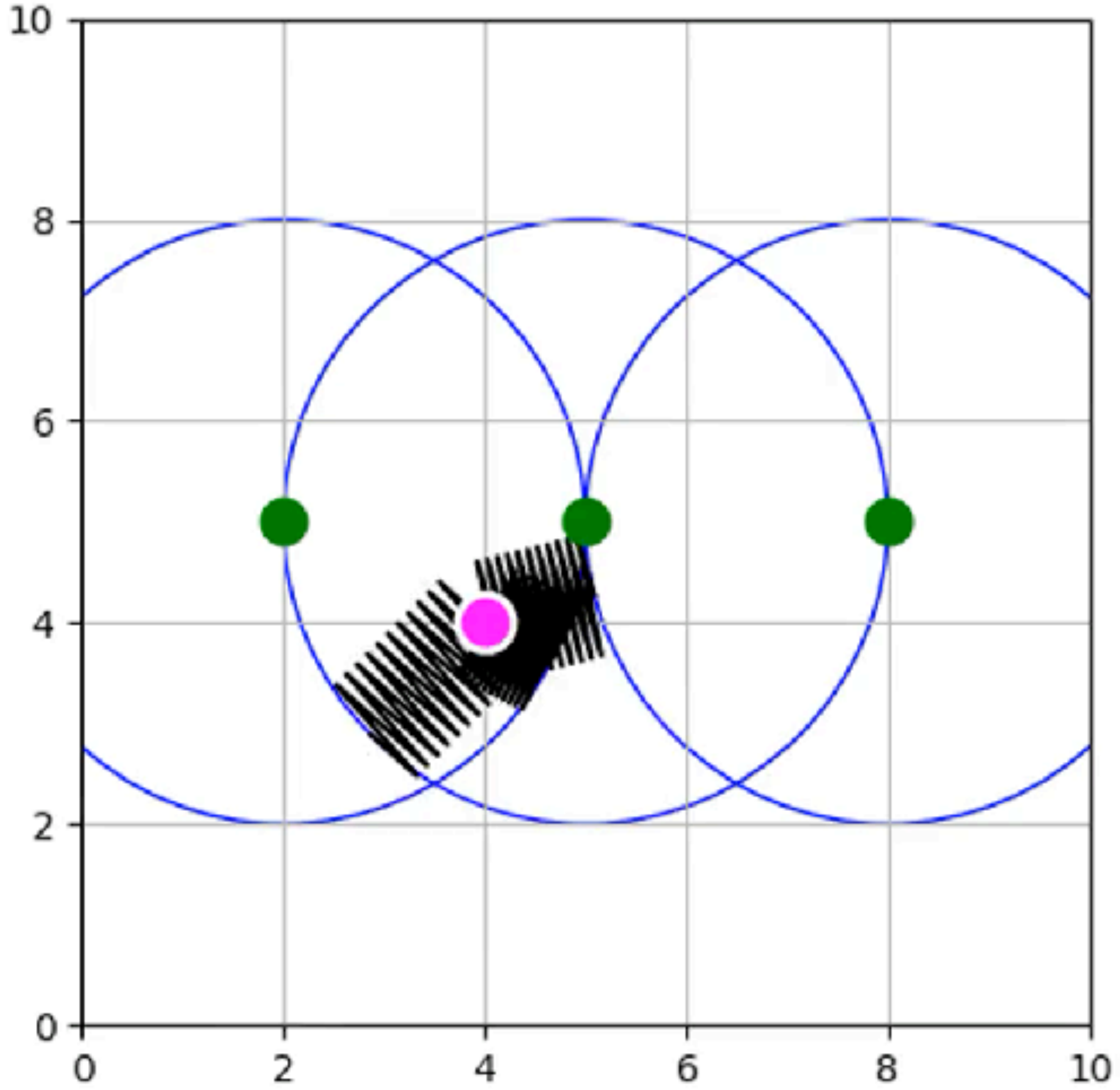
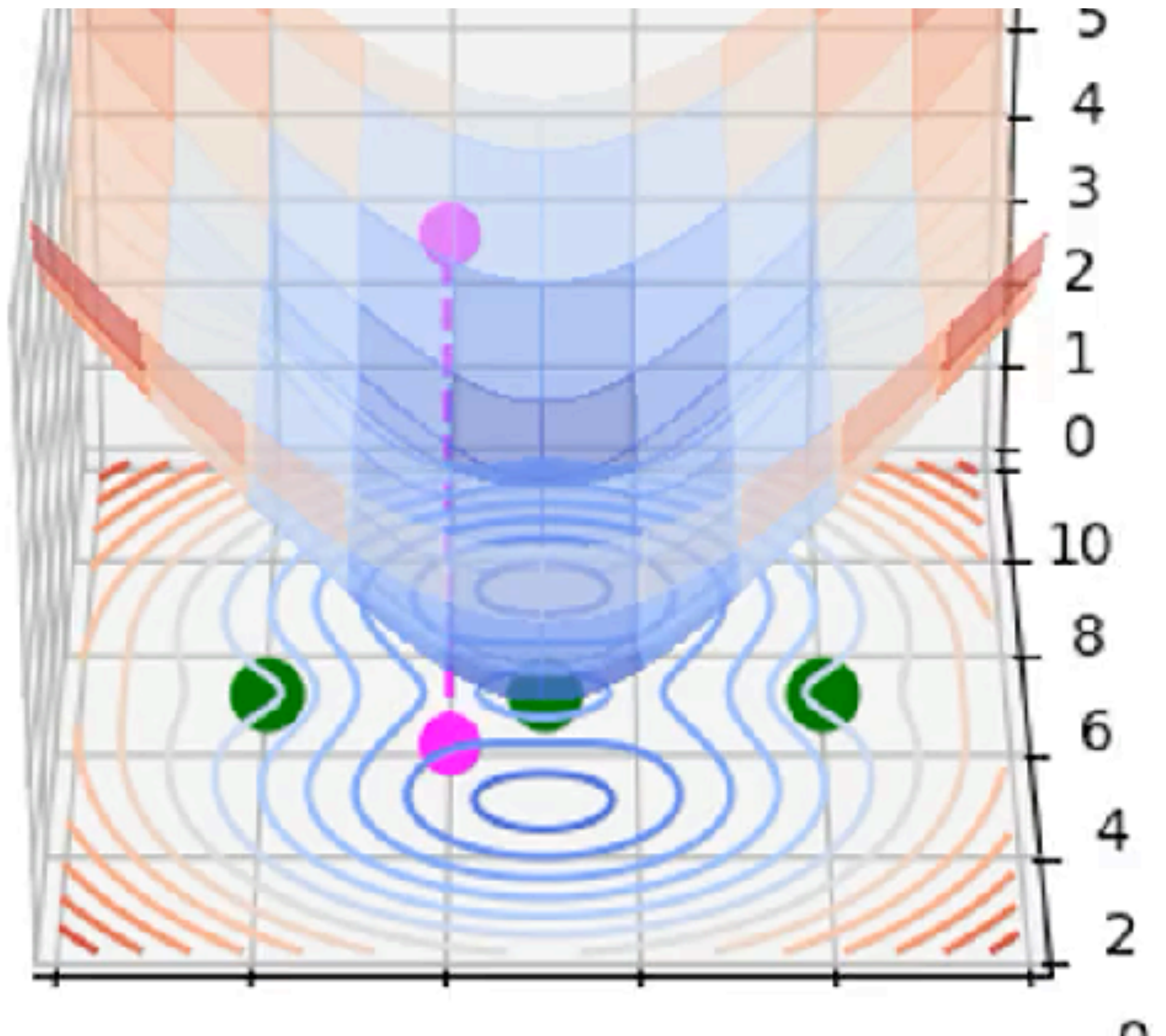
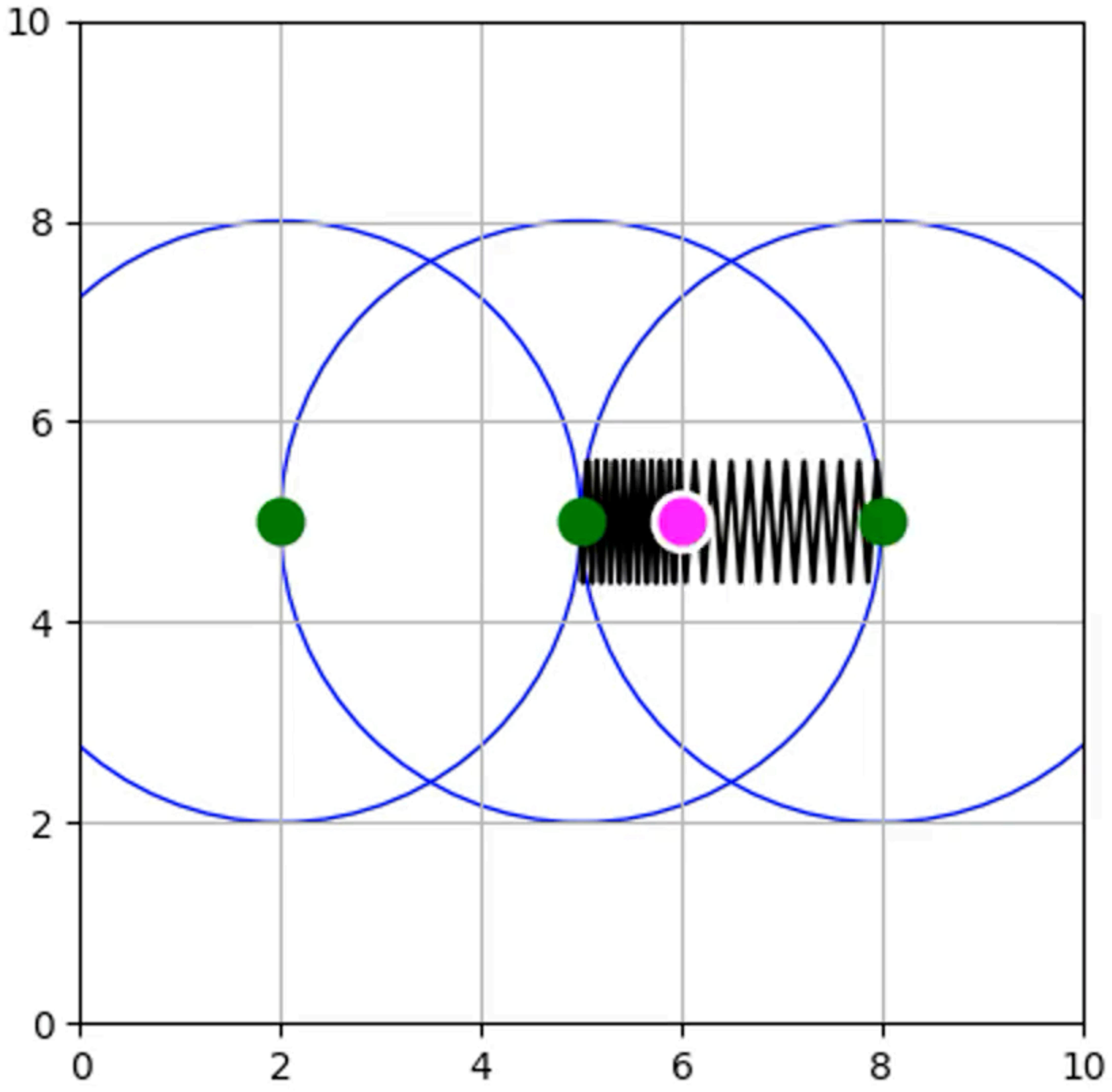


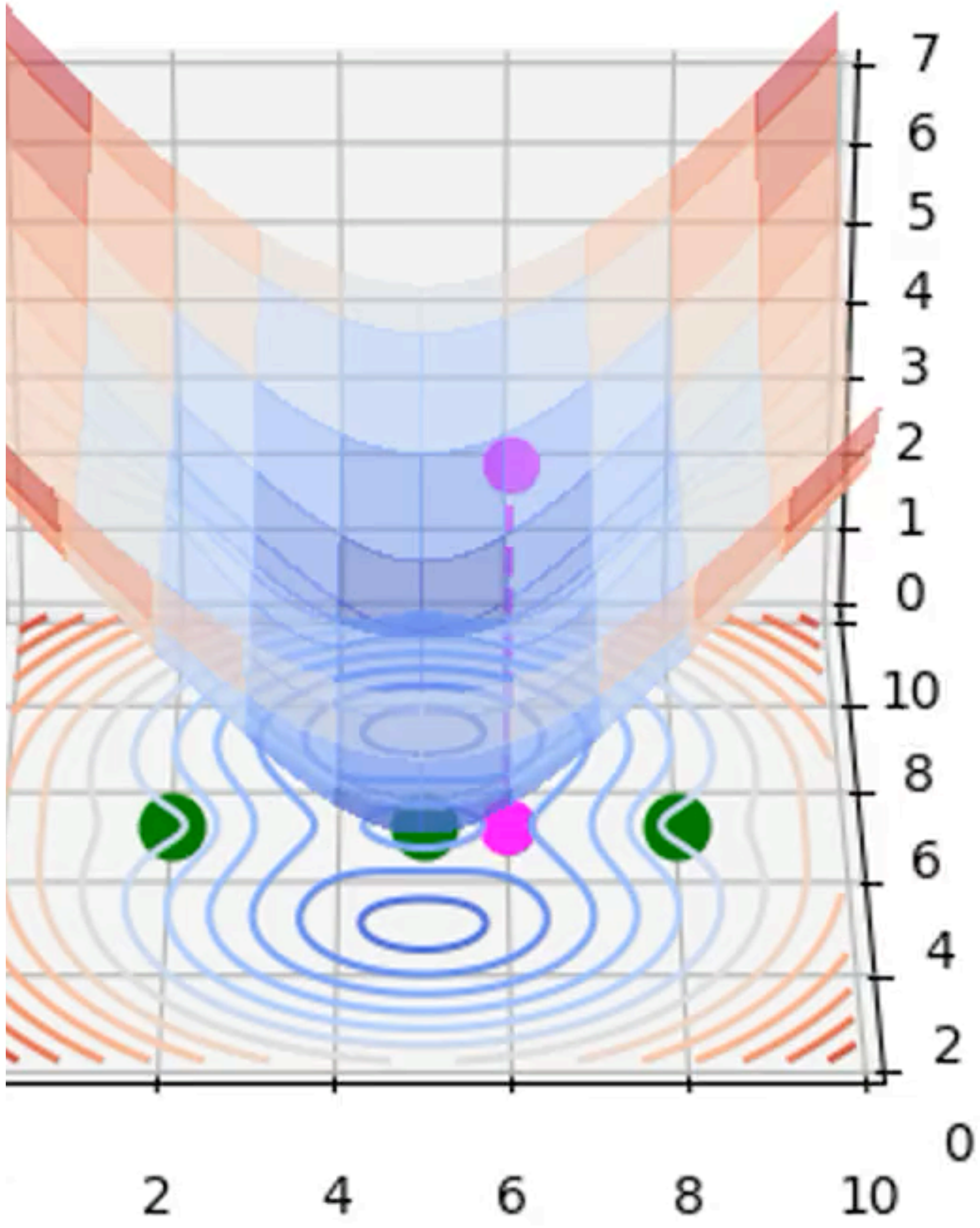






Saddle points





UWB

$$\mathbf{x}^{\star} = \arg\min_{\mathbf{x}_t, \mathbf{m}_i} \sum_{i,t} \| \|\mathbf{x}_t - \mathbf{m}_i\| - \mathbf{z}_t^{UWB_i} \|^2_{\Sigma_t^{UWB_i}}$$

UWB SLAM

What is the dimensionality of zero-loss subspace?

"8-dim space" - "3 DOF from measurements" = "5-dim"

Straightforward extensions

GPS — odometry — 3D marker(s)

$$= \arg\min_{\substack{\mathbf{x}_0,\ldots\mathbf{x}_T \\ \mathbf{m}^1\ldots\mathbf{m}^J}} \sum_t \|\mathbf{x}_t - \mathbf{z}_t^{gps}\|^2_{\Sigma_t^{gps}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t) - \mathbf{z}^v\|^2_{\Sigma_t^v} + \sum_{t,j} \|\mathrm{w2r}(\mathbf{m}^j, \mathbf{x}_t) - \mathbf{z}\|^2_{\Sigma_t^{m^j}}$$

priors — loop-closures

$$+ \sum_t \|\mathbf{x}_t - \mathbf{x}_t^{prior}\|^2_{\Sigma_t^{prior}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_0, \mathbf{x}_T)\|^2_{\Sigma_t^{lc}}$$

motion model — UWB

$$+ \sum_t \|g(\mathbf{x}_{t-1}, \mathbf{u}_t) - \mathbf{x}_t\|^2_{\Sigma_t^g} + \sum_{i,t} \| \|\mathbf{x}_t - \mathbf{m}_i\| - \mathbf{z}_t^{UWB_i}\|^2_{\Sigma_t^{UWB_i}}$$

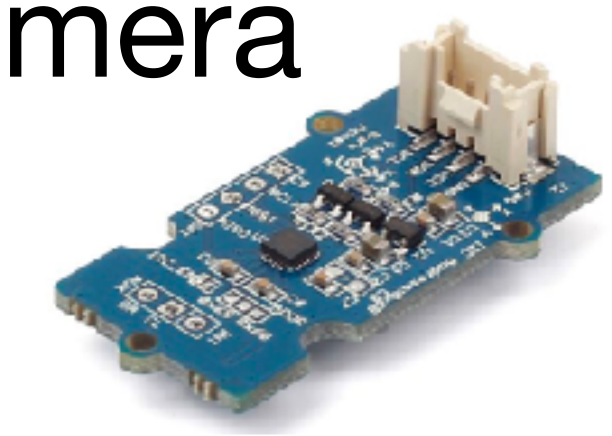
+ 2D marker(s)

???

e.g. camera detections

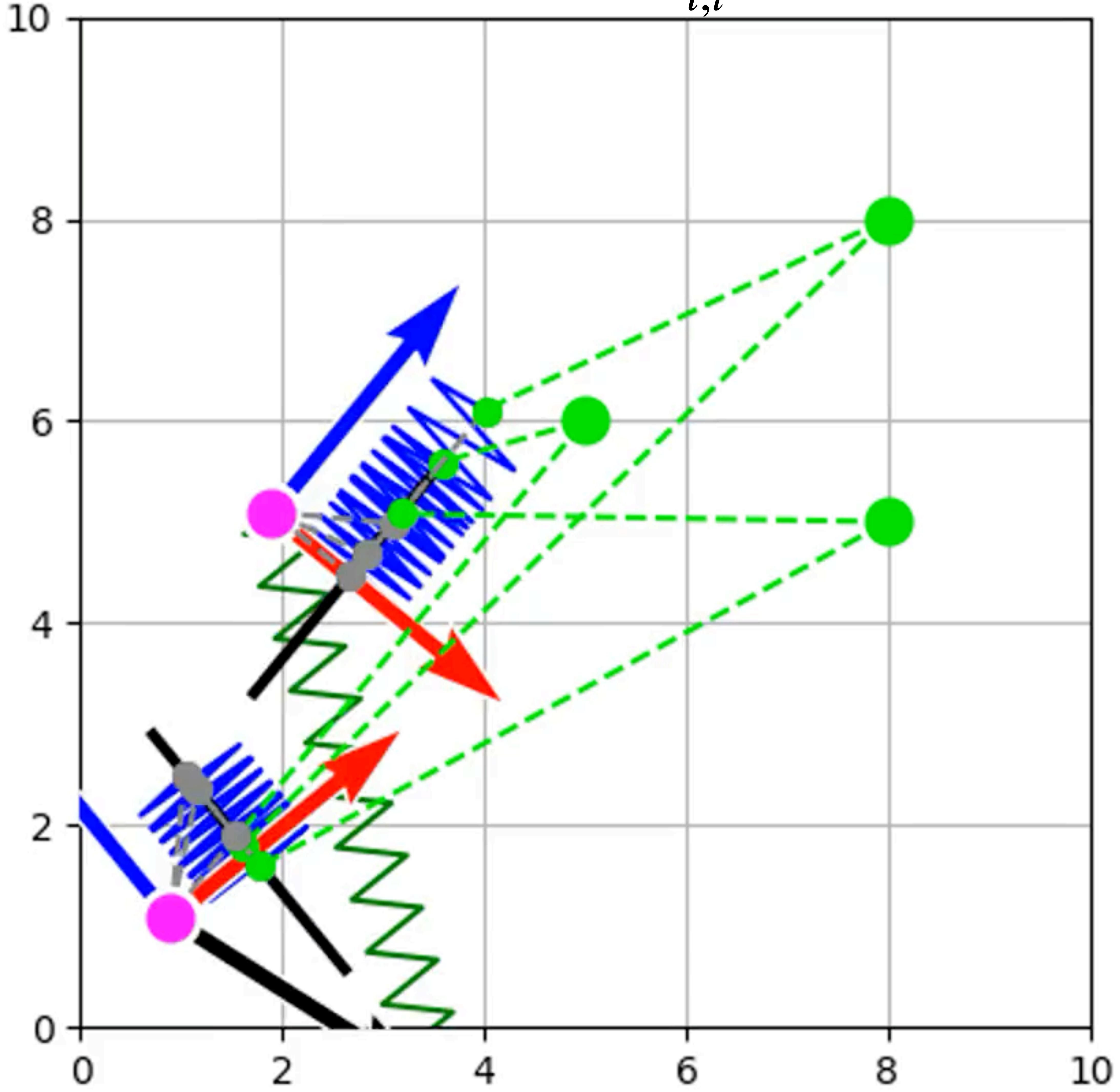
Localization from camera

2D marker detector (RGB camera) +



Odometry (IMU)

$$\mathbf{x}^\star = \arg\min_{\mathbf{x}_t} \sum_{i,t} \|\text{w2cam}(\mathbf{m}_i, \mathbf{x}_t) - \mathbf{z}_t^{\mathbf{m}_i}\|^2 + \|\text{w2r}(\mathbf{x}_2, \mathbf{x}_1) - \mathbf{z}_{12}^{odom}\|^2$$



$\mathbf{x}_t$ …robot poses

$\mathbf{m}_i$ …known marker positions

$\mathbf{z}_t^{\mathbf{m}_i}$ …marker measurements

$\sum_{i,t} \|\text{w2r}(\mathbf{m}_i, \mathbf{x}_t) - \mathbf{z}_t^{\mathbf{m}_i}\|^2$ … marker loss

$\|\text{w2r}(\mathbf{x}_2, \mathbf{x}_1) - \mathbf{z}_{12}^{odom}\|^2$ … odom loss

camera coordinate frame + img. plane

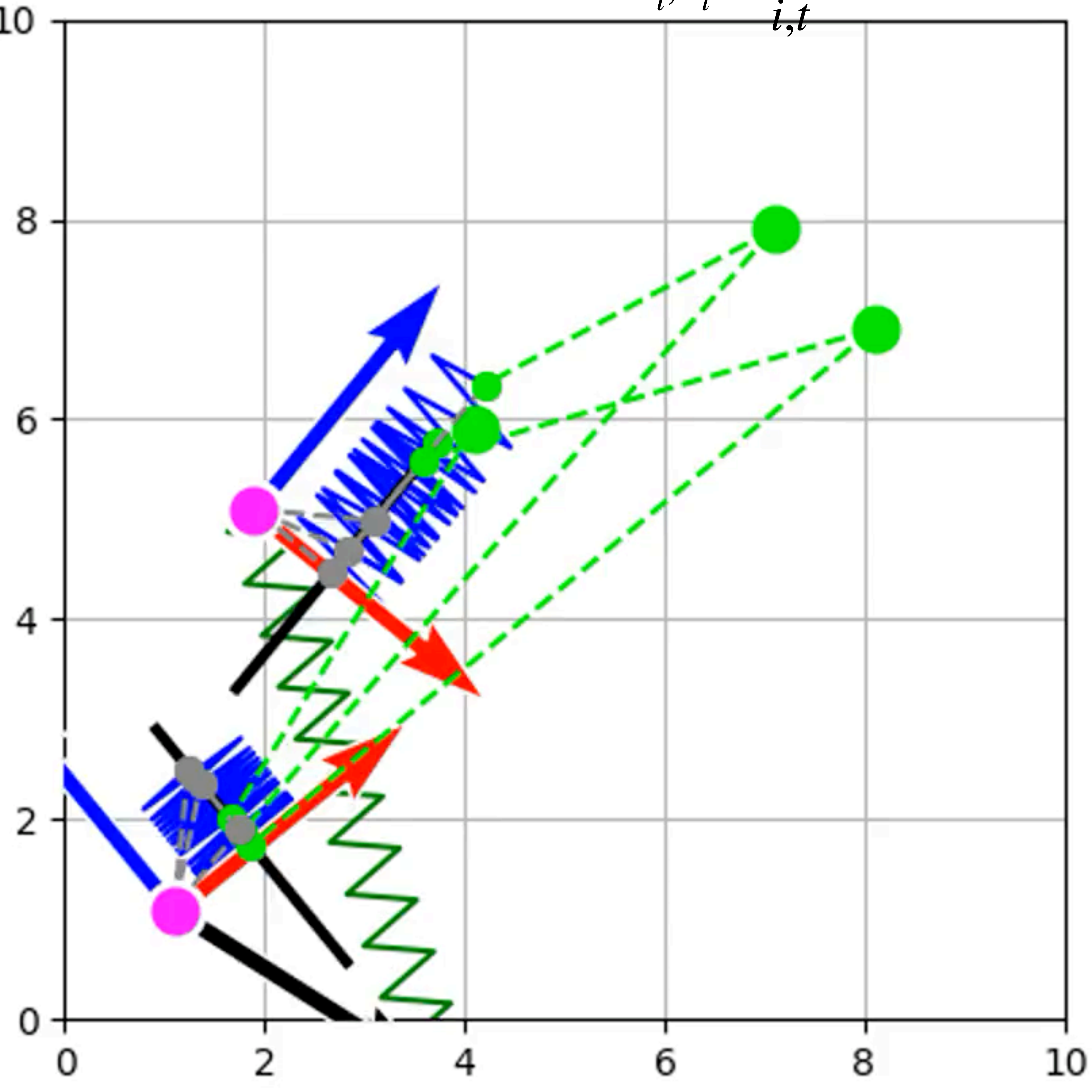
odometry

SLAM from camera (bundle adjustment)

2D marker detector (RGB camera) +

Odometry (IMU)

$$\mathbf{x}^\star = \arg\min_{\mathbf{m}_i, \mathbf{x}_t} \sum_{i,t} \|\text{w2cam}(\mathbf{m}_i, \mathbf{x}_t) - \mathbf{z}_t^{\mathbf{m}_i}\|^2 + \|\text{w2r}(\mathbf{x}_2, \mathbf{x}_1) - \mathbf{z}_{12}^{odom}\|^2$$



$\mathbf{x}_t$ ...robot poses

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$\mathbf{z}_t^{\mathbf{m}_i}$ ...marker measurements

$\sum_{i,t} \|\text{w2r}(\mathbf{m}_i, \mathbf{x}_t) - \mathbf{z}_t^{\mathbf{m}_i}\|^2$ ... marker loss

$\|\text{w2r}(\mathbf{x}_2, \mathbf{x}_1) - \mathbf{z}_{12}^{odom}\|^2$ ... odom loss

camera coordinate frame + img. plane

odometry

Straightforward extensions

GPS — odometry — 3D marker(s)

$$= \arg\min_{\substack{\mathbf{x}_0,\ldots\mathbf{x}_T \\ \mathbf{m}^1\ldots\mathbf{m}^J}} \sum_t \|\mathbf{x}_t - \mathbf{z}_t^{gps}\|^2_{\Sigma_t^{gps}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_{t+1}, \mathbf{x}_t) - \mathbf{z}^v\|^2_{\Sigma_t^v} + \sum_{t,j} \|\mathrm{w2r}(\mathbf{m}^j, \mathbf{x}_t) - \mathbf{z}\|^2_{\Sigma_t^{m^j}}$$

priors — loop-closures

$$+ \sum_t \|\mathbf{x}_t - \mathbf{x}_t^{prior}\|^2_{\Sigma_t^{prior}} + \sum_t \|\mathrm{w2r}(\mathbf{x}_0, \mathbf{x}_T)\|^2_{\Sigma_t^{lc}}$$

motion model — UWB

$$+ \sum_t \|g(\mathbf{x}_{t-1}, \mathbf{u}_t) - \mathbf{x}_t\|^2_{\Sigma_t^g} + \sum_{i,t} \|\|\mathbf{x}_t - \mathbf{m}_i\| - \mathbf{z}_t^{UWB_i}\|^2_{\Sigma_t^{UWB_i}}$$

2D marker(s)

$$+ \sum_{i,t} \|\mathrm{w2cam}(\mathbf{m}_i, \mathbf{x}_t) - \mathbf{z}_t^{\mathbf{m_i}}\|^2$$

e.g. camera detections

Problems for students

Show that: $\mathbf{z}^w = \begin{bmatrix} R(\theta_t) & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{z}^r + \mathbf{x}_t$ => $\mathbf{z}^r = \begin{bmatrix} R(\theta_t)^\top & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix} (\mathbf{z}^w - \mathbf{x}_t)$

When does it matter, in which coordinate frame (rcf/wcf) the residual is measured?

Does the (non)-linear measurement and/or motion function always imply (non)-convex criterium?

Given problem with measurement and motion function with gaussian noise:

- Derive maximum likelihood estimate
- Draw corresponding factorgraph
- Write down criterion function
- Discuss its non-convexity