Bayes filter

Karel Zimmermann

Complete states

Complete states: $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_t \in \mathcal{R}^n$

Def: A is *conditionally independent* on C given B iff $p(A \mid B, C) = p(A \mid B)$

Def: State $\mathbf{x}_{t-1}$ is *complete* iff future $\mathbf{x}_t$ is conditionally independent on past given $\mathbf{x}_{t-1}$

Consequences:

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

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measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

Factor graph

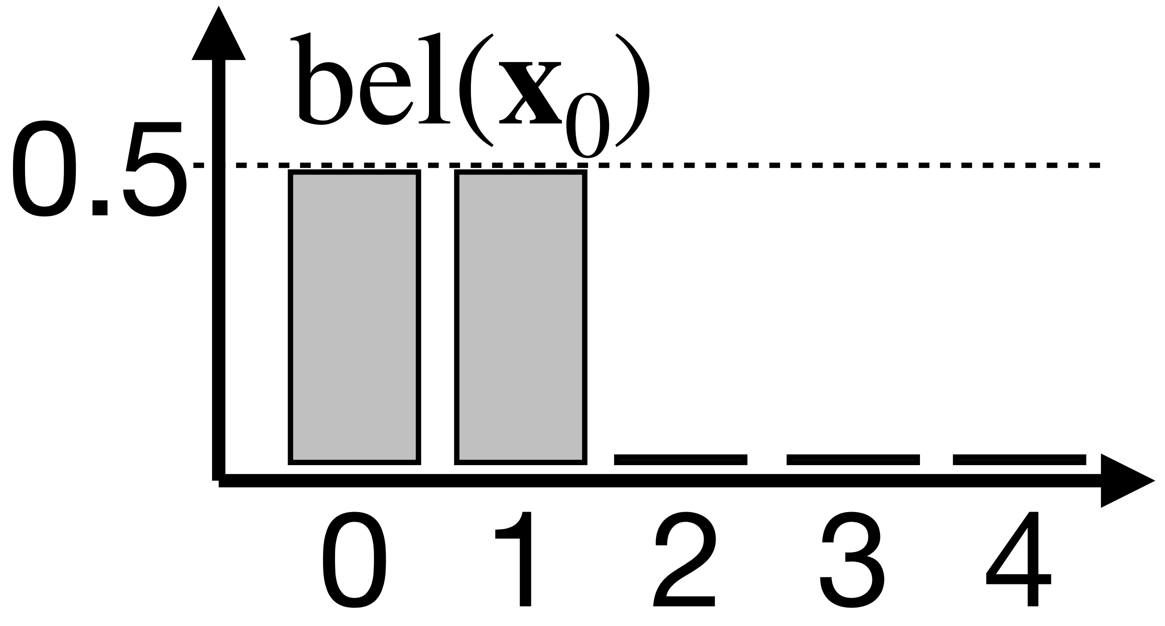
measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

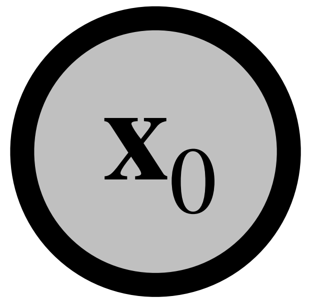
state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

Can I get the optimal $\mathbf{x}_3$ from this factor graph?

Bayes filter

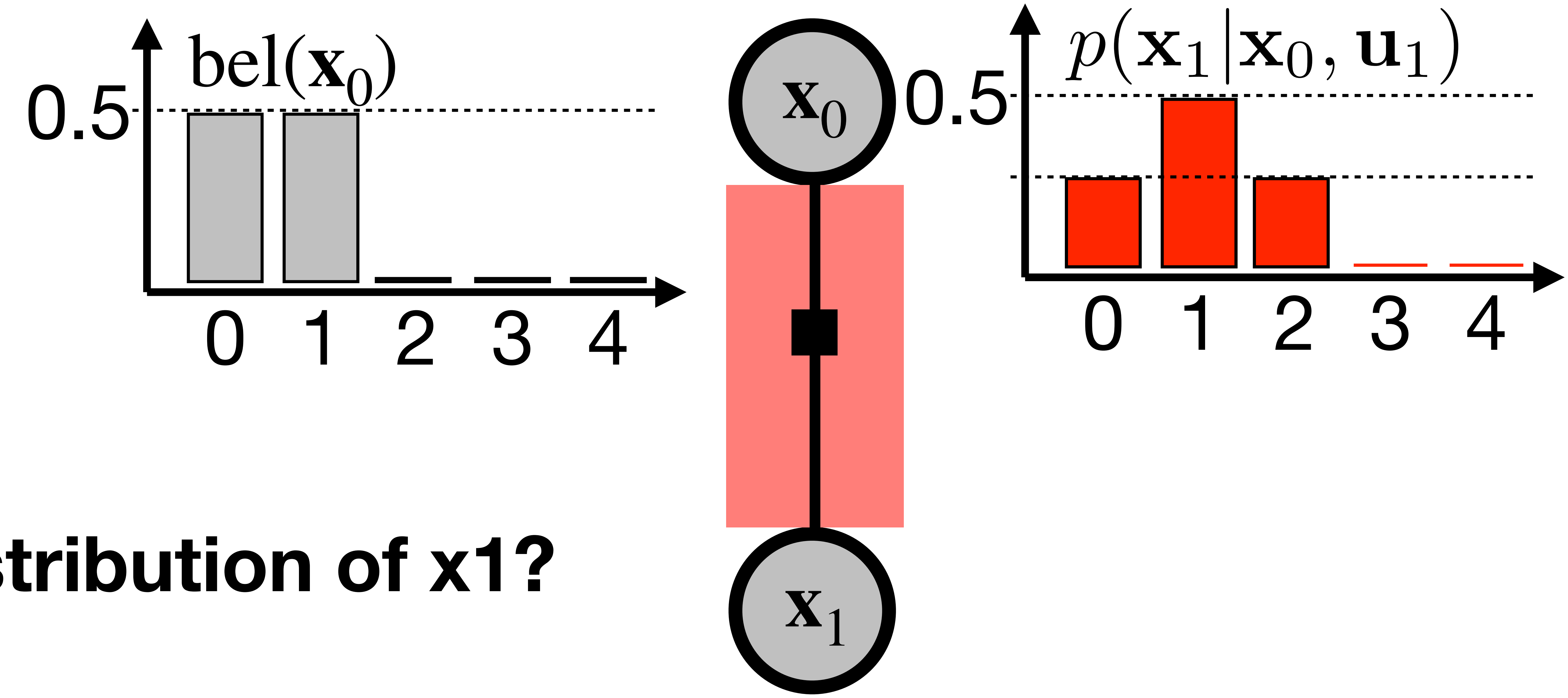
Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$





Bayes filter

Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$



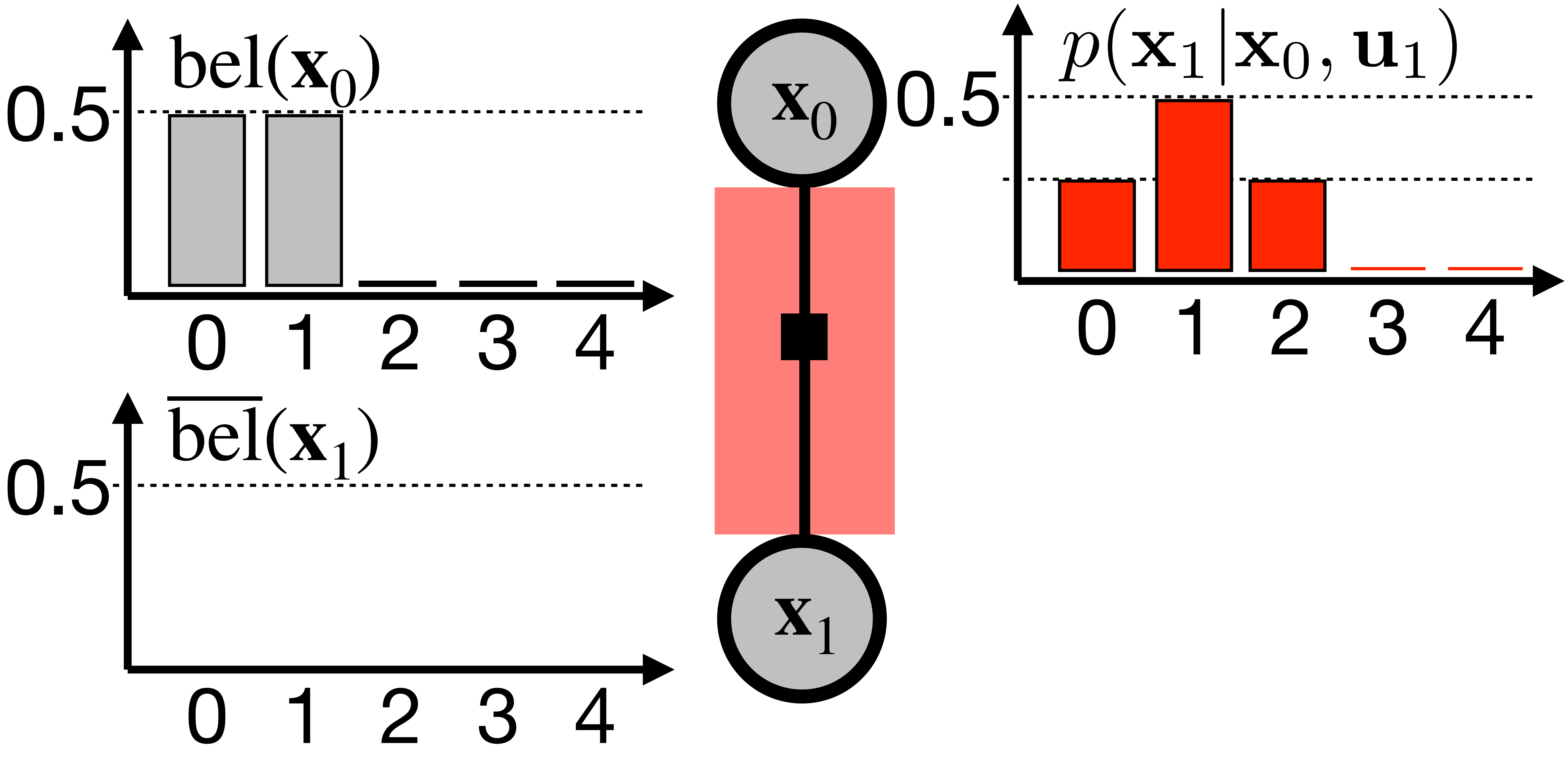
How do you estimate probability distribution of x1?

Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$



Bayes filter

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Prediction step (action $\mathbf{u}_t$ performed):

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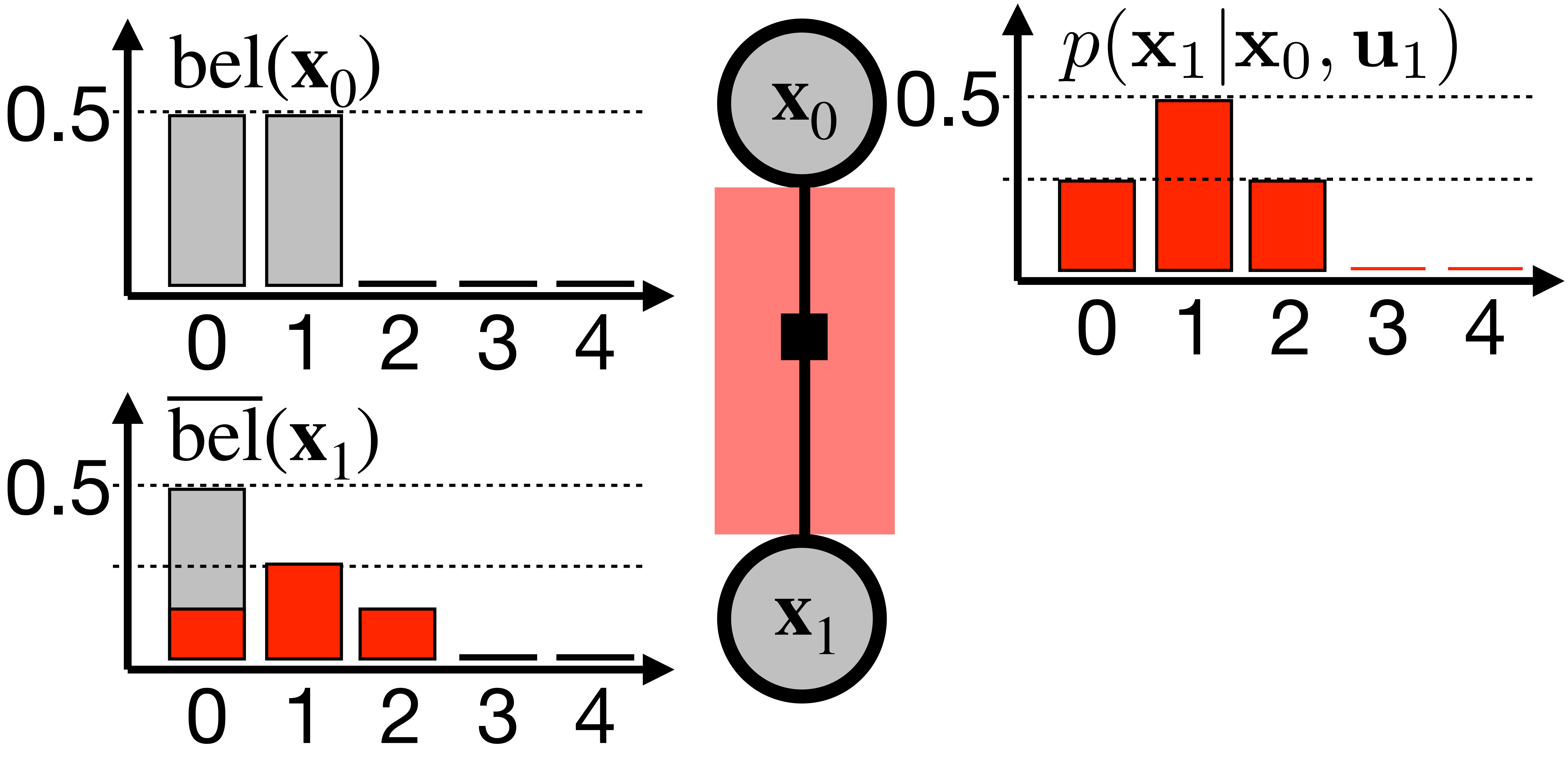

Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

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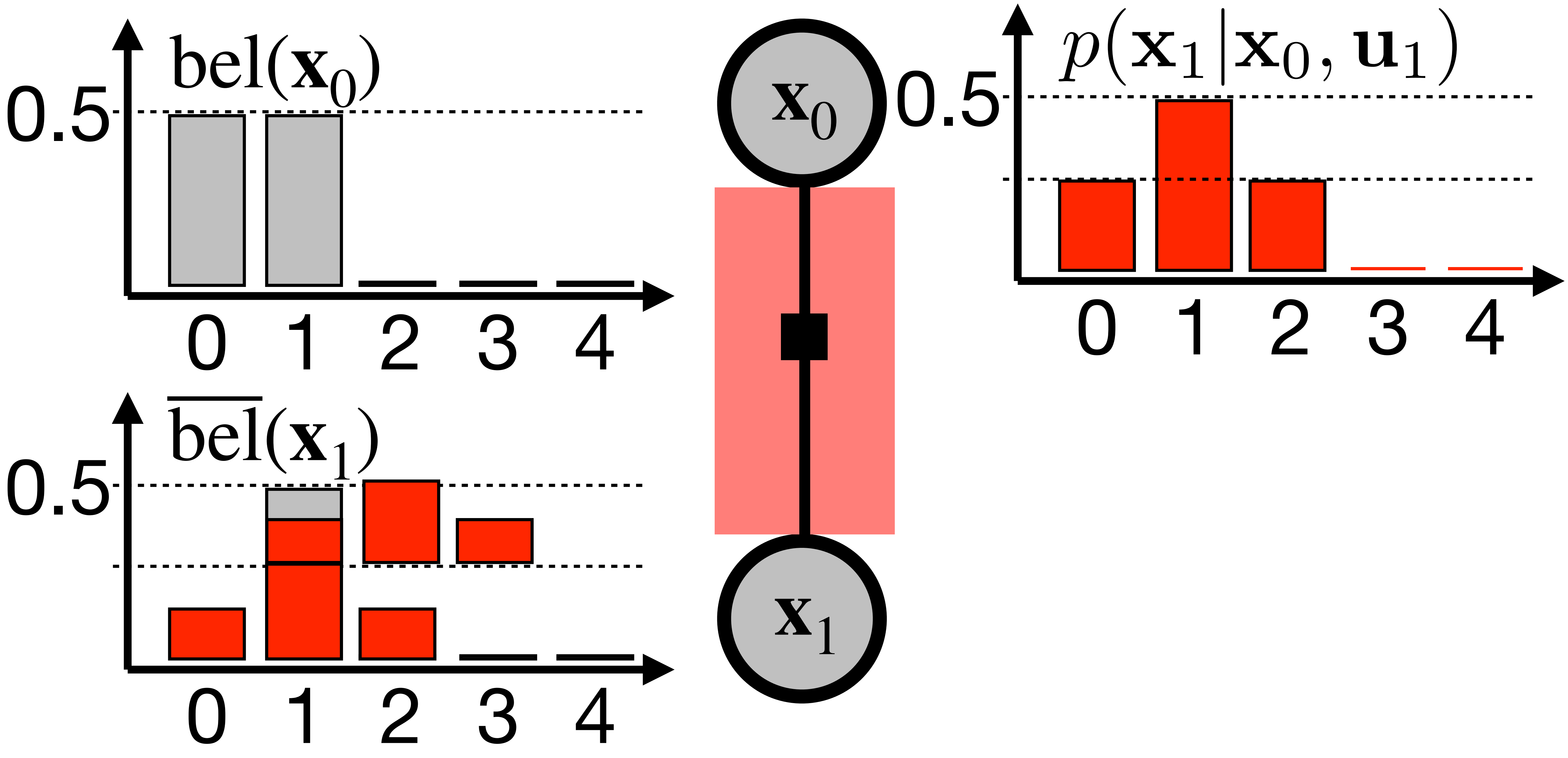

Bayes filter

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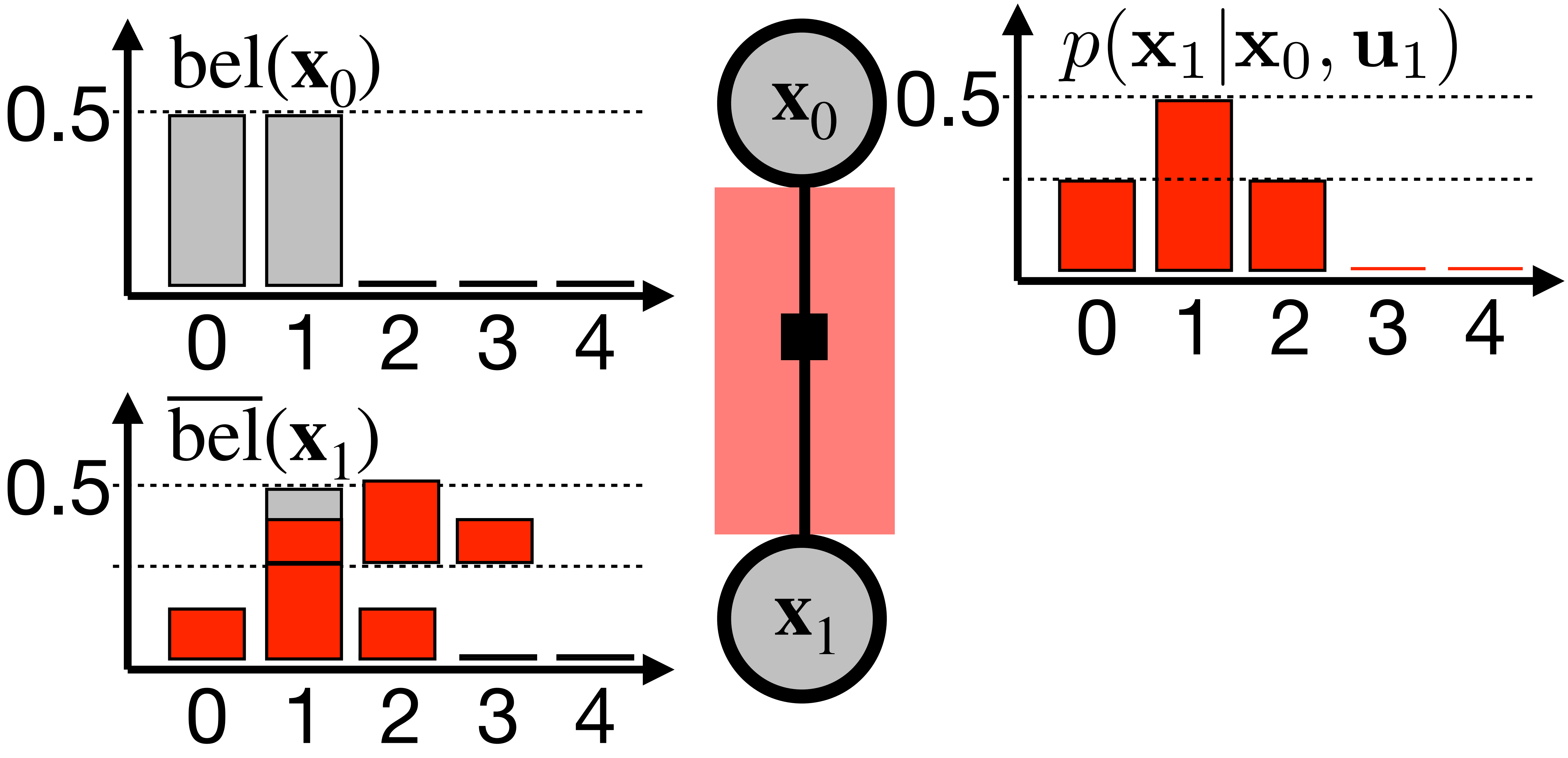
Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \,|\, \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$



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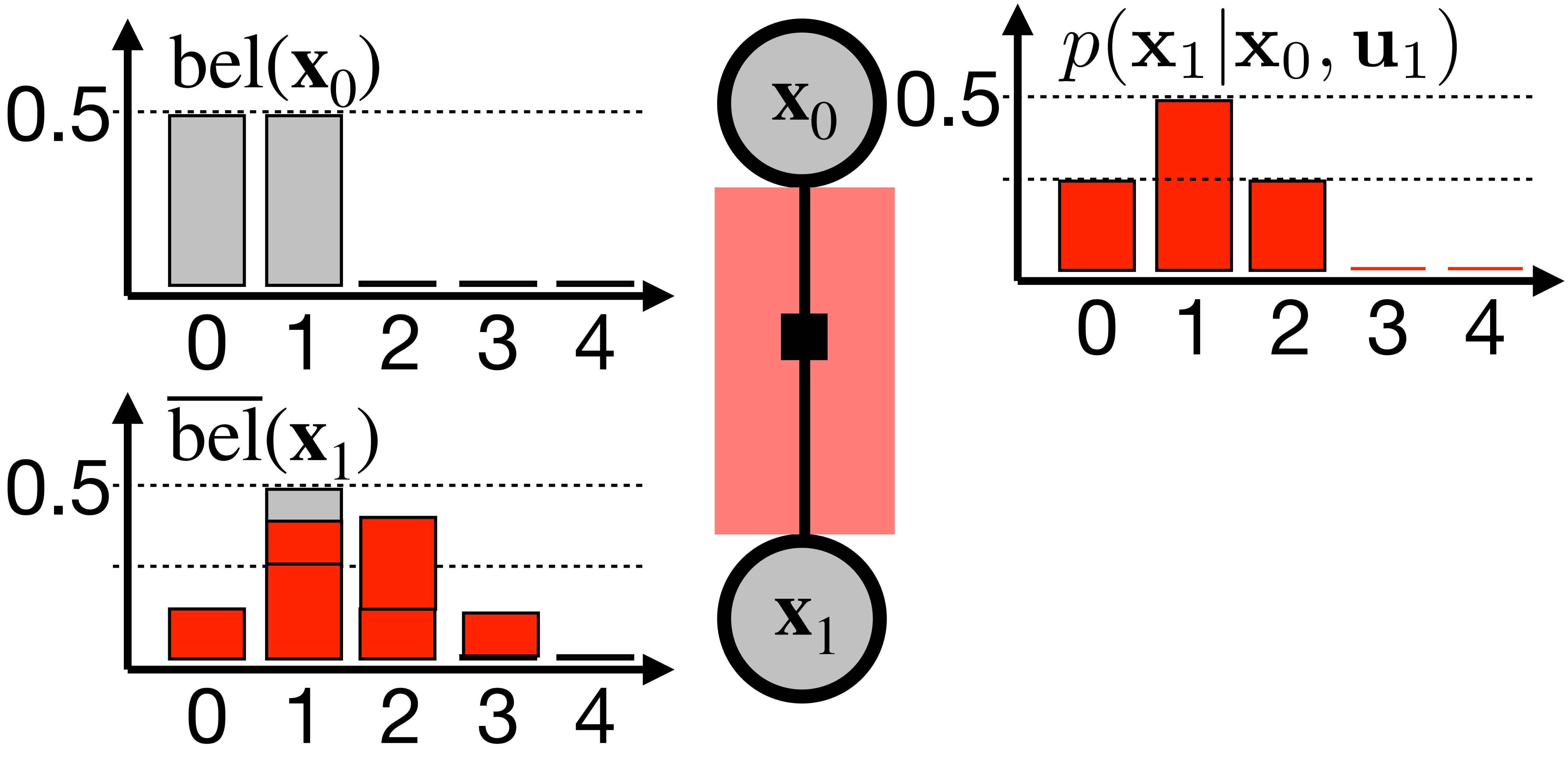

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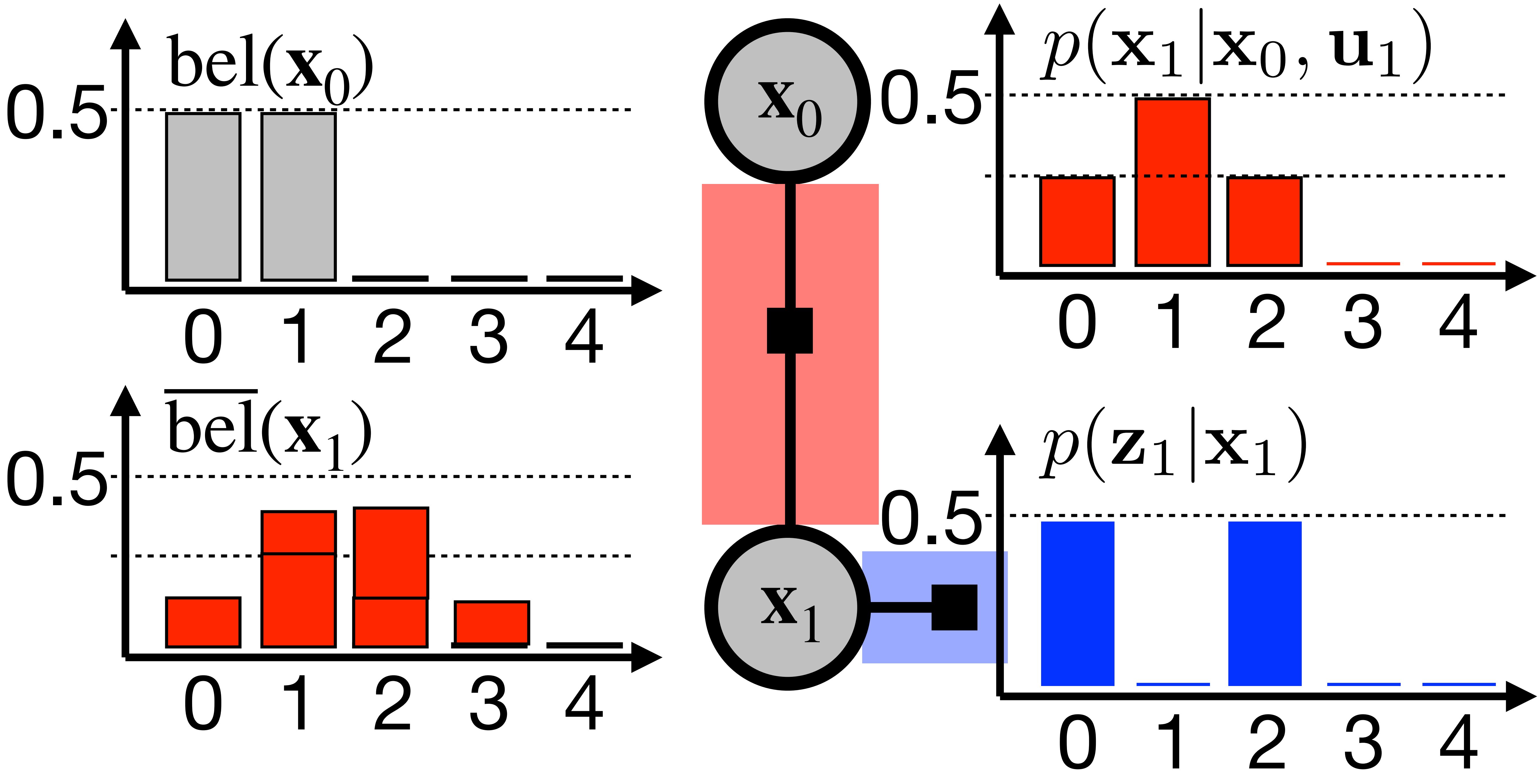


How do you update probability distribution of x1 after the blue measurement?

Bayes filter

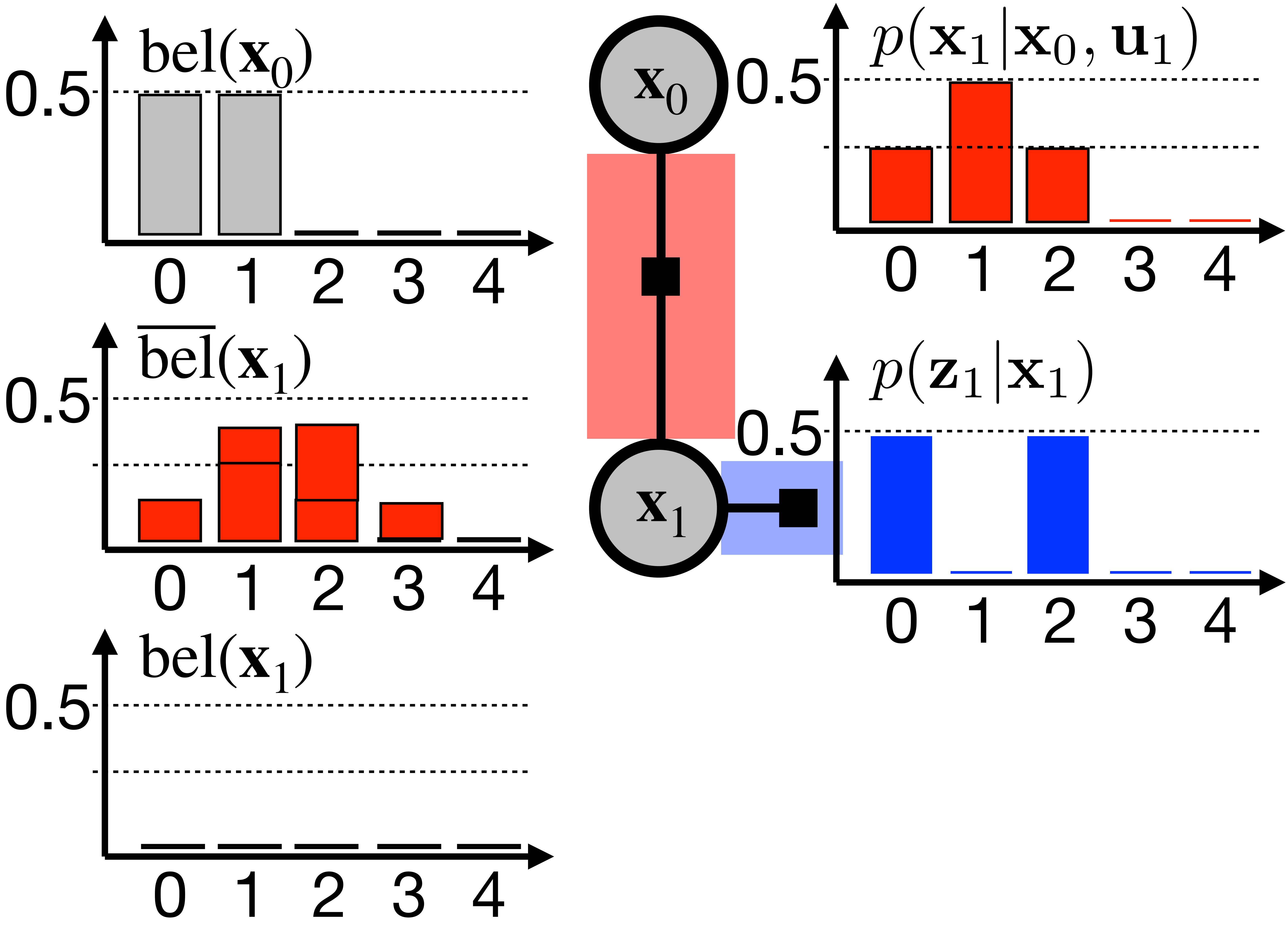
Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \,|\, \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

Measurement update (new $\mathbf{z}_t$ received):

$$\mathrm{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\mathrm{bel}}(\mathbf{x}_t)$$



Bayes filter

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Prediction step (action $\mathbf{u}_t$ performed):

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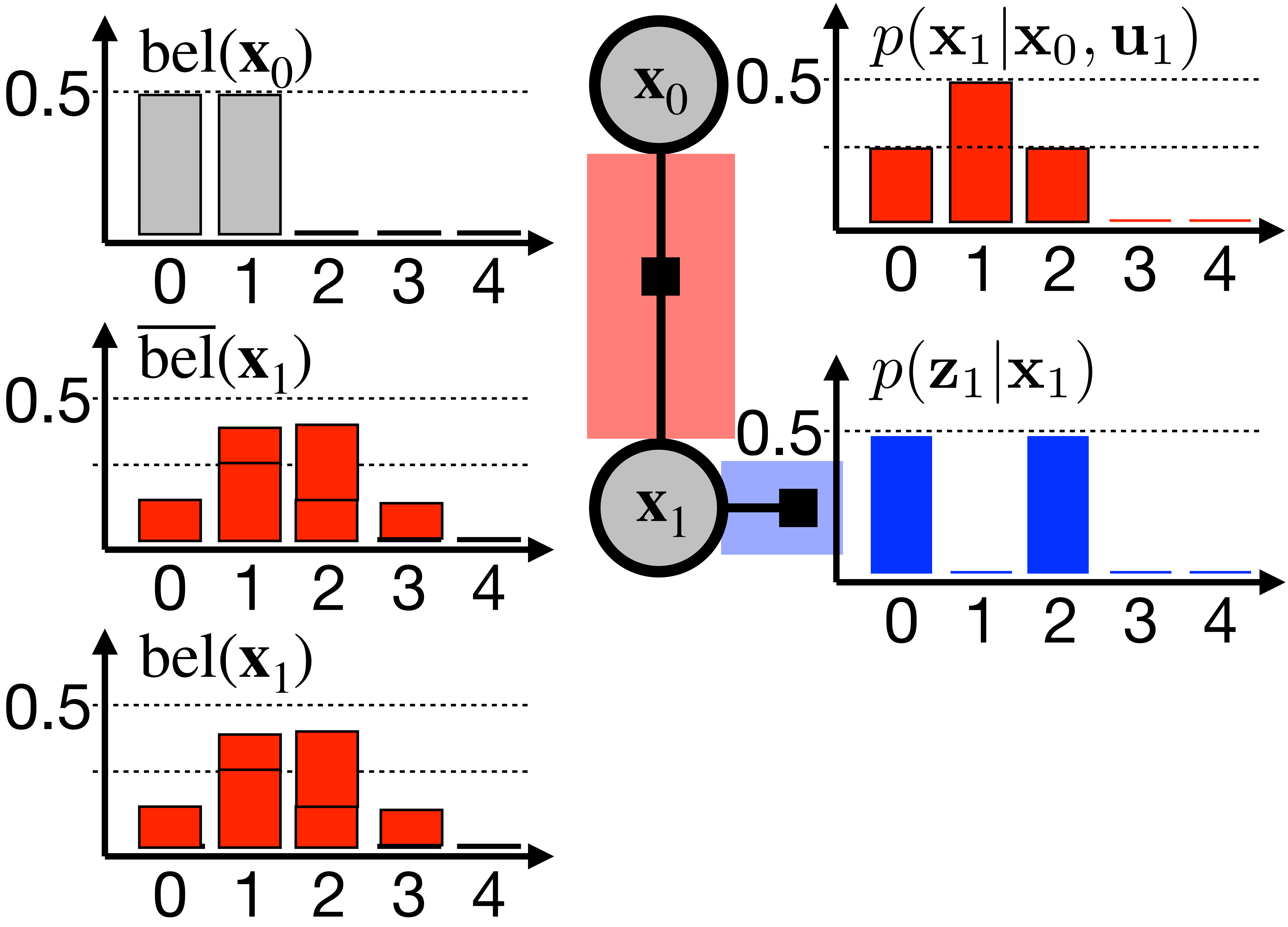

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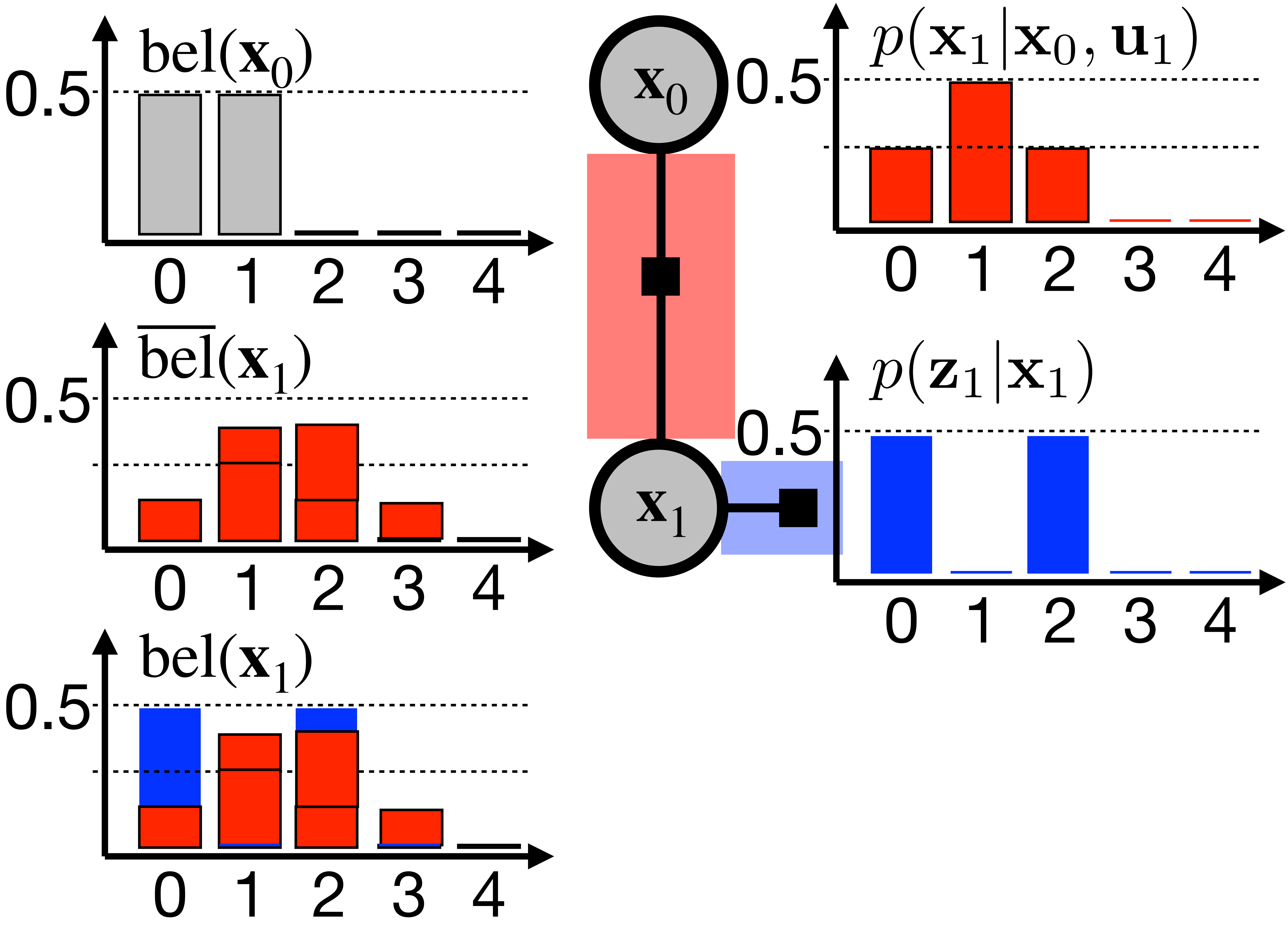


Bayes filter

Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$

Prediction step (action $\mathbf{u}_t$ performed):

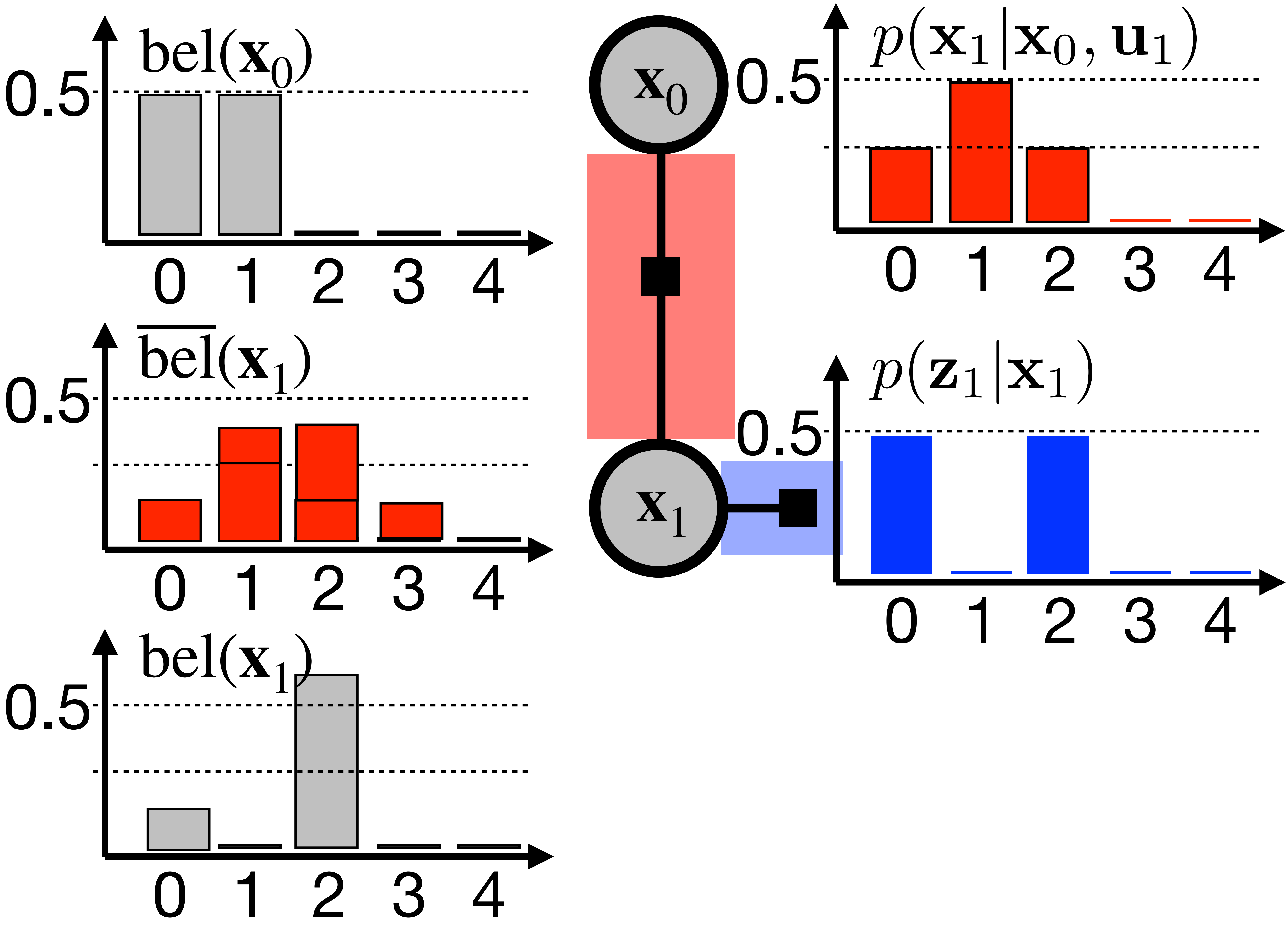
$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \,|\, \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

Measurement update (new $\mathbf{z}_t$ received):

$$\mathrm{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\mathrm{bel}}(\mathbf{x}_t)$$

Repeat forever

$t = t + 1$



$\overline{\mathrm{bel}}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$ … prior belief

(prob. distr. of current state **without** considering the current measurement $\mathbf{z}_t$)

Bayes filter

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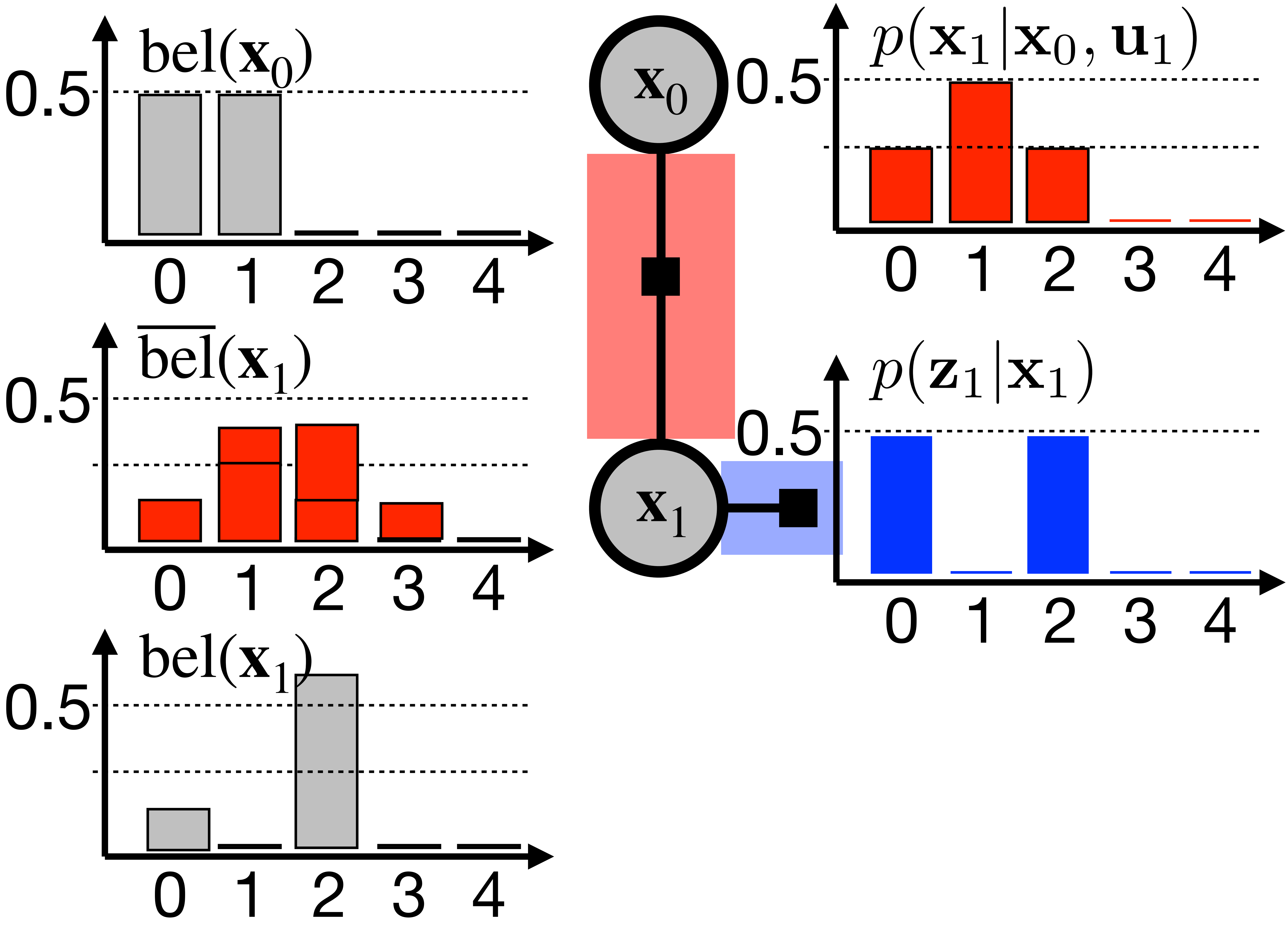


$\overline{\mathrm{bel}}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$ … prior belief (prob. distr. of current state **without** considering the current measurement $\mathbf{z}_t$)

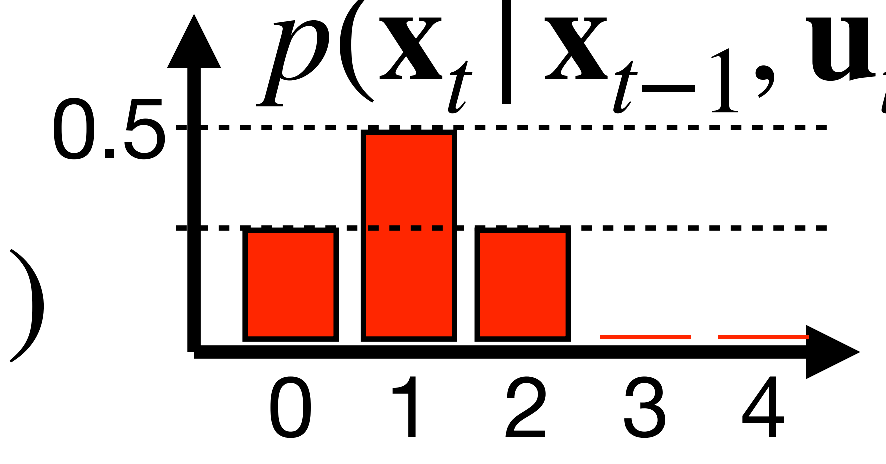
$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ … posterior belief (prob. distr. of current state **with** considering the current measurement $\mathbf{z}_t$)

Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \,|\, \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$



3. Measurement update (new $\mathbf{z}_t$ received):

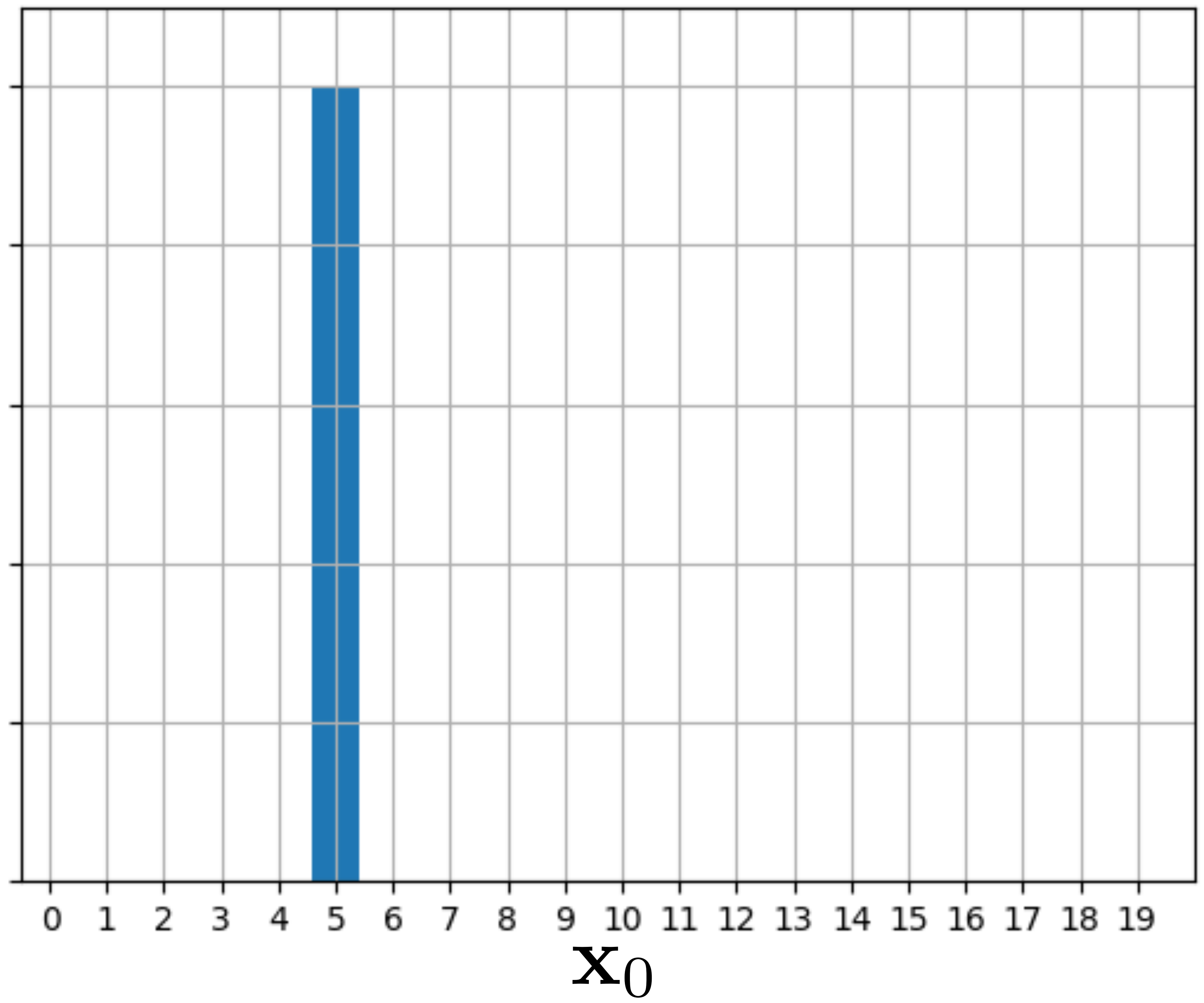
   For all $\mathbf{x}_t$

$$\mathrm{bel}(\mathbf{x}_t) = \overline{\mathrm{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$

$\mathrm{bel}(\mathbf{x}_0)$



Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
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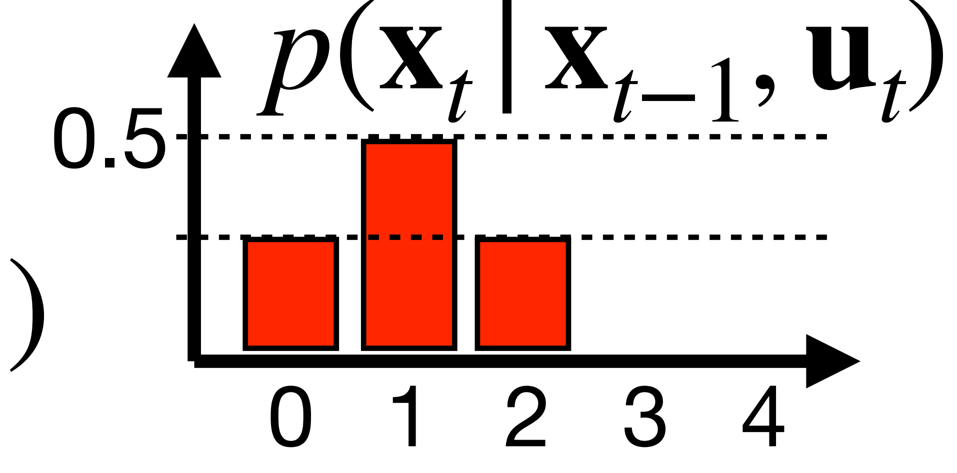

3. Measurement update (new $\mathbf{z}_t$ received):

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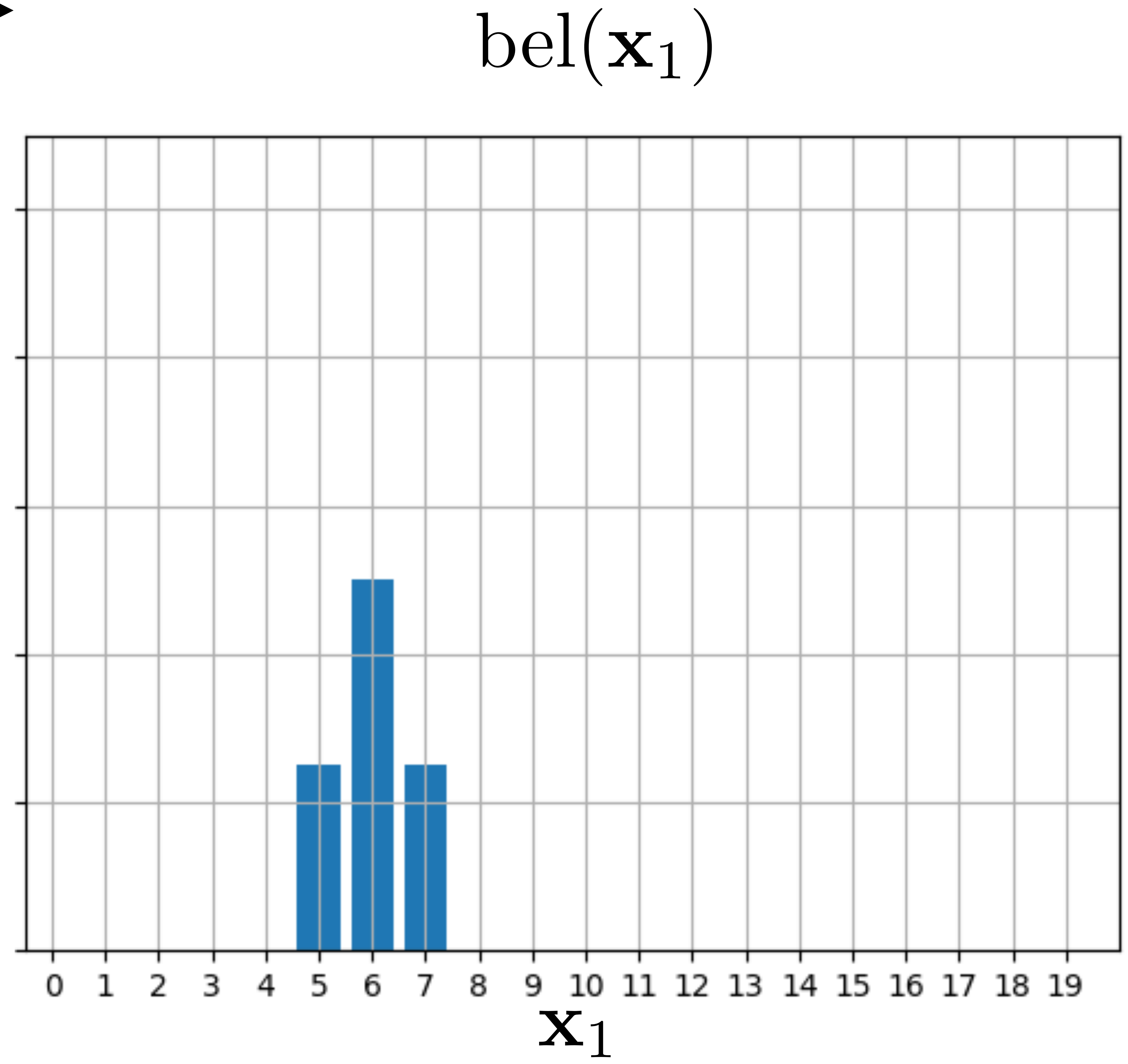
4. Repeat from 2:

   $t = t + 1$



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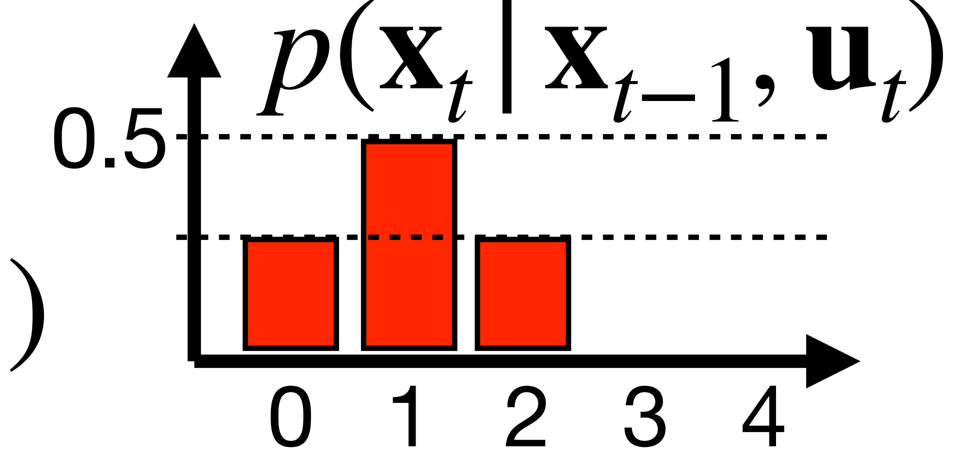

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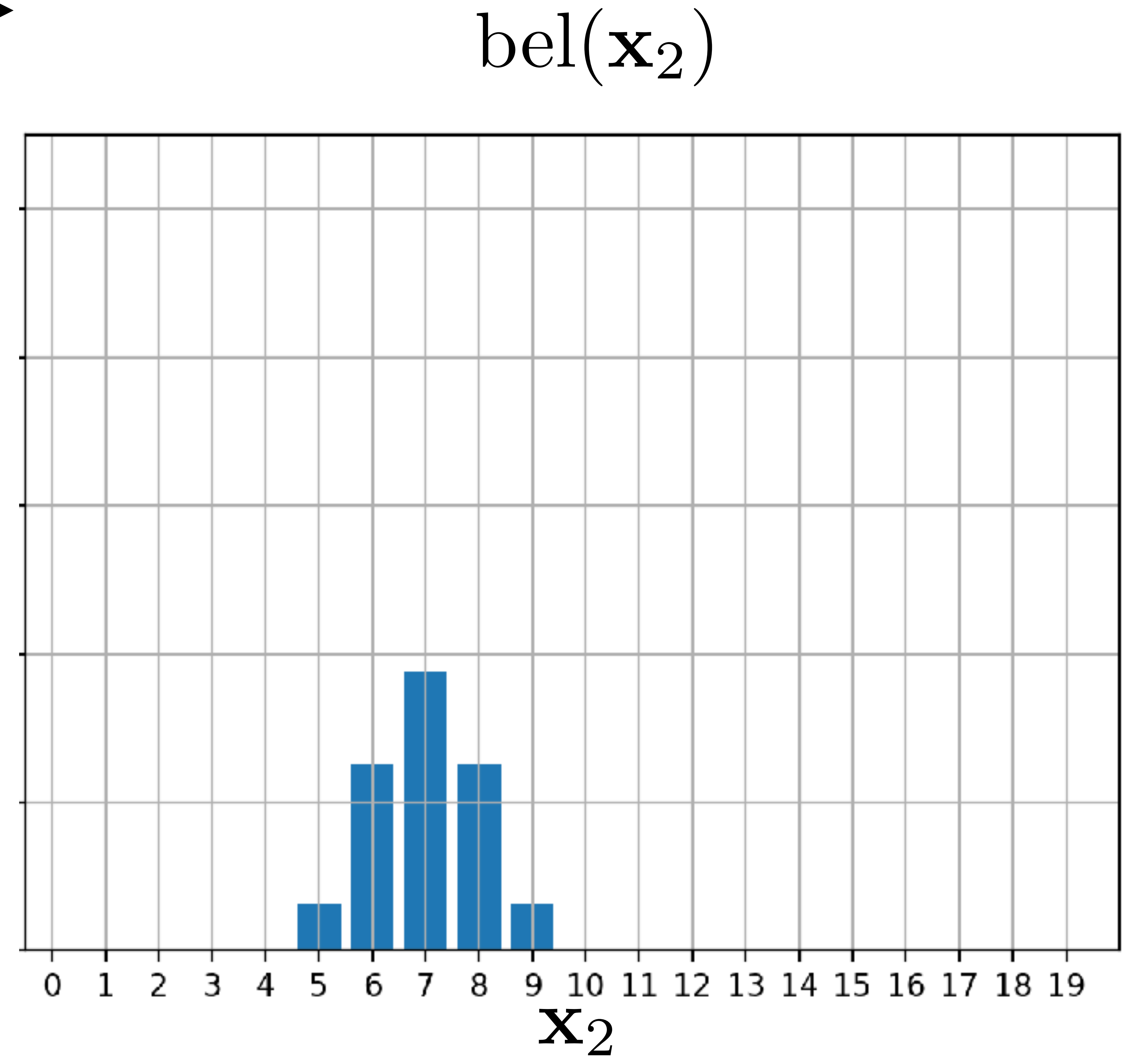

Bayes filter

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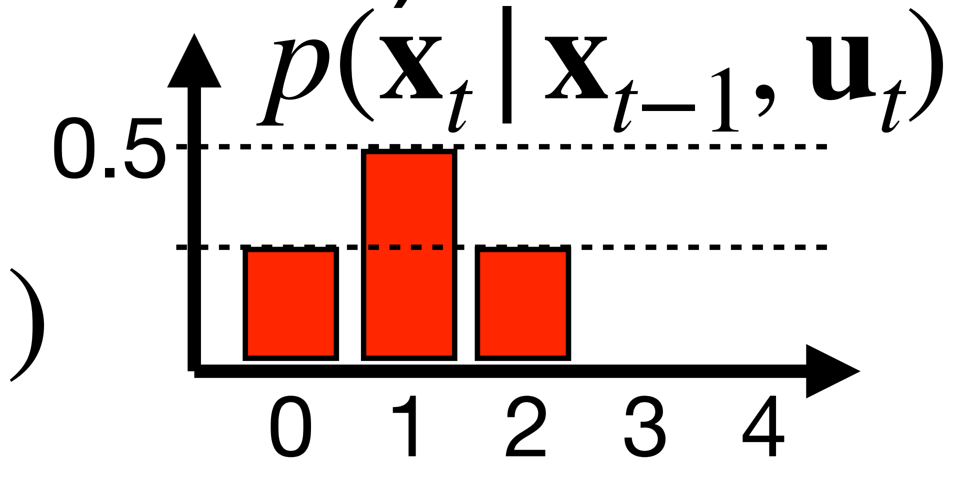


3. Measurement update (new $\mathbf{z}_t$ received):

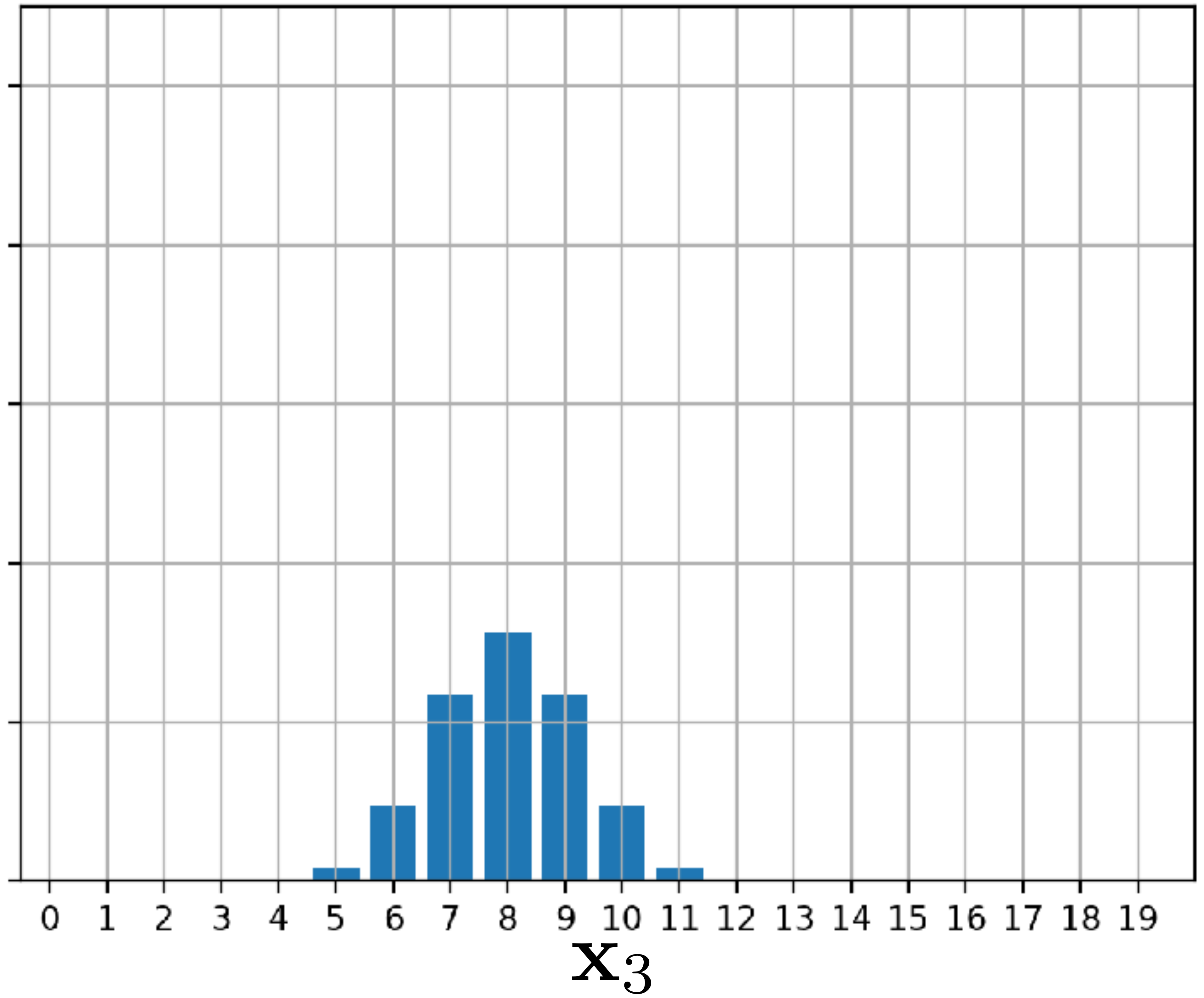
   For all $\mathbf{x}_t$

$$\text{bel}(\mathbf{x}_t) = \overline{\text{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$

$\text{bel}(\mathbf{x}_3)$



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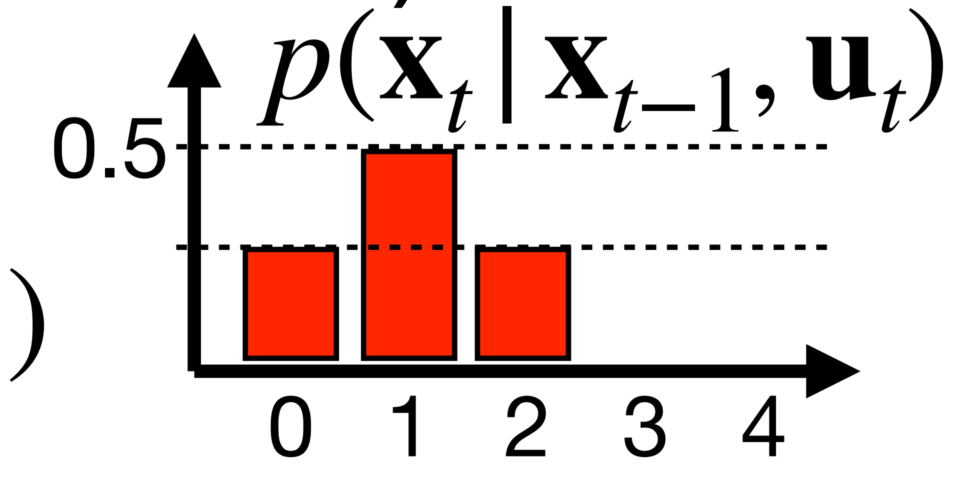


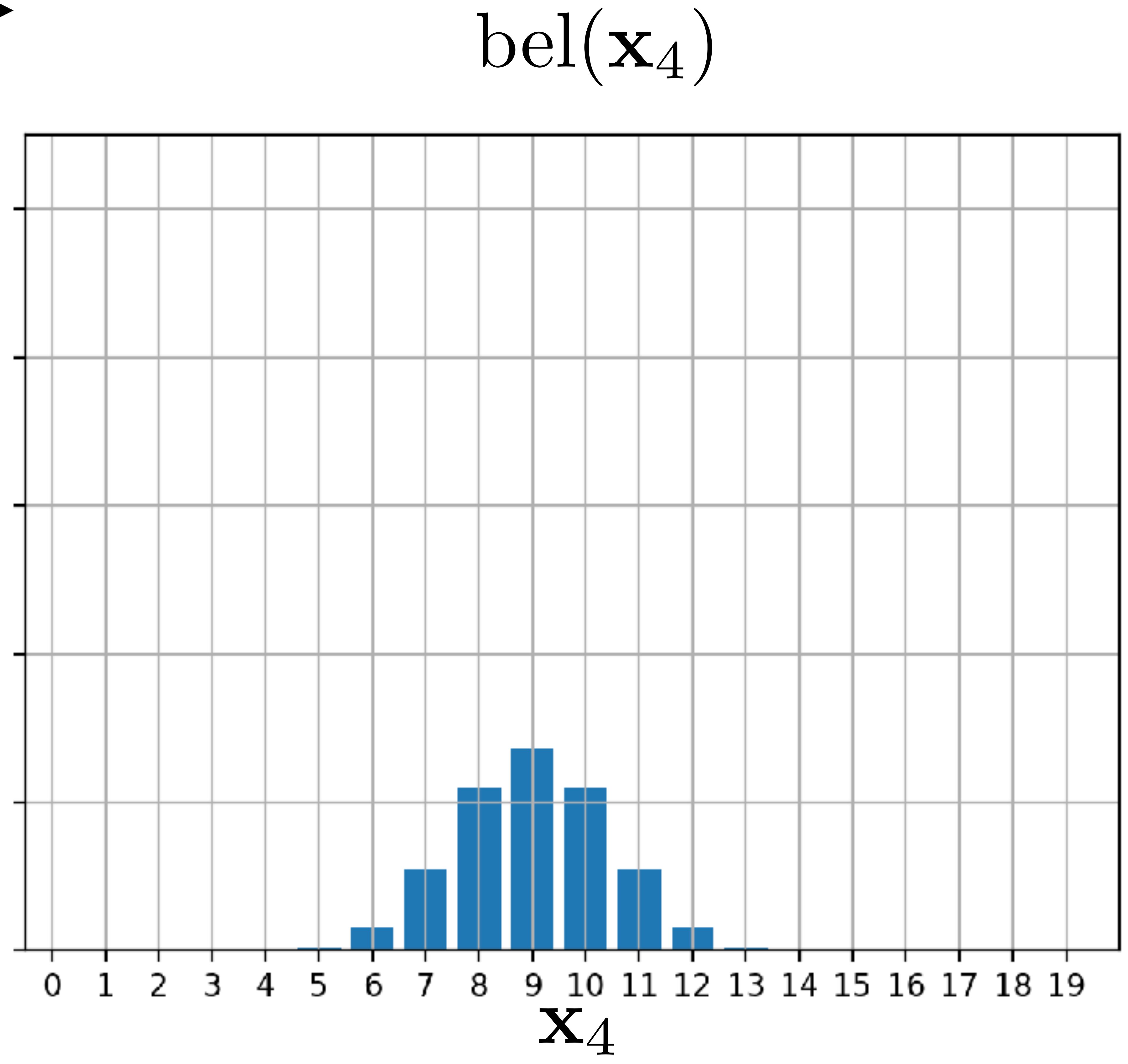
3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \overline{\mathrm{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$



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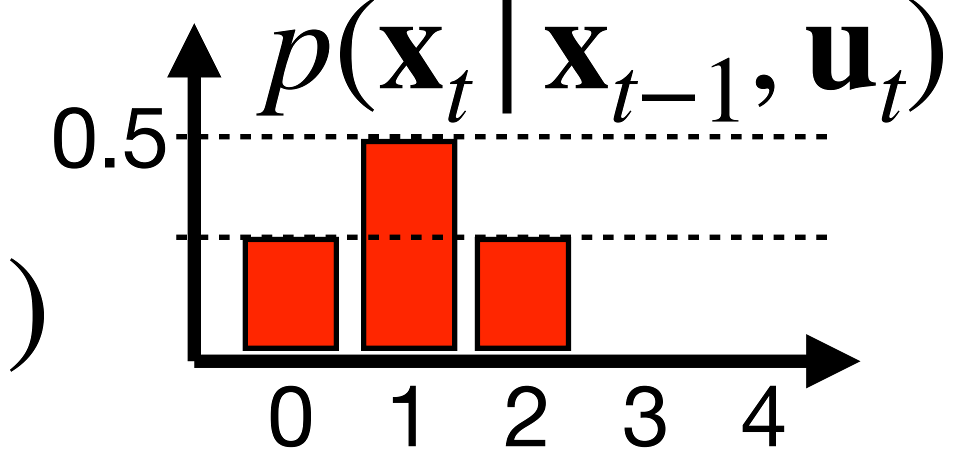


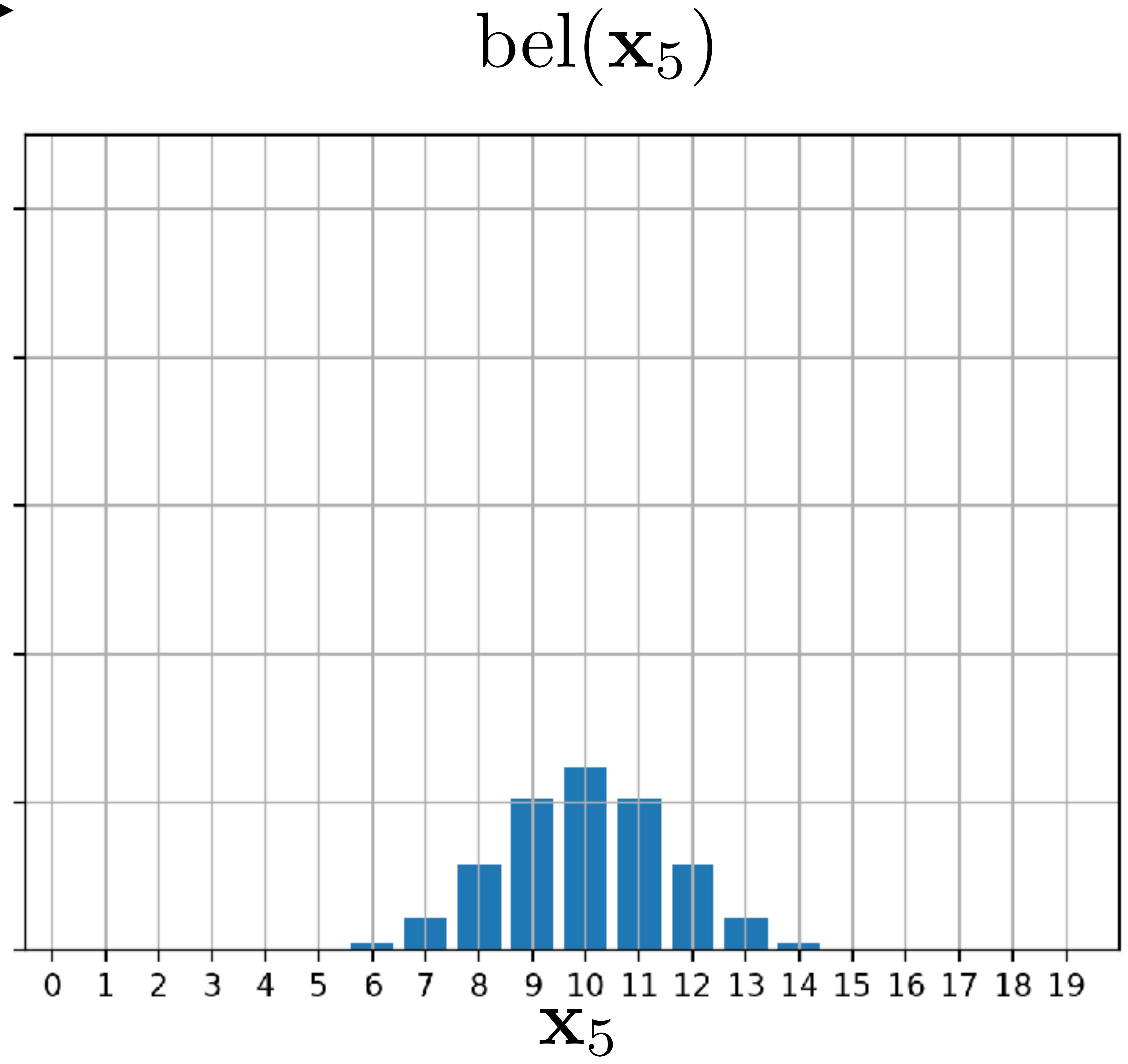
3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \overline{\mathrm{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$



Bayes filter

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   For all $\mathbf{x}_t$

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3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\text{bel}}(\mathbf{x}_t)$$

   $$s = s + \text{bel}(\mathbf{x}_t)$$

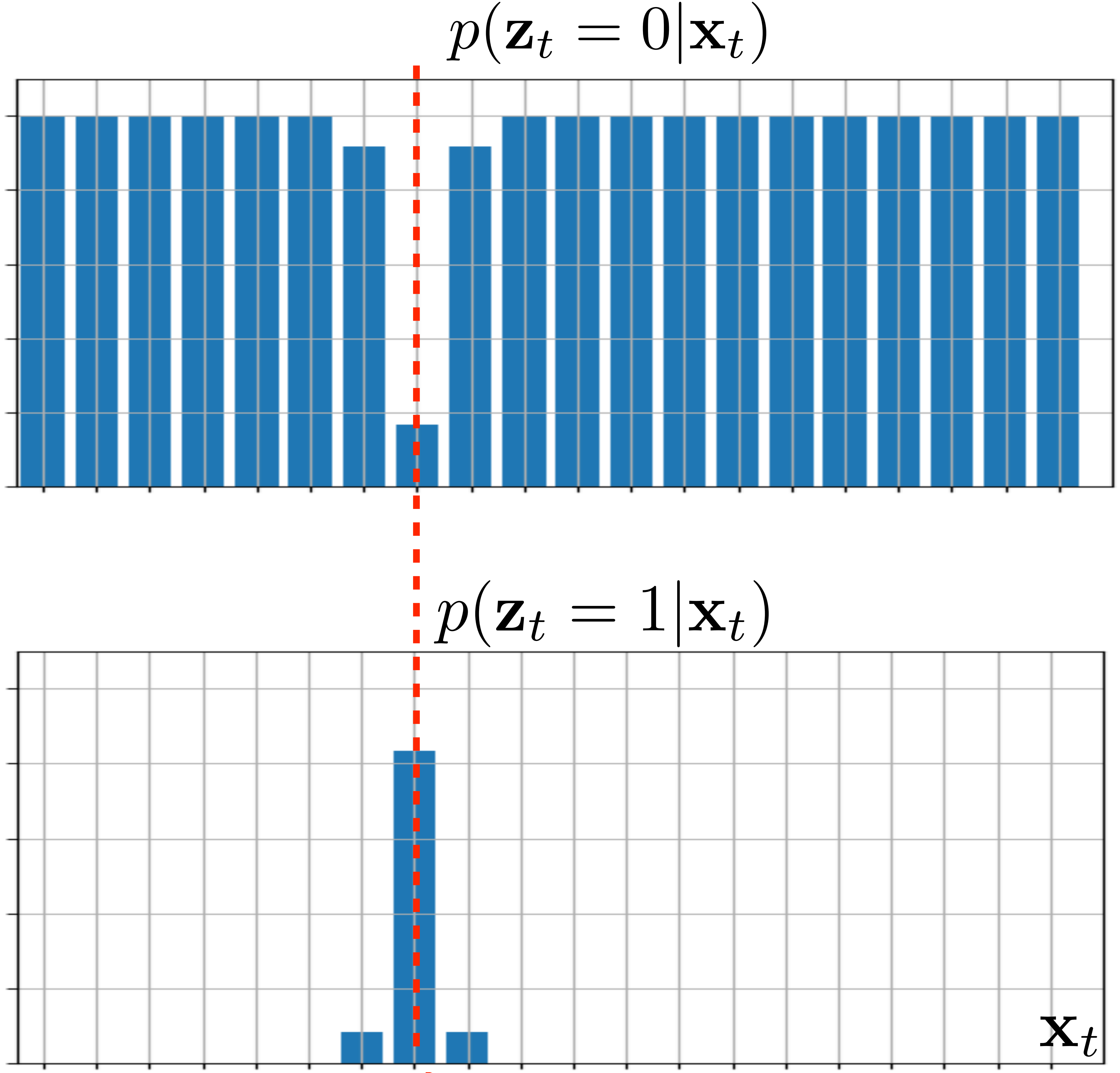
   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = \frac{\text{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$

Let's add measurements !



one marker at known locations
+
inaccurate sensor

Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \,|\, \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \,|\, \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

   For all $\mathbf{x}_t$

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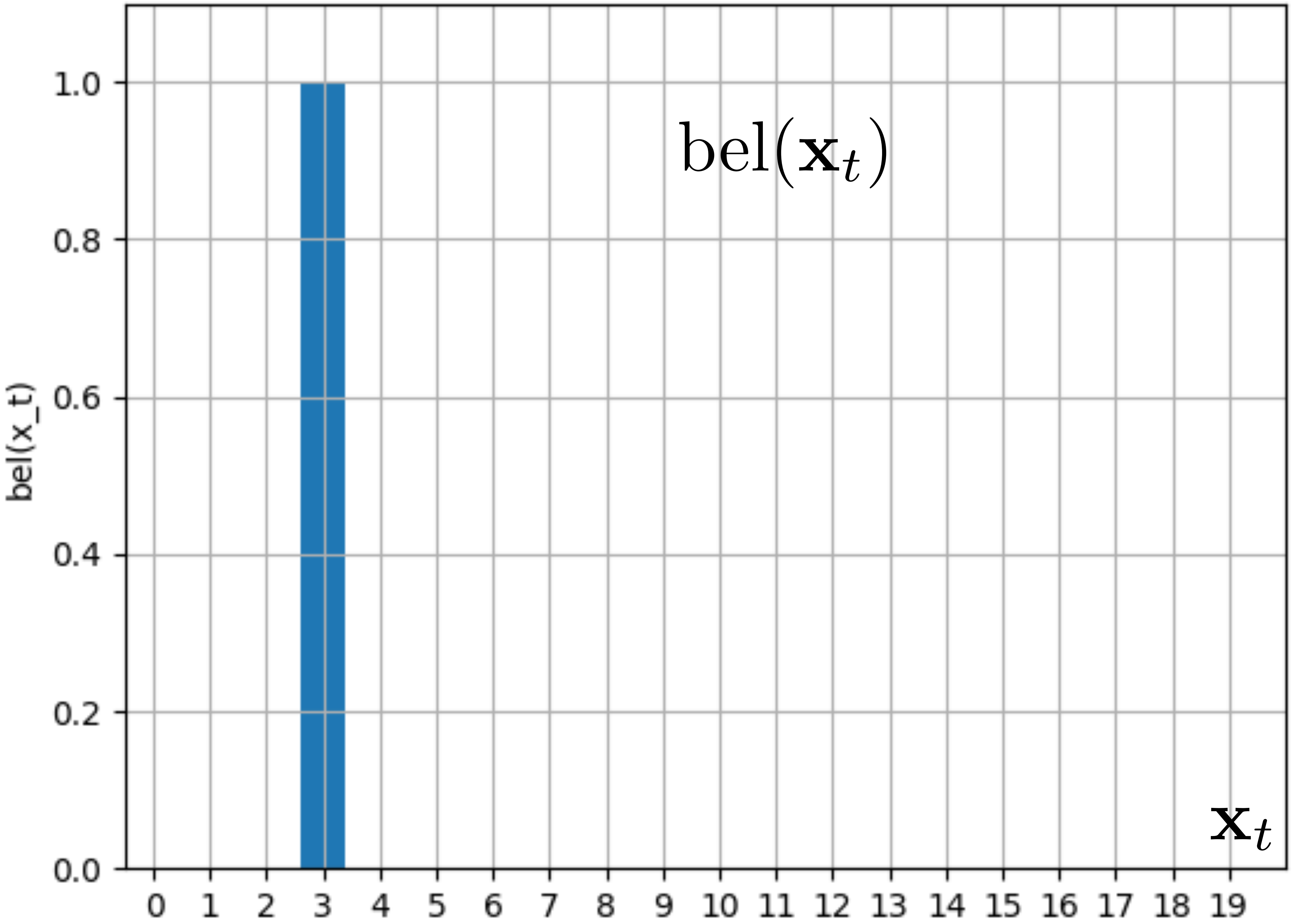
4. Repeat from 2:

   $t = t + 1$



Bayes filter

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2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

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3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

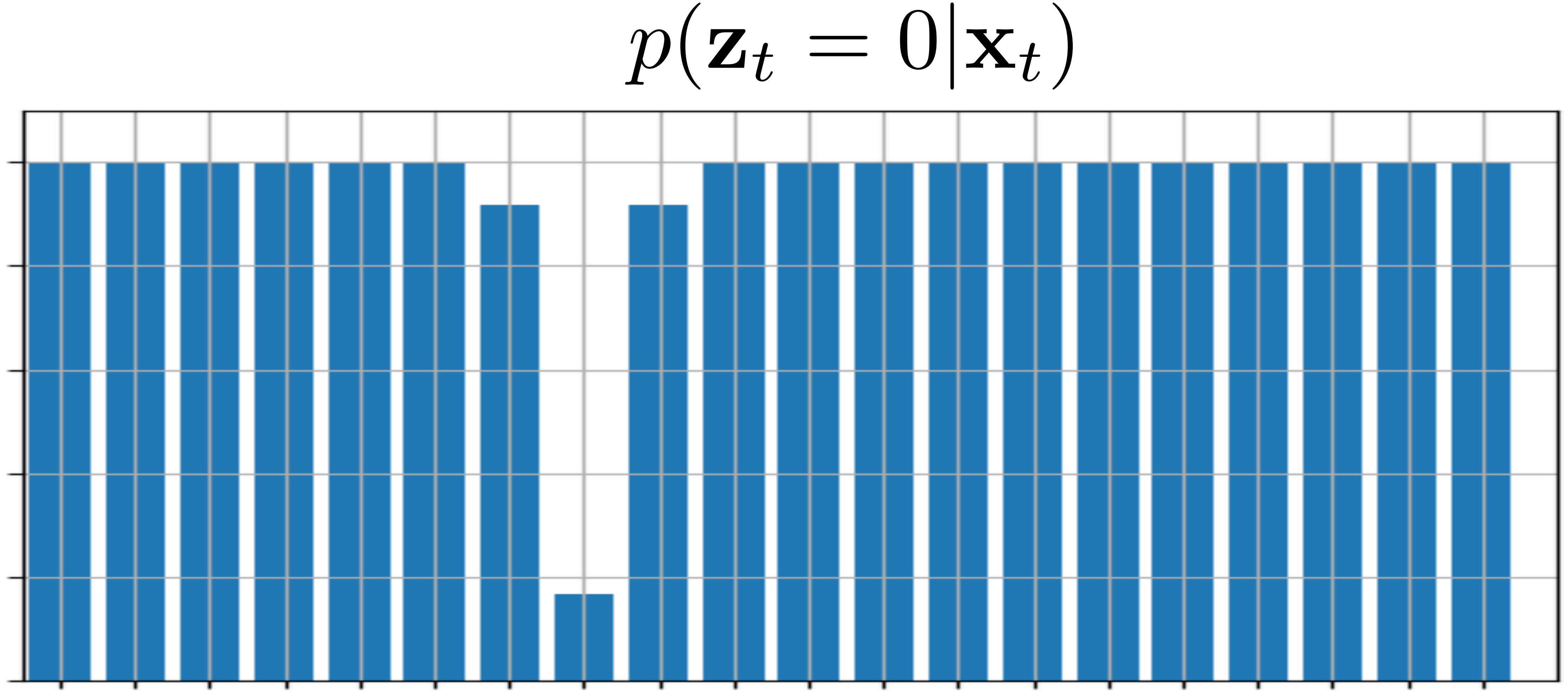
   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

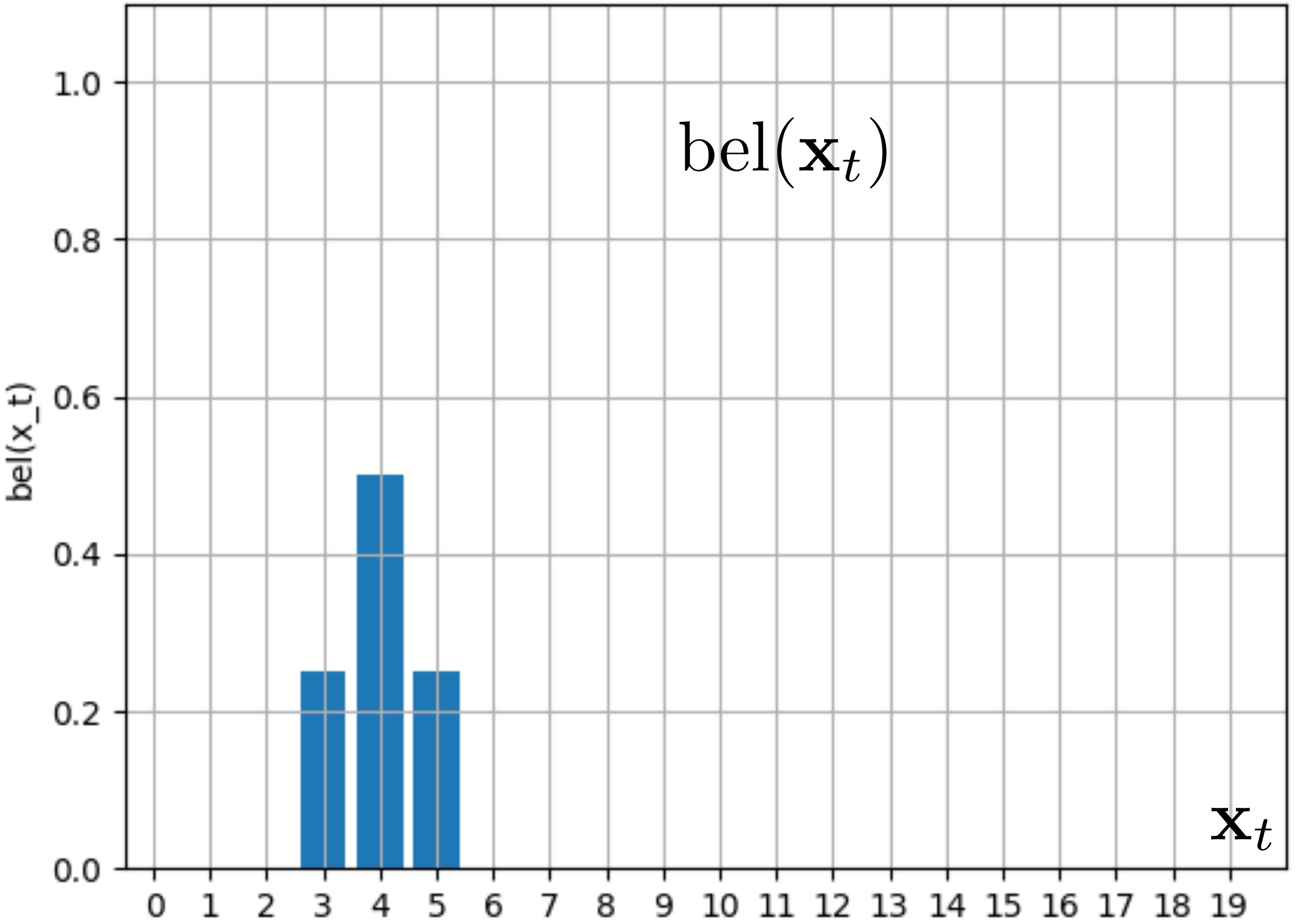
   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$





Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
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   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

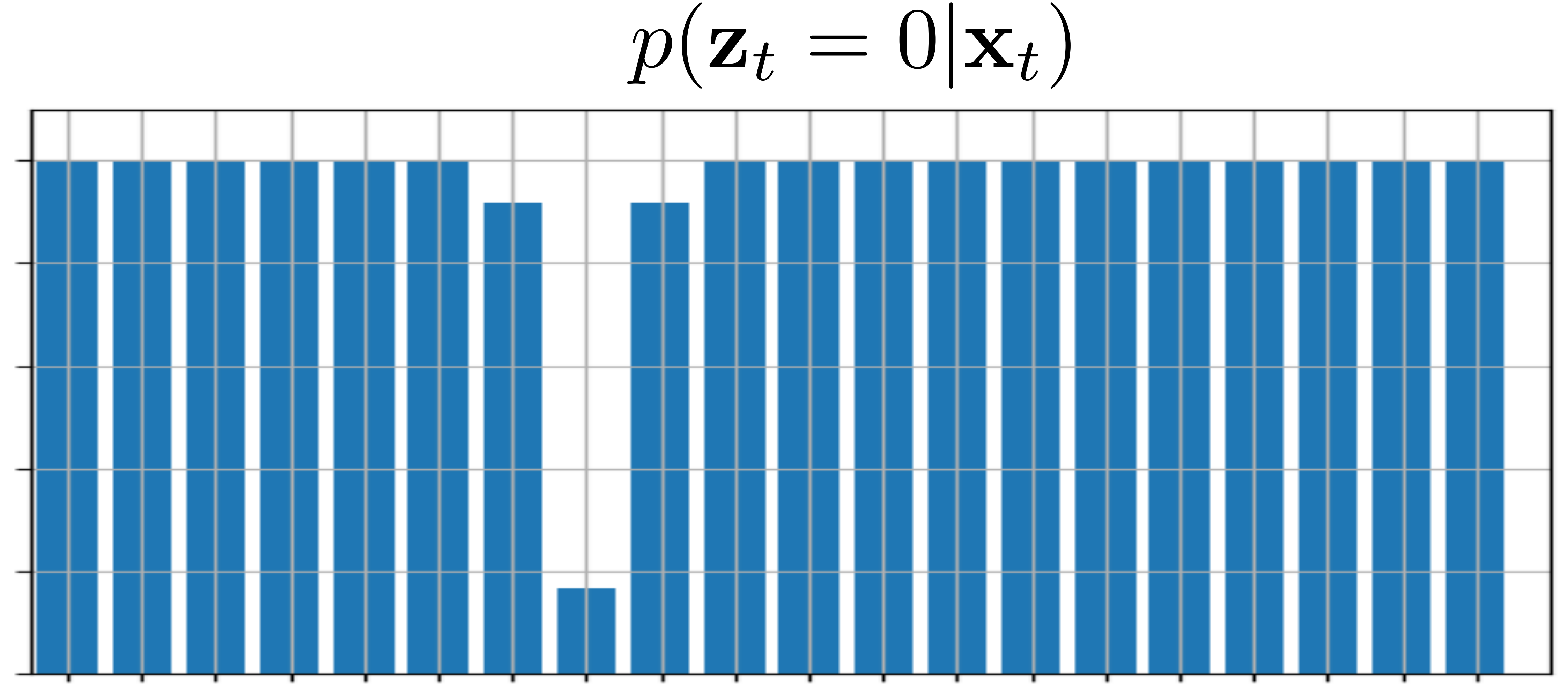
   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

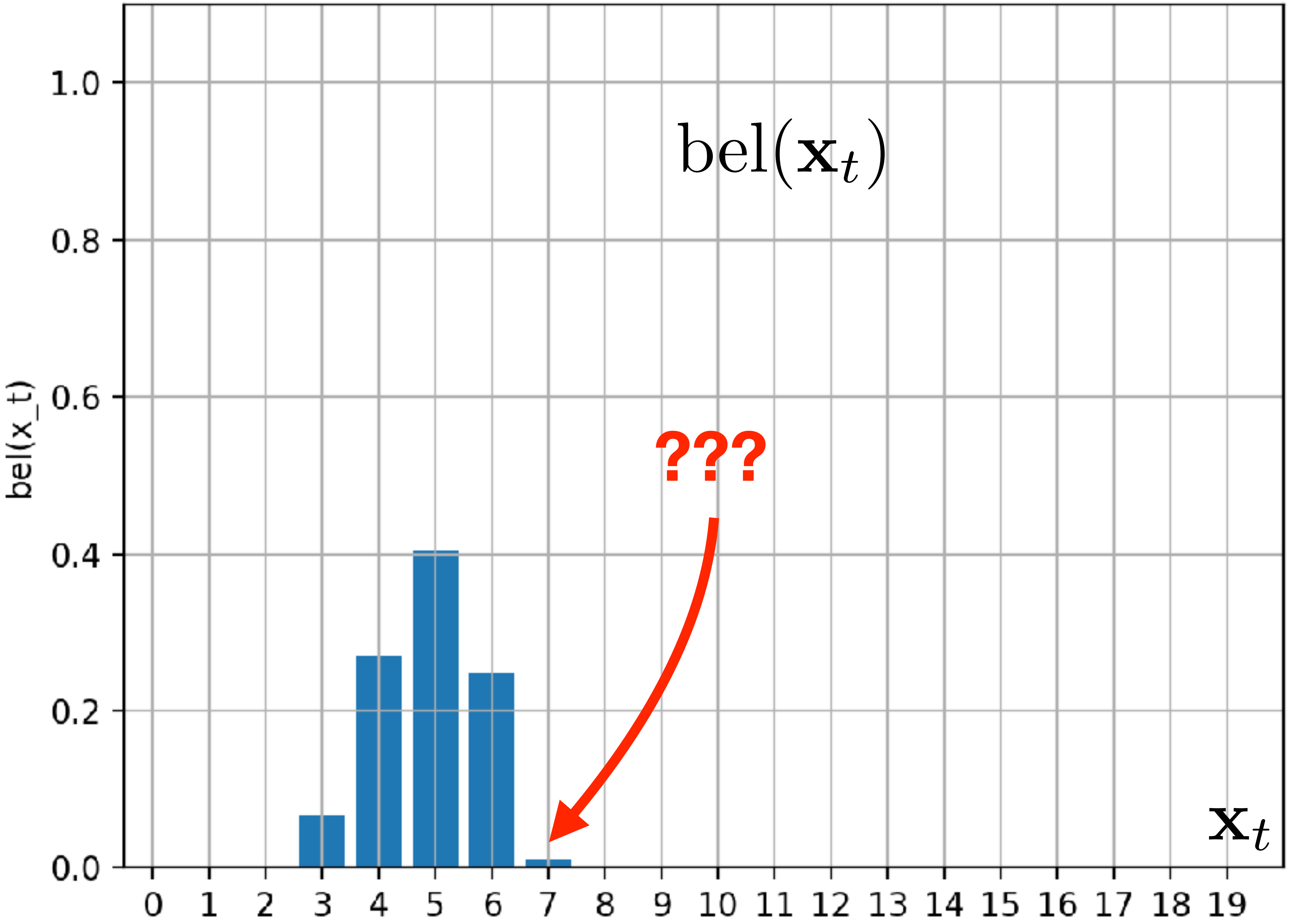
   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$





Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \,|\, \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \,|\, \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

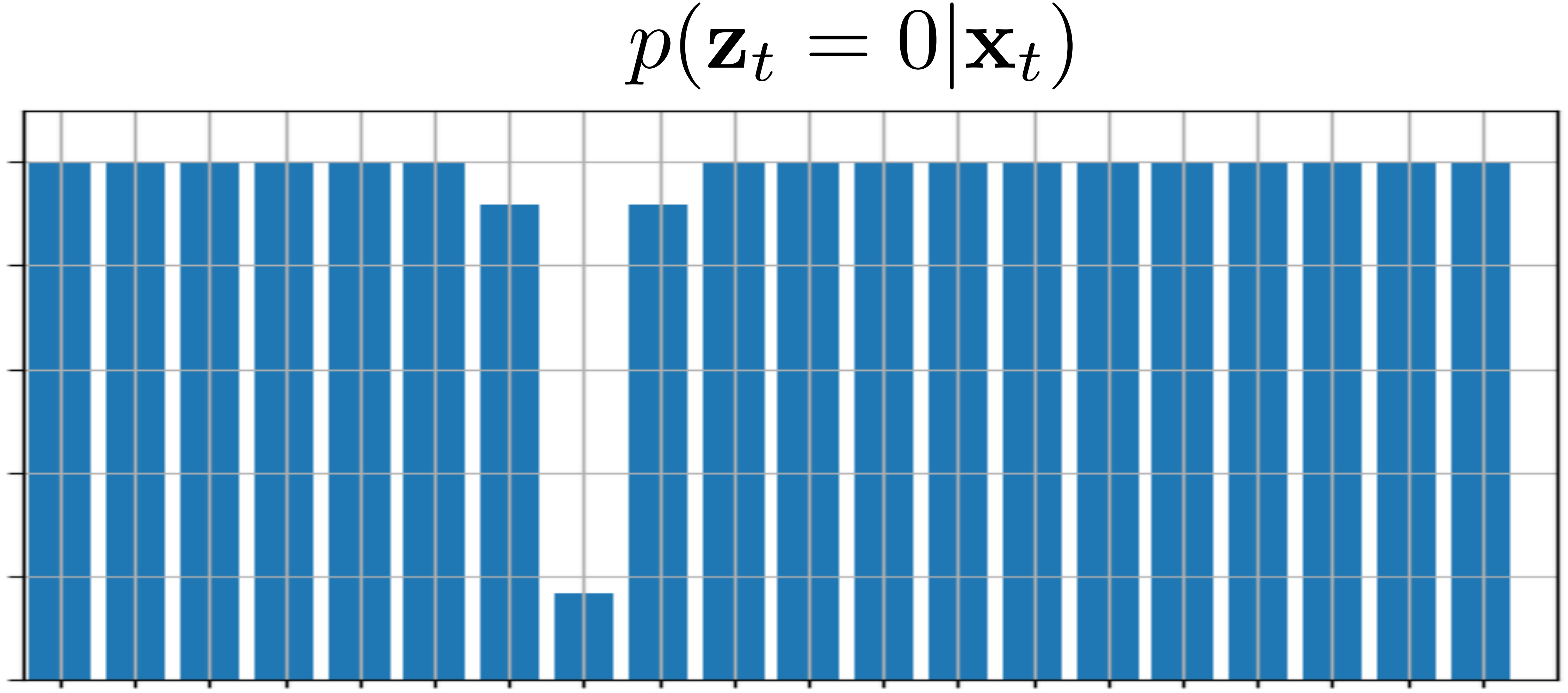
   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

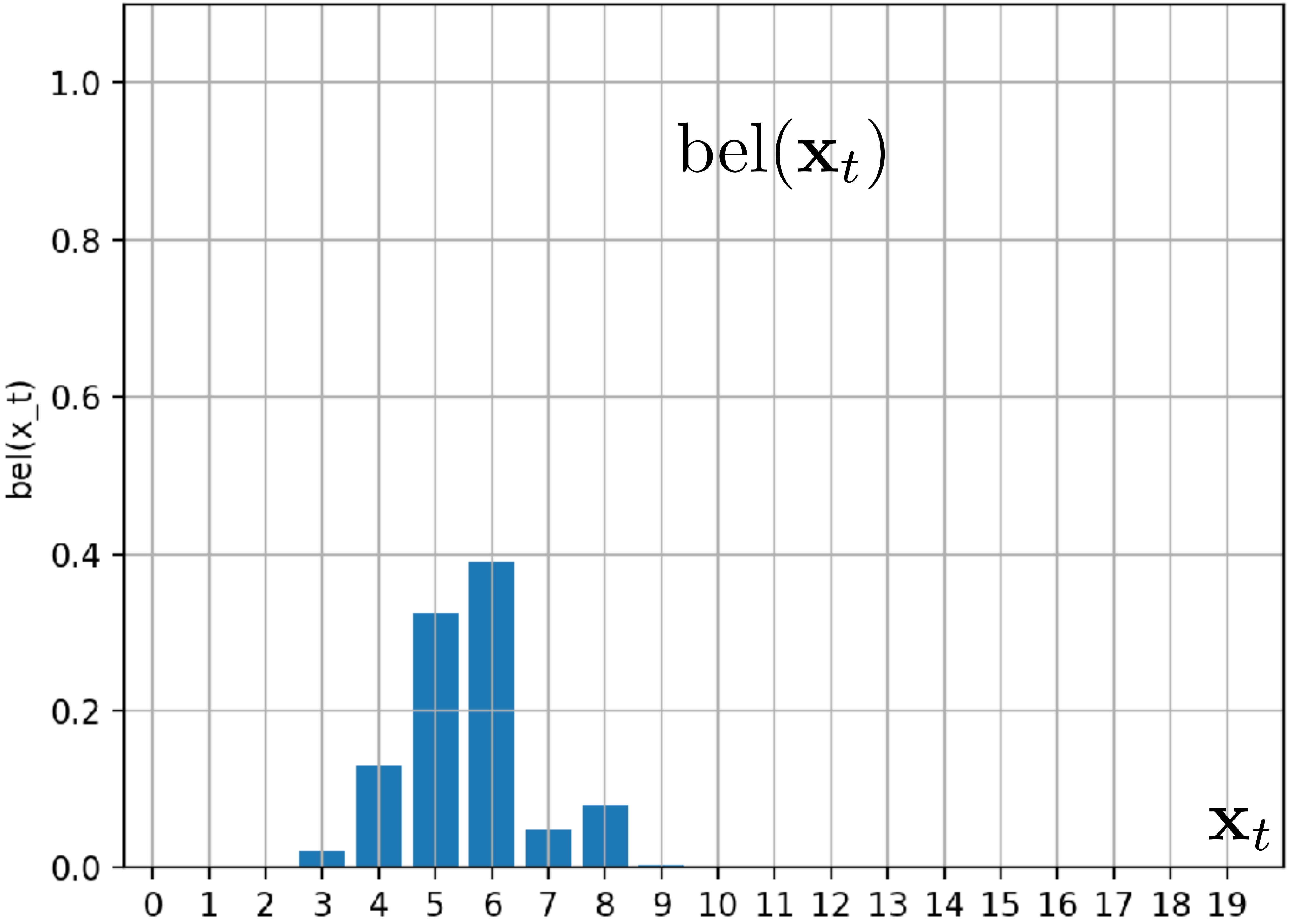
   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$





Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

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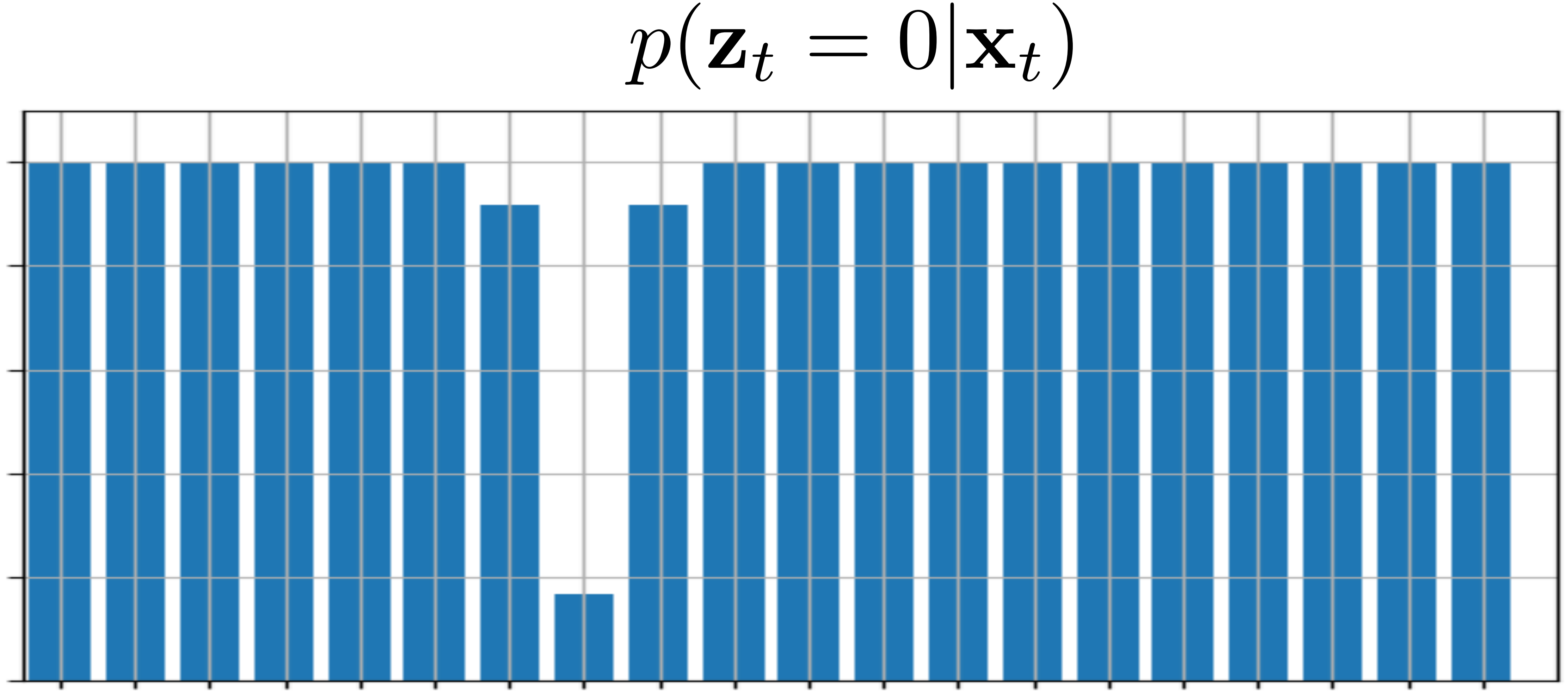

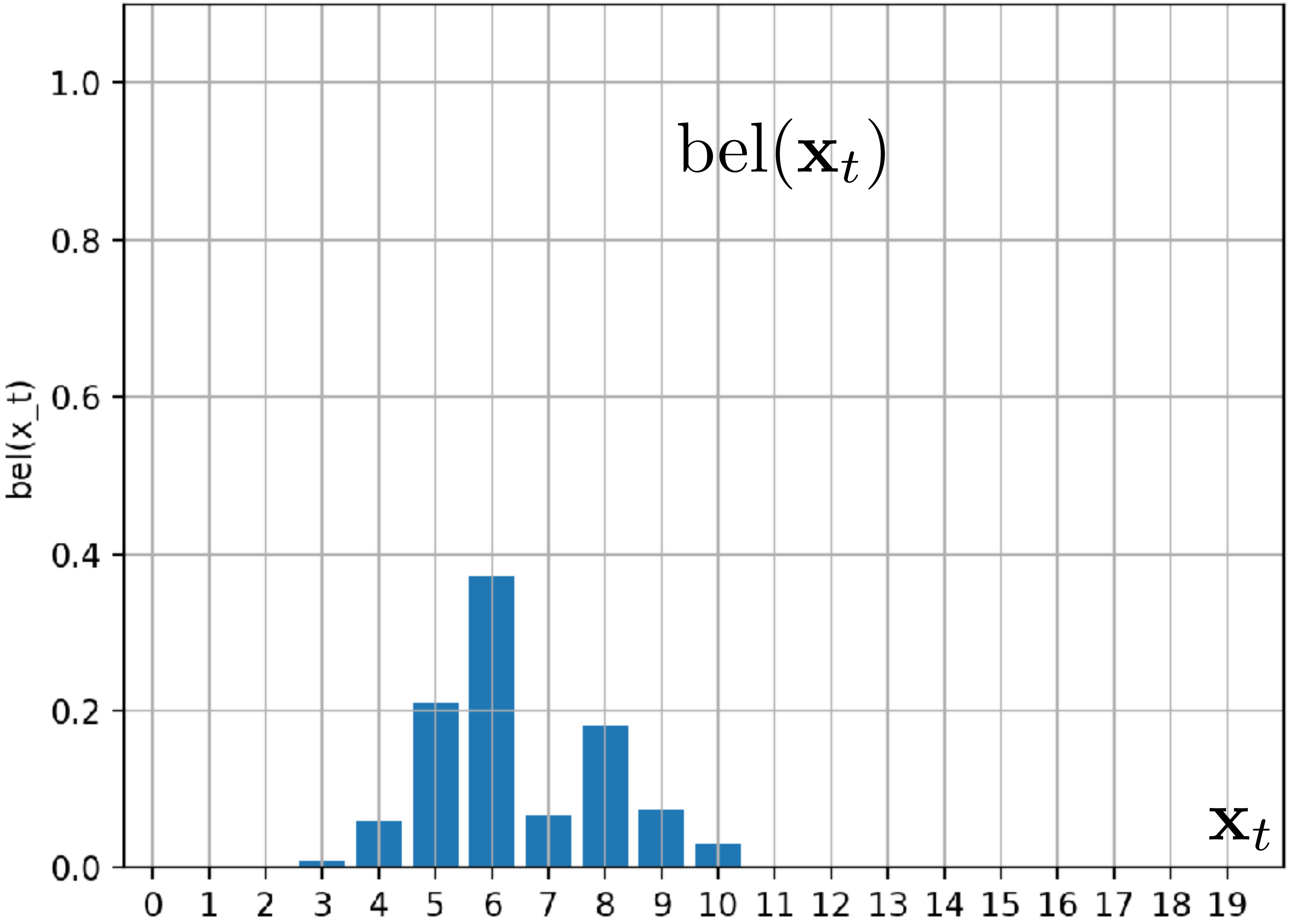

Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

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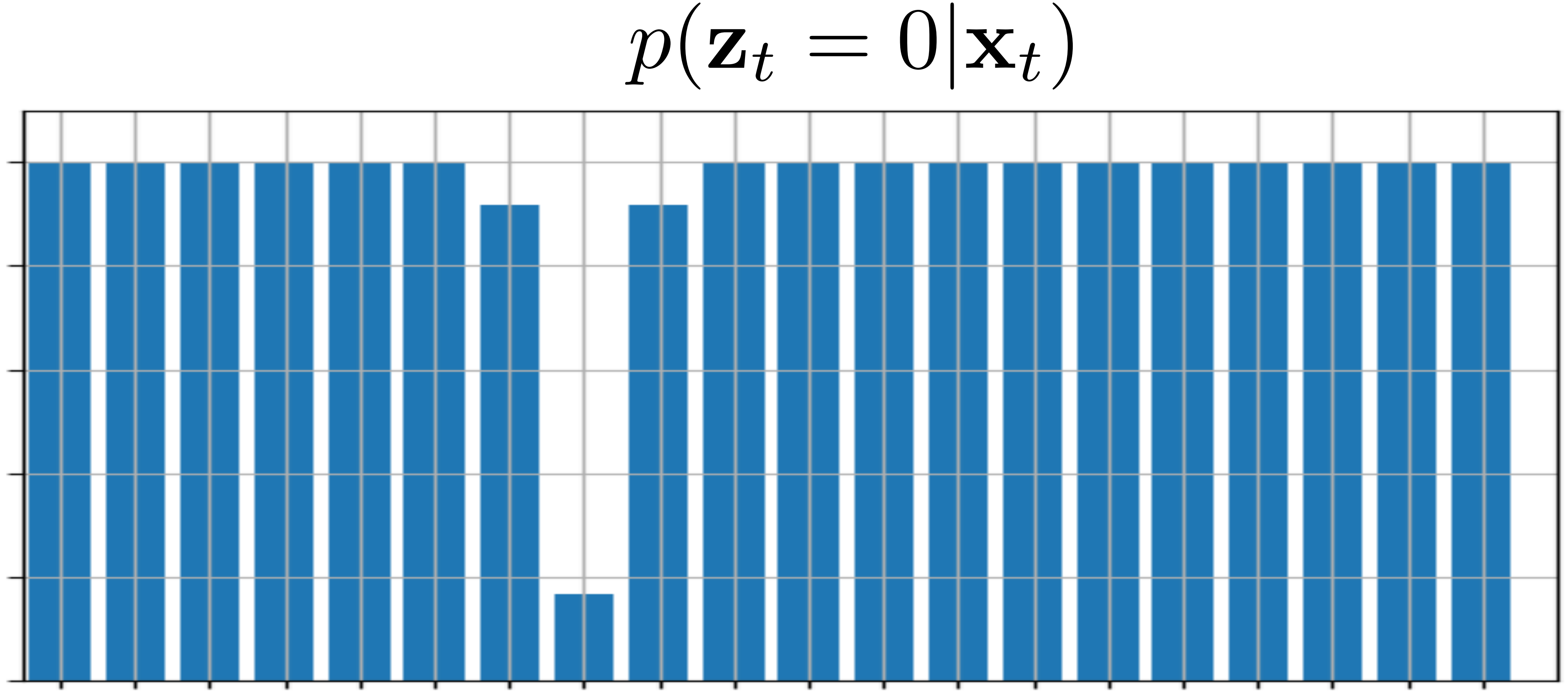

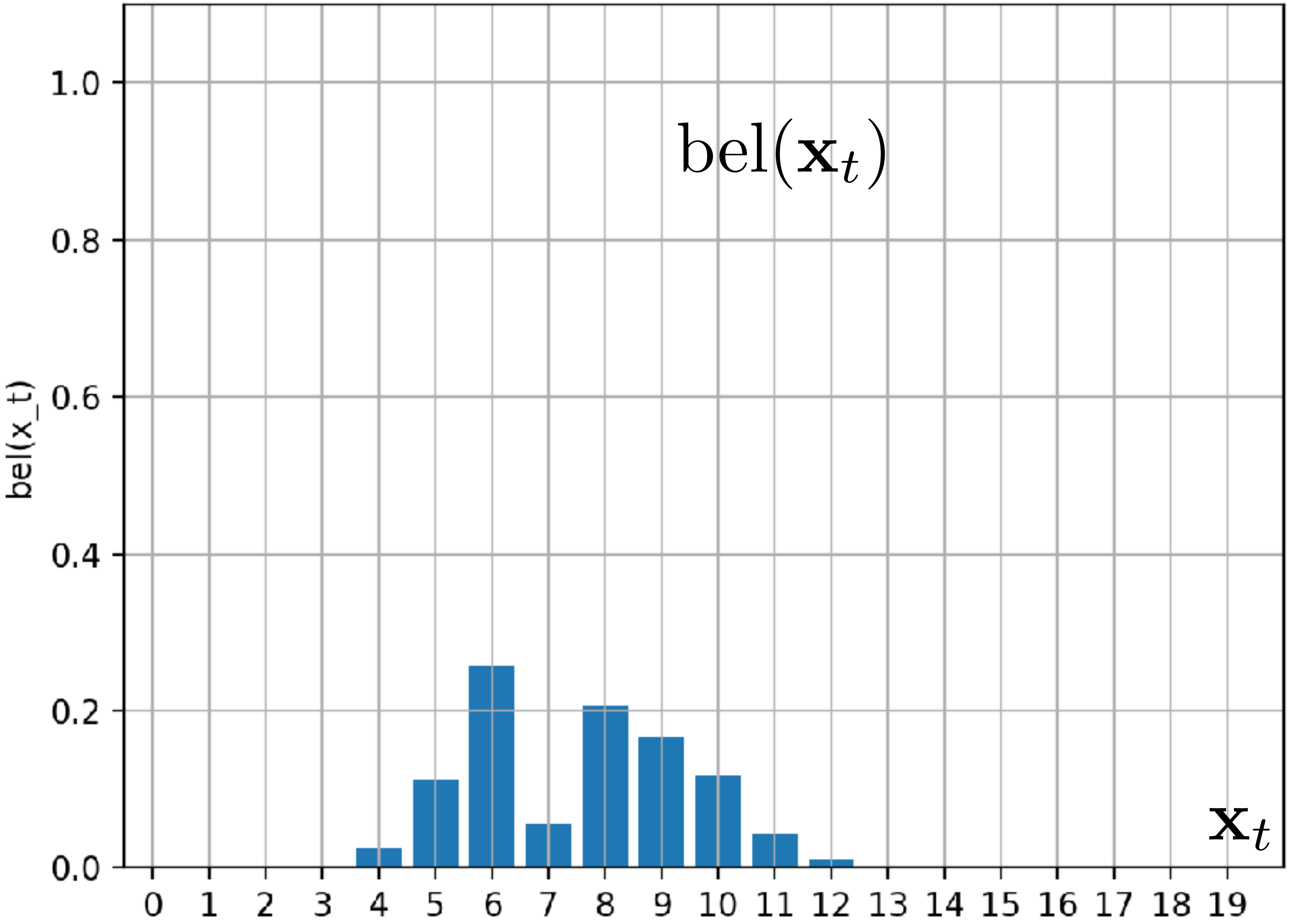

Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

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   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

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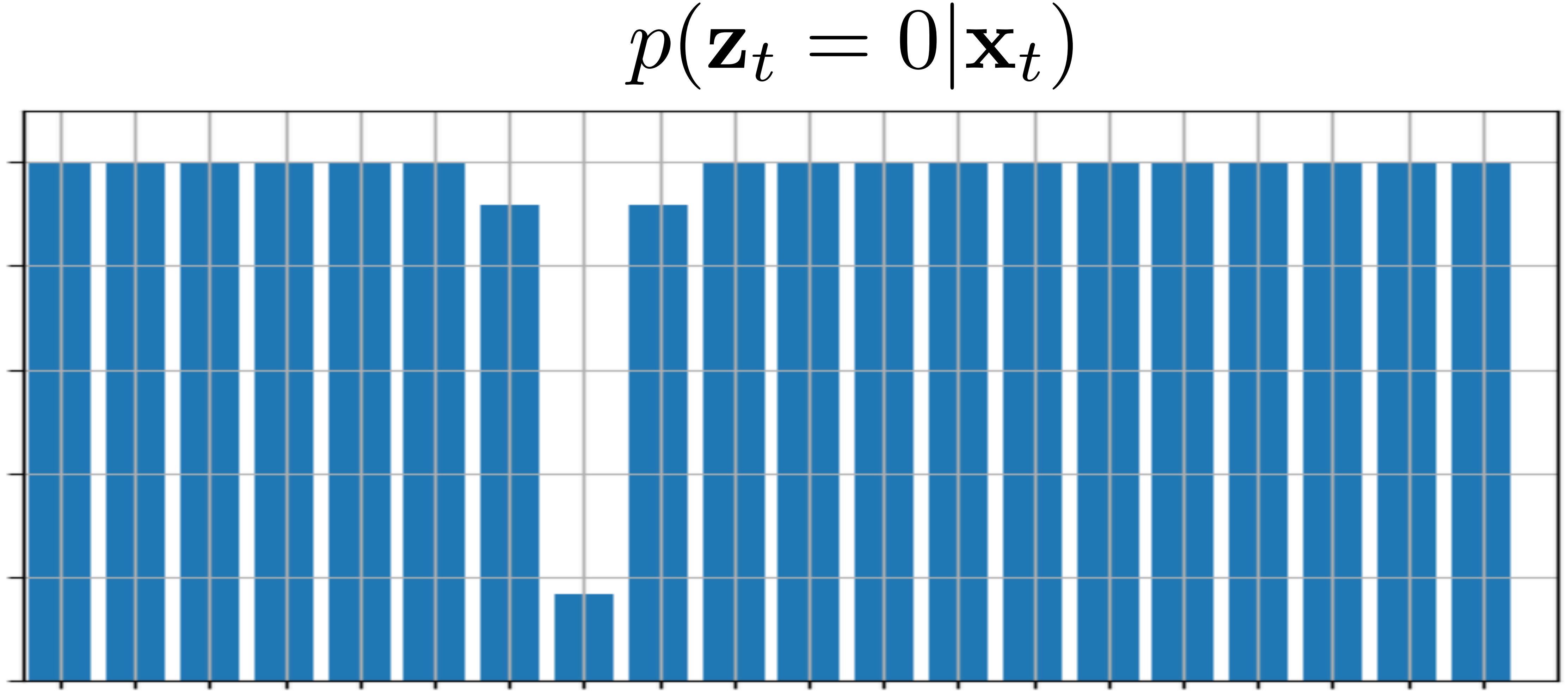
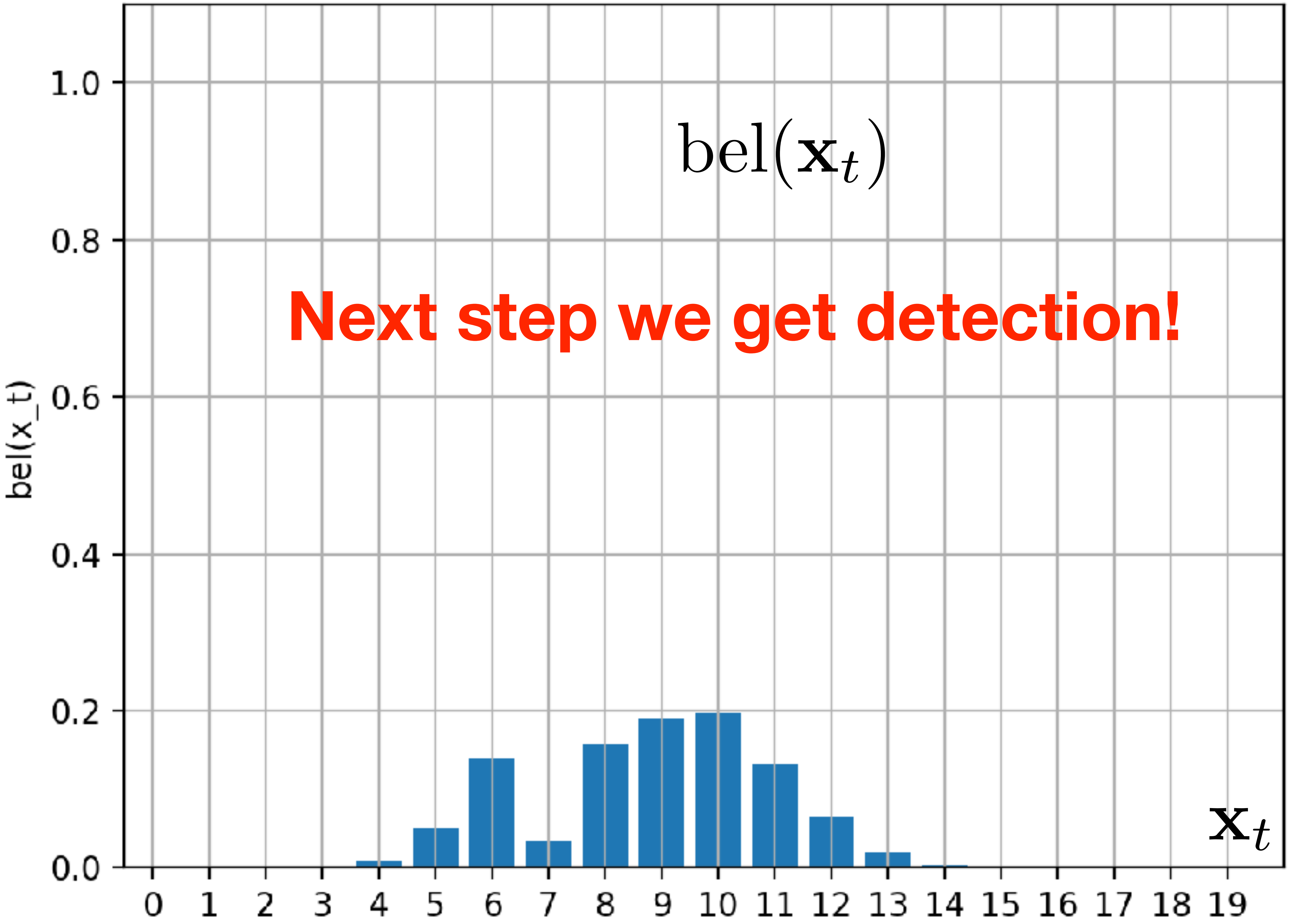
   $t = t + 1$





Bayes filter

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\text{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \text{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\text{bel}}(\mathbf{x}_t)$$

   $$s = s + \text{bel}(\mathbf{x}_t)$$

   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = \frac{\text{bel}(\mathbf{x}_t)}{s}$$

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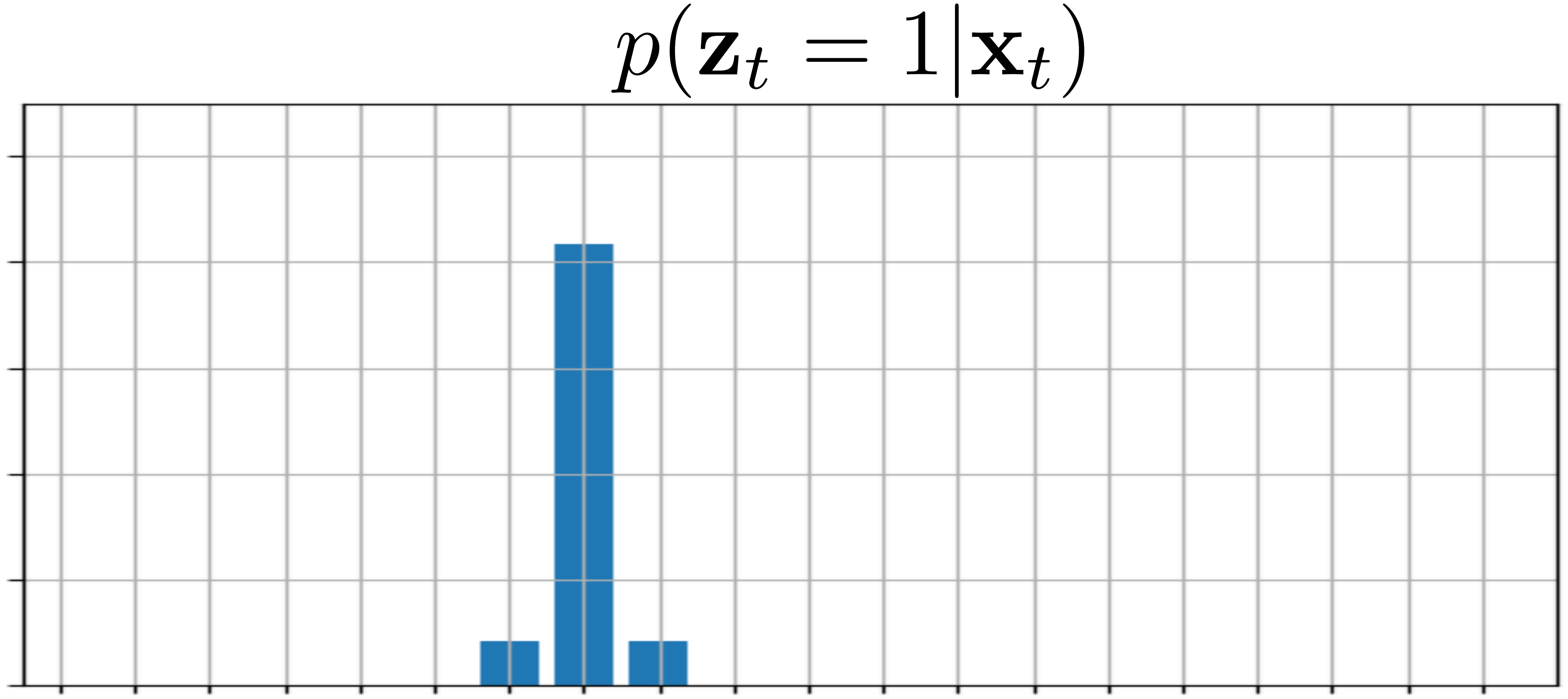
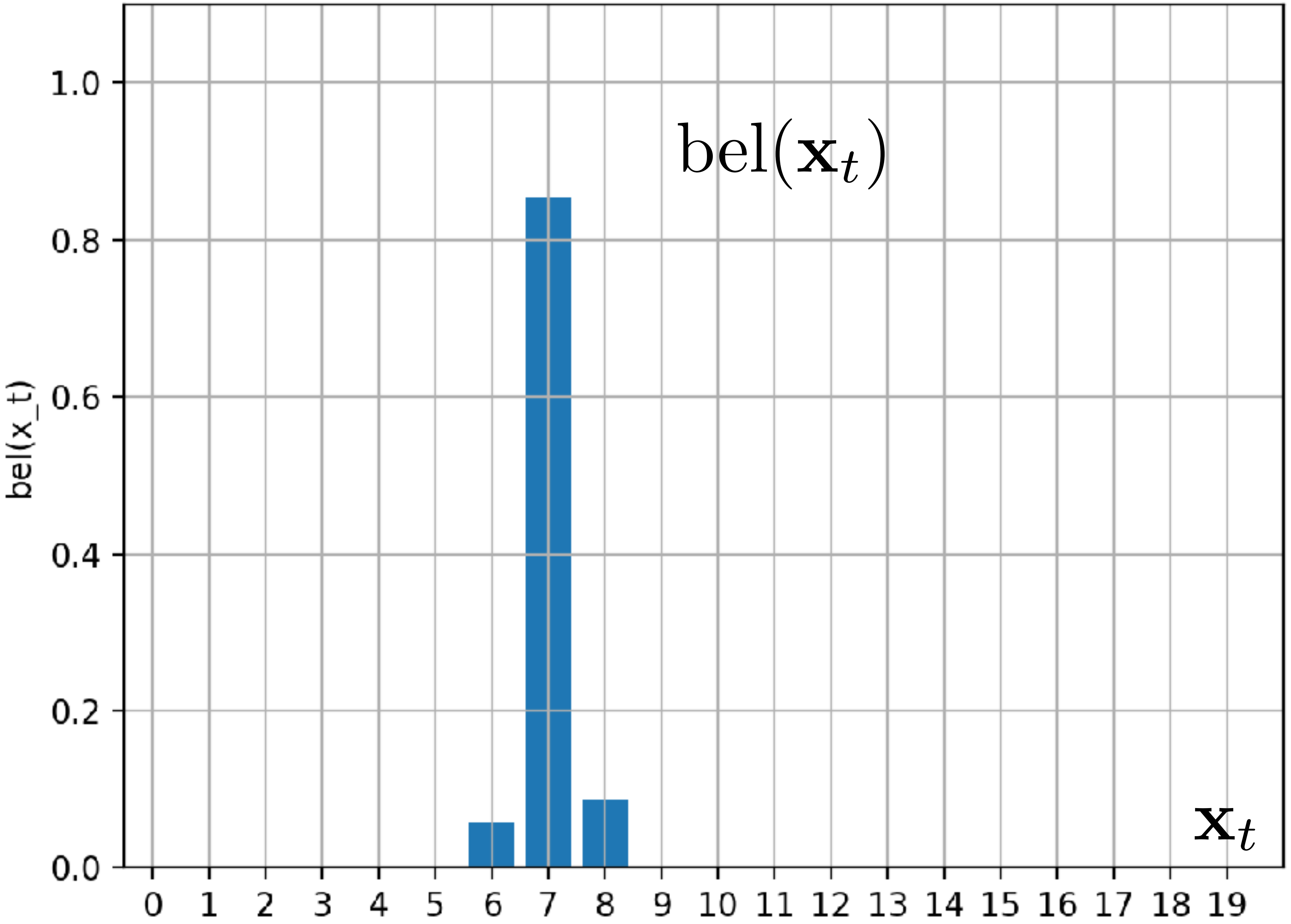
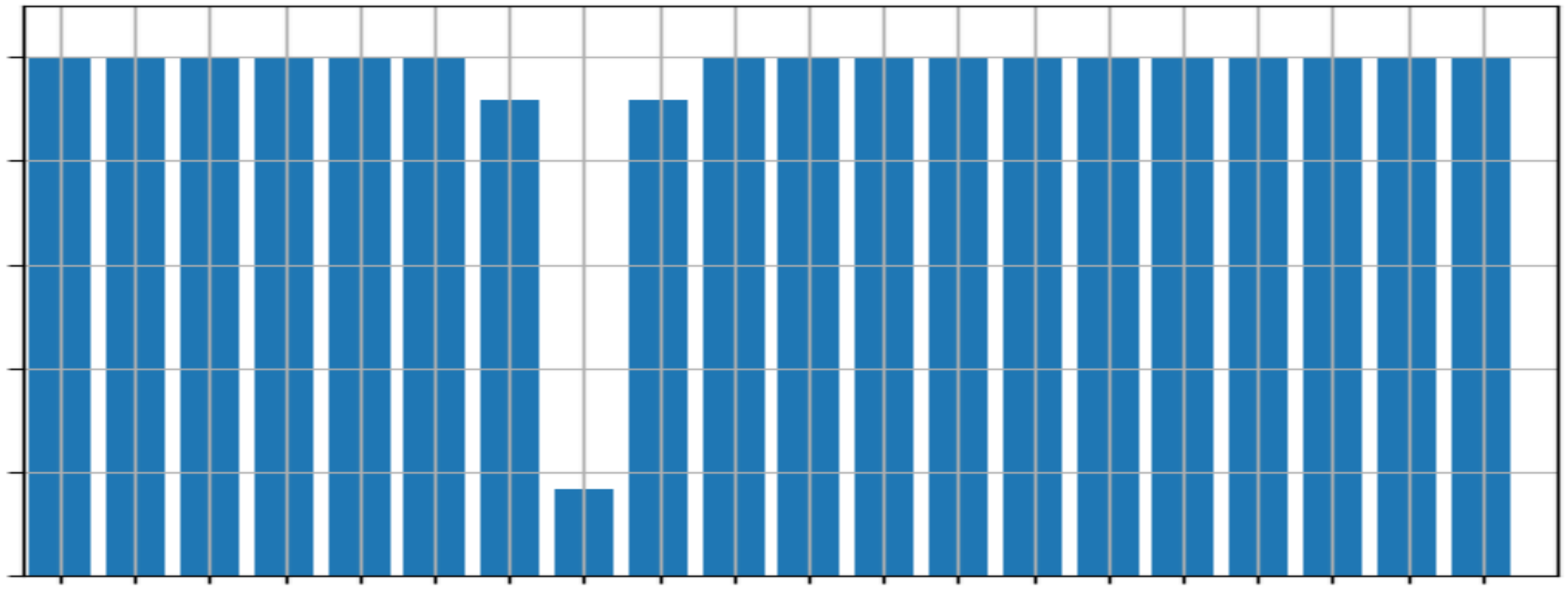




Bayes filter



1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \,|\, \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \,|\, \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

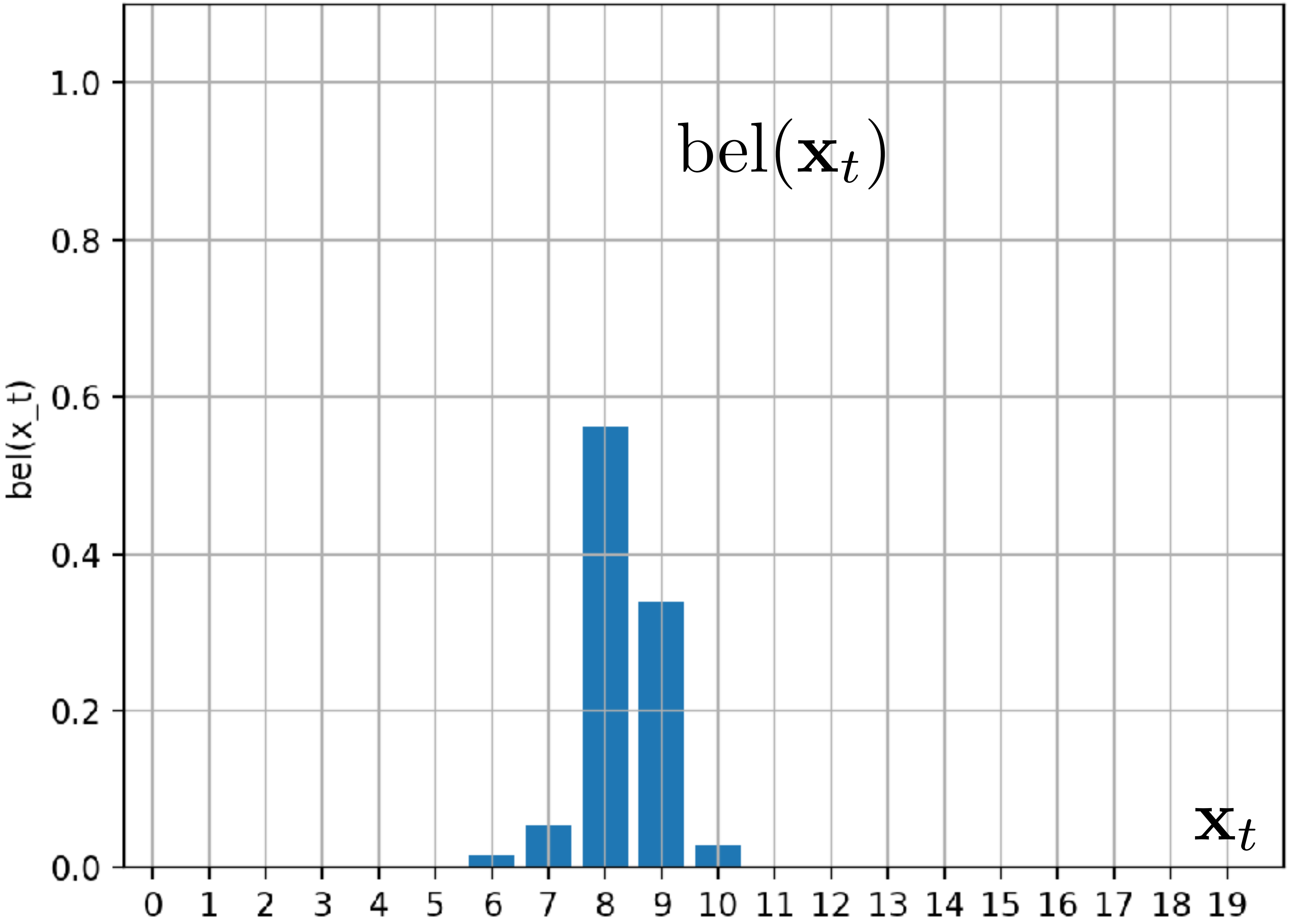
   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$



Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

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   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

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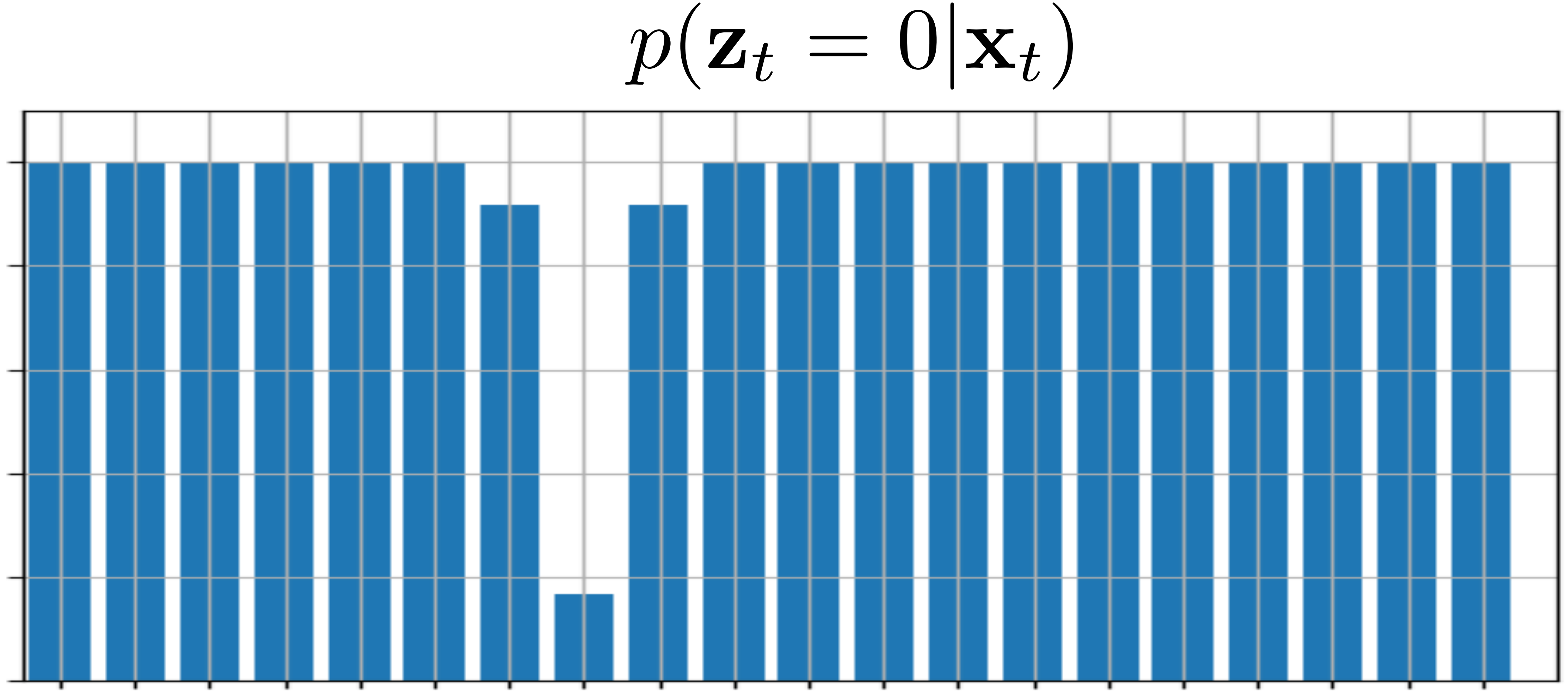
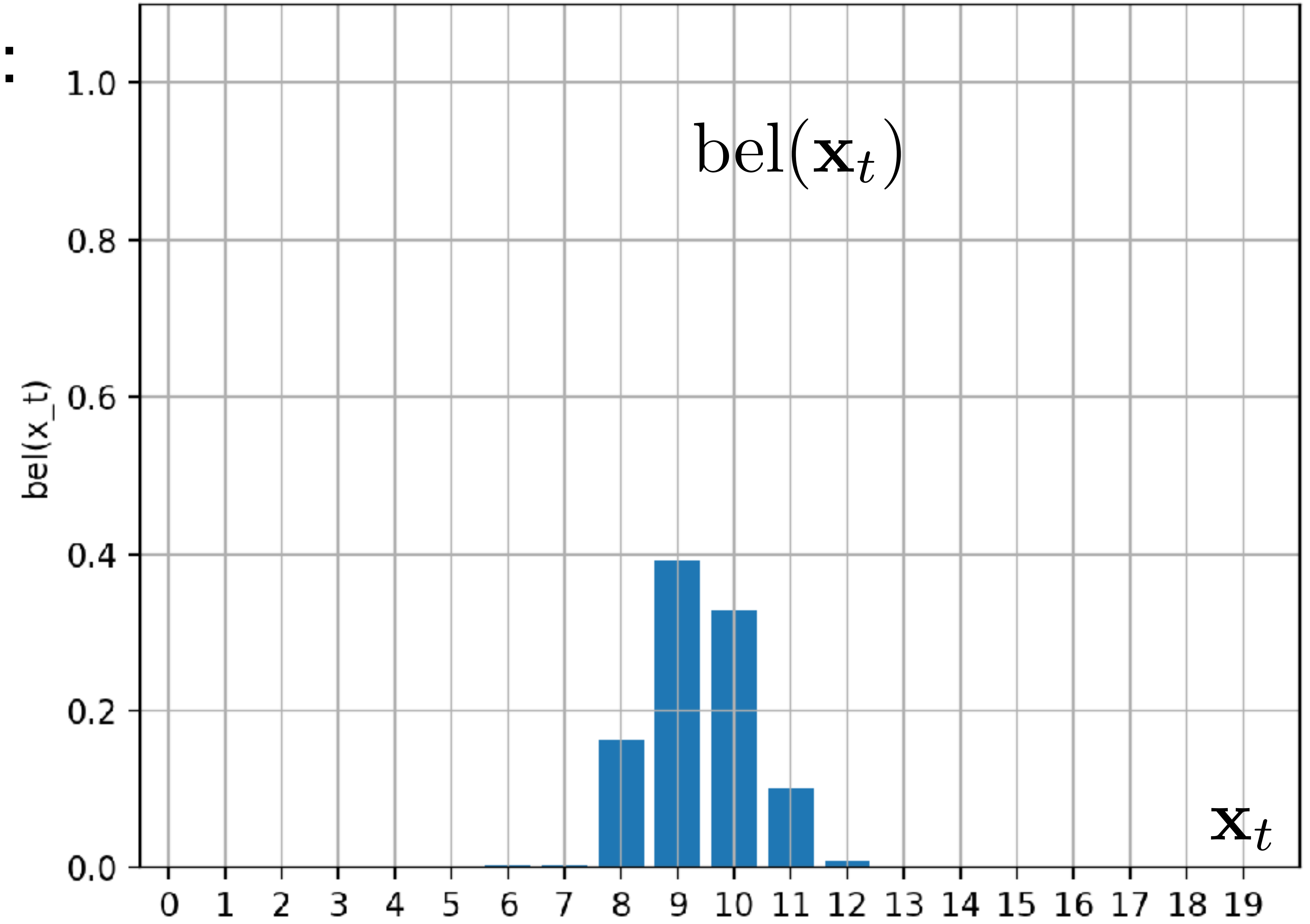
   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$





Bayes filter

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\text{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \text{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\text{bel}}(\mathbf{x}_t)$$

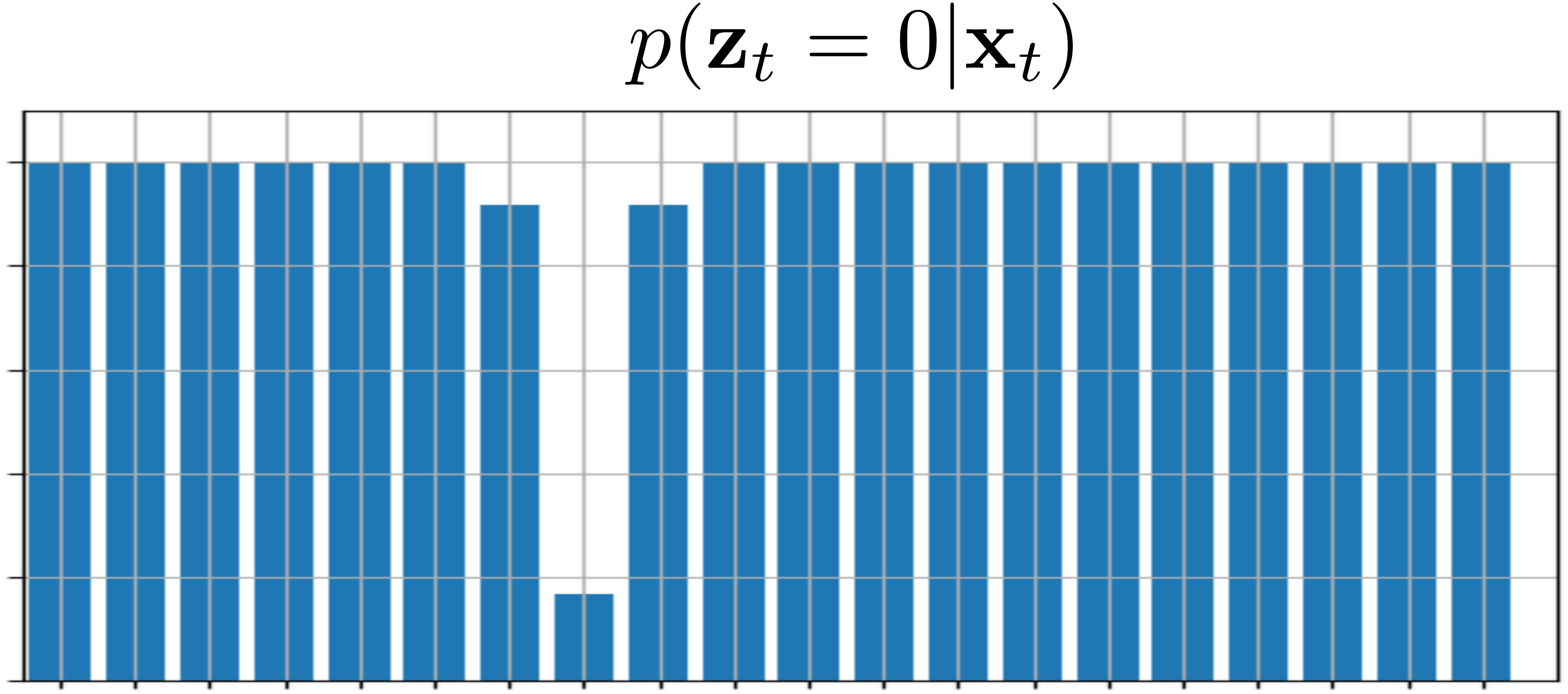
   $$s = s + \text{bel}(\mathbf{x}_t)$$

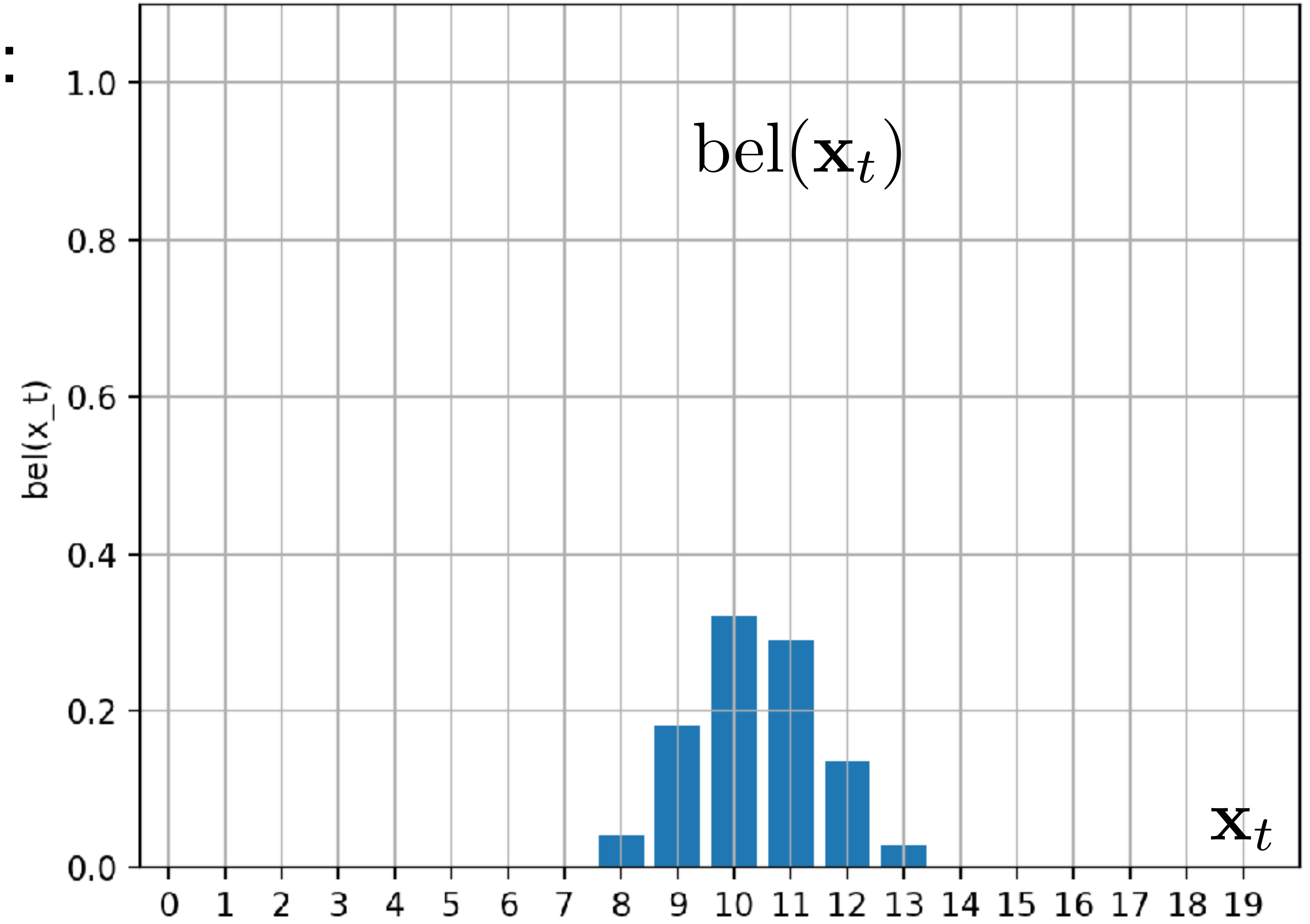
   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = \frac{\text{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$





Bayes filter: Kidnapped robot problem

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

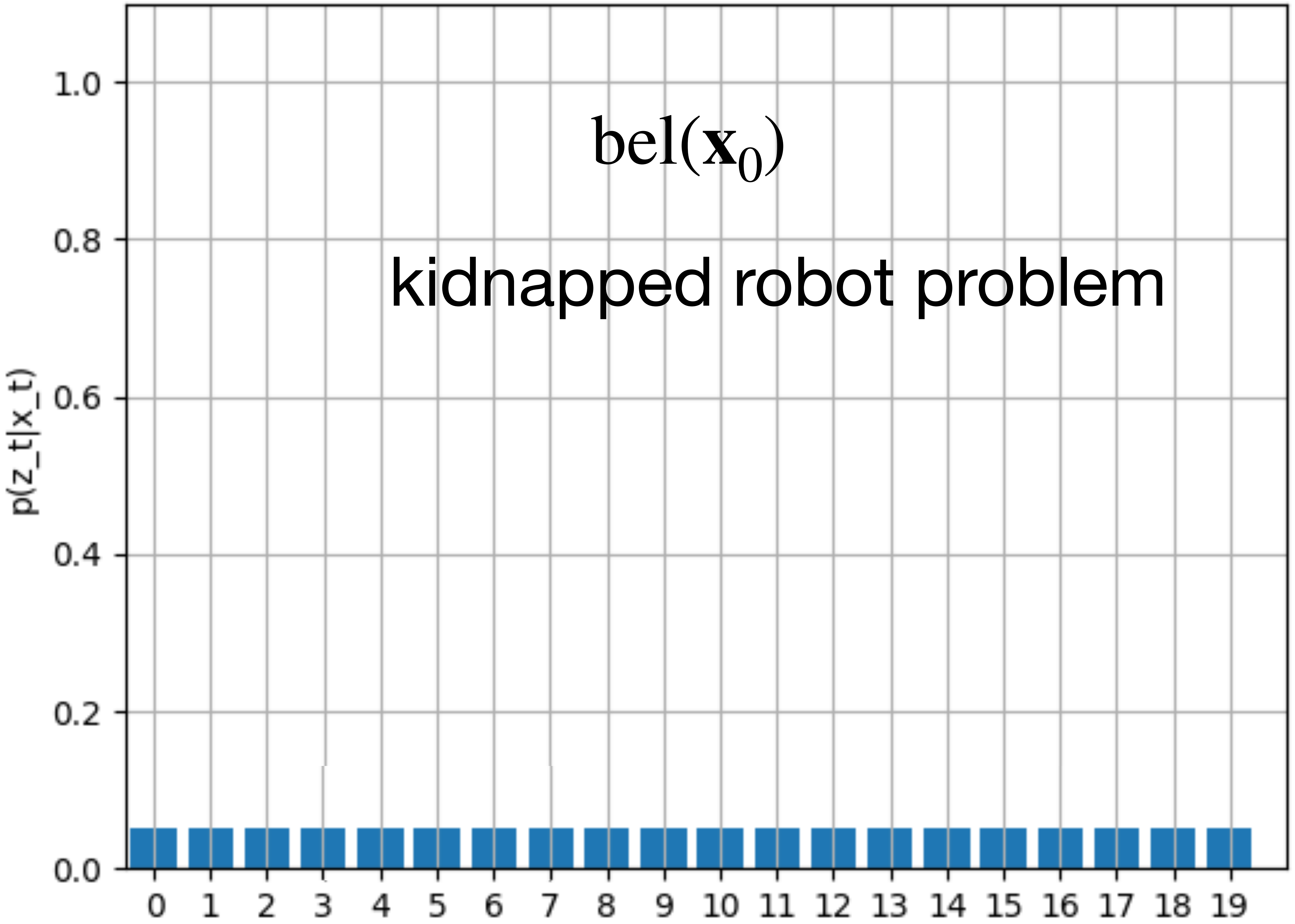
   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$



Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

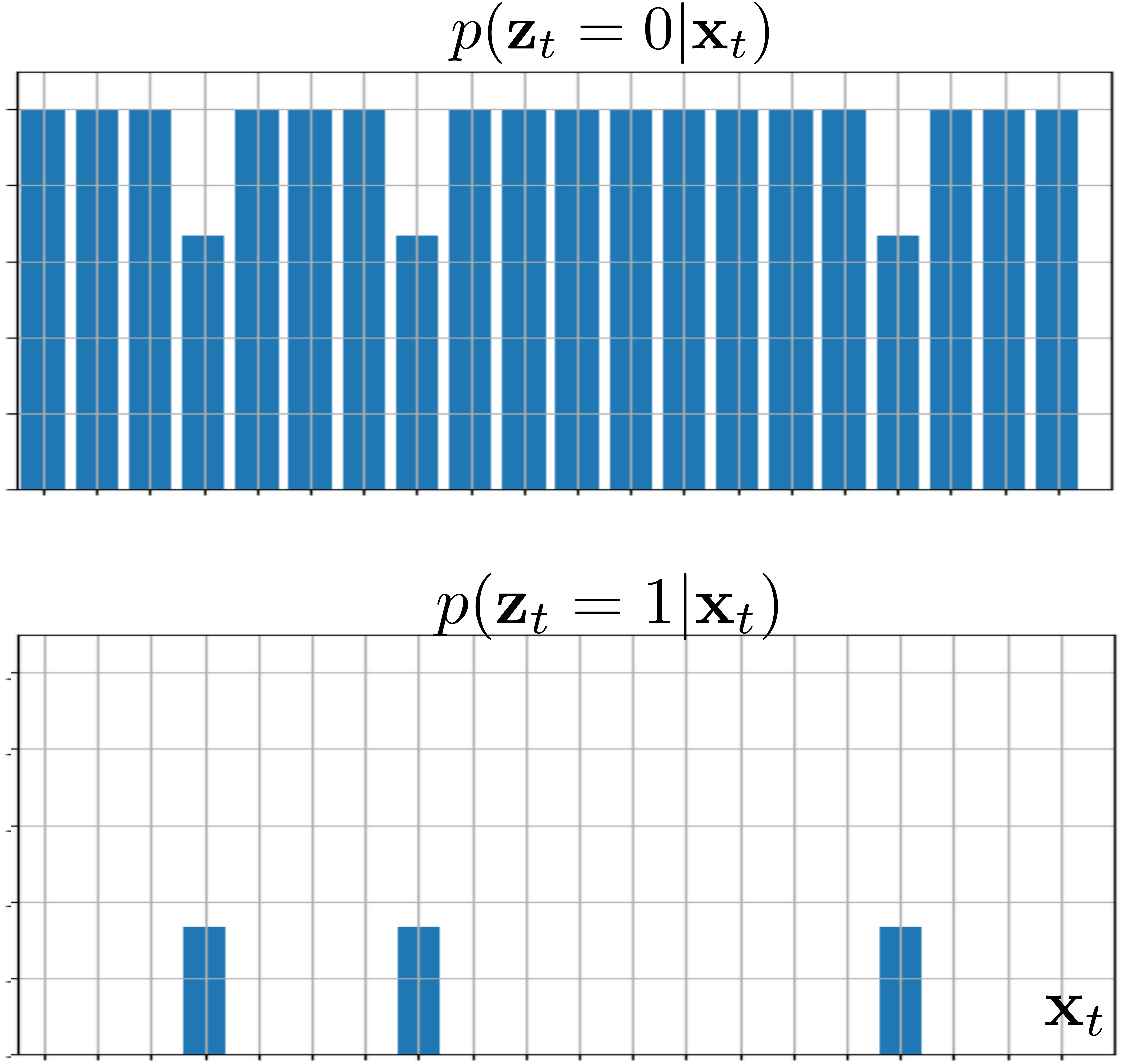
   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$



3 undistinguishable markers

Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

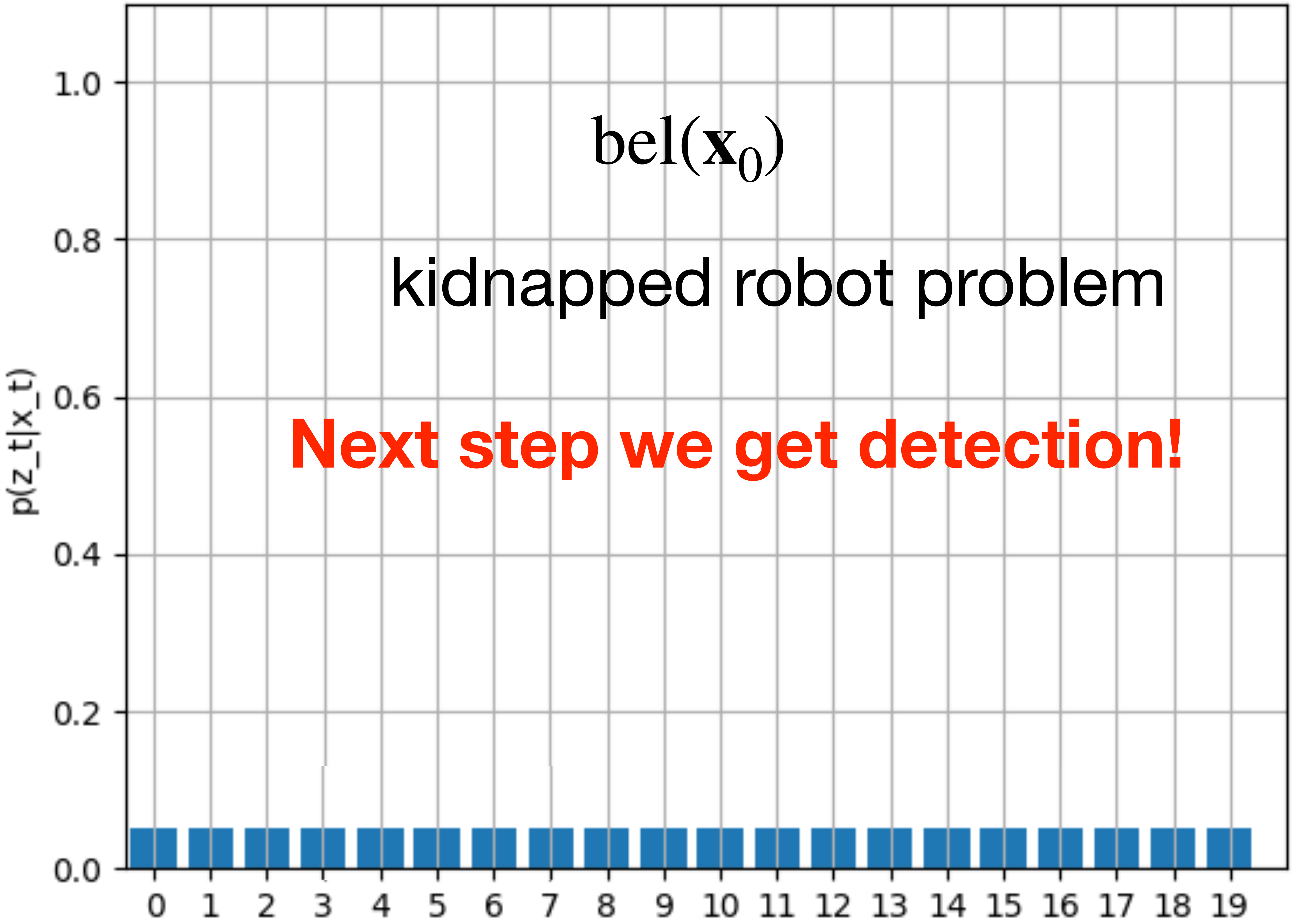
   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$



Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

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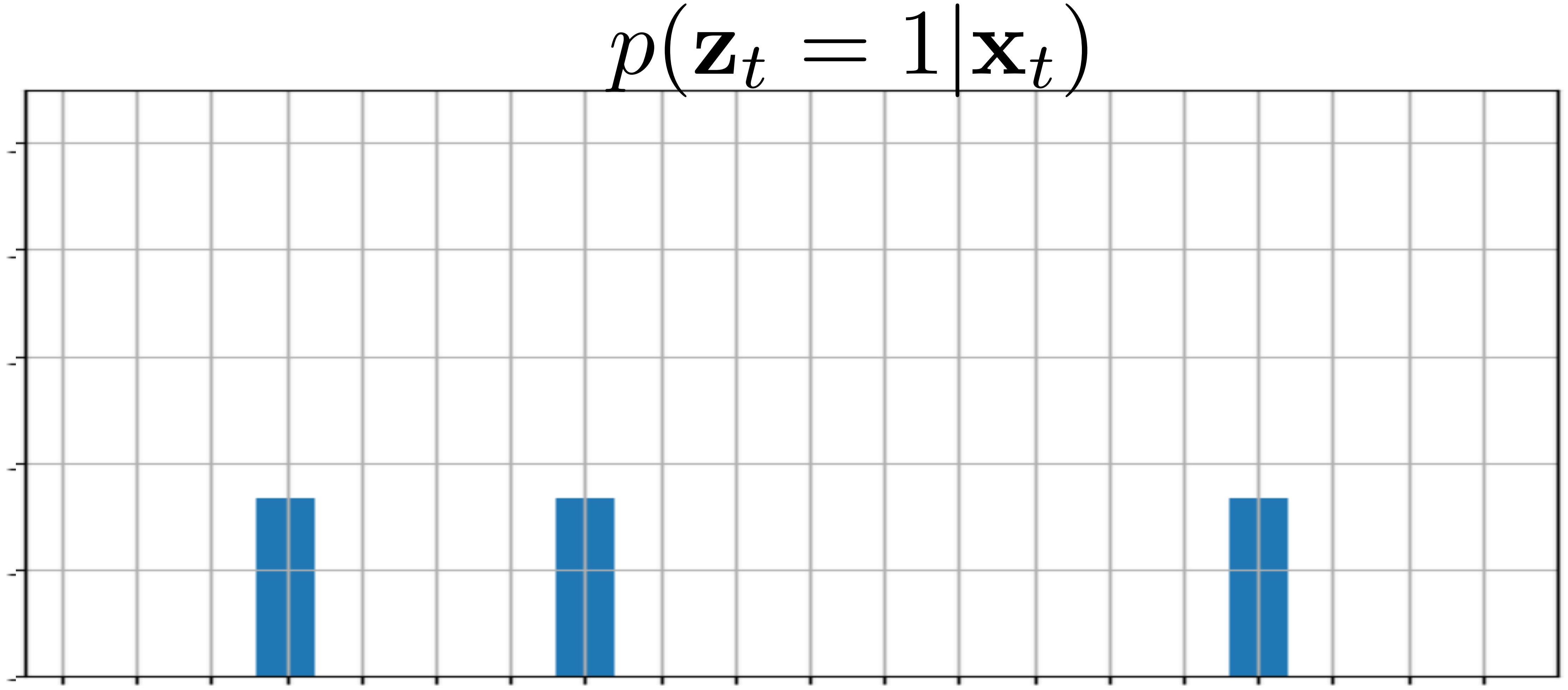

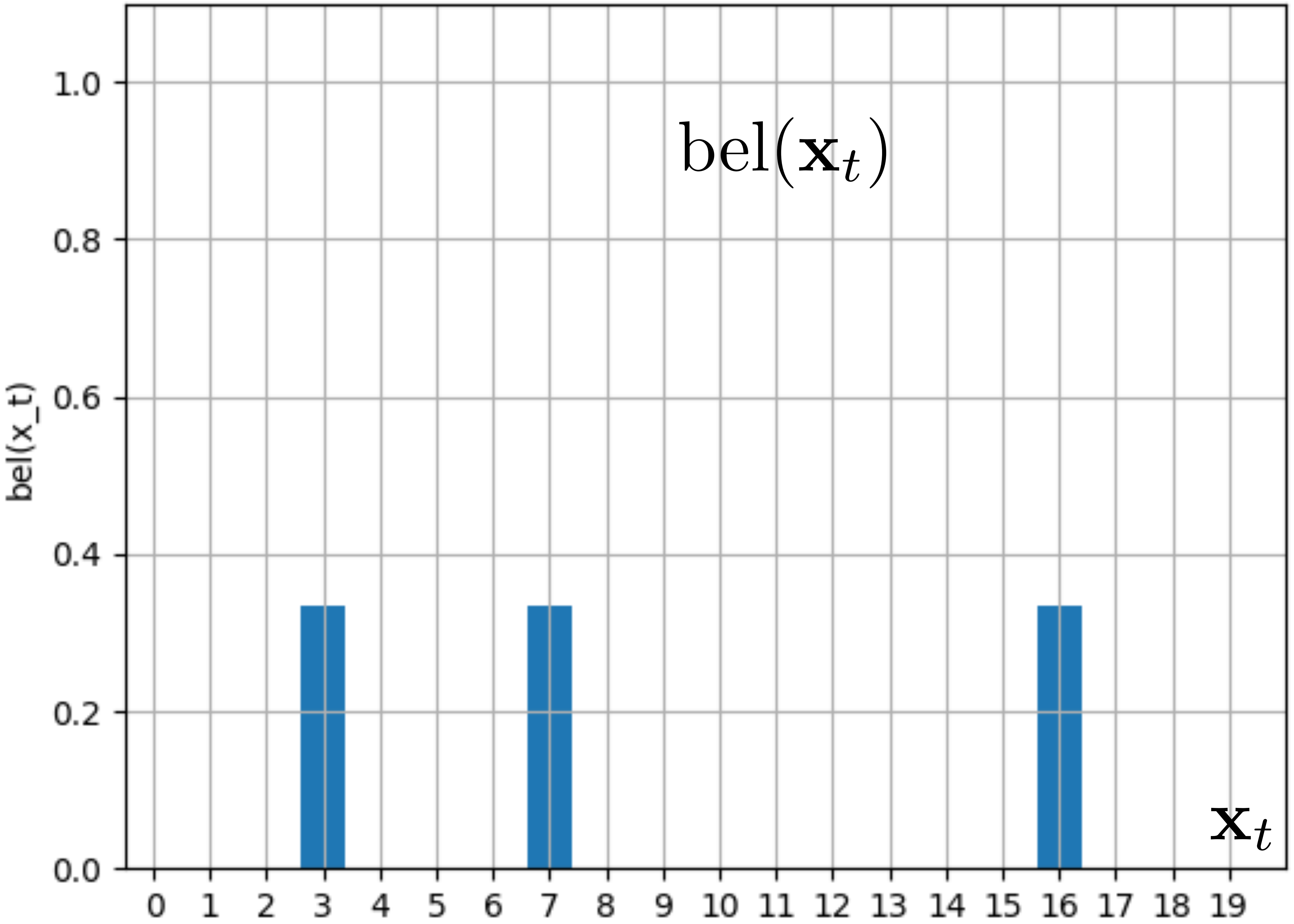

Bayes filter

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\text{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \text{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\text{bel}}(\mathbf{x}_t)$$

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   $$\text{bel}(\mathbf{x}_t) = \frac{\text{bel}(\mathbf{x}_t)}{s}$$

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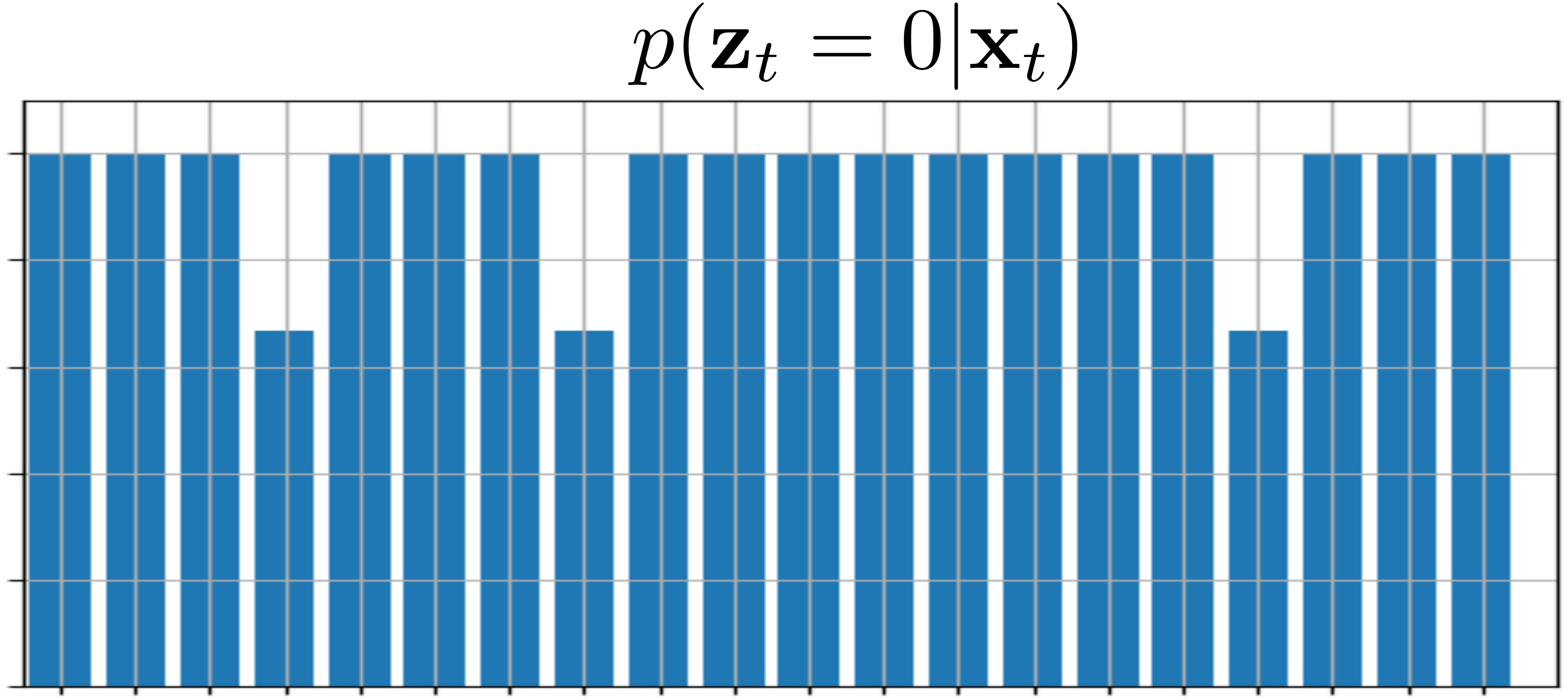

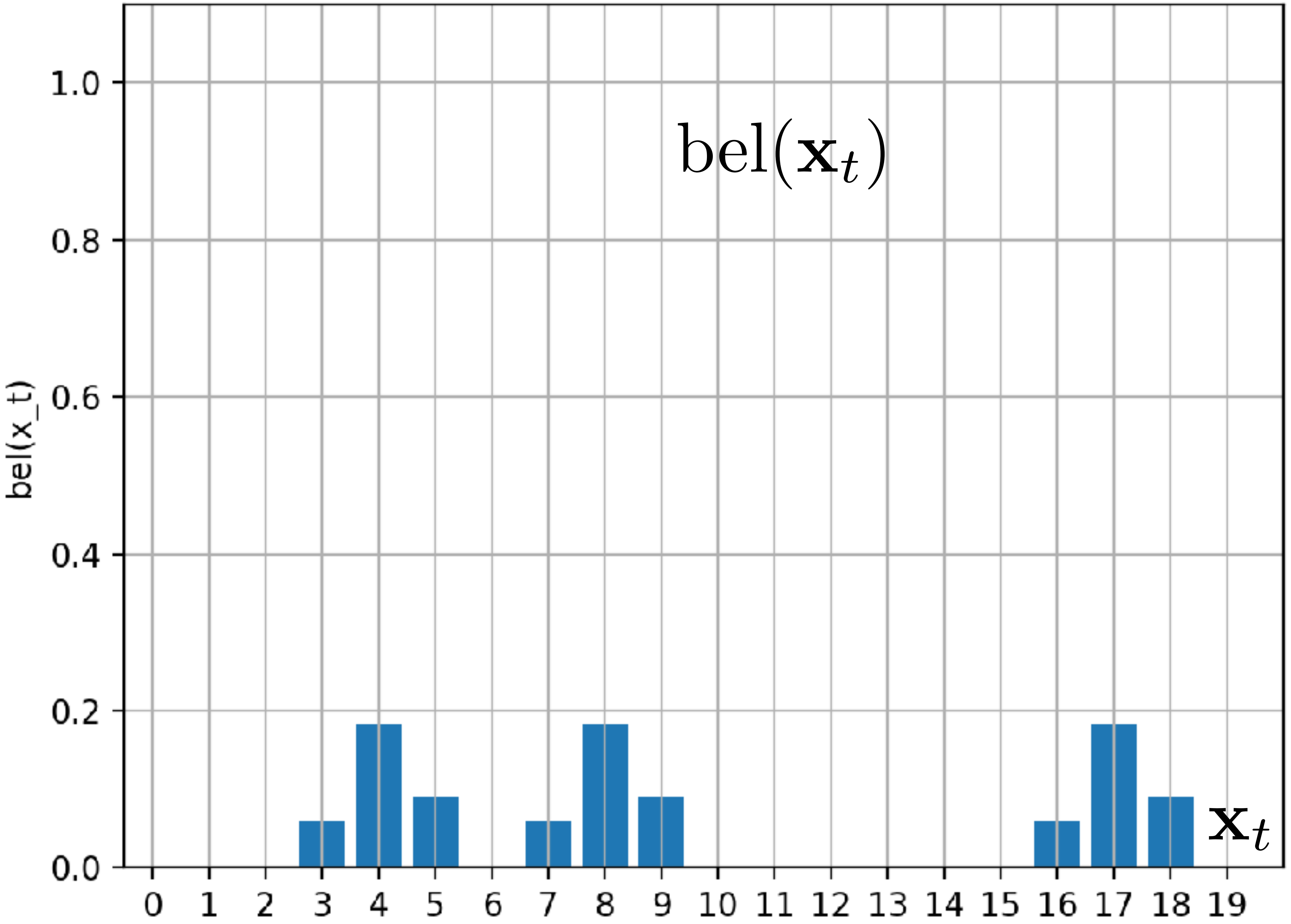

Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

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   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

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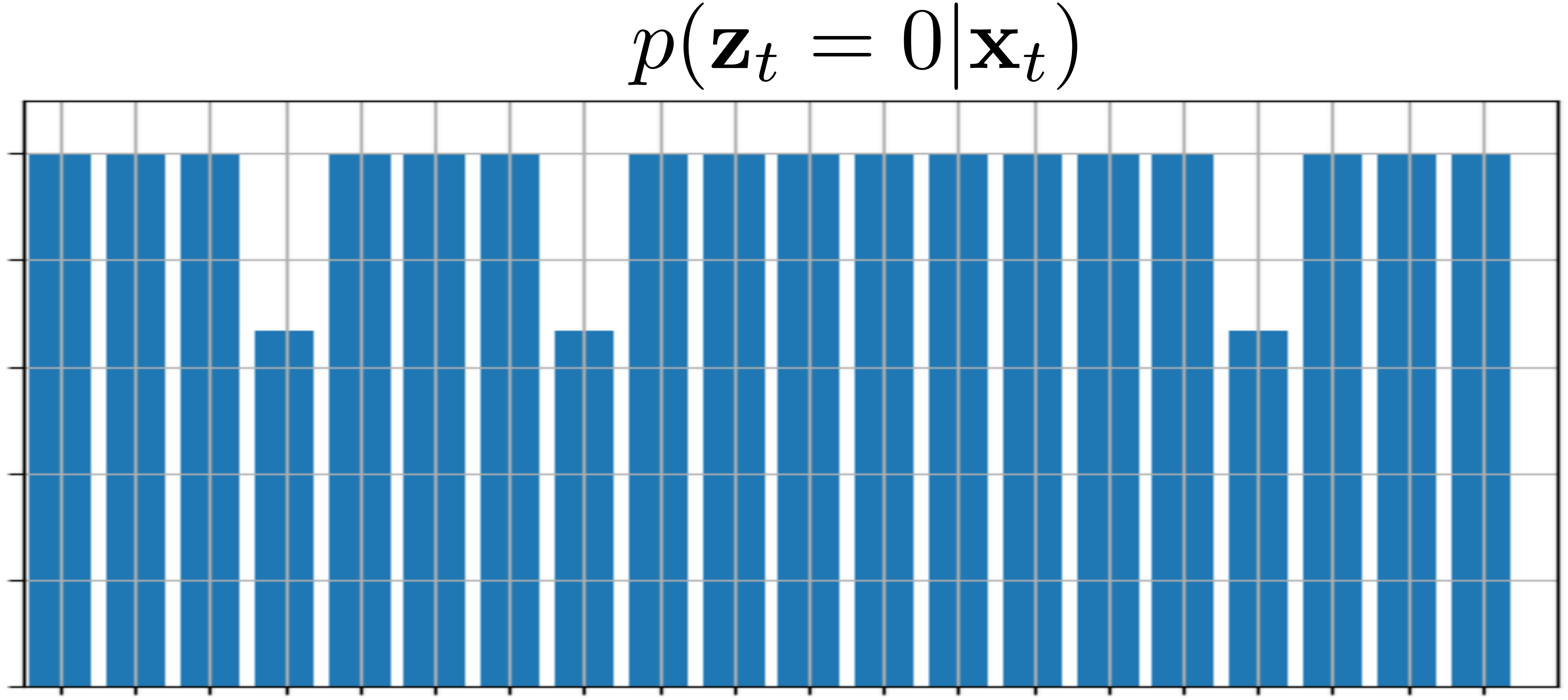
   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$



Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

   breakpoint

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

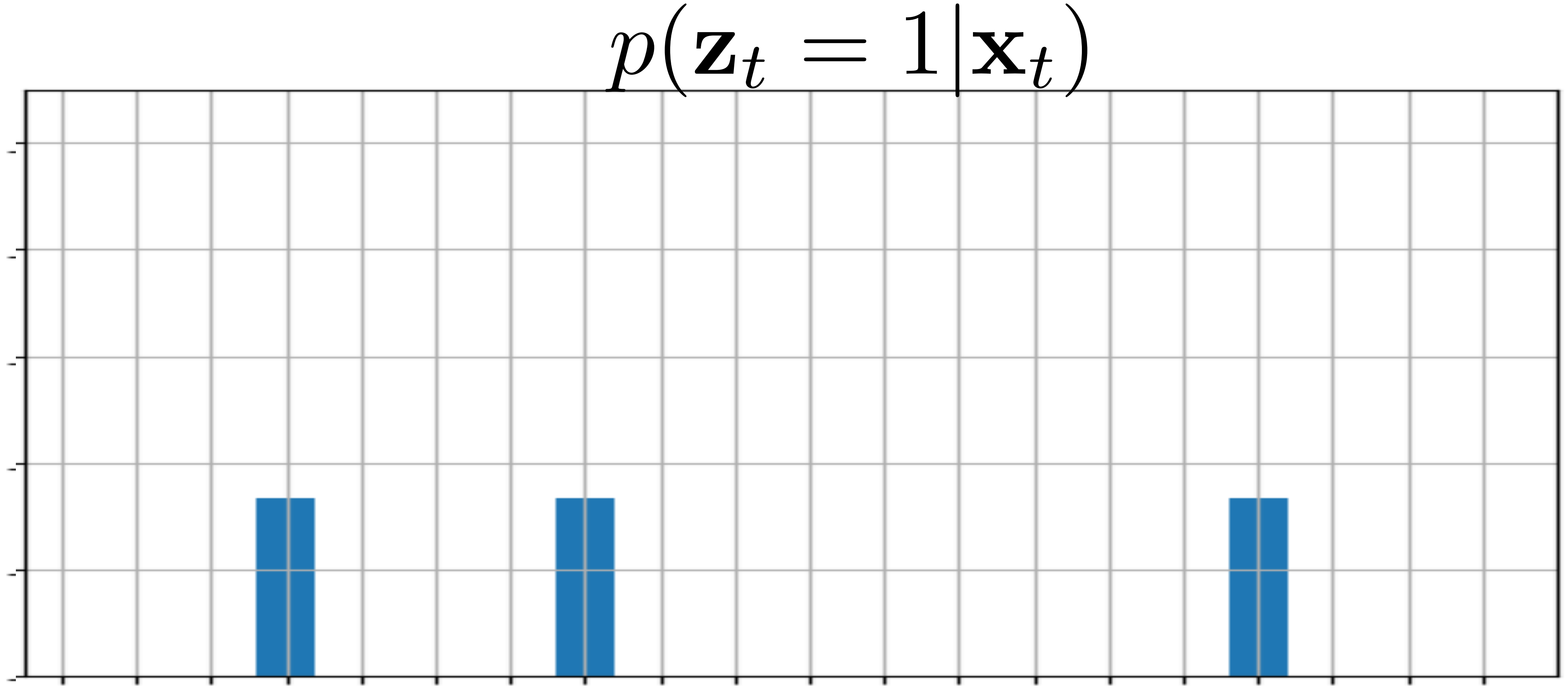
   For all $\mathbf{x}_t$

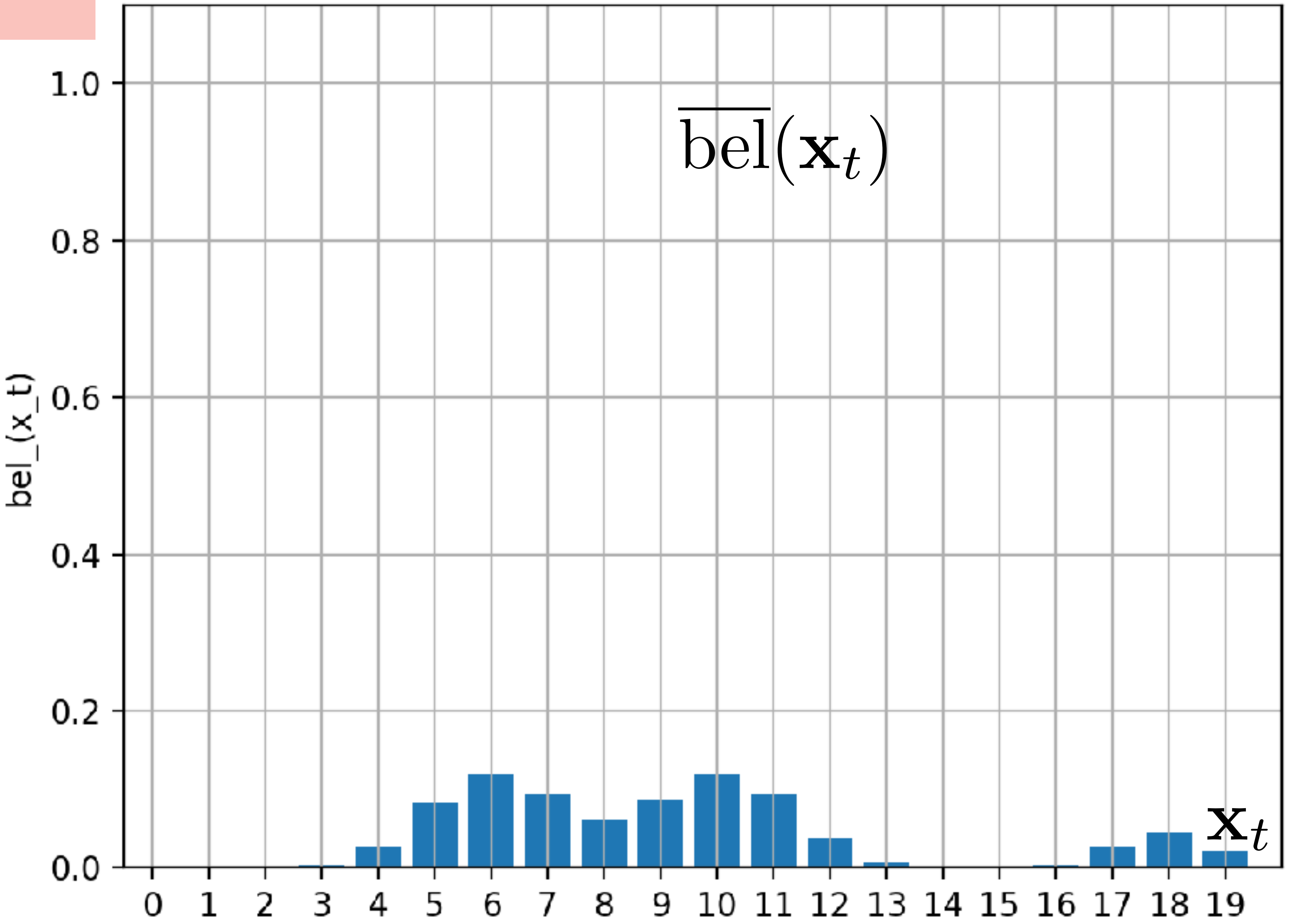
   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

   = ???

4. Repeat from 2:

   $t = t + 1$





Bayes filter

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2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

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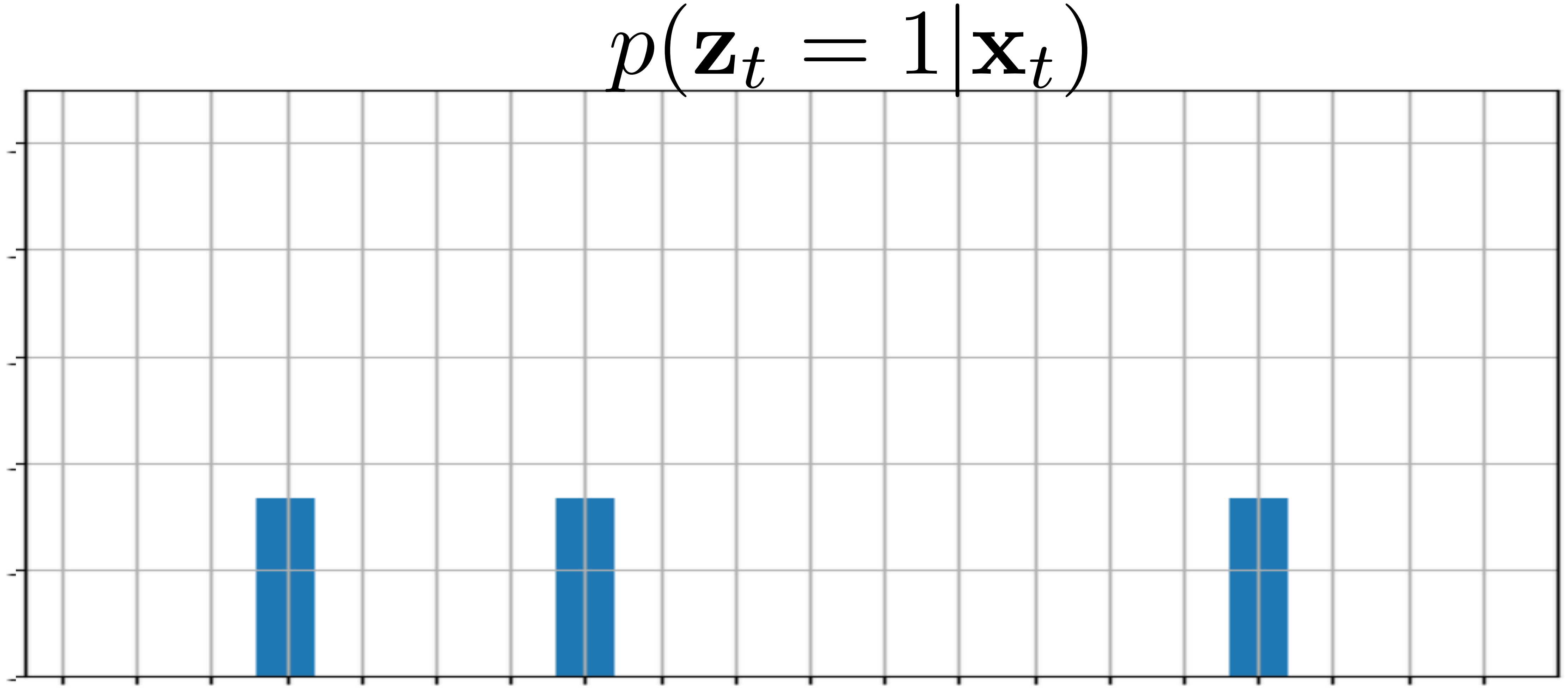
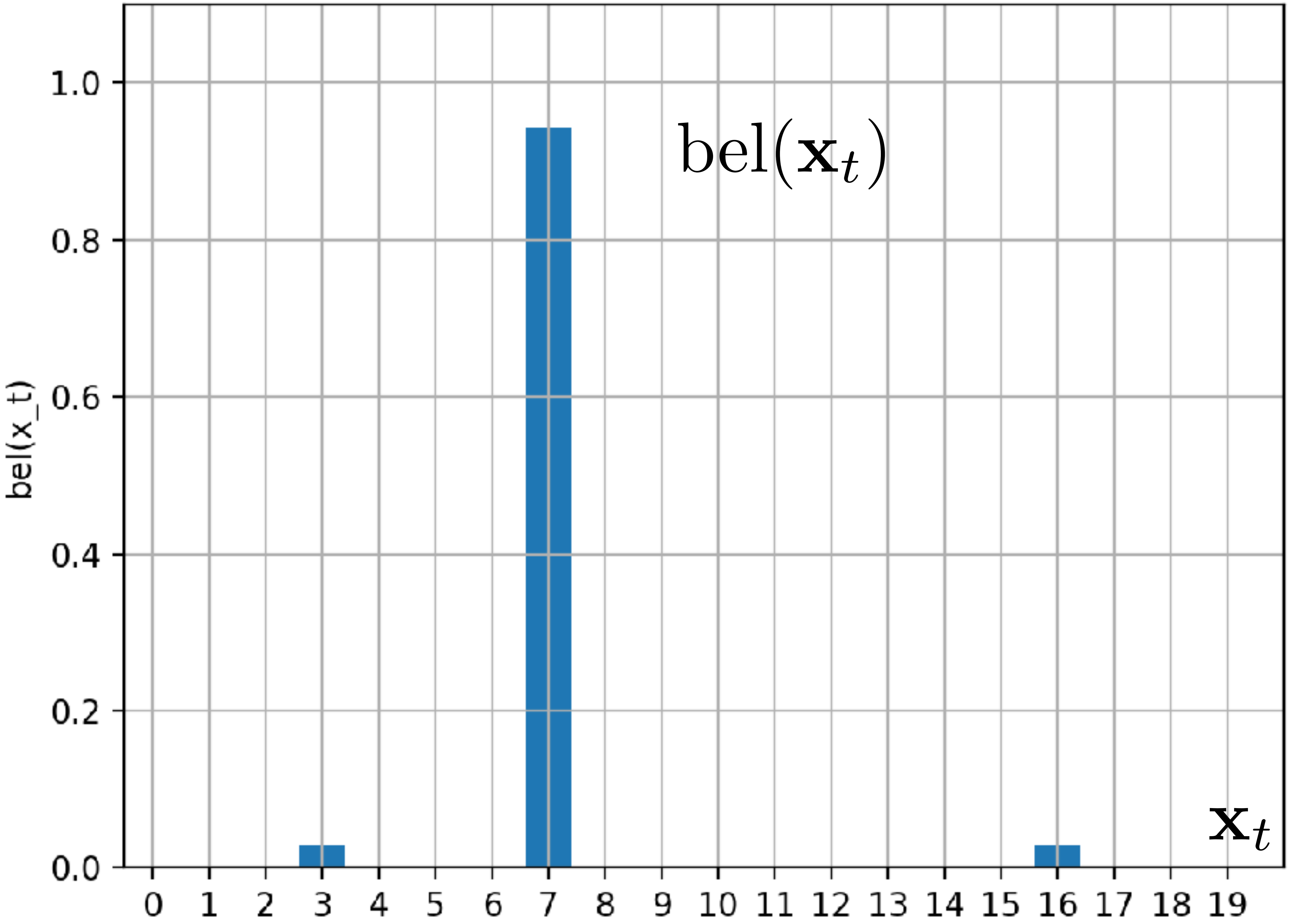
   breakpoint

4. Repeat from 2:

   $t = t + 1$





Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

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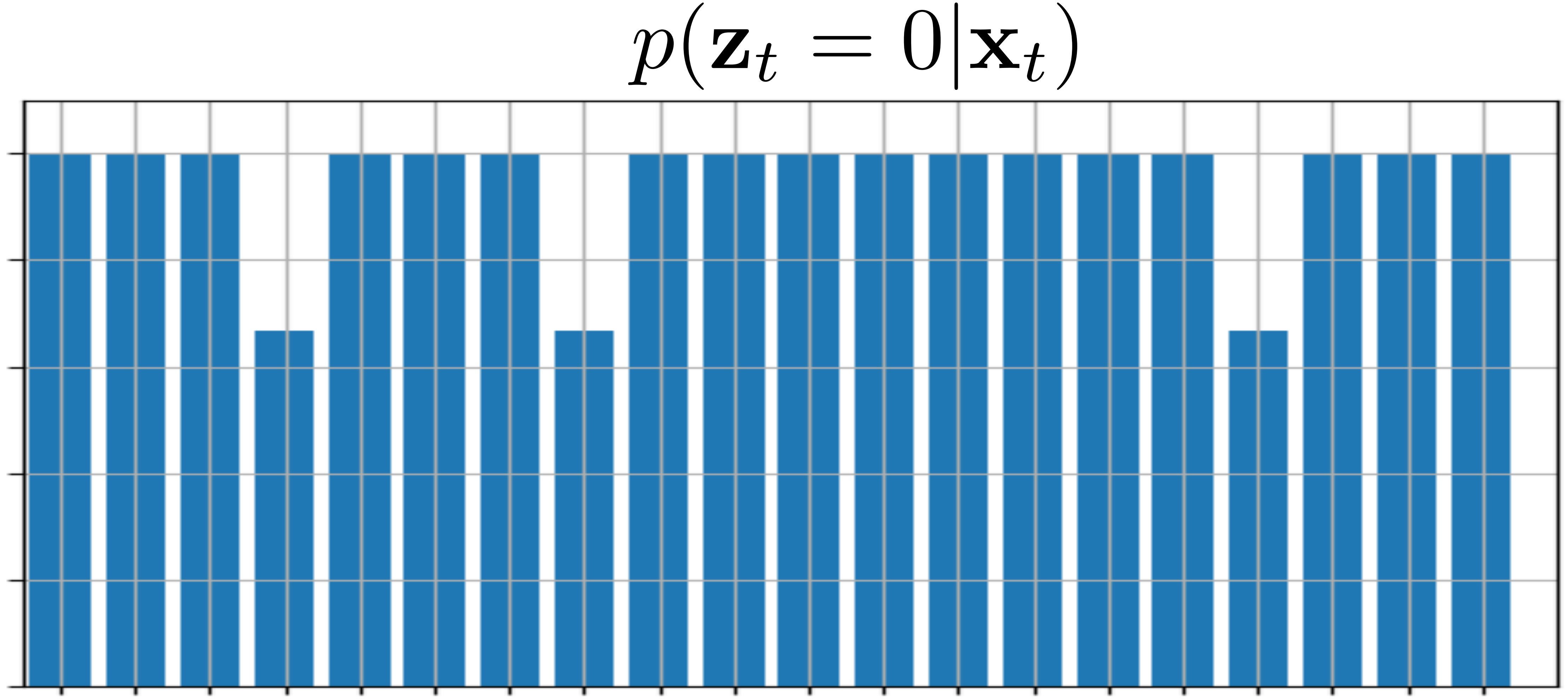
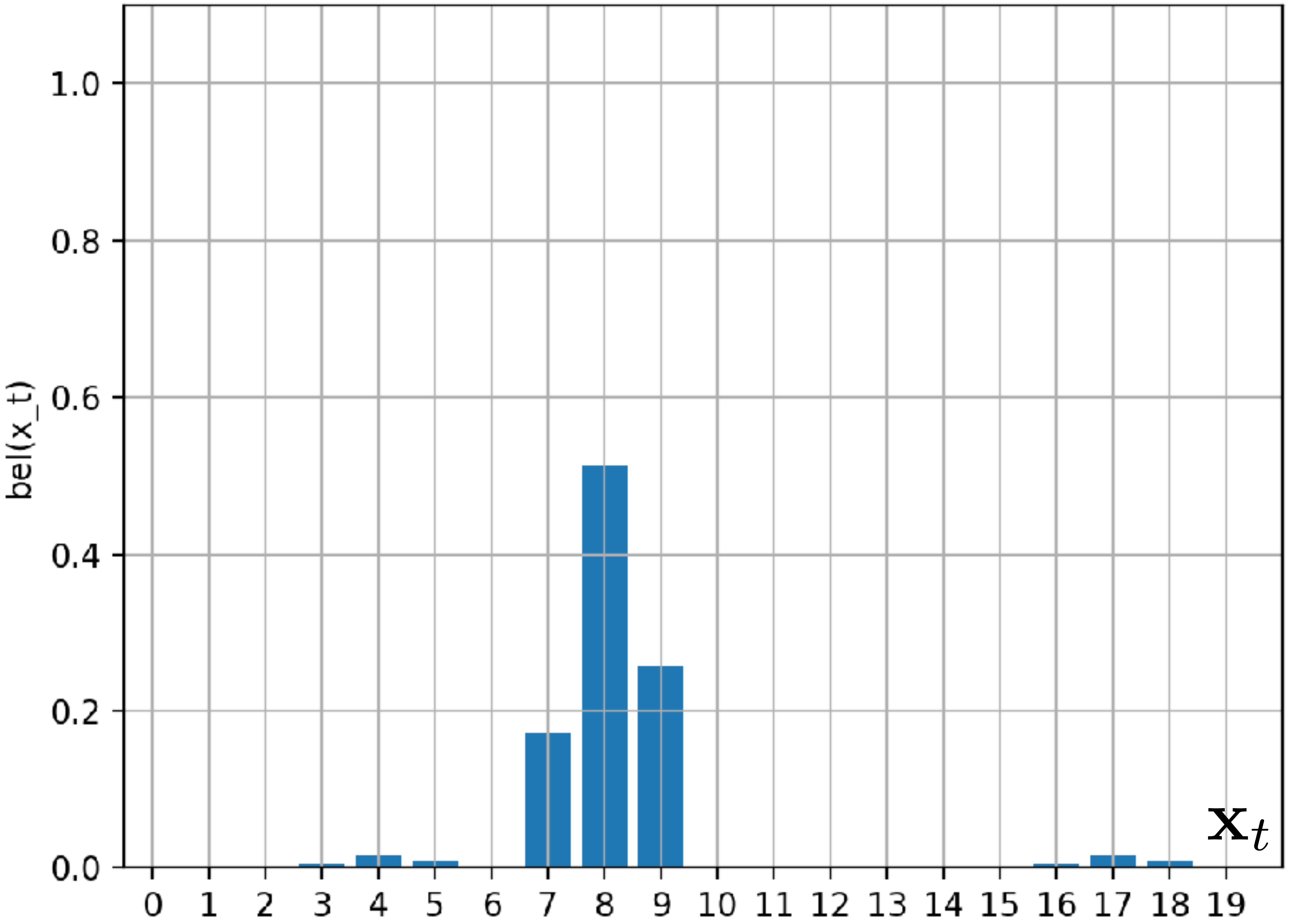




Bayes filter

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\text{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \text{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\text{bel}}(\mathbf{x}_t)$$

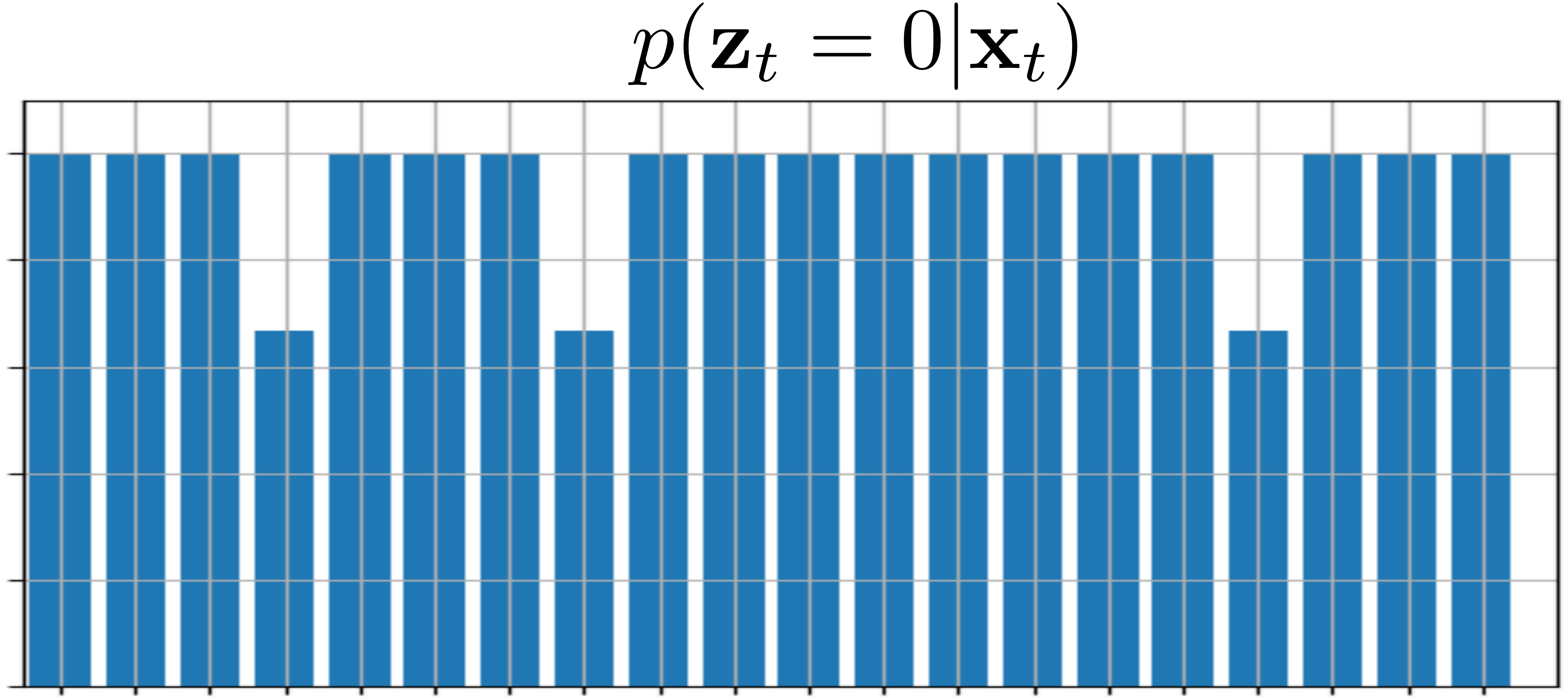
   $$s = s + \text{bel}(\mathbf{x}_t)$$

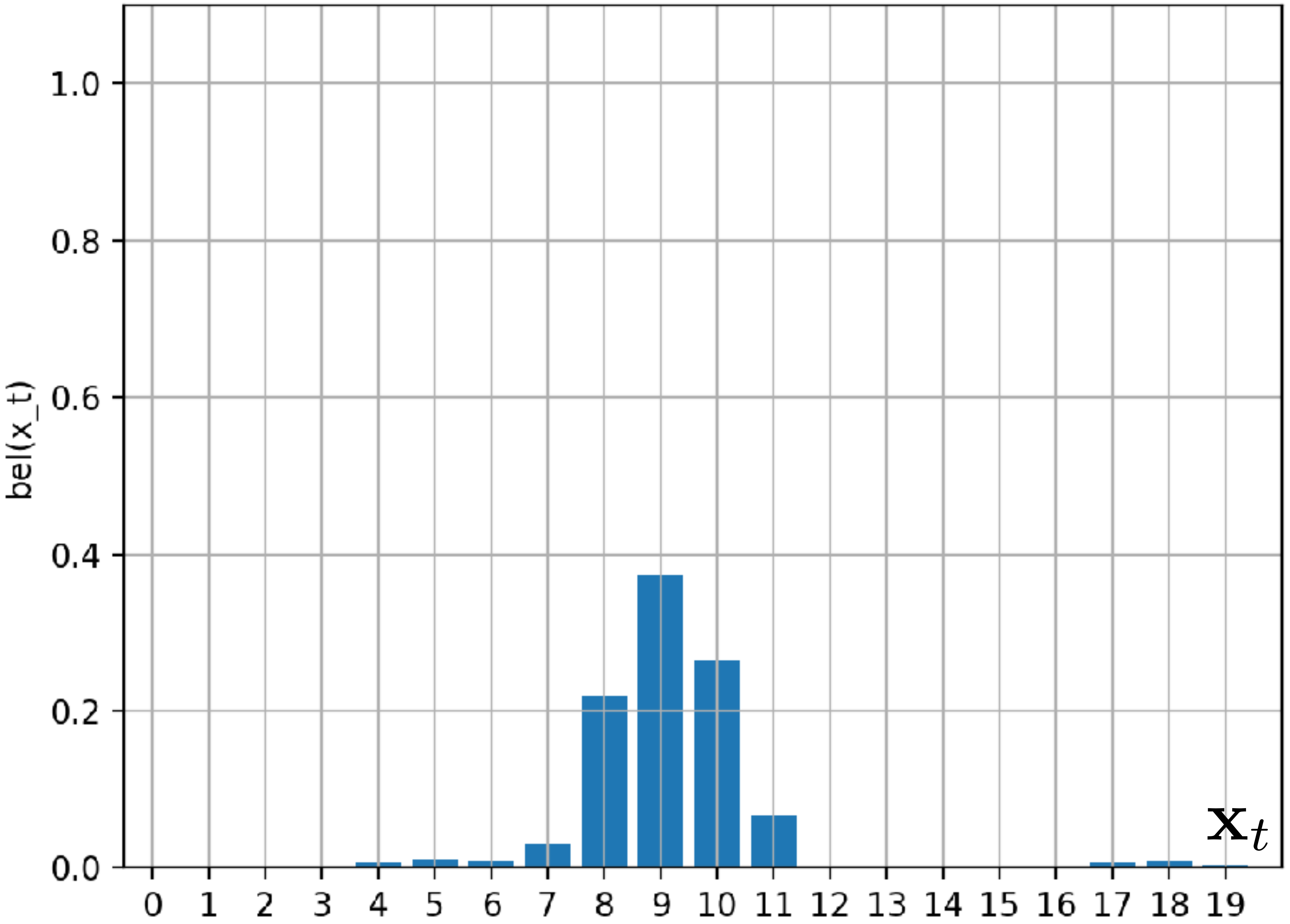
   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = \frac{\text{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$





Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \sum_{\mathbf{x}_{t-1}} p(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t \mid \mathbf{x}_t) \cdot \overline{\mathrm{bel}}(\mathbf{x}_t)$$

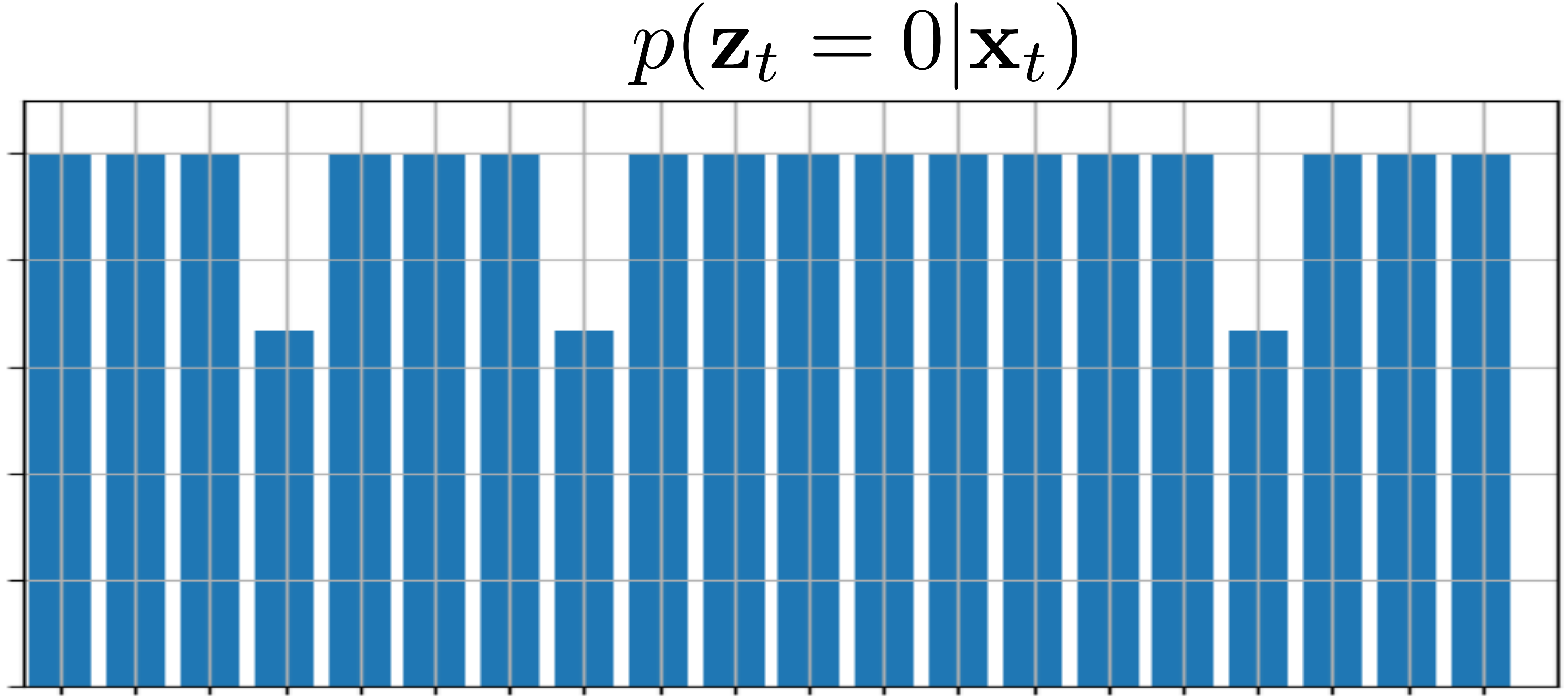
   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

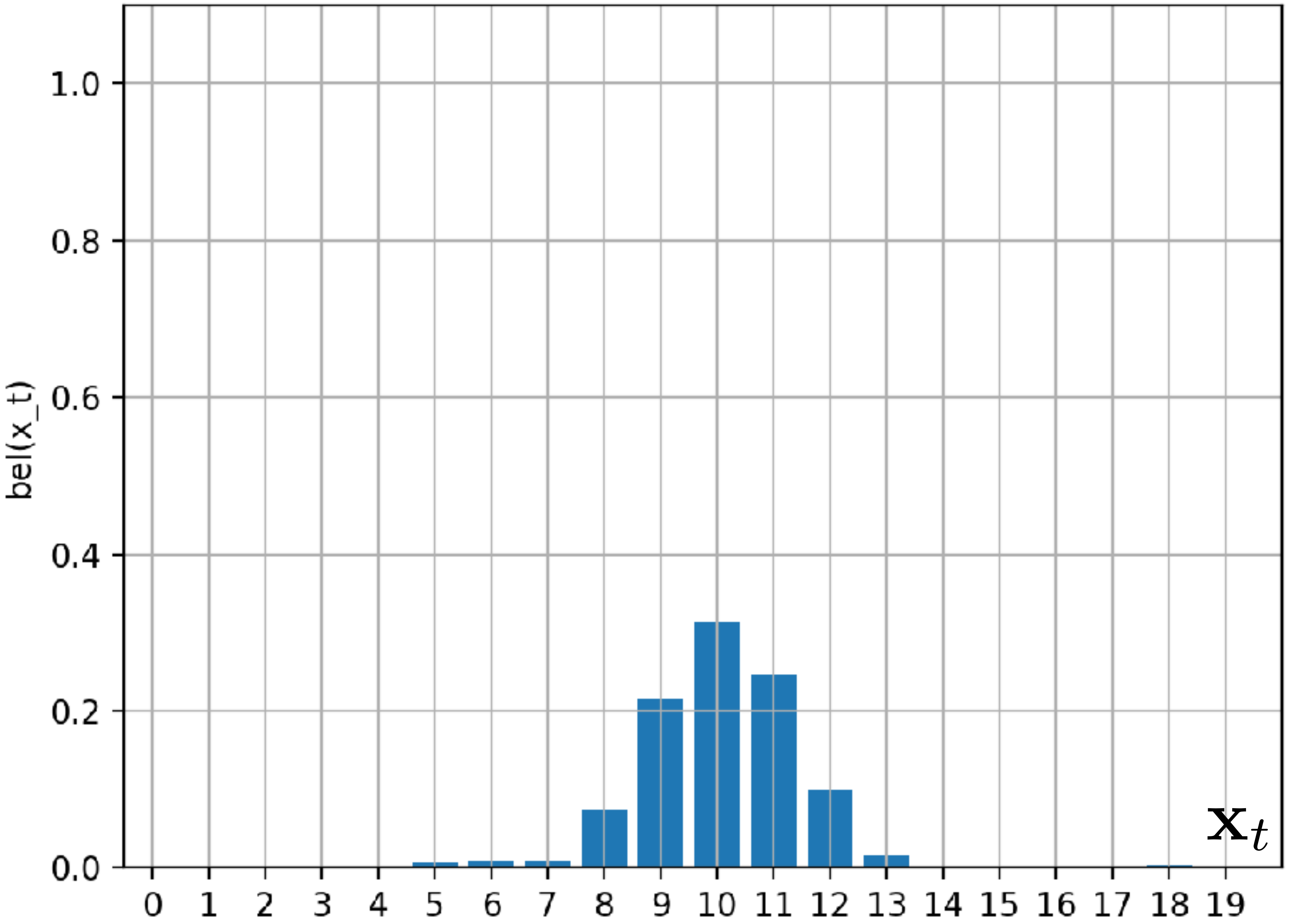
   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$





$$\text{bel}(\mathbf{x}_t) = \underbrace{\eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)}\cdot \underbrace{\int p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t)\cdot \text{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}}_{\overline{\text{bel}}(\mathbf{x}_t)}$$

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t)\text{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

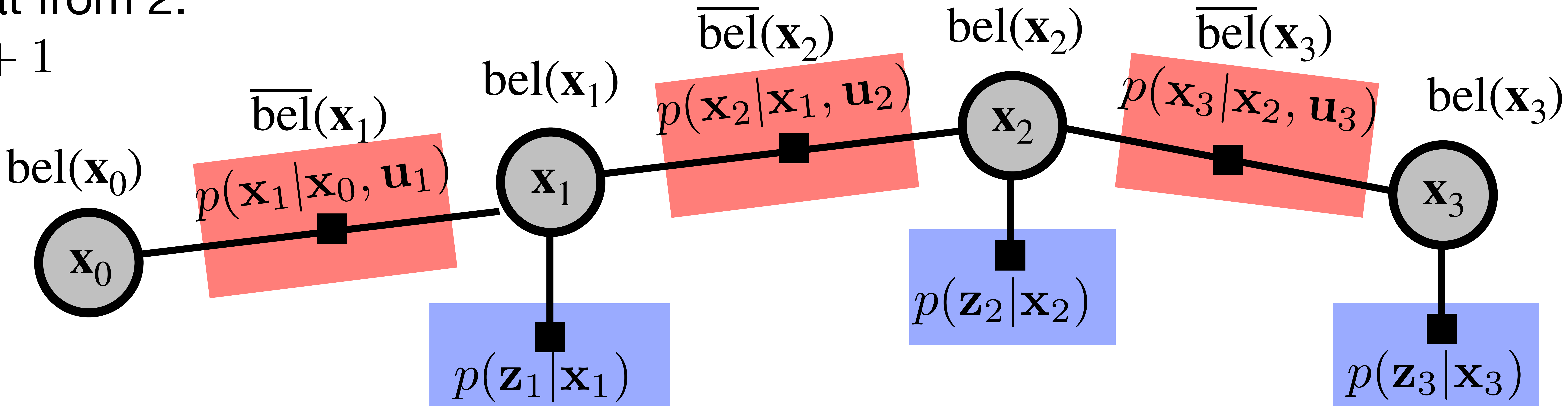
$$\text{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t)\overline{\text{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:
   $t = t + 1$

Is there any obvious limitation of dicrete prob. distribution?

Discrete probability distribution will suffer from curse of dimensionality

=> Let's return to Gaussians in continuous space



Kalman filter

Gaussian is preserved !!!

Linear system with Gaussian noise:

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}(\mathbf{A}_t\mathbf{x}_{t-1} + \mathbf{B}_t\mathbf{u}_t,\ \mathbf{R}_t)$$

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}(\mathbf{C}_t\mathbf{x}_t,\ \mathbf{Q}_t)$$

Bayes filter:

$$\mathrm{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \underbrace{\int p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) \cdot \mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}}_{\overline{\mathrm{bel}}(\mathbf{x}_t)}$$

Kalman filter:

Prediction step

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$

$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t\boldsymbol{\Sigma}_{t-1}\mathbf{A}_t^\top + \mathbf{R}_t$$

Measurement update step

$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top(\mathbf{C}_t\overline{\boldsymbol{\Sigma}}_t\mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$

$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t\overline{\boldsymbol{\mu}}_t)$$

$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{C}_t)\overline{\boldsymbol{\Sigma}}_t$$

Kalman filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t \boldsymbol{\mu}_{t-1} + \mathbf{B}_t \mathbf{u}_t$$

$$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t \boldsymbol{\Sigma}_{t-1} \mathbf{A}_t^\top + \mathbf{R}_t$$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$

3. Measurement update (new $\mathbf{z}_t$ received):

$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t \mathbf{C}_t^\top (\mathbf{C}_t \overline{\boldsymbol{\Sigma}}_t \mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$

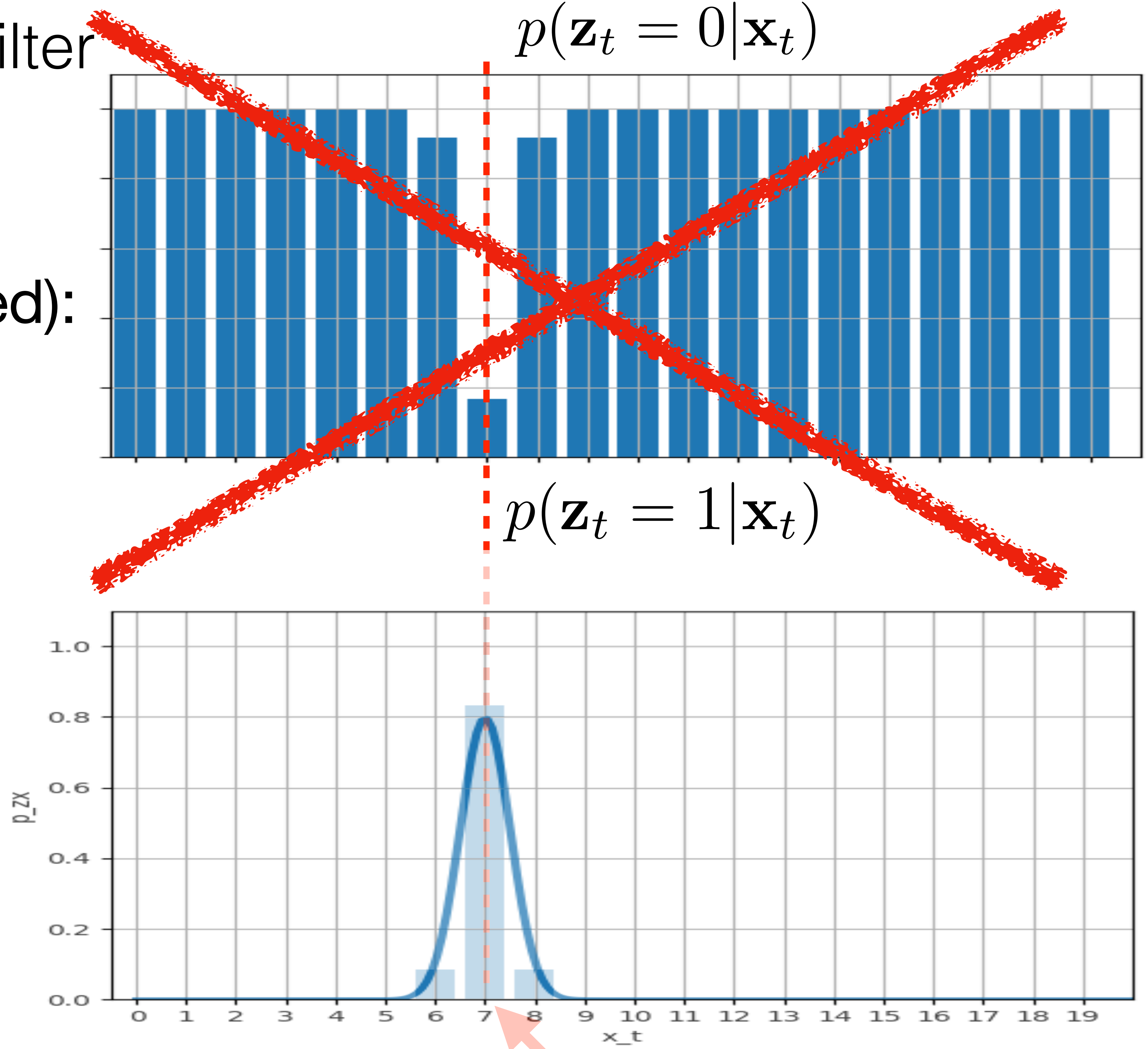
$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t \overline{\boldsymbol{\mu}}_t)$$

$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{C}_t) \overline{\boldsymbol{\Sigma}}_t$$

$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

4. Repeat from 2:

$$t = t + 1$$



one marker at known locations
+
inaccurate sensor

Kalman filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):
   $$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t \boldsymbol{\mu}_{t-1} + \mathbf{B}_t \mathbf{u}_t$$
   $$\overline{\boldsymbol{\Sigma}}_t = \mathbf{A}_t \boldsymbol{\Sigma}_{t-1} \mathbf{A}_t^\top + \mathbf{R}_t$$
   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$
3. Measurement update (new $\mathbf{z}_t$ received):
   $$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t \mathbf{C}_t^\top (\mathbf{C}_t \overline{\boldsymbol{\Sigma}}_t \mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$
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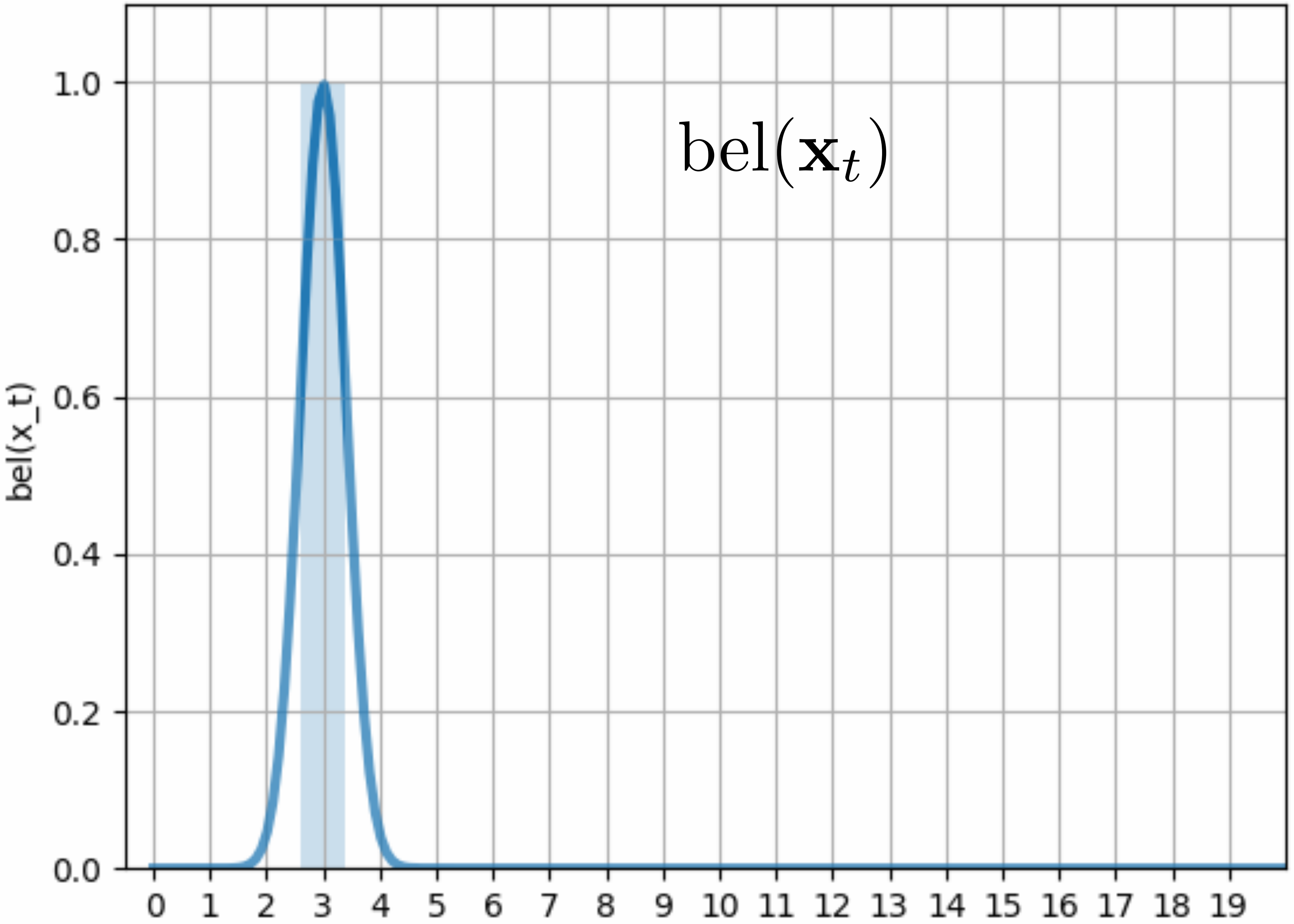

Kalman filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t \boldsymbol{\mu}_{t-1} + \mathbf{B}_t \mathbf{u}_t$$
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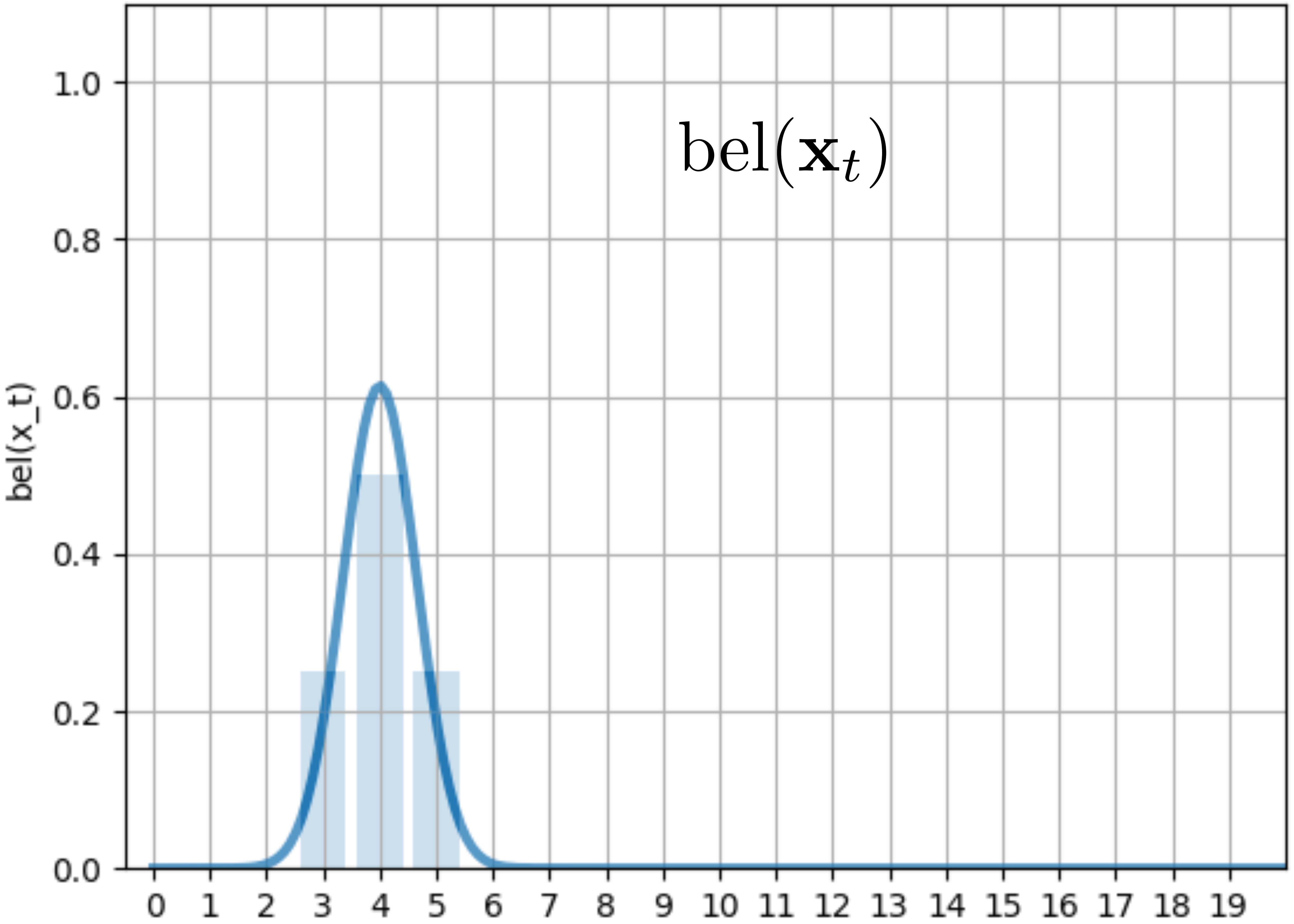

Kalman filter

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$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t \boldsymbol{\mu}_{t-1} + \mathbf{B}_t \mathbf{u}_t$$
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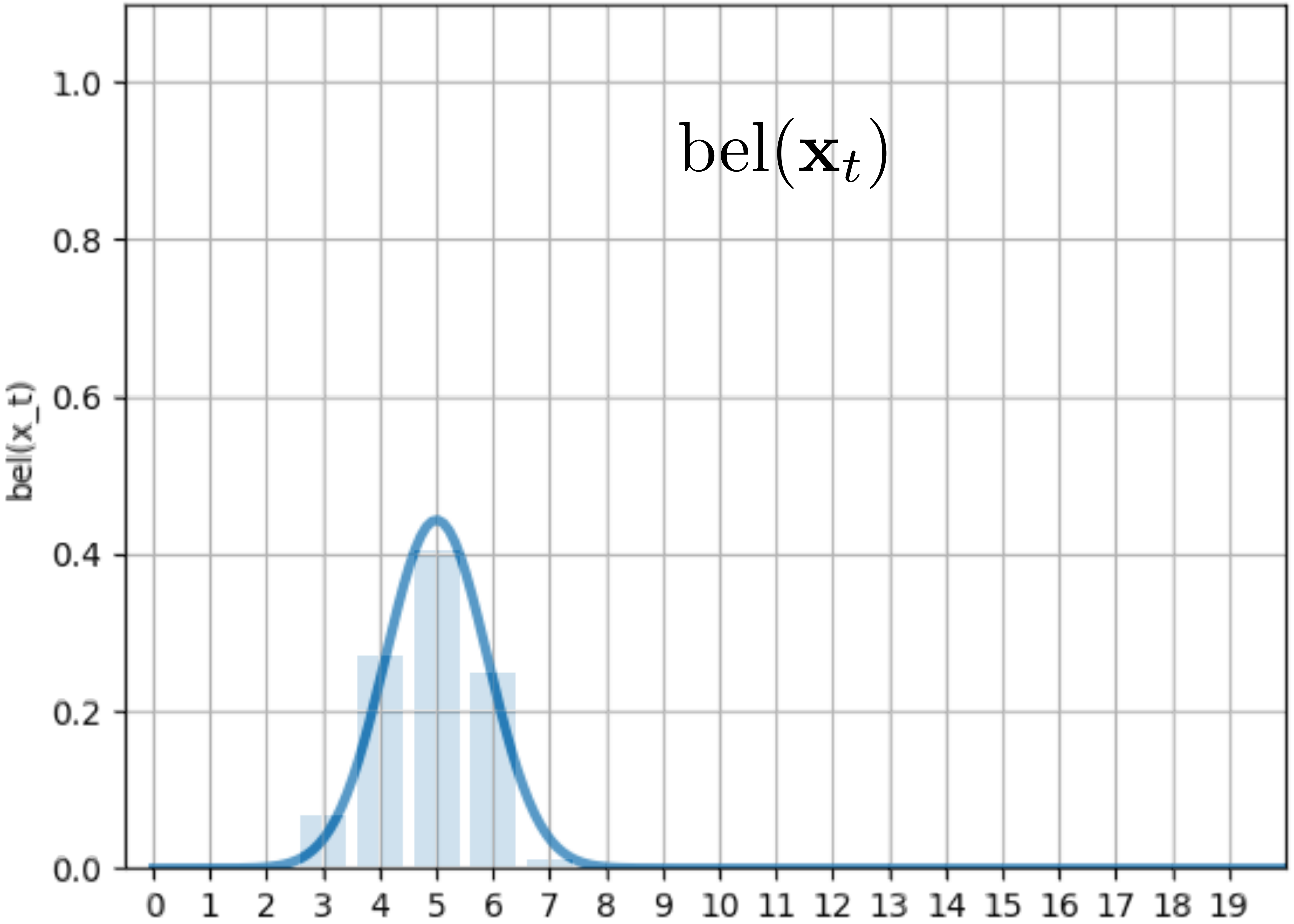

Kalman filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t \boldsymbol{\mu}_{t-1} + \mathbf{B}_t \mathbf{u}_t$$
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$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$

3. Measurement update (new $\mathbf{z}_t$ received):

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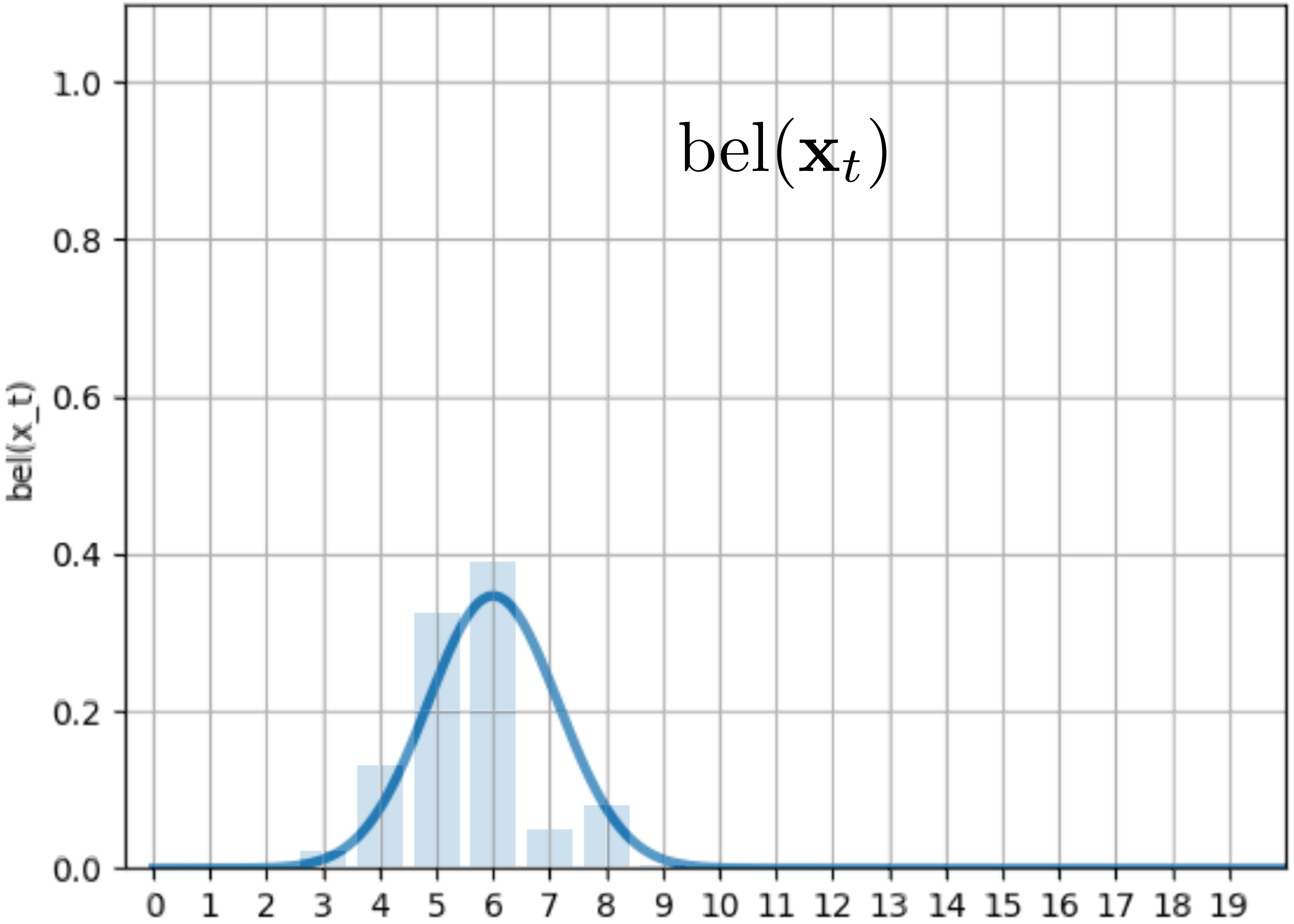

Kalman filter

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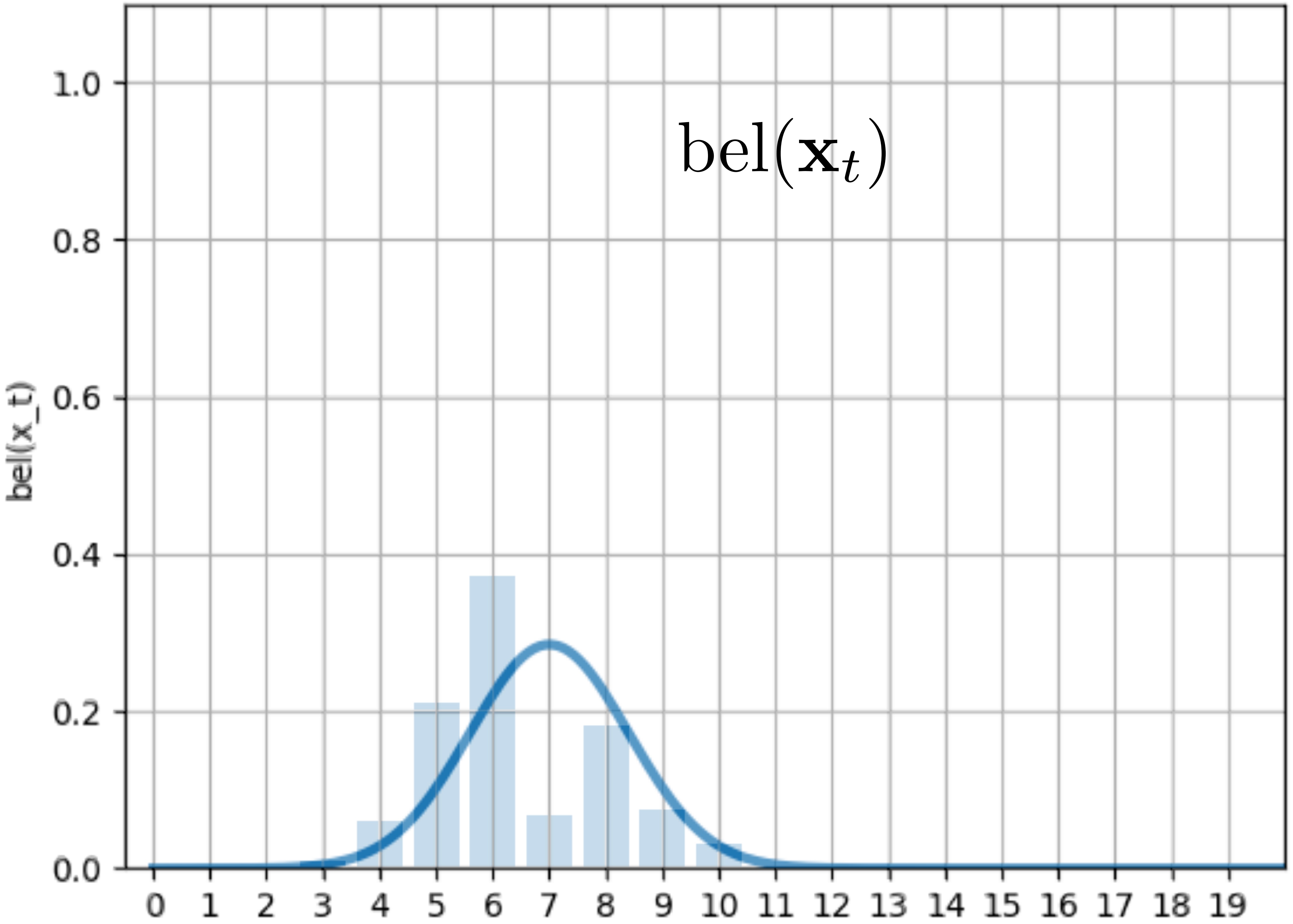

Kalman filter

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2. Prediction step (new action $\mathbf{u}_t$ performed):

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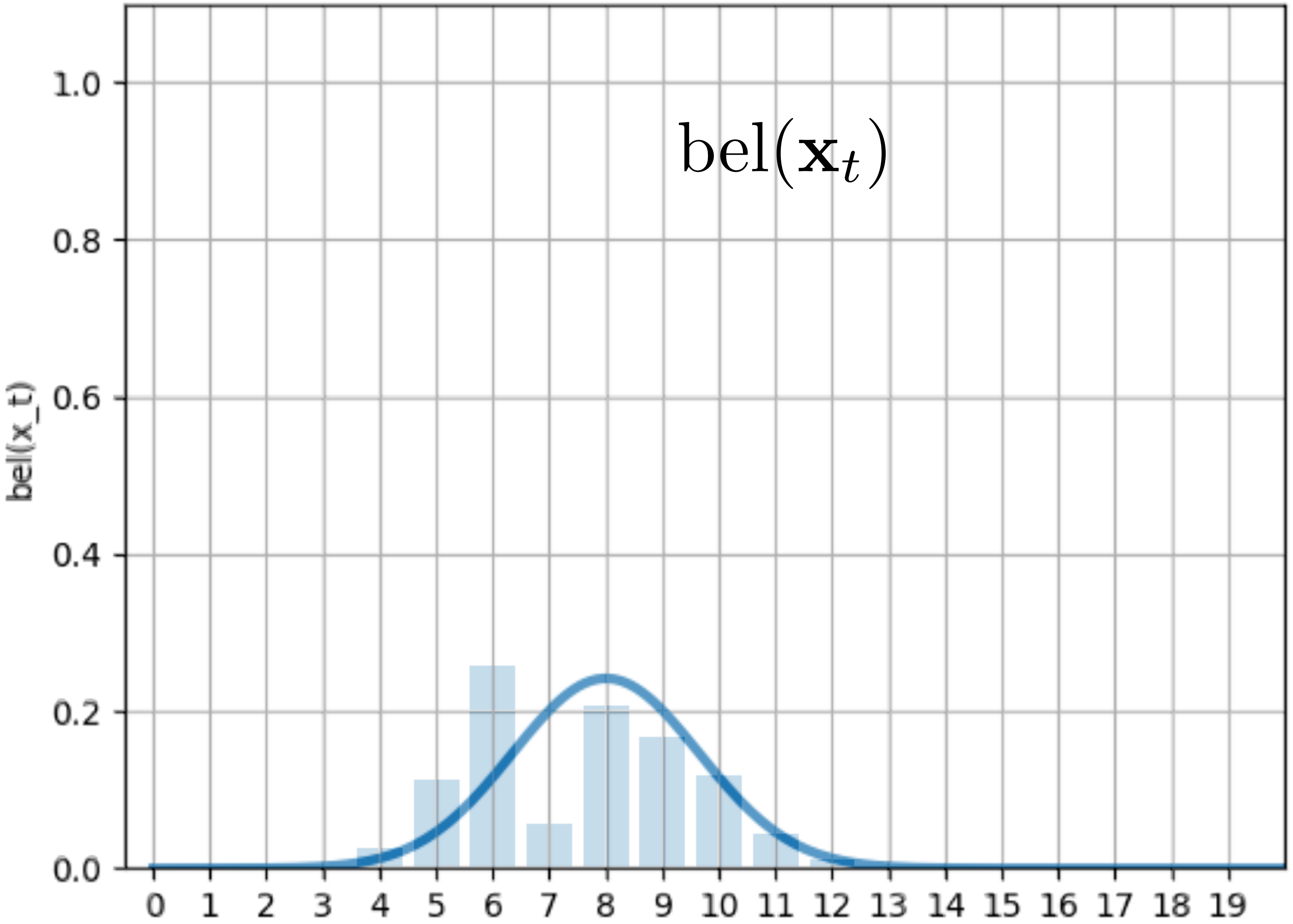

Kalman filter

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2. Prediction step (new action $\mathbf{u}_t$ performed):

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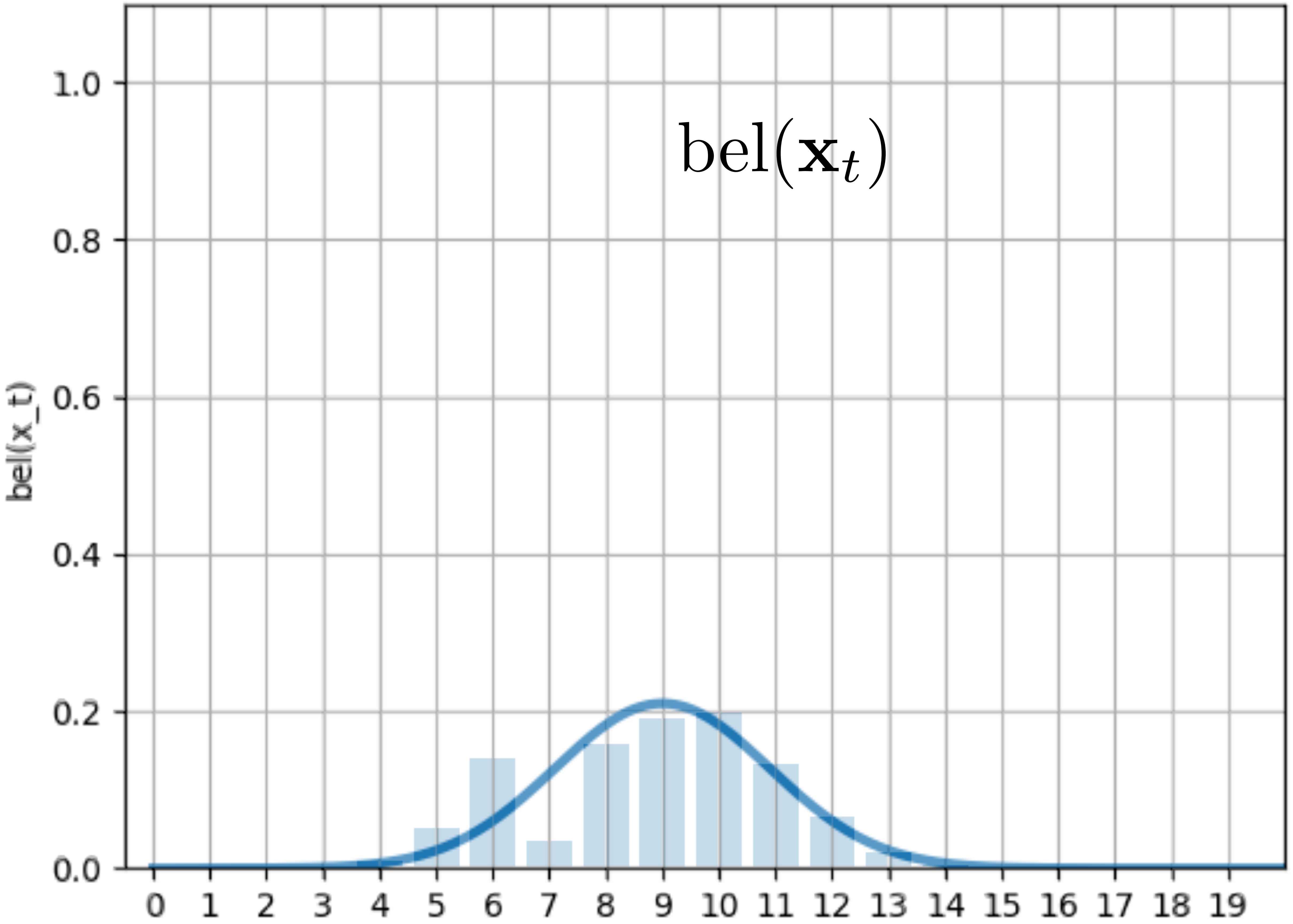
$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

4. Repeat from 2:

$$t = t + 1$$

Detection will come next!



Kalman filter

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2. Prediction step (new action $\mathbf{u}_t$ performed):

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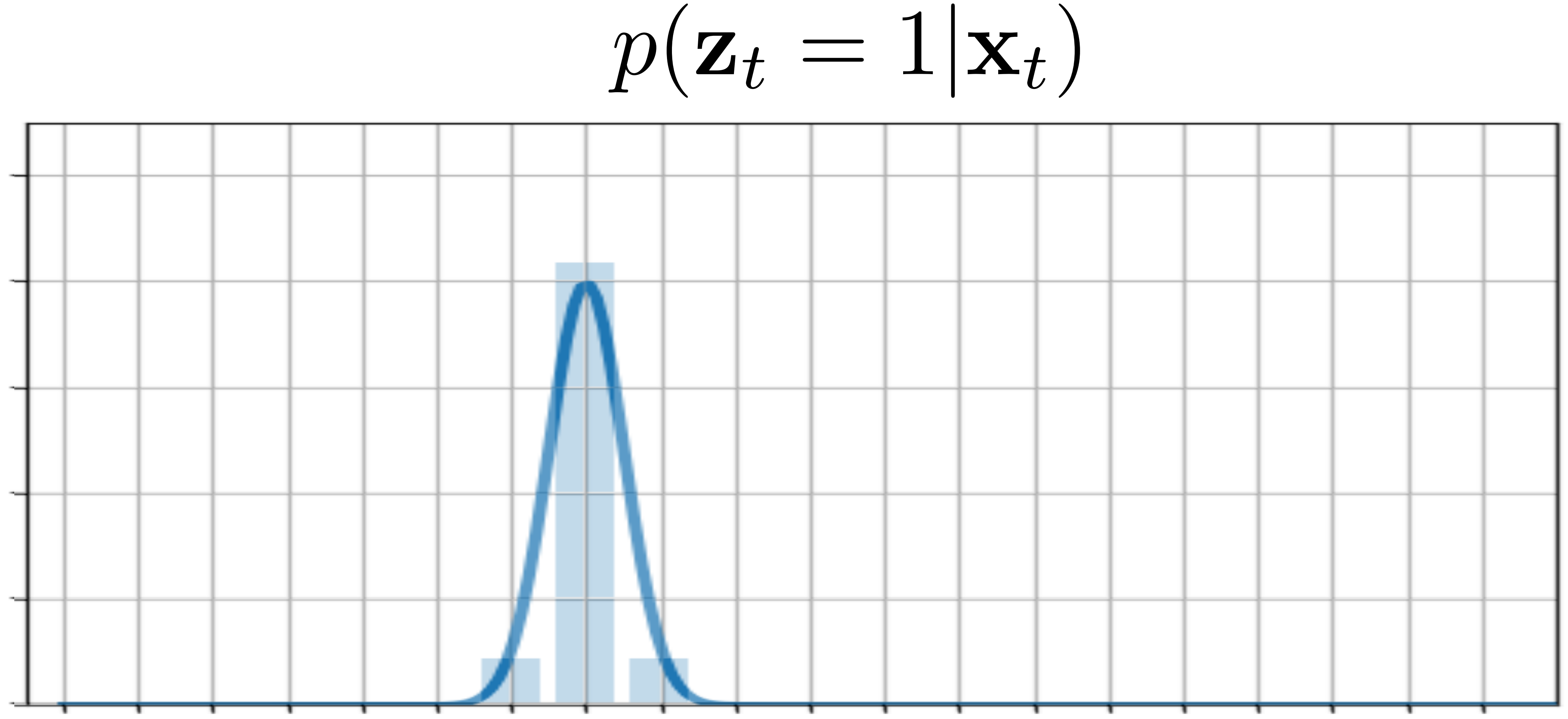
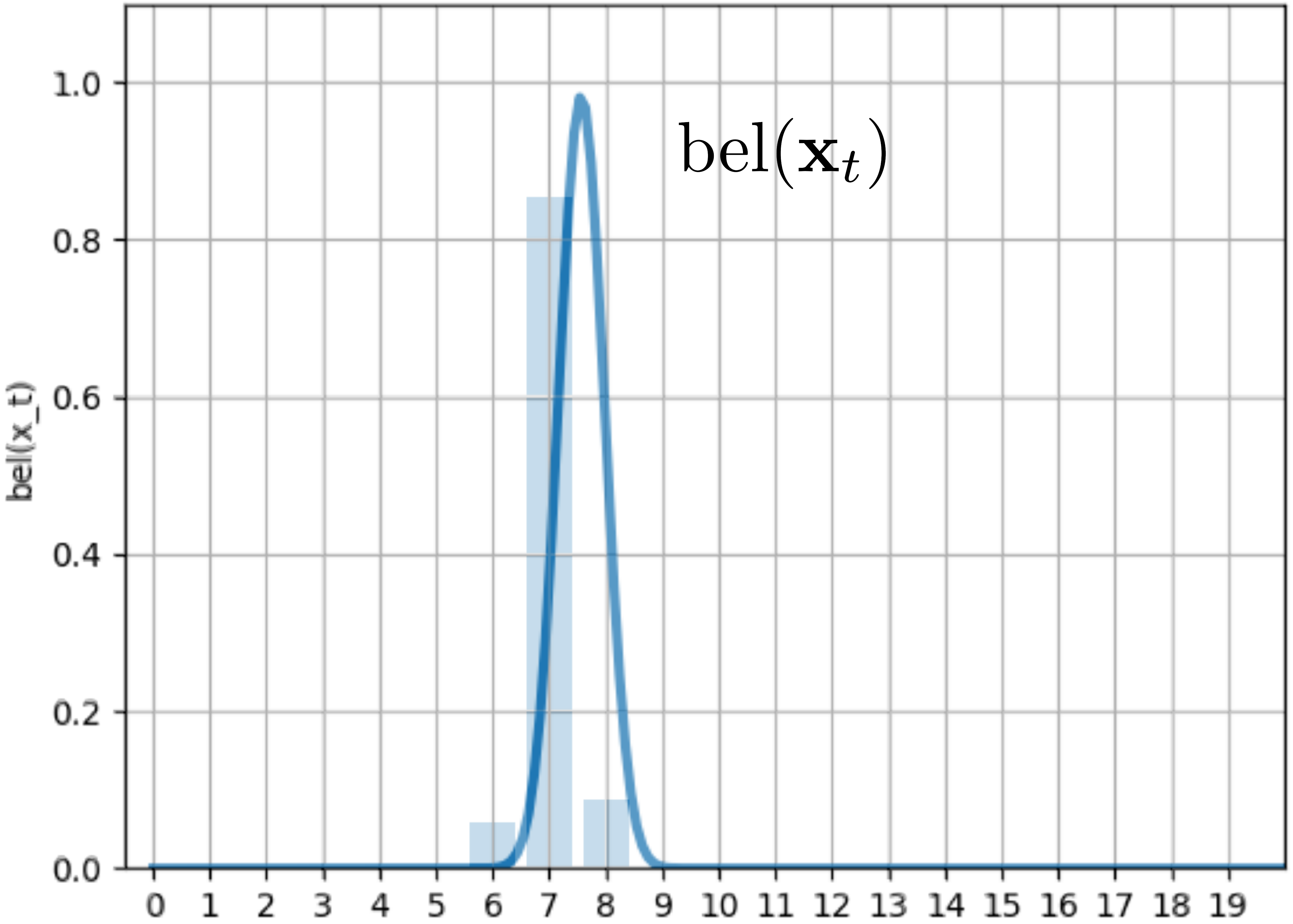
$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

4. Repeat from 2:

$$t = t + 1$$





Kalman filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

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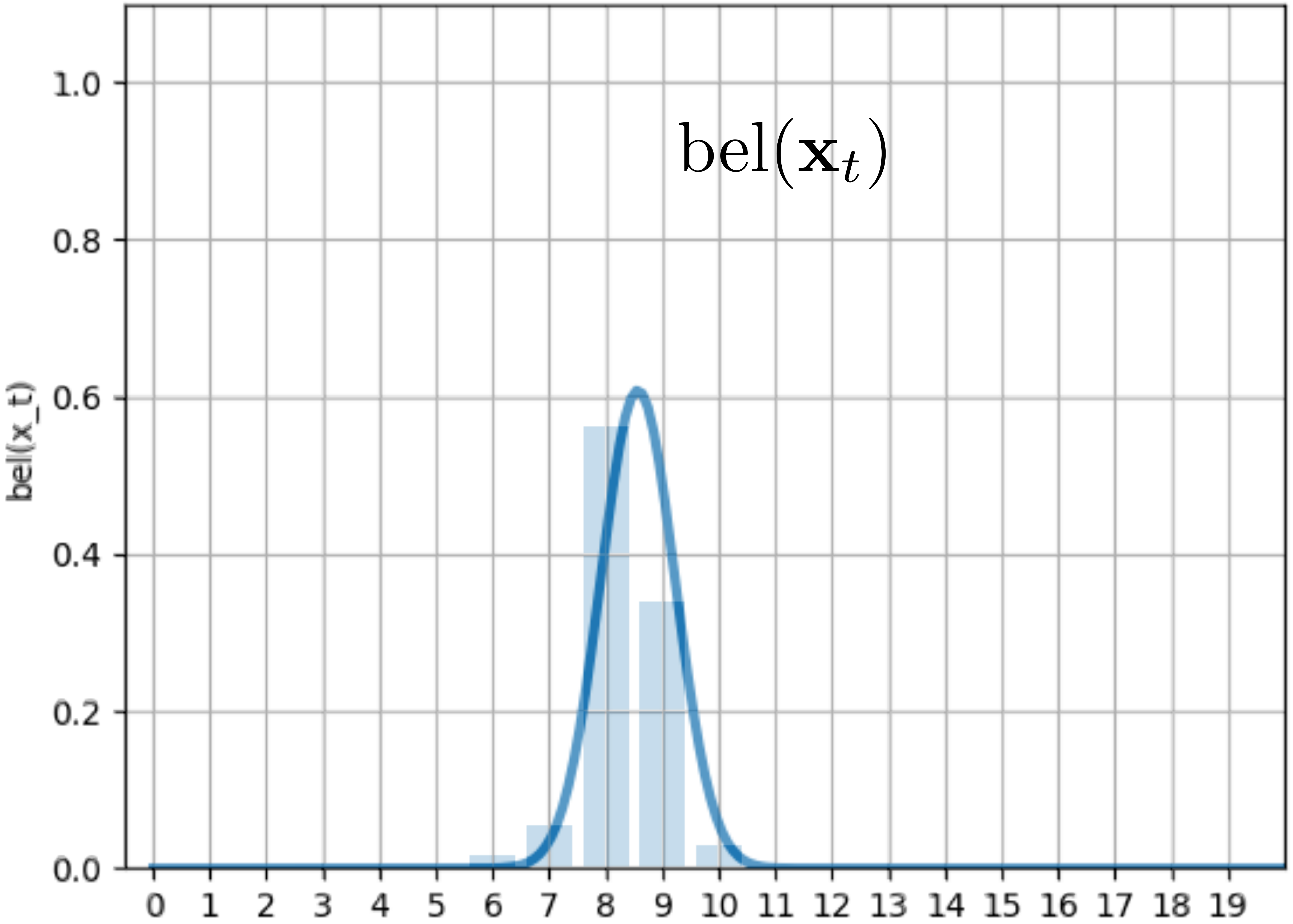

Kalman filter

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4. Repeat from 2:

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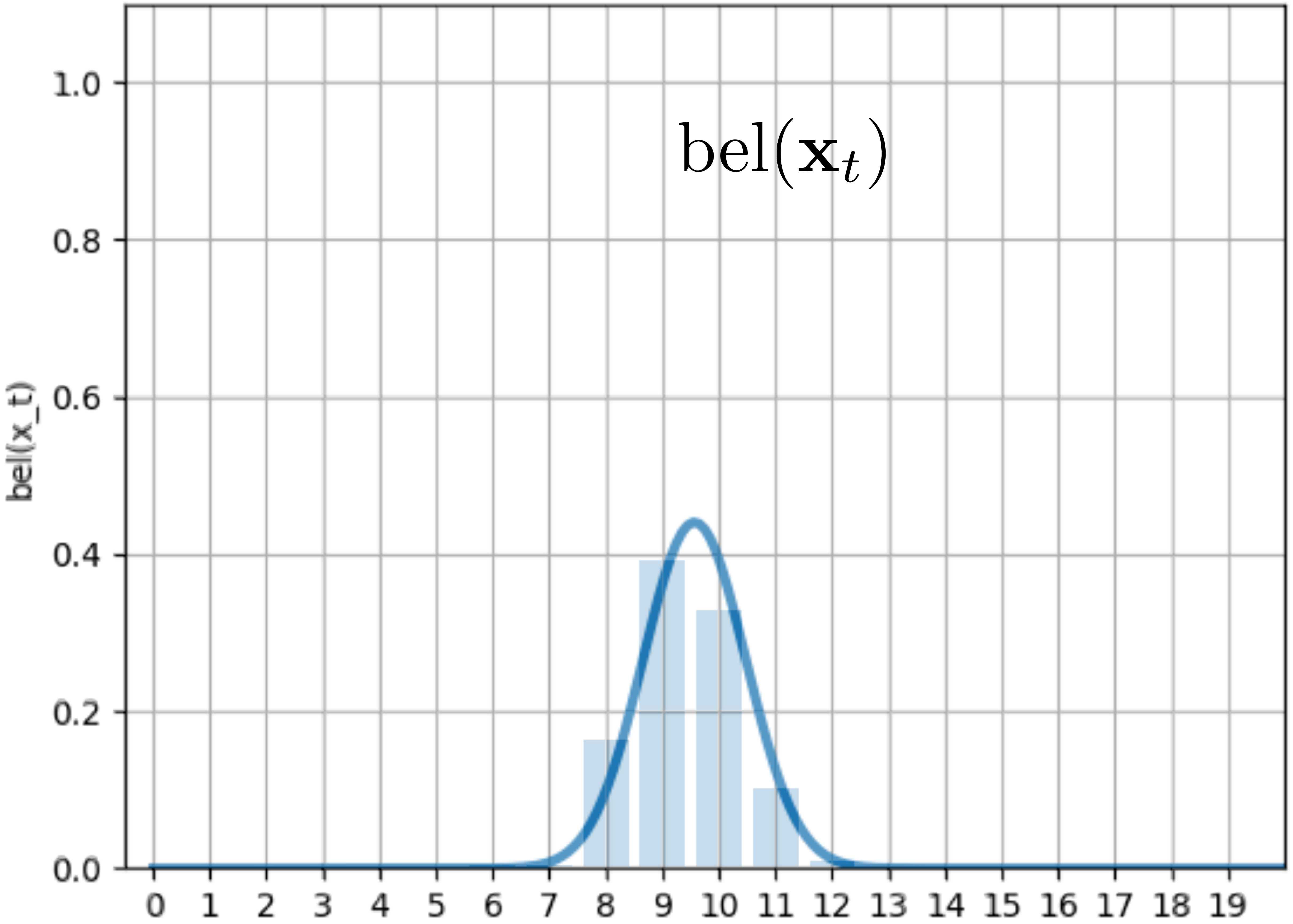

Kalman filter

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2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t \boldsymbol{\mu}_{t-1} + \mathbf{B}_t \mathbf{u}_t$$

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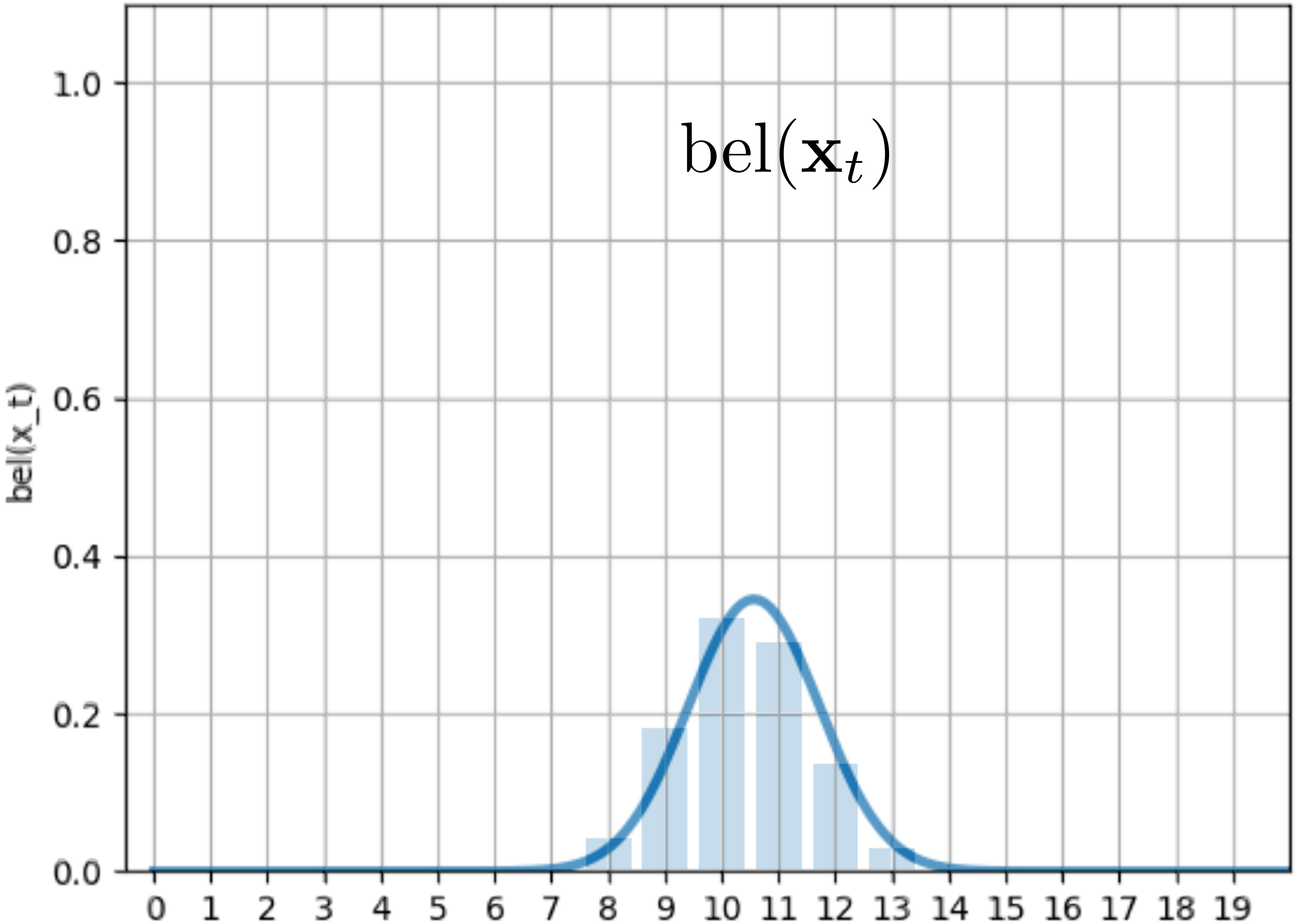

Kalman filter

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$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$

3. Measurement update (new $\mathbf{z}_t$ received):

$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t \mathbf{C}_t^\top (\mathbf{C}_t \overline{\boldsymbol{\Sigma}}_t \mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$

$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - \mathbf{C}_t \overline{\boldsymbol{\mu}}_t)$$

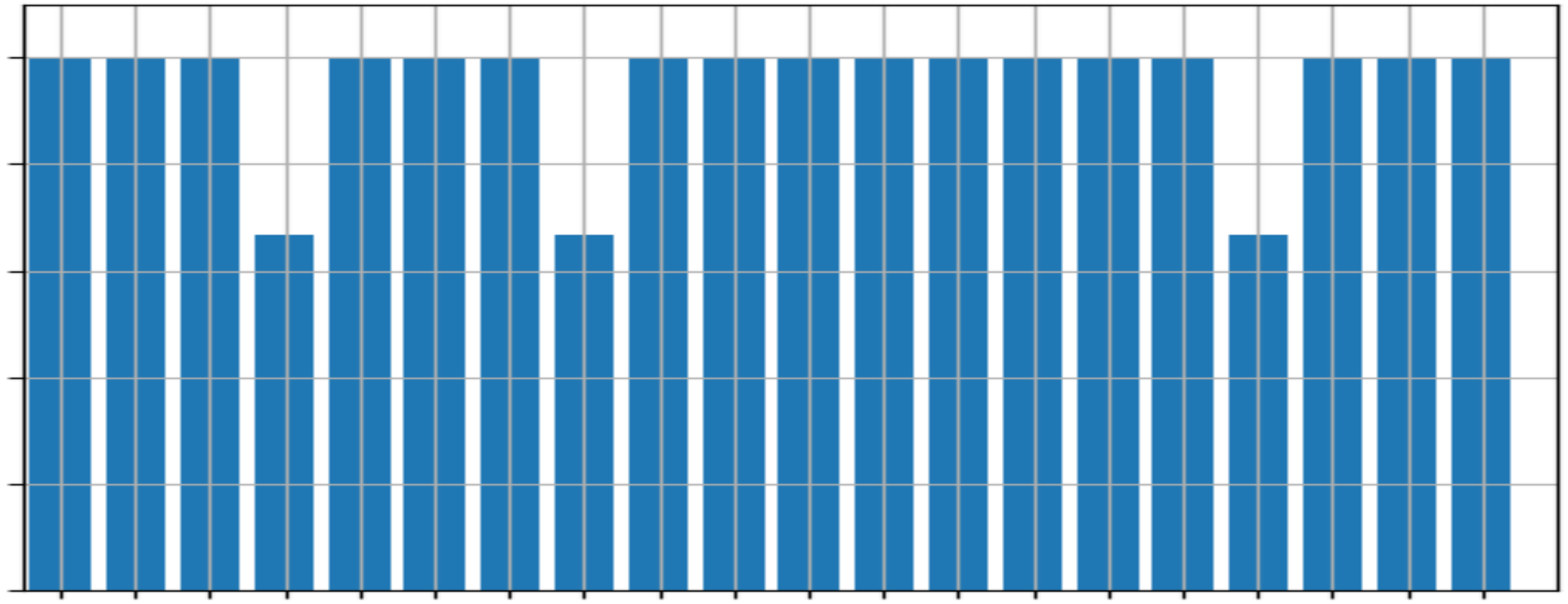
$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{C}_t) \overline{\boldsymbol{\Sigma}}_t$$

$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

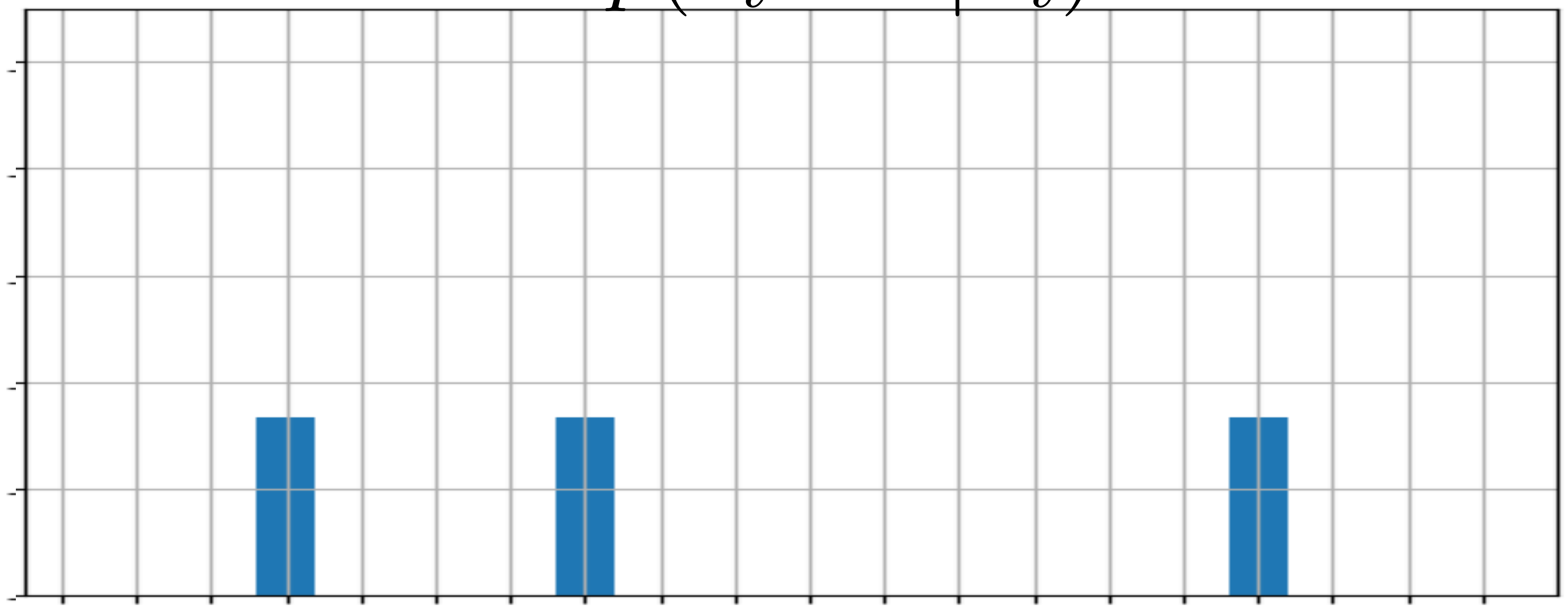
4. Repeat from 2:

$$t = t + 1$$

$p(\mathbf{z}_t = 0 | \mathbf{x}_t)$



$p(\mathbf{z}_t = 1 | \mathbf{x}_t)$



3 undistinguishable markers

Can we replicate the experiment with 3 markers???

Discrete Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

For all $\mathbf{x}_t$

$\overline{\mathrm{bel}}(\mathbf{x}_t) =$ … $\cdot\, \mathrm{bel}(\mathbf{x}_{t-1})$

3. Measure… w $\mathbf{z}_t$ received):

For all $\mathbf{x}$…

$\mathrm{bel}(\mathbf{x}_t)$ … $\mathrm{l}(\mathbf{x}_t)$

4. Repeat f…

$t = t + 1$

https://cs.wikipedia.org/wiki/Thomas_Bayes

Kalman filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$ $t = 1$
2. Prediction step:

$\overline{\boldsymbol{\mu}}_t$ …

$\overline{\boldsymbol{\Sigma}}_t$ …

3. M… :

$\mathbf{K}_t$ … $+ \mathbf{Q}_t)^{-1}$

$\boldsymbol{\mu}_t$ … $\overline{\boldsymbol{\mu}}_t)$

$\boldsymbol{\Sigma}_t$ …

https://cs.wikipedia.org/wiki/Rudolf_Emil_K%C3%A1lm%C3%A1n