Exam from ARO, May 26, 2022 **Name:** ____________________
Number of submitted sheets: ____________________

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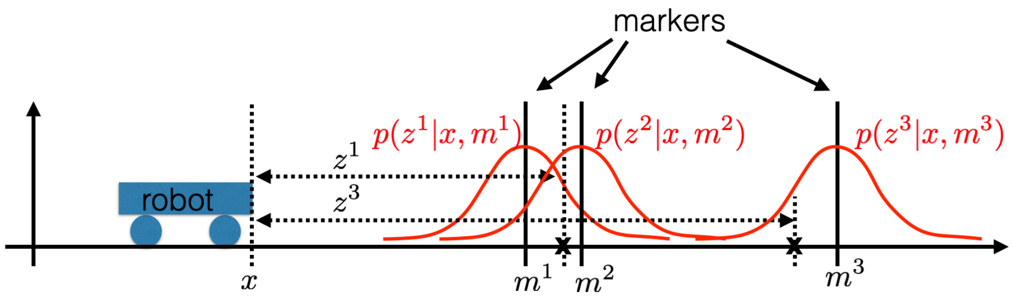
1. **Pose from known correspondences:** Consider the problem of 1D-robot localization from known marker positions. Robot measures the distance from the markers (robot is assumed to operate only on the left side of the markers as shown on the image). Markers are known to be located at positions $m^1 = 5, m^2 = 8, m^3 = 12$. The robot at position x detects only two markers: marker m^1 at distance $z^1 = 3$ and marker m^3 at distance $z^3 = 5$ (marker m^2 is not detected). Each measurement is assumed to have zero-mean Gaussian noise with the same variance σ^2 and measurements are assumed to be independent on each other.



a) What is the probability distribution from which the first measurement z^1 has been generated (feel free to use normalizing constant K instead of $1/\sigma\sqrt{2\pi}$)? Substitute all known variables.

$p(z^1|x, m^1) =$

b) Derive what is the maximum likelihood estimate of the robot pose x?

$x =$

c) Assume that we have replaced the sensor by a new one. The new sensor provided the same measurements ($z^1 = 3, z^3 = 5$), however, we know that the measurement noise n comes from Laplace distribution $\frac{1}{2}\exp(-|n|)$ (instead of the normal). What is the **set** of all maximum likelihood estimates of its pose?

$x \in$

2. **Pose from unknown correspondences:** Let us assume that everything is the same as in the previous example: sensor has Gaussian noise, 3 markers are at the same locations. The only difference is that correspondences are unknown, i.e. you measured the two distances (3m and 5m) from markers but the markers are indistinguishable from each other, therefore you need to figure out also the correspondences between markers and measurements.

   a) What is the maximum likelihood estimate of its pose?
   Hint: If you have troubles to guess the globally-optimal correspondences, feel-free to use the ICP-like gradient minimization initialized in pose $x = 2$.

   $x =$

3. **Complete state:** Robot (manipulator) is supposed to build a tower from construction blocks. There are two types of blocks: smaller cube (e.g. $1 \times 1 \times 1$ cm) and larger cubes (e.g. $2 \times 2 \times 2$ cm). Robot can perform three types of actions: (i) pull out a cube from the box (the result is random), (ii) through away the selected cube, (iii) put the cube on top of the tower. The outcome of this actions is always successful. It is not allowed to put a larger cube on a smaller cube or demolish the existing tower. Robot can observe only the type of selected cube (small or large). The algorithm maximizes the height of the resulting tower. You know that the box contains 5 smaller cubes, and 5 larger cubes.

   a) Suggest a low dimensional representation of the state that is complete. Make the answer as compact as possible (rewriting any definition from slides has no influence on the number of points you get from the answer).

   b) How does the usage of complete state simplify the state estimation? (Answer is 1 sentence).

4. **Extended Kalman filter:** Let us assume a robot operating on 1-dimensional trolley that lies in 2-dimensional plane. Its state $\mathbf{x}_t = [x_t, y_t]^\top$ in time t is described by its x, y-coordinates in the plane. Its state-transition probability is non-linear system with Gaussian noise:

$$p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}(g(\mathbf{x}_{t-1}, \mathbf{u}_t),\ \mathbf{R}_t),$$

where

$$g(\mathbf{x}_{t-1}, \mathbf{u}_t) = \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} x_{t-1} + u_t \\ 1 - (x_{t-1} + u_t)^2 \end{bmatrix}$$

and $\mathbf{R}_t$ is identity matrix.

a) Linearize the system around the mean $\mu_{t-1} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$ of its belief from time t-1.

Hint: The linearization is $g(\mathbf{x}_{t-1}, \mathbf{u}_t) \approx g(\boldsymbol{\mu}_{t-1}, \mathbf{u}_t) + \mathbf{G}_t(\mathbf{x}_{t-1} - \boldsymbol{\mu}_{t-1})$, where $\mathbf{G}_t = \frac{\partial g(\mathbf{x}=\boldsymbol{\mu}_{t-1}, \mathbf{u}=\mathbf{u}_t)}{\partial \mathbf{x}}$ is Jacobian of g in point μ_{t-1}.

$\mathbf{G}_t =$

$g(\mathbf{x}_{t-1}, \mathbf{u}_t) \approx$

b) What kind of probability distribution is the belief $\overline{\mathrm{bel}}(\mathbf{x}_t)$ after the prediction step of the EKF (i.e. with the linearized model)? Justify your answer by 1 sentence.

c) Assume that no measurements are provided, and robot keeps moving forever. What happens with the EKF-estimated belief?

5. **Project 3D point to camera:** Consider a perspective camera with the following intrinsic camera parameters K, camera rotation matrix R that corresponds to the rotation of 90° around x-axis and 0° around y-axis and z-axis, and translation vector t:

$$\mathbf{K} = \begin{bmatrix} 300 & 0 & 300 \\ 0 & 300 & 150 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{t} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix},$$

a) Project point

$$\mathbf{q} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

into the camera. What are pixel coordinates $\mathbf{u} \in \mathcal{R}^2$ of the projection?

$u_1 =$

$u_2 =$

b) You are given 10 2D-3D correspondences $(\mathbf{u}_i, \mathbf{q}_i), : i = 1 \ldots 10$. Consider camera calibration problem as the solution of the following constrained least squares problem:

$$\arg\min_{\mathbf{p}} \{ \|\mathbf{A}\mathbf{p}\| \mid \mathbf{p}^\top \mathbf{p} = 1 \}$$

What is the dimensionality $(rows \times cols)$ of matrix $\mathbf{A}$?

6. How are defined obstacles in the configuration space $\mathcal{C}$? Let $A(q)$ is the robot geometry at configuration $q \in \mathcal{C}$, $\mathcal{O}$ are the obstacles in workspace.

   - $\mathcal{C}_{obs} = \{q \in \mathcal{C} | A(q) \cap \mathcal{O} \neq \emptyset\}$
   - $\mathcal{C}_{obs} = \{q \in \mathcal{C} | A(q) \cap \mathcal{O} = \emptyset\}$
   - $\mathcal{C}_{obs} = \mathcal{O}$
   - $\mathcal{C}_{obs} = \{q \in \mathcal{C} | A(q) \cup \mathcal{O} \neq \emptyset\}$

7. Let's assume a 3D solid object that can freely move in a 3D workspace. How many dimensions has the corresponding configuration space?

8. Describe what is completeness of path planning algorithms.

9. Describe what is probabilistic completeness of path planning algorithms.

10. Let's assume a point robot moving in a 2D map with obstacles. Which **complete** path planning methods are suitable for this scenario?

11. Describe (using text/sketch/pseudocode .. what you prefer) how rewiring in RRT* works. If the procedure consists of multiple steps, make clear what is their order.

12. Describe (using text/sketch/pseudocode ... what you prefer) how planning using Visibility Graph works. If the method consists of multiple steps, make clear what is their order. What are pros/cons of the method? For what systems (robots) is the method suitable? What kind of map/obstacle/robot geometry representation is required for it? What is the time complexity of the method?

13. What is "clearance" of a path? Which planners provide paths with the best clearance?

14. Describe (using text/sketch/pseudocode ... what you prefer) how planning via potential field works. If the method consists of multiple steps, make clear what is their order. What are pros/cons of the method?

15. What is the narrow passage problem? Which planners are affected by it? What the narrow passage problem practically means for planning?

16. Describe (using text/sketch/pseudocode . . . what you prefer) how planning via Rapidly-exploring Random Tree works. If the method consists of multiple steps, make clear what is their order. What are pros/cons of the method?

17. What distribution of random samples is used in the basic sampling-based planners? In which space are the samples generated?

18. What is the time complexity of finding the nearest neighbor to a query point if we have n points stored in KD-tree?

19. What is the difference between 'collision detection' and 'collision determination'?

20. Describe what is OBB (Oriented Bounding box) and AABB (Axis Aligned Bounding Box). How are OBB/AABB used in path planning? Which planners or supporting algorithms are using OBB or AABB?