# Quicksilver: fast deep learning registration 

 Yang et al, Neuroimage 2017Jan Kybic

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## Key features

- Deep learning-based
- Prediction/correction network
- Fast (11s on 1GPU for a 3D volume)
- Diffeomorphic transformation
- Large deformations (large deformation diffeomorphic metric mapping - LDDMM)
- Patch-based, patch pruning
- Uncertainty quantification
- Multimodal registration


## Formulation

$$
E(\Phi)=\operatorname{Reg}[\Phi]+\frac{1}{\sigma^{2}} \operatorname{Sim}\left[I_{0} \circ \Phi^{-1}, I_{1}\right] .
$$

LDDMM is a non-parametric registration method which represents the transformation via spatio-temporal velocity fields. In particular, the sought-for mapping, $\Phi$, is obtained via an integration of a spatio-temporal velocity field $v(x, t)$ for unit time, where $t$ indicates time and $t \in[0,1]$, such that $\Phi_{t}(x, t)=v(\Phi(x, t), t)$ and the sought-for mapping is $\Phi(x, 1)$. To single-out desirable velocity-fields, non-

$$
\begin{aligned}
E(v)= & \int_{0}^{1}\|v\|_{L}^{2} d t+\frac{1}{\sigma^{2}}\left\|M \circ \Phi^{-1}(1)-T\right\|^{2} \\
& \text { s.t. } \Phi_{t}(x, t)=v(\Phi(x, t), t), \Phi(x, 0)=\mathrm{id}
\end{aligned}
$$

Differential formulation $\Phi_{t}^{-1}+D \Phi^{-1} v=0$.

## Classical solution

- forward transformation - follow a particle in v. Ensures diffeomorphy
- optimization
- current mismatch
- solve (adjoint) system backward
- gradient of the velocity field at all $t$
- update $v$


## Shooting formulation

- find the shortest path (geodesics) between images
- geodesic parameterized by initial $\Phi^{-1}$ and momentum $m=L v$
- $m$ supported mainly on image edges, $m(x, t)=\lambda(x, t) \nabla I(x, t)$
- $v$ is a smoothed momentum, $v=L^{-1} m$


## Method

- predict $m$ patch-by-patch
- train network to predict $m$
- training data - $m$ found by numerical optimization
- $m$ well predicted from patches, does not have to be smooth, zero in homogeneous regions

- large stride, drop background patches


## Network structure


encoder/decoder, $I_{1}$ loss function on $m$ (not $E$ ),
3 decoders (easier to train)

## Probabilistic network

- Instead of $\boldsymbol{y}=f(\boldsymbol{x})$, predict $p(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{X}, \boldsymbol{Y})$ for training data $\boldsymbol{X}, \boldsymbol{Y}$
- variational inference for network weights $\boldsymbol{W}$,minimize KL divergence of $q(\boldsymbol{W})$ and $p(\boldsymbol{W} \mid \boldsymbol{X}, \boldsymbol{Y})$

$$
q\left(\mathbf{W}_{i}\right)=\mathbf{M}_{i} \cdot \operatorname{diag}\left(\left[z_{i, j}\right]_{j=1}^{K_{i}}\right), \quad z_{i, j} \sim \operatorname{Bernoulli}(d)
$$

- $\rightarrow$ dropout with probability 0.2

$$
p\left(\mathbf{y}^{\prime} \mid \mathbf{x}^{\prime}, \mathbf{X}, \mathbf{Y}\right) \approx \frac{1}{T} \sum_{t=1}^{T} \hat{f}\left(\mathbf{x}^{\prime}, \hat{\mathbf{w}}\right)
$$

- result=mean, variance $\rightarrow$ uncertainty estimate


## Patch number reduction

- skip all background patches
- large voxel stride ( 14 for $15 \times 15 \times 15$ )


## Prediction/correction



- trained sequentially
- Mand $T \circ \Phi$ are in the same coordinate space, can be added


## Datasets

- T1, T2 MR images
- training $m$ obtained from T1 images $\rightarrow$ learn also multimodal T1-T2 registration


## Atlas-to-image example


blue - low uncertainty

## Prediction/correction experiments



## Quantitative results

|  | Deformation Error for each voxel $[\mathrm{mm}]$ |  |  |  |  |  |  | $\operatorname{det} \boldsymbol{J}>\mathbf{0}$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Data percentile for all voxels | $0.3 \%$ | $5 \%$ | $25 \%$ | $50 \%$ | $75 \%$ | $95 \%$ | $99.7 \%$ |  |
| Affine | 0.0613 | 0.2520 | 0.6896 | 1.1911 | 1.8743 | 3.1413 | 5.3661 | $\mathrm{~N} / \mathrm{A}$ |
| D, velocity, stride 5 | 0.0237 | 0.0709 | 0.1601 | 0.2626 | 0.4117 | 0.7336 | 1.5166 | $100 \%$ |
| D, velocity, stride 14 | 0.0254 | 0.075 | 0.1675 | 0.2703 | 0.415 | 0.743 | 1.5598 | $100 \%$ |
| D, deformation, stride 5 | 0.0223 | 0.0665 | 0.1549 | 0.2614 | 0.4119 | 0.7388 | 1.5845 | $56 \%$ |
| D, deformation, stride 14 | 0.0242 | 0.0721 | 0.1671 | 0.2772 | 0.4337 | 0.7932 | 1.6805 | $0 \%$ |
| P, momentum, stride 14, 50 samples | 0.0166 | 0.0479 | 0.1054 | 0.1678 | 0.2546 | 0.4537 | 1.1049 | $100 \%$ |
| D, momentum, stride 5 | 0.0129 | 0.0376 | 0.0884 | 0.1534 | 0.2506 | 0.4716 | 1.1095 | $100 \%$ |
| D, momentum, stride 14 | 0.013 | 0.0372 | 0.0834 | 0.1359 | 0.2112 | 0.3902 | 0.9433 | $100 \%$ |
| D, momentum, stride 14, 40 epochs | 0.0119 | 0.0351 | 0.0793 | 0.1309 | 0.2070 | 0.3924 | 0.9542 | $100 \%$ |
| D, momentum, stride 14 + correction | $\mathbf{0 . 0 1 0 4}$ | $\mathbf{0 . 0 3 0 9}$ | $\mathbf{0 . 0 7 0 4}$ | $\mathbf{0 . 1 1 6 7}$ | $\mathbf{0 . 1 8 5}$ | $\mathbf{0 . 3 4 7 8}$ | $\mathbf{0 . 8 4 1}$ | $100 \%$ |

## Target overlap



## Multimodal registration



Figure 10: Example test case for multi-modal image-to-image tests. (a): T1w moving image; (b): T2w target image; (c): T1w-T1w LDDMM optimization (L0) result; (d)-(f): deformation prediction+correction (LPC) result using (d) T1w-T1w data; (e) T1w-T2w data; (f) T1w-T2w data using only 10 images as training data.

