Diffeomorphic registration

Jan Kybic

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Avants et al: Symmetric Diffeomorphic Image Registration with Cross-Correlation... MIA 2008

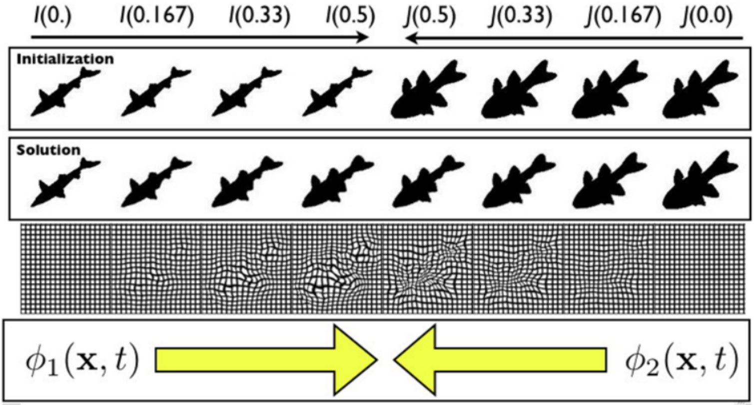
Symmetric image normalization (SyN)

- Dense deformation field
- Large deformation
- Symmetric formulation
- Diffeomorphic = invertible and differentiable with differentiable inverse
- Computationally expensive
- Implementation available

Deformation field and velocity

- Deformation
  $$\varphi I = I \circ \varphi(\mathbf{x}, t = 1).$$
- Velocity
  $$\phi(\mathbf{x},1)=\phi(\mathbf{x},0)+\int_0^1 v(\phi(\mathbf{x},t),t)dt$$
- Distance
  $$D(\phi(\mathbf{x},0),\phi(\mathbf{x},1))=\int_0^1 \|v(\mathbf{x},t)\|_L dt$$
- Kernel
  $$L = a\nabla^2 + b\mathbf{Id}$$

Geodesic path between images



Symmetric formulation

$$\phi_1(\mathbf{x},1)I = J,$$

$$\phi_2^{-1}(\phi_1(\mathbf{x},t),1-t)I = J,$$

$$\phi_2(\phi_2^{-1}(\phi_1(\mathbf{x},t),1-t),1-t)I = \phi_2(\mathbf{z},(1-t)J,$$

$$\phi_1(\mathbf{x},t)I = \phi_2(\mathbf{z},1-t)J,$$

Minimization problem

$$E_{sym}(I,J) = \inf_{\phi_1} \inf_{\phi_2} \int_{t=0}^{0.5} \left\{ \left\| \upsilon_1(\mathbf{x},t) \right\|_L^2 + \left\| \upsilon_2(\mathbf{x},t) \right\|_L^2 \right\} dt + \int_\Omega \left| I(\phi_1(0.5)) - J(\phi_2(0.5)) \right|^2 d\Omega.$$

Subject to each $\phi_i \in \mathit{Diff}_0$ the solution of:

$$d\phi_i(\mathbf{x},t)/dt = \upsilon_i(\phi_i(\mathbf{x},t),t) \text{ with}$$

$$\phi_i(\mathbf{x},0) = \mathbf{Id} \text{ and } \phi_i^{-1}(\phi_i) = \mathbf{Id}, \phi_i(\phi_i^{-1}) = \mathbf{Id}.$$

Cross correlation

- $I_1 = I(\phi_1(x, 0.5)), J_2 = J(\phi_2(x, 0.5))$,
  $\bar{I}(\mathbf{x}) = I_1(\mathbf{x}) - \mu_{I_1}(\mathbf{x})$ and $\bar{J}(\mathbf{x}) = J_2 - \mu_{J_2}(\mathbf{x})$

  $$CC(\bar{I},\bar{J},\mathbf{x}) = \frac{<\bar{I},\bar{J}>^2}{<\bar{I}><\bar{J}>} = A^2/BC,$$

- Minimize

  $$E_{CC}(\bar{I},\bar{J}) = \inf_{\phi_1} \inf_{\phi_2} \int_{t=0}^{\frac{1}{2}} \left\{ \|v_1(\mathbf{x},t)\|_L^2 + \|v_2(\mathbf{x},t)\|_L^2 \right\} dt + \int_\Omega CC(\bar{I},\bar{J},\mathbf{x}) d\Omega.$$

Euler-Langrange equations

$$\nabla_{\phi_1(\mathbf{x},0.5)} E_{cc}(\mathbf{x}),=2L\upsilon_1(\mathbf{x},0.5)+\frac{2A}{BC}(\bar{J}(\mathbf{x}) - \frac{A}{B}\bar{I}(\mathbf{x}))\ |D\phi_1\ |\nabla\bar{I}(\mathbf{x}),$$

$$\nabla_{\phi_2(\mathbf{x},0.5)} E_{cc}(\mathbf{x}),=2L\upsilon_2(\mathbf{x},0.5)+\frac{2A}{BC}(\bar{I}(\mathbf{x}) - \frac{A}{C}\bar{J}(\mathbf{x}))\ |D\phi_2\ |\nabla\bar{J}(\mathbf{x}).$$

with Jacobian determinant $|D\phi_i|$

Algorithm 1 : Computing Cross-Correlation Derivatives

(1) Deform I by $\varphi_1(0.5)$ and J by $\varphi_2(0.5)$.

(2) Calculate $\bar{I}$ and $\bar{J}$ from the result of step (1).

(3) Calculate and store images representing A, B and C.

Deformation from speed

- Gradients give us update of v_1, v_2
- Diffeomorphism ϕ is generated by

$$d\varphi(\mathbf{x}, t)/dt = \boldsymbol{v}(\varphi(\mathbf{x}, t), t)$$

- Discretize as

$$\phi(\mathbf{x}, t+\Delta t) \leftarrow \phi(\mathbf{x}, t) + \Delta t \; v(\phi(\mathbf{x}, t), t).$$

Inversion method

- Enforce $\phi\phi^{-1} = \mathrm{Id}$
- Start with $\varphi(x) = y$, $\psi^{-1}(\tilde{y}) = x, \tilde{y} \neq x$

1: **while** $\|\psi^{-1}(\varphi(\mathbf{x})) - \mathbf{x}\|_\infty > \varepsilon_2 r$ **do**

2: Compute $\boldsymbol{v}^{-1}(\mathbf{x}) = \psi^{-1}(\varphi(\mathbf{x})) - \mathbf{x}$.

3: Find scalar γ such that $\|\boldsymbol{v}^{-1}\|_\infty = 0.5r$.

4: Integrate ψ^{-1} s.t. $\psi^{-1}(\tilde{\mathbf{y}}, t) += \gamma\, \boldsymbol{v}^{-1}(\psi^{-1}(\tilde{\mathbf{y}}, t))$.

5: **end while**

Symmetric normalization algorithm

(1) Initialize $\phi_1 = \mathbf{Id} = \phi_1^{-1}$ and $\phi_2 = \mathbf{Id} = \phi_2^{-1}$.

(2) Repeat the following steps until convergence:

(3) Compute the cross correlation as described in algorithm 1.

(4) Compute each $\boldsymbol{v}_\mathrm{i}$ by smoothing the result of step (3) in this table. One may also us modified midpoint method for each velocity, as in the LPF algorithm [39], to give smo in time.

(5) Update each φ_i by $\boldsymbol{v}_i$ through the *o.d.e.* as described in equation 8. This step auton adjusts the time step-size such that the maximum length of the updates to the φ_i is su and approximately constant over iterations. We explicitly guarantee $\|\boldsymbol{v}_1(\cdot, t)\| = \|\boldsymbol{v}_2(\cdot$ also update the estimate to the geodesic distance by trapezoidal rule, as in the LPF m

(6) Use algorithm 2 to get the inverses of the φ_i.

(7) Generate the time 1 solutions from $\phi_1(1) = \phi_2^{-1}(\phi_1(\mathbf{x}, 0.5), 0.5)$ and $\phi_1^{-1}(1) = \phi_2(1) = \phi_1^{-1}(\phi_2(\mathbf{x}, 0.5), 0.5)$.

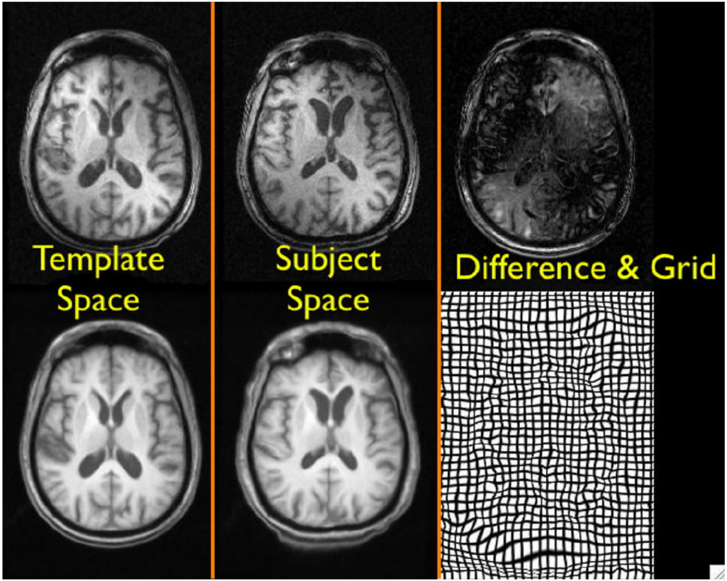
Implementation details and performance

- uses multiresolution, 100 iterations per level
- 5×slower than demons, 30%slower than ITK elastic registration
- robust and accurate
- can handle both large and small deformations

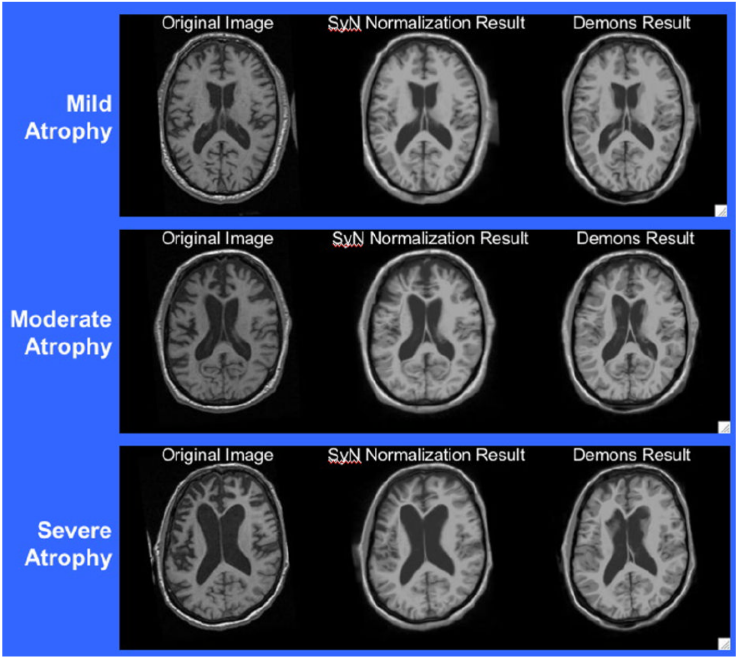
Experiments

- 20 T_1MRI from 10 normal elderly and 10 frontotemporal dementia patients
- register normal and patients to atlas to find volumetric differences

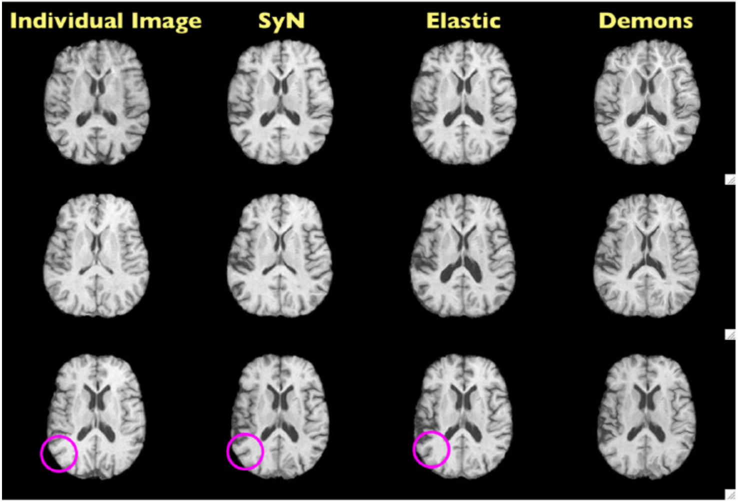
MRI Bias example



Atlas registration

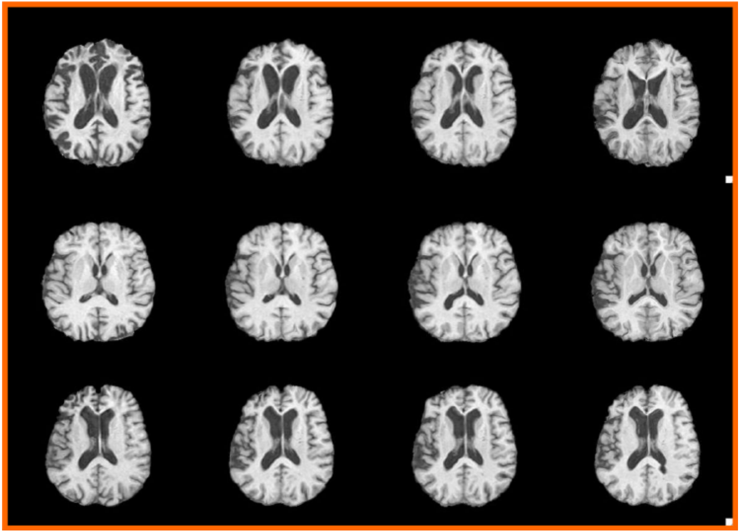


Atlas registration (controls)

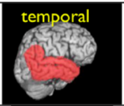
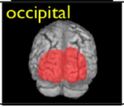


images should look similar to first column

Atlas registration (patients)



Dice coefficient

Structure	Demons	Elastic XCor > Demons	SyN XCor > Elastic
	Mean+-Sigma: 0.76 +- 0.021	0.81 +- 0.02	0.84 +- 0.019
	Min - Max : [0.69-0.79]	[0.76-0.84]	[0.79-0.87]
	Significance:	$p < 0.0001$	$p < 0.0001$
	0.69 +- 0.034	0.74 +- 0.03	0.78 +- 0.027
	[0.62-0.73]	[0.68-0.79]	[0.70-0.83]
	-	$p < 0.0001$	$p < 0.0001$
	0.78 +- 0.030	0.79 +- 0.024	0.83 +- 0.022
	[0.72-0.82]	[0.73-0.84]	[0.78-0.87]
	-	$p < 0.011$	$p < 0.0001$

Volume measurements