PKR Test-03 EN

1. Find all rotation matrices that transform vectors

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ to vectors } \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 0 \end{bmatrix}$$

2. Let us assume a motion understood as a mapping of a general point X into a point Y given by

$$\vec{y}_\beta = \mathtt{R}\,\vec{x}_\beta + \vec{o}\,'_\beta\,,$$

where $\vec{x}_\beta$, resp. $\vec{y}_\beta$, are coordinates of the vector, that represents point X, resp. point Y, in the coordinate system with orthonormal basis β. Matrix $\mathtt{R}$ and vector $\vec{o}\,'$ are given as follows

$$\mathtt{R} = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ -2 & -1 & 2 \end{bmatrix}, \quad \vec{o}\,'_\beta = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

   (a) Write down the matricidal equation that determines the points on the axis of motion.

   (b) Find all points of the axis of motion.

3. Consider two rotations. The first rotation $\vec{y} = \mathtt{R}_1(\vec{x})$ is given by matrix

$$\mathtt{R}_1 = \frac{1}{3}\begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{bmatrix}$$

   The second rotation $\vec{y} = \mathtt{R}_2(\vec{x})$ is given by angle-axis parameterization

$$\theta_2 = \frac{\pi}{3}, \quad \vec{v}_2 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

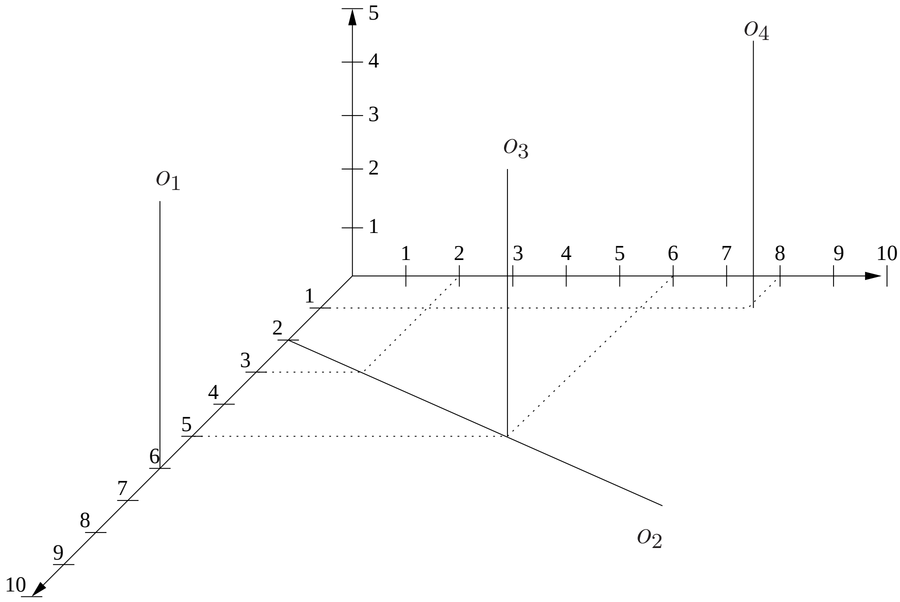
   Find a unit quaternion representing the composite rotation

$$\mathtt{R}_2(\mathtt{R}_1(\vec{x}))$$

4. Costruct a lexicographic Groebner basis w.r.t. the variable ordering $x > y$ of the following polynomial system and find all its complex solutions

$$\begin{aligned} x\,y + x + 1 &= 0 \\ x\,y + y + 1 &= 0 \end{aligned}$$

5. Let us have a manipulator with four axes of motion as shown in the figure.



(a) Draw the coordinates system of the links as defined by the Denavit-Hartenberg convention into the figure;

(b) Compute the Denavit-Hartenberg parameters of the manipulator.

6. Let us have a manipulator whose forward kinematics is defined by the equations

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f_1(\theta_1, d_2) \\ f_2(\theta_1, d_2) \end{bmatrix} = \begin{bmatrix} l \cos\theta_1 \\ d_2 - l \sin\theta_1 \end{bmatrix}, \quad l > 0$$

Determine the sets of singular configurations and singular poses for this manipulator.