Auctions

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Introduction to Auctions

Auctions: Traditional

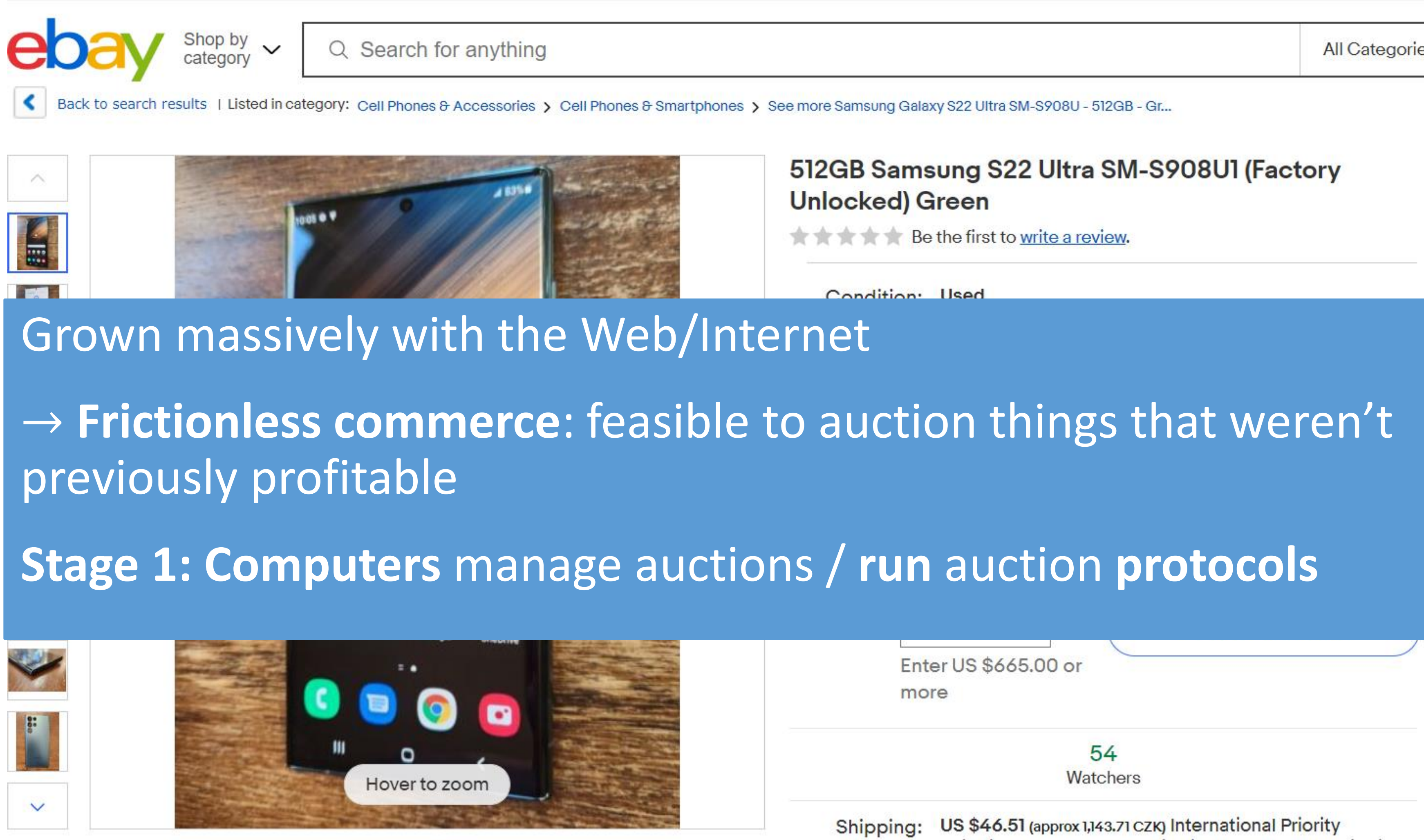
Auctions used in Babylon as early as 500 B.C.

Stage 0: No automation

Tuna Fish Auction

Property Auction

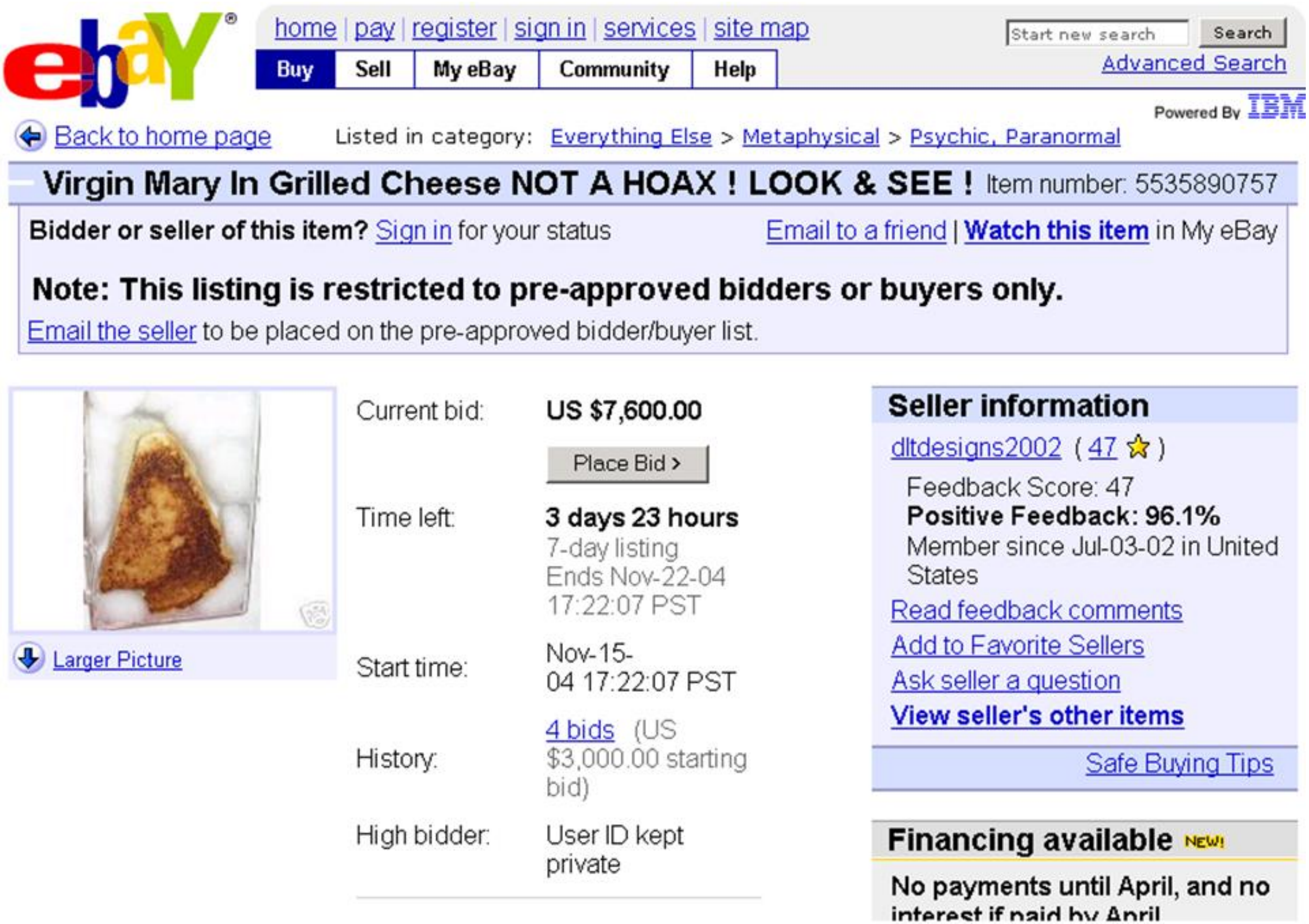
Auctions: Partial Automation



Grown massively with the Web/Internet

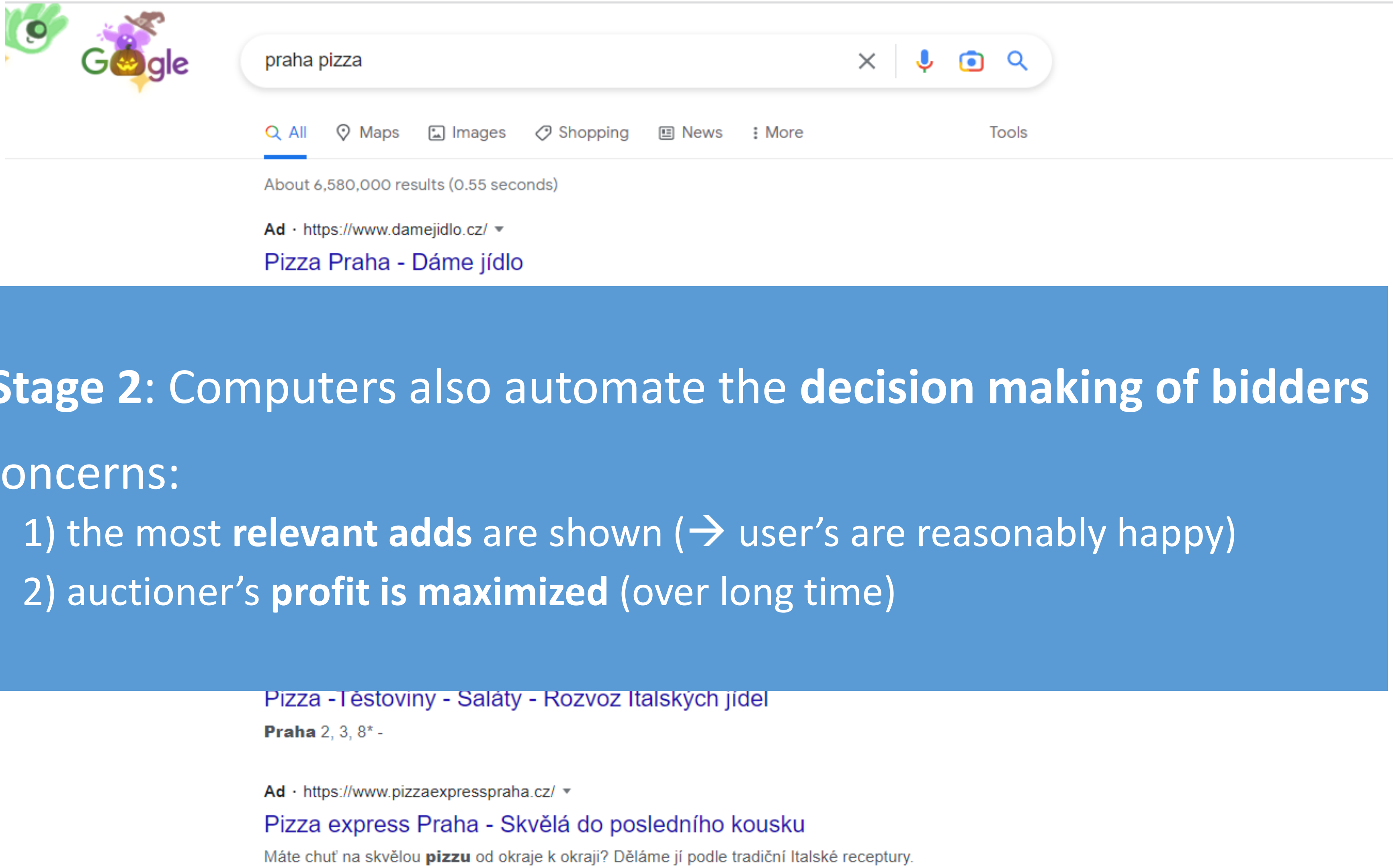
→ **Frictionless commerce**: feasible to auction things that weren't previously profitable

Stage 1: Computers manage auctions / **run** auction **protocols**


ebay
home | pay | register | sign in | services | site map
Start new search
Search
Advanced Search
Buy
Sell
My eBay
Community
Help
Powered By IBM
Back to home page
Listed in category: Everything Else > Metaphysical > Psychic, Paranormal
Virgin Mary In Grilled Cheese NOT A HOAX ! LOOK & SEE ! Item number: 5535890757
Bidder or seller of this item? Sign in for your status
Email to a friend | Watch this item in My eBay
Note: This listing is restricted to pre-approved bidders or buyers only.
Email the seller to be placed on the pre-approved bidder/buyer list.
Larger Picture
Current bid: US $7,600.00
Place Bid >
Time left: 3 days 23 hours
7-day listing
Ends Nov-22-04 17:22:07 PST
Start time: Nov-15-04 17:22:07 PST
History: 4 bids (US $3,000.00 starting bid)
High bidder: User ID kept private
Seller information
dltdesigns2002 (47)
Feedback Score: 47
Positive Feedback: 96.1%
Member since Jul-03-02 in United States
Read feedback comments
Add to Favorite Sellers
Ask seller a question
View seller's other items
Safe Buying Tips
Financing available NEW!
No payments until April, and no interest if paid by April

Silent Auction

Auctions: (Almost) Full automation



Stage 2: Computers also automate the **decision making of bidders**

Concerns:

1) the most **relevant adds** are shown (→ user's are reasonably happy)
2) auctioner's **profit is maximized** (over long time)

What is an Auction?

*An **auction** is a protocol that allows **agents** (=bidders) to indicate their **interests** in one or more **resources** (=items or goods) and that uses these indications of interest to determine both an **allocation** of the resources and a set of **payments** by the agents.* [Shoham & Leyton-Brown 2009]

Auctions use employ **cardinal preferences** to express interest .

Auctions are mechanisms **with money**.

Auctions can be viewed as (Baysian) **games** of a specific structure.

Why Auctions?

Market-based price setting: for objects of unknown value, the value is dynamically assessed by the market!

Flexible: any object type can be allocated

Can be **automated**

- use of simple rules reduces complexity of negotiations
- well-suited for computer implementation

Revenue-maximising and **efficient allocations** are achievable

Auctions Rules

Auctions are **structured negotiations** governed by **auction rules** **(→** *rules of the game)*

Bidding rules

How **offers (bids)** are made:

- by whom
- when
- what their content is

Clearing rules

Who gets which goods (**allocation**) and what money changes hands (**payment**).

Information rules

What information about the state of the negotiation is *revealed* **to whom** and **when**.

Lots of Applications

Industrial procurement

Transport and logistics

Energy markets

Cloud and grid computing

Internet auctions

(Electromagnetic spectrum allocation)

... and counting!

Types of Auctions

multi-unit
multiple indistinguishable items (A, A, A)

single-good

multi-item
multiple different items (A,B,C)

Combinatorial auctions

multi-attribute
items characterized by multiple attributes (A=a_1, a_2, a_3)

Single-Item Auctions

Basic Auction Mechanisms

English

Japanese

Dutch

First-Price

Second-Price

(All pay auction)

English Auction

1. Auctioneer starts the bidding (at some **reservation price**)
2. Bidders then shout out **ascending** prices (with minimum increments)
3. Once bidders stop shouting, the *high bidder* gets the good at that price

Japanese Auctions

Same as an English auction except that the auctioneer calls out the prices

1. All bidders start out **standing**
2. When the price reaches a level that a bidder is not willing to pay, that bidder **sits down;** once a bidder sits down, they **can't get back up.**
3. The **last** person **standing** gets the good

Dutch Auction

1. The auctioneer starts a clock at some high value; it descends
2. At some point, a bidder shouts “mine!" and gets the good at the price shown on the clock

Good when items need to be sold **quickly** (similar to Japanese)

No information is revealed during auction

First-, Second-Price Sealed Bid Auctions

First-price sealed bid auction

- bidders write down bids on pieces of paper
- auctioneer **awards** the good to the bidder with the **highest bid**
- that bidder pays the amount of **his bid**

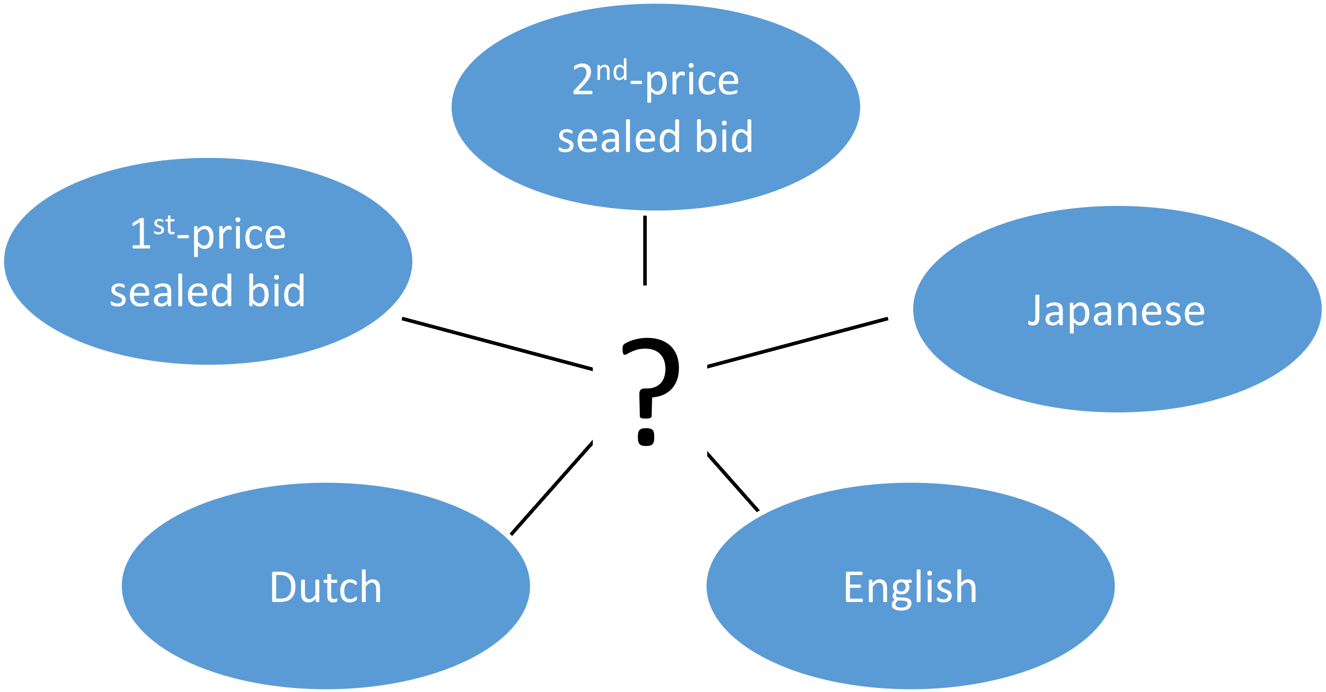
Second-price sealed bid auction (**Vickerey** auction)

- bidders write down bids on pieces of paper
- auctioneer **awards** the good to the bidder with the **highest bid**
- that bidder pays the amount bid by **the second-highest** bidder

Intuitive Comparison

	English	Dutch	Japanese	1^{st}-Price	2^{nd}-Price
Duration	#bidders, increment	starting price, clock speed	#bidders, increment	fixed	fixed
Info Revealed	2^{nd}-highest val; bounds on others	winner's bid	all val's but winner's	none	none
Jump bids	yes	n/a	no	n/a	n/a
Price Discovery	yes	no	yes	no	no

Analysing Auctions



Are there fundamental similarities / differences between mechanisms described?

Two Problems

Analysis of auction mechanisms	**Design** of auction mechanisms
• determine the properties of a given auction mechanism • methodology: treat auctions as (extended-form) *Bayesian games* and analyse players' (i.e. bidders') strategies	• design the auction mechanism (i.e. the game for the bidders) with the desirable properties • methodology: apply mechanism design techniques

Payoff

Agent's payoff
from participating in an auction

If winner

payoff* =
valuation of the item −
price paid for the item

* Also termed **surplus**

If *not* winner

payoff=**zero**[1]

Individual rationality: the agent never bids higher than its valuation.

[1] not true for the all pay auction

Risk Attitudes

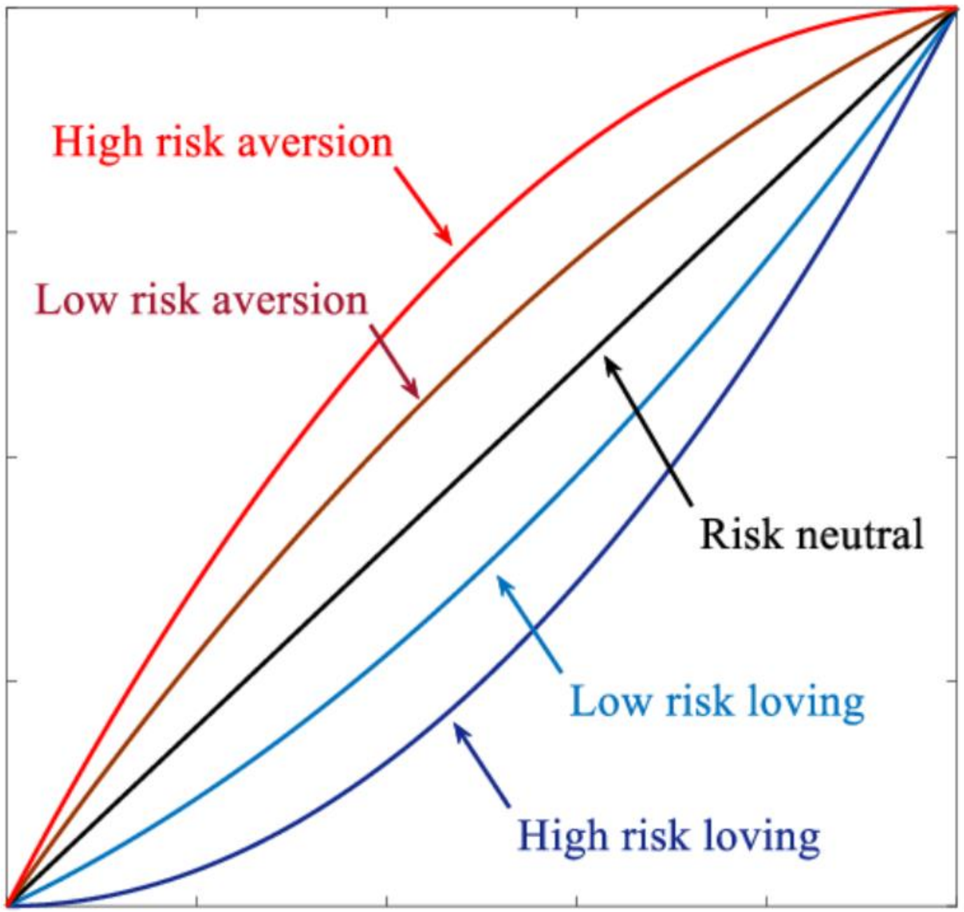
Risk neutrality: the payoff is a *linear function* of the difference between the item's valuation and the price paid

Risk seeking (also **risk loving)**: the payoff is a *convex* function of the difference (aggressively seeking high gains is prioritized)

Risk aversion: the payoff is a *concave* function of the difference (conservatively ensuring at least some gains is prioritized)

Risk Attitudes

payoff
(as a function of surplus)



surplus

Valuation Models

Independent private value (IPV)	Correlated value
An agent A's valuation of the good is **independent from other agent's** valuation of the good (e.g. a taxi ride to the airport).	Valuations of the good are **related between agents** (typically the more other agents are prepared to pay, the more the agent A prepared to pay – e.g. purchase of items for later resale).

Bayesian Game

Definition (Bayesian Game)

A Bayesian game is a tuple $\langle N, A, \Theta, p, \boldsymbol{u} \rangle$ where

- N is the set of **players**
- $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, Θ_i is the **type space** of player i
- $A = A_1 \times A_2 \times \cdots \times A_n$ where A_i is the **set of actions** for player i
- $p: \Theta \mapsto [0,1]$ is a **common prior over types**
- $\boldsymbol{u} = (u_1, \dots, u_n)$, where $u_i: \mathrm{A} \times \Theta \mapsto \mathbb{R}$ is the **utility function** of player i

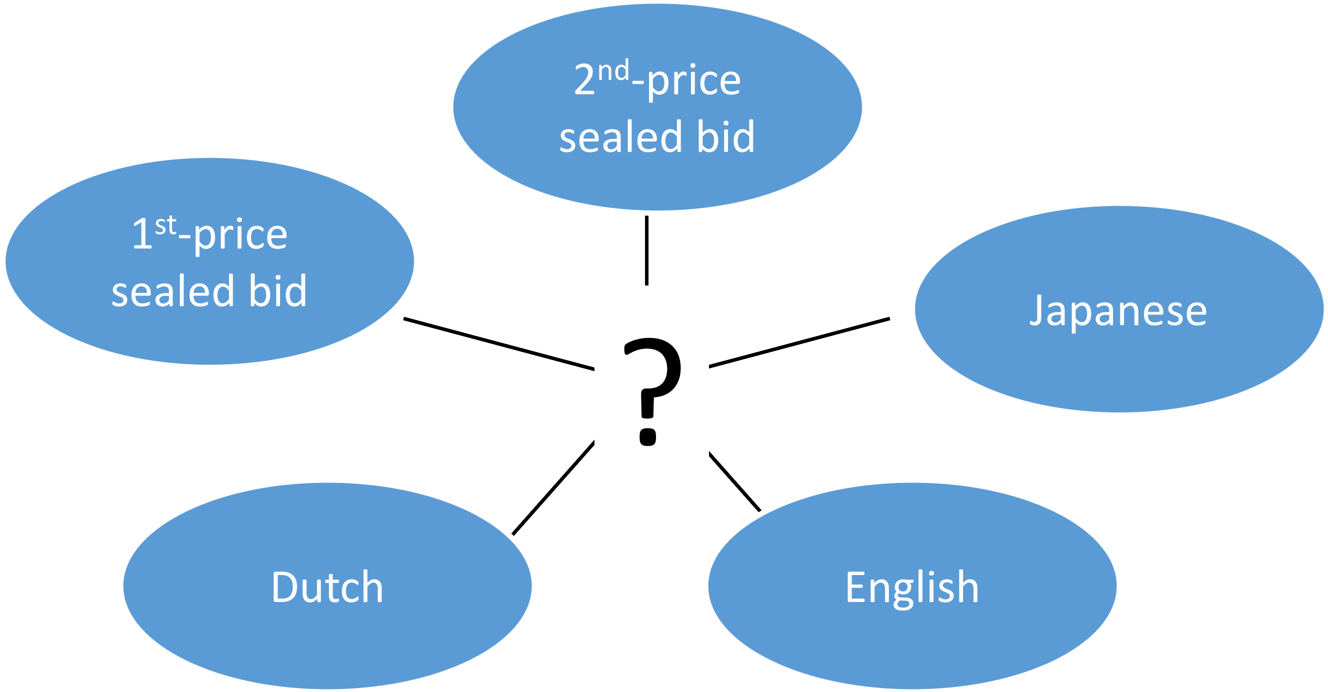
We assume that all of the above is **common knowledge** among the players, but the type of an agent **is private** (i.e. only known by that agent).

Relation to (sealed bid) Auctions

Sealed bid auction under IPV is a Bayesian game in which

- player i's **actions** correspond to its **bids** $\widehat{v}_i$
- player types Θ_i correspond to players' **private valuations** v_i over the auctioned item(s)
- the **payoff** of player i corresponds to: ***i*'s valuation of the item $\boldsymbol{v_i}$ – price paid** (if winner); zero otherwise.

Similar analogies for more complicated auction mechanisms.



Are there fundamental similarities / differences between mechanisms described?

Bidding in Second-Price Sealed Bid Auction

Bidding in Second Price Sealed Bid Auction

How should agents bid in the second-price sealed bid auctions?

Bidding in Second Price Sealed Bid Auction

Theorem

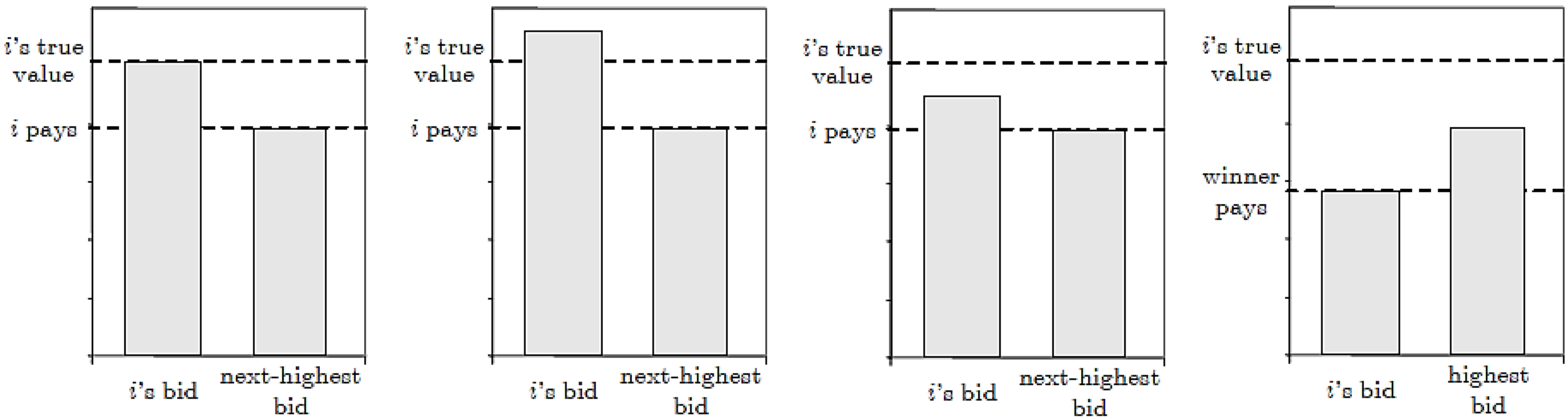
Truth-telling is a **dominant strategy** in a second-price sealed bid auction (assuming independent private values – IPV).

Proof: Assume that the other bidders bid in some arbitrary way. We must show that i's best response is always to bid truthfully. We'll break the proof into two cases:

- Bidding honestly, i would win the auction
- Bidding honestly, i would lose the auction

Second-Price Sealed Bid Proof

Bidding honestly, i is the winner



If i bids higher, he will still win and still pay the same amount

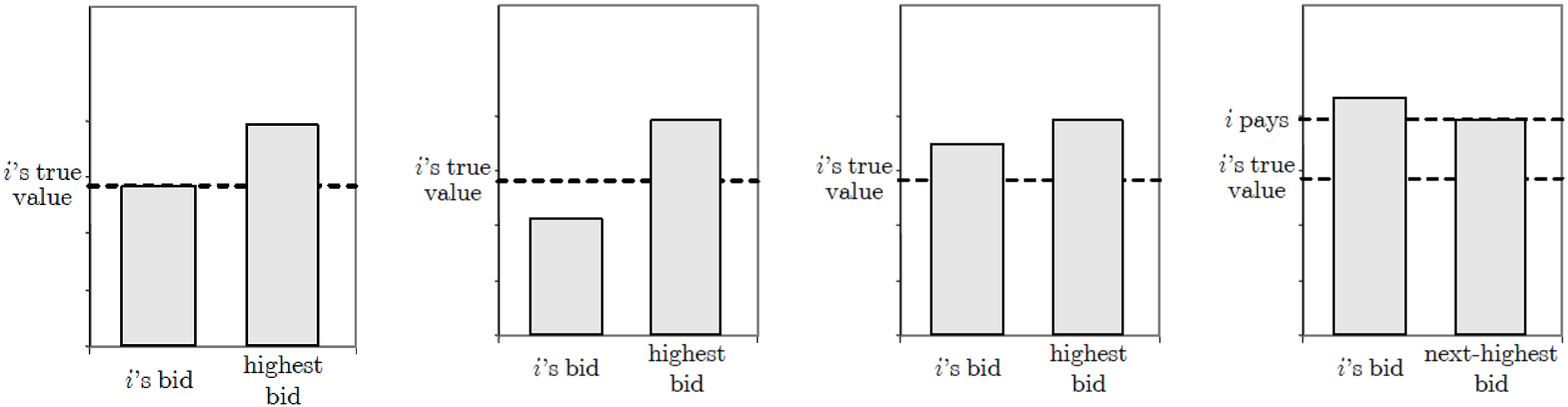
If i bids lower, he will either still win and still pay the same amount. . .

... or lose and get the payoff of zero.

➔ *There is a disadvantage bidding lower and no advantage bidding higher*

Second-Price Sealed Bid Proof

Bidding honestly, i is not the winner



If i bids lower, he will still lose and still pay nothing

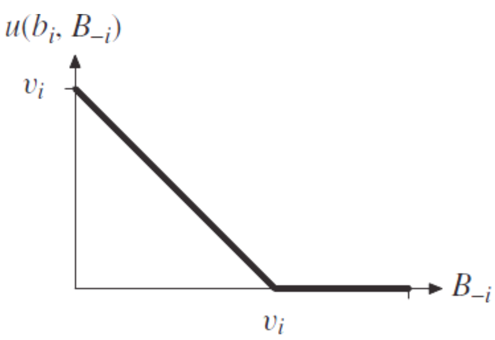
If i bids higher, he will either still lose and still pay nothing...

... or win and pay more than his valuation ($\Rightarrow$ negative payoff).

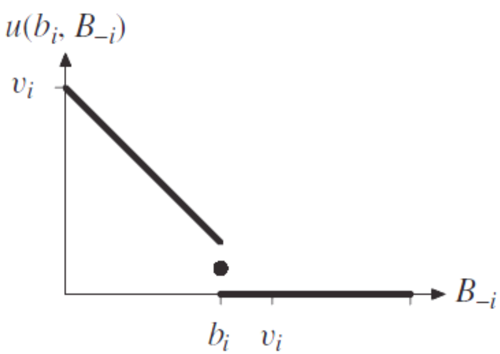
➔ *There is a disadvantage bidding higher and no advantage bidding lower*

Second-Price Sealed Bid Proof (alternative)

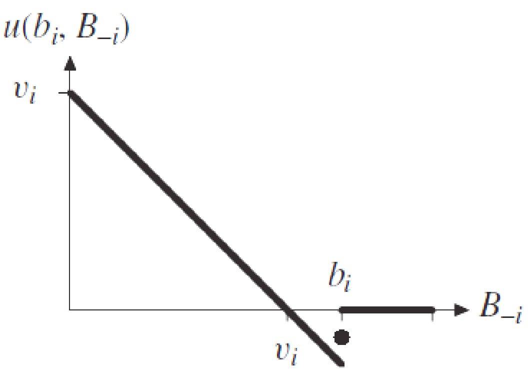
$u(b_i, B_{-i})$ … i's bidder payoff when bidding b_i and when the highest of the other bidders (except i) is B_{-i}.



$b_i = v_i$
(i.e. i bids its valuation)



$b_i < v_i$
(i.e. i bids lower than its valuation)



$b_i > v_i$
(i.e. i bids higher than its valuation)

(from Maschler, page 93 and 94)

Second-Price Sealed Bid

Advantages:

- **Truthful** bidding is dominant strategy
- No incentive for counter-speculation
- Computational efficiency

Disadvantages:

- Lying auctioneer
- Bidder collusion self-enforcing
- Reveals true valuations
- Not revenue maximizing

Bidding in First-Price Sealed Bid Auctions

Dutch and First-price Sealed Bid

Strategically equivalent: an agent bids without knowing about the other agents’ bids (i.e. difference are technical implementation)

- a bidder must decide on the amount he's willing to pay, conditional on having placed the highest bid

Differences

- First-price auctions can be held **asynchronously**
- Dutch auctions are **fast**, and require **minimal communication**
  - only **one bit** needs to be transferred from the bidders to the auctioneer

Bidding in Dutch / First Price Sealed Bid

How should bidders bid in these auctions?

- Note: bidding **true** valuation results in **zero** surplus

There's a **trade-off** between **probability of winning** vs. **amount paid** upon winning (and thus the winner's surplus)

➔ Bidders don't have a **dominant strategy** anymore.

Individually optimal strategy depends on the **assumptions** about **others' valuations.**

We have a Bayesian game → **Bayes-Nash equilibrium.**

Equilibrium Strategy

Assume a **first-price auction** with **two risk-neutral bidders** whose valuations are drawn independently and **uniformly** at random from the interval [0, 1] - what is the equilibrium strategy?

$\rightarrow \left(\frac{1}{2}v_1, \frac{1}{2}v_2\right)$ is the Bayes-Nash equilibrium strategy profile

Interim expected utility

Given a Bayesian game (N, A, Θ, p, u) with *finite* sets of players, actions, and types, player i's ***interim* expected utility** with respect to *type* θ_i and a *mixed strategy profile* s is

$$EU_i(s|\theta_i) = \sum_{\theta_{-i}\in\Theta_{-i}} p(\theta_{-i}|\theta_i) \sum_{a\in A} \left(\prod_{j\in N} s_j\left(a_j|\theta_j\right) \right) u_i(a, \theta_i, \theta_{-i})$$

Proof

Assume that bidder 2 bids $\frac{1}{2}v_2$, and that bidder 1 bids s_1.

The following outcomes are possible*

1. Bidder 1 **wins** when $\frac{1}{2}v_2 < s_1$, gaining payoff $u = v_1 - s_1$.
2. Bidder 1 **loses** when $\frac{1}{2}v_2 > s_1$ and then gets payoff $u = 0$.

$$EU_1(s|v_1) = \int_0^1 u(s)dv_2 =$$

$$= \int_0^{2s_1} (v_1 - s_1)dv_2 + \int_{2s_1}^1 (0)dv_2 =$$

$$= (v_1 - s_1)v_2|_0^{2s_1} = 2v_1s_1 - 2s_1^2$$

* we can ignore the case where the agents have the same valuation, because this occurs with probability zero).

Proof Continued

We can determine bidder 1's best response to bidder 2' strategy by taking the derivative and setting it to zero:

$$\frac{\partial}{\partial s_1}(2v_1 s_1 - 2s_1^2) = 0$$

$$2v_1 - 4s_1 = 0$$

$$s_1 = \frac{1}{2}v_1$$

Thus, when player 2 is bidding half her valuation, player 1's best reply is to **bid half his valuation** (and analogously for player 2, given the symmetry of the game).

Equilibrium in Dutch / First Price Sealed Bid Auctions

Theorem

In a first-price sealed bid auction with n **risk-neutral** agents whose valuations $v_1, v_2, \ldots, v_n$ are **independently** drawn from a **uniform distribution** on the **same** bounded **interval** of the real numbers, the **unique** symmetric **equilibrium** is given by the **strategy profile** $\left(\frac{n-1}{n} v_1, \ldots, \frac{n-1}{n} v_n\right)$.

The more players, **the harder to win** and the lower the expected surplus.

$\Rightarrow$ Dutch / FPSB auctions **not incentive compatible,** i.e., there are incentives to **counter-speculate**.

Equilibrium in Dutch / First Price Sealed Bid Auctions

For non-uniform valuation distributions: Each bidder should bid **the expectation of the second-highest valuation**, conditioned on the assumption that his own valuation is the highest.

Equilibrium in more general cases?

Note we only **verified** the equilibrium.

What about more general assumptions?

→ We need to guess the equilibrium and it gets more complicated as we relax the assumptions about the distributions of valuations (non-uniformity, no symmetry etc.).

Even determining a Nash equilibrium exists gets difficult.

This because auctions are **non-continuous games**: even a small variation in the bid amount can lead to not-winning and thus large changes in the payoff.

Bidding in English and Japanese Auctions

English and Japanese Auctions Analysis

A much more complicated **strategy space**

- extensive-form game
- bidders are able to condition their bids on information revealed by others
- in the case of English auctions, the ability to place jump bids

Intuitively, though, the **revealed information** does not make any **difference** in the **independent-private value** (IPV) setting.

English and Japanese Auctions Analysis

Theorem

Under the IPV model, it is a **dominant strategy** for bidders to bid **up to** (and not beyond) their valuations in both Japanese and English auctions.

In correlated-value auctions, it can be worthwhile to counter-speculate.

Seller’s Revenue

Which auction should the seller choose?

Expected Seller's Revenue (First Price Sealed Bid Auction)

$$\mathrm{E}\left(\max\left\{\frac{V_1}{2},\frac{V_2}{2}\right\}\right)=\frac{1}{2}\mathrm{E}(\max\{V_1,V_2\}=?$$

Let $Z=\max\{V_1,V_2\}$

$$F_Z(z)=\mathrm{P}(Z\le z)=\mathrm{P}(\max\{V_1,V_2\}\le \mathrm{z})=\mathrm{P}(V_1\le z)\cdot\ \mathrm{P}(V_2\le z)=\mathrm{z}^2$$

$$\Rightarrow f_Z(z)=\begin{cases}2z & \text{if } 0\le z\le 1\\ 0 & \text{otherwise}\end{cases}$$

$$\frac{1}{2}E(Z)=\frac{1}{2}\int_0^1 zf_z(z)dz=\int_0^1 z^2dz=\frac{1}{3}z^3\Big|_0^1=\mathbf{\frac{1}{3}}$$

Expected Seller's Revenue

$\mathrm{E}(\min\{V_1, V_2\}) = ?$

Note:

$$\min\{V_1, V_2\} + \max\{V_1, V_2\} = V_1 + V_2$$

Hence:

$$\mathrm{E}(\min\{V_1, V_2\}) + \mathrm{E}\,(\max\{V_1, V_2\}) = \mathrm{E}(V_1) + \mathrm{E}(V_2) = \frac{1}{2} + \frac{1}{2} = 1$$

We already calculated (previous slide):

$$\mathrm{E}\,(\max\{V_1, V_2\}) = \mathrm{E}(Z) = \frac{2}{3}$$

Hence:

$$\mathrm{E}\,(\min\{V_1, V_2\}) = \mathbf{\frac{1}{3}}$$

Seller's Revenue

Corrolary (Expected Seller's Revenue)

In the symmetric case with two risk-neutral bidders and IPV, the **expected seller's revenue** from the first-price and second-price sealed bid auction is **the same**.

Somewhat surprising and far from self-evident.

Revenue Equivalence

In fact, holds in more general.

Theorem (Revenue Equivalence)

Assume that each of n **risk-neutral** agents has an **independent private valuation** for a single good at auction, drawn from a **common cumulative distribution** $F(v)$ that is **strictly increasing** and **atomless** on $[\underline{v}, \overline{v}]$. Then any auction mechanism in which

1. the good will be allocated to the agent with the highest valuation; and
2. any agent with valuation $\underline{v}$ has an expected utility of zero

yields the **same expected revenue**, and hence results in any bidder with valuation v making the **same expected payment**.

Revenue Equivalence

Informally: As long as two mechanism **allocate in the same way** and they **do not charge anything** to the agent with the lowest valuation, the **rest of payment functions** is the same.

You cannot get **extra money** from bidder without changing the allocation function or the payment to the lowest-valued bidder.

In fact, the revenue equivalence holds beyond IPV and single good.

Assuming bidders are risk neutral and have independent private valuations, **all the auctions** we have spoken about so far—English, Japanese, Dutch, and all sealed bid auction protocols—are **revenue equivalent**.

Applying Revenue Equivallence

Expected value of the k^{th}-largest of n IID draws* from $[0, v_{max}]$:

$$\frac{n+1-k}{n+1} v_{max}$$

Expected seller's revenue in the second-price auction (with IID valuations):

$$\frac{n-1}{n+1} v_{max}$$

* termed *k-th order statistics*

Applying Revenue Equivallence

Both second-price and first-price auction **satisfies** the conditions of the revenue equivalence theorem.

Thus, a bidder in the **first-price auction** must **bid his expected payment** conditional on being the **winner of a second price auction**.

If v_i is the high value, there are $n-1$ other values drawn from the uniform distribution on $[0, v_i]$. The expected value of the second-highest bid is therefore the first-order statistics of $n-1$ draws from $[0, v_i]$, which is

$$\frac{n-1}{n} v_{max}$$

We still need to verify the above is an equilibrium

(the revenue equivalence theorem does not state that every revenue-equivalent strategy profile is an equilibrium)

Auctions Summary

Auctions are mechanisms for **allocating scarce resource** among **self-interested agent**

Mechanism-design and game-theoretic perspective

Many auction mechanisms: English, Dutch, Japanese, First-price sealed bid, Second-price sealed bid

Desirable properties: truthfulness, efficiency, optimality, ...

Rapidly expanding list of **applications** worth billions of dollars

Reading:

- [Shoham] – Chapter 11