FSM Learning

Radek Mařík

Czech Technical University
Faculty of Electrical Engineering
Department of Telecommunication Engineering
Prague CZ

December 13, 2023

Outline

1. FSM Learning
   - FSM Learning Overview
   - Angluin's Algorithm
   - Example
2. Hidden Markov Model
   - A Brief Overview
3. Markov Decision Process
   - Introduction
   - Utility Function, Policy
   - Value Iteration
   - Policy Iteration
   - Conclusions

Outline

1. FSM Learning
   - FSM Learning Overview
   - Angluin's Algorithm
   - Example
2. Hidden Markov Model
   - A Brief Overview
3. Markov Decision Process
   - Introduction
   - Utility Function, Policy
   - Value Iteration
   - Policy Iteration
   - Conclusions

Finite State Machine

A **finite-state machine** is a sextuple $(S, \Sigma, \Gamma, s_0, \delta, \lambda)$, where

- S is a finite nonempty set of states,
- Σ is an input alphabet (a finite nonempty set of symbols),
- Γ is an output alphabet (a finite nonempty set of symbols),
- s_0 is an initial state, $s_0 \in S$,
- δ is a state-transition function: $\delta : S \times \Sigma \to S$,
- λ is an output function: $\lambda : S \times \Sigma_\epsilon \to \Gamma_\epsilon$.

Additional designations:

- Σ^* is the set of all strings (words) over the input alphabet,
- Γ^* is the set of all strings (words) over the output alphabet,
- Alphabet X^* always contains ϵ and $\forall x \in X^* : \epsilon \cdot x = x = x \cdot \epsilon$.
- Thus X^* is always nonempty and it is also countable because X is countable.

Goal

- A system trying to figure out the effects its actions have on its environment...
  - It performs actions.
  - It gets observations.
  - It tries to make an internal model of what is happening.
- Let's model the world as a DFA.

Applications

- Communication protocol learning,
- Hidden process learning,
- WWW application learning,
- Black box proprietary behavior identification,
- Software implementation identification.

Learning a Language

- Inferring finite automata is analogous to learning a language
- There is no way to distinguish between two automata that recognize the same language, without examining the state structure.
- We focus on finding the minimum equivalent automata.
- It has been shown that the only classes of languages that can be learned from **positive data only** are classes which include no infinite language.

Active Learning [Hon13]

- **Passive learning** - a set X is given and we cannot modify it.
    - NP problem
- **Active learning** - a set X can be selected and it can be modified during a learning process.
    - P problem

Outline

1. FSM Learning
   - FSM Learning Overview
   - Angluin's Algorithm
   - Example
2. Hidden Markov Model
   - A Brief Overview
3. Markov Decision Process
   - Introduction
   - Utility Function, Policy
   - Value Iteration
   - Policy Iteration
   - Conclusions

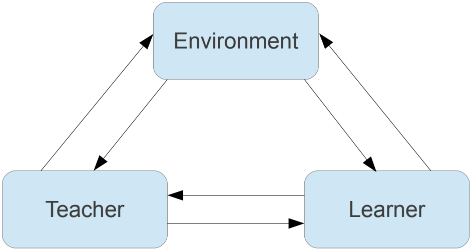
Teacher [Hon13]

The teacher has to be able to answer two kinds of queries

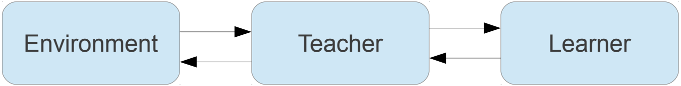
- **Membership query** - Yes/No.
  - In a membership query the learner selects a word $w \in \Sigma^*$ and
  - the teacher gives the answer whether or not $w \in L$.
- **Equivalence query (counterexamples)** - Yes/a counterexample string.
  - In an equivalence query the learner selects a hypothesis automaton $\mathcal{H}$, and the teacher answers whether or not L is the language of $\mathcal{H}$.
  - If yes, then the algorithm terminates.
  - If no, then the teacher gives a counterexample, i.e., a word in which L differs from the language of $\mathcal{H}$.

An issue of whether or not we have a **reset** button.

Active Learning with a Teacher [Hon13]



A learning architecture with a minimally adequate teacher.



An architecture with a degraded teacher working as an interface.

Angluin's Algorithm - Top Level View

- Iteratively, the algorithm builds a DFA using membership queries, then presents the teacher with the DFA as a solution.
- If the DFA is accepted, the algorithm is finished. Otherwise, the teacher responds with a counter-example, a string that the DFA presented would either accept or reject incorrectly.
- The algorithm uses the counter-example to refine the DFA, going back to the first step.

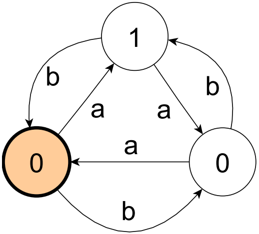
Angluin's Algorithm - Control Structures

States and Experiments

The algorithm uses two sets,

- S for states,
  - S ... access sequences to states
  - $S \bullet A$ ... sequences to exercise all transitions
- E for experiments (distinguishing sequences), and
- one observation table, T, where
  - elements of $S \cup S \bullet A$ form rows, and
  - elements of E form columns – the values of each cell is the outcome of a membership test for the concatenation of the row and column strings.

Observation Table [Ang86, Sha08, Hon13]



Definition 1.1

Let $\mathcal{E} = (A, \mathrm{accept})$ be an accepting environment.

Observation table of environment $\mathcal{E}$ is an ordered triple $OT = (S, E, \mathrm{T})$, where

- $S \subseteq A^*$, $S \neq \emptyset$, S finite, S is prefix closed.
- $E \subseteq A^*$, $E \neq \emptyset$, E finite, E is suffix closed.
- T is a function $(S \cup S \bullet A) \times E \to \{0, 1\}$.

- The set S is called *input set*.
- E is *a distinguishing set*.

		E	
		ϵ	a
S	ϵ	0	1
	a	1	0
	b	0	0
$S \bullet A$	aa	0	0
	ab	0	1
	ba	0	1
	bb	1	0

Initial Observation Table [Hon13]

L^* algorithm initialization:

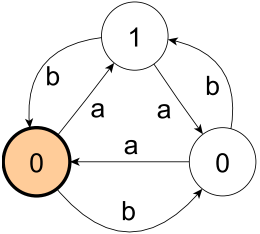
- Init the observation table $OT = (S, E, \mathrm{T})$, where $S = \{\epsilon\}$, $E = \{\epsilon\}$.
- Create *a queue of membership queries*: all pairs $s \cdot e$, where $s \in S \cup S \cdot A$ and $e \in E$.
- Get the answers from the set of $\{0, 1\}$ provided by the teacher, if $s \cdot e$ belongs to the learned language. Insert the answer value to the place $\mathrm{T}(s, e)$ in the observation table.
- Different rows in the section S of the table define states of the a possible automaton.

		E
		ϵ
S	ϵ	1
$S \cdot A$	a	0
	b	0

Observation Table - Closeness, Consistency [Hon13]

$$(s_1, s_2 \in S)$$

$$(s_1 \overset{E}{\sim} s_2) \iff (\forall e \in E)(\lambda(s_1, e) = \lambda(s_2, e))$$



		E	
		ϵ	a
S	ϵ	0	1
	a	1	0
	b	0	0
$S \bullet A$	aa	0	0
	ab	0	1
	ba	0	1
	bb	1	0

Definition 1.2

An observation table $OT = (S, E, \mathrm{T})$ **is closed**, if $(\forall t \in S \cdot A)(\exists s \in S)(s \overset{E}{\sim} t)$.

The table is consistent, if
$(\forall s, t \in S, s \overset{E}{\sim} t) \implies (\forall a \in A)(s \cdot a \overset{E}{\sim} t \cdot a)$.

- The closeness and consistency checking is performed when the queue of queries becomes empty.

Observation Table - Modifications [Hon13]

- If $OT = (S, E, \mathrm{T})$ is not closed, then
  1. Search for $t \in S \cdot A$, so that $s \overset{E}{\not\sim} t$ for all $s \in S$.
  2. This t is added to the set S and the queue of membership queries is extended with $t \cdot a \cdot e$ for all $a \in A$ and $e \in E$.
- If OT is not consistent,
  1. Search for $s, t \in S$, $e \in E$ and $a \in A$, so that $s \overset{E}{\sim} t$, but $T(s \cdot a, e) \neq T(t \cdot a, e)$.
  2. The word $a \cdot e$ is added to the distiguishing set E.
  3. The queue of membership queries is extended with $s' \cdot e$ for all $s' \in S \cup S \cdot A$.
  4. It is obvious that $s \overset{E}{\sim} t$ is not satisfied in the new observation table.

L^* algorithm [Ang86, Sha08, Hon13]

1. Init the observation table $OT = (S, E, \mathrm{T})$.
2. Fill the observation table using the membership query queue.
3. Check if OT is closed and consistent:
    1. If OT is not closed, extend the set S with $t \in S \cdot A$, so that $s \overset{E}{\nsim} t$ for all $s \in S$. Extend the queue of membership queries and continue to 2.
    2. If OT is not consistent, extend the set E with the word $a \cdot e$, $e \in E$, and $a \in A$ so that there are $s, t \in S$, that $s \overset{E}{\sim} t$, but $\mathrm{T}(s \cdot a, e) \neq \mathrm{T}(t \cdot a, e)$. Extend the queue of membership queries and continue to 2.
4. Make the conjecture $\mathcal{A}$ and ask the teacher for its correctness.
5. If the teacher returns a counterexample $c \in A^+$,
   delete the conjecture $\mathcal{A}$,
   add all elements of the set $\mathrm{pref}(c)$ to the set S,
   extend the queue of membership queries and continue to 2.
6. Accept the conjecture $\mathcal{A}$ as the automaton
   modeling the environment $\mathcal{E}$.

FSM Conjecture Example [Ang86, Ang87]

- An acceptor $M(S, E, T)$
  - over the alphabet A,
  - with state set Q,
  - initial state q_0,
  - accepting states F, and
  - transition function δ:

$$Q = \{\text{row}(s) : s \in S\}, \tag{1}$$

$$q_0 = \text{row}(\epsilon), \tag{2}$$

$$F = \{\text{row}(s) : s \in S \text{ and } T(s) = T(s \bullet \epsilon) = 1\}, \tag{3}$$

$$\delta(\text{row}(s), a) = \text{row}(s \bullet a). \tag{4}$$

- $S = \{\epsilon, a, b, bb\}$, $E = \{\epsilon, a\}$

T_4		E: ϵ	E: a
S	ϵ	1	0
	a	0	1
	b	0	0
	bb	1	0
$S \bullet A$	aa	1	0
	ab	0	0
	ba	0	0
	bba	0	1
	bbb	0	0

M_2/δ	a	b
q_0	q_1	q_2
q_1	q_0	q_2
q_2	q_2	q_0

Outline

1 FSM Learning
- FSM Learning Overview
- Angluin's Algorithm
- Example

2 Hidden Markov Model
- A Brief Overview

3 Markov Decision Process
- Introduction
- Utility Function, Policy
- Value Iteration
- Policy Iteration
- Conclusions

L^* Algorithm - Example I [Ang87]

Example 1

The unknown regular automaton accepts the set of all strings over $\{a, b\}$ with an even number of a's and an even number of b's.

The initial observation table, $S = E = \{\epsilon\}$

T_1		E
		ϵ
S	ϵ	1
$S \cdot A$	a	0
	b	0

- The observation table T_1 is consistent, but not closed, since row(a) is distinct from row(ϵ).
- L^* chooses to move the string a to the set S and then queries the strings aa and ab to construct the observation table T_2.

L^* Algorithm - Example II [Ang87]

Example 2

The unknown regular automaton accepts the set of all strings over $\{a, b\}$ with an even number of a's and an even number of b's.

$S = \{\epsilon, a\}$, $E = \{\epsilon\}$

T_2		E: ϵ
S	ϵ	1
	a	0
$S \bullet A$	b	0
	aa	1
	ab	0

M_1/δ	a	b
q_0	q_1	q_1
q_1	q_0	q_1

- The observation table T_2 is consistent and closed.
- L^* makes a conjecture of the acceptor M_1.
- The initial state of M_1 is q_0 and the final state is also $q0$.
- The teacher selects a counterexample bb (rejected by M_1).

L^* Algorithm - Example III [Ang87]

$S = \{\epsilon, a, b, bb\},\ E = \{\epsilon\}$

T_3		E: ϵ
S	ϵ	1
	a	0
	b	0
	bb	1
$S \bullet A$	aa	1
	ab	0
	ba	0
	bba	0
	bbb	0

- The observation table T_3 is closed, but not consistent, since row(a) = row(b) but row$(aa) \neq$ row(ba).
- L^* adds the string a to E and queries the strings aaa, aba, baa, $bbaa$, and $bbba$ to construct the table T_4.

L^* Algorithm - Example IV [Ang87]

$S = \{\epsilon, a, b, bb\}$, $E = \{\epsilon, a\}$

T_4		E: ϵ	E: a
S	ϵ	1	0
	a	0	1
	b	0	0
	bb	1	0
$S \bullet A$	aa	1	0
	ab	0	0
	ba	0	0
	bba	0	1
	bbb	0	0

M_2/δ	a	b
q_0	q_1	q_2
q_1	q_0	q_2
q_2	q_2	q_0

- The observation table T_2 is consistent and closed.
- L^* makes a conjecture of the acceptor M_2.
- The initial state of M_2 is q_0 and the final state is also $q0$.
- The teacher selects a counterexample abb (accepted by M_1, but not in U).

L^* Algorithm - Example V [Ang87]

T_5		ϵ	a
S	ϵ	1	0
	a	0	1
	b	0	0
	bb	1	0
	ab	0	0
	abb	0	1
$S \bullet A$	aa	1	0
	ba	0	0
	bba	0	1
	bbb	0	0
	aba	0	0
	$abba$	1	0
	$abbb$	0	0

$S = \{\epsilon, a, b, bb, ab, abb\}$
$E = \{\epsilon, a\}$

- The observation table T_5 is closed but not consistent since row$(b) =$ row(ab) but row$(bb) \neq$ row(abb).
- L^* adds the string b to E and queries the strings aab, bab, $bbab$, $bbbb$, $abab$, $abbab$, and $abbbb$ to construct the table T_6.

L^* Algorithm - Example VI [Ang87]

T_6		E		
		ϵ	a	b
S	ϵ	1	0	0
	a	0	1	0
	b	0	0	1
	bb	1	0	0
	ab	0	0	0
	abb	0	1	0
$S \bullet A$	aa	1	0	0
	ba	0	0	0
	bba	0	1	0
	bbb	0	0	1
	aba	0	0	1
	$abba$	1	0	0
	$abbb$	0	0	0

$S = \{\epsilon, a, b, bb, ab, abb\}$
$E = \{\epsilon, a, b\}$

M_3/δ	a	b
q_0	q_1	q_2
q_1	q_0	q_3
q_2	q_3	q_0
q_3	q_2	q_1

- The observation table T_2 is consistent and closed.
- L^* makes a conjecture of the acceptor M_2.
- The initial state of M_3 is q_0 and the final state is also $q0$.
- The teacher replies to this conjecture with *yes*.
  - M_3 is a correct acceptor for the language U.

L^* Algorithm Performance

- The example:
  - # MQ: 25
  - # EQ: 3
- Real protocols

Protocol	States	Letters	MQ	EQ
Abp-lossy	3	3	22	2
Buff3	9	3	202	5
Dekker-2	2	3	7	1
Sched2	13	6	691	7
VMnew	11	4	513	7

- Synthetic data

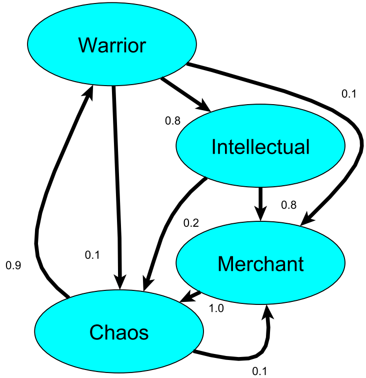
States	Letters	MQ	EQ
100	25	40000	15

- At present up to 1000 states.

Outline

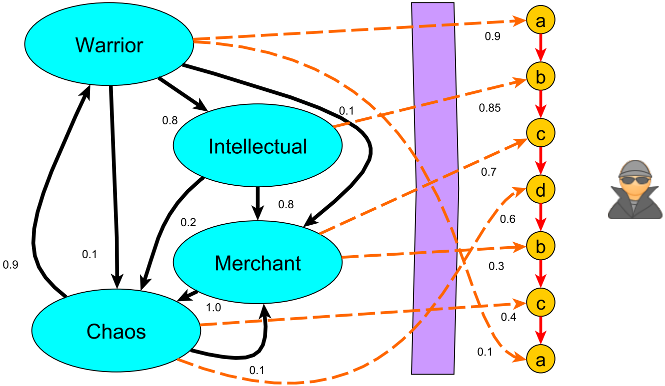
1. FSM Learning
   - FSM Learning Overview
   - Angluin's Algorithm
   - Example
2. Hidden Markov Model
   - A Brief Overview
3. Markov Decision Process
   - Introduction
   - Utility Function, Policy
   - Value Iteration
   - Policy Iteration
   - Conclusions

Hidden Markov Model (HMM) - Overview



1. Many observation sequences → FSM model learning
   - Iterative Baum-Welch algorithm [BP66] - Expectation-Maximization (EM)
2. FSM Model + an observation sequence
   → the probability of the state sequence
   - The Viterbi algorithm
3. FSM Model + a sequence part → the most probable states

Hidden Markov Model (HMM) - Overview



1. Many observation sequences → FSM model learning
   - Iterative Baum-Welch algorithm [BP66] - Expectation-Maximization (EM)
2. FSM Model + an observation sequence → the probability of the state sequence
   - The Viterbi algorithm
3. FSM Model + a sequence part → the most probable states

Outline

1 FSM Learning
- FSM Learning Overview
- Angluin's Algorithm
- Example

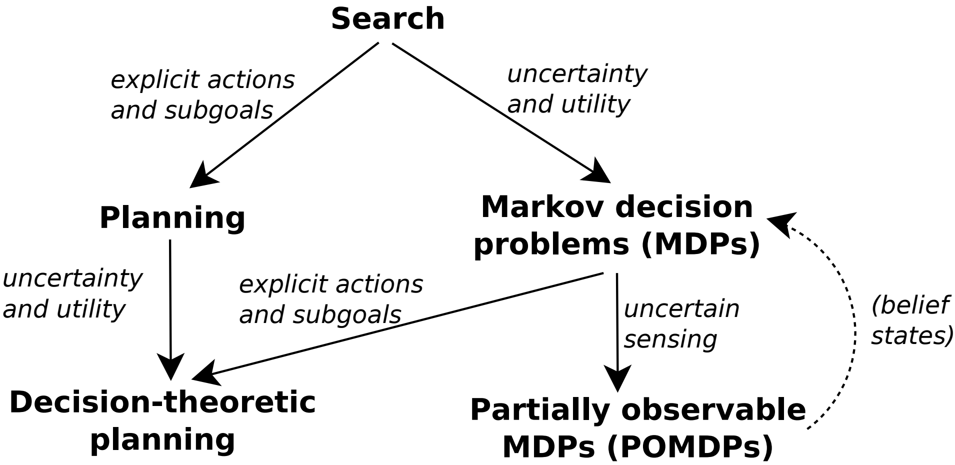
2 Hidden Markov Model
- A Brief Overview

3 Markov Decision Process
- Introduction
- Utility Function, Policy
- Value Iteration
- Policy Iteration
- Conclusions

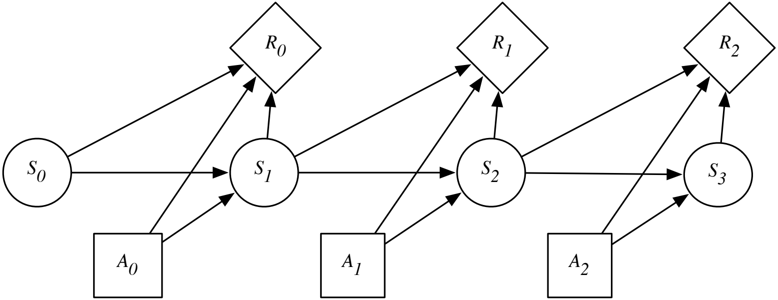
Sequential Decisions [RN10]

- Achieving agent's objectives often requires multiple steps.
- A rational agent does not make a multi-step decision and carry it out without considering revising it based on future information.
    - Subsequent actions can depend on what is observed
    - What is observed depends on previous actions
- Agent wants to maximize reward accumulated along its course of action
- What should the agent do if environment is non-deterministic?
    - Classical planning will not work
    - Focus on state sequences instead of action sequences

Sequential Decision Problems [Jak10]



Markov Decision Process [PM10]



Markov Decision Process [PM10]

Definition (Markov Decision Process)

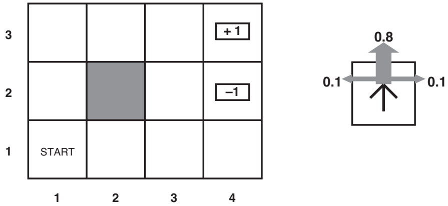
A **Markov Decision Process (MDP)** is a 5-tuple $\langle S, A, T, R, s_0 \rangle$ where

- S is a set of states
- A is a set of actions
- $T(S, A, S')$ is the transition model
- $R(S)$ is the reward function
- s_0 is the initial state

- Transitions are Markovian

$$P(S_n|A, S_{n-1}) = P(S_n|A, S_{n-1}, S_{n-2}, \ldots, S_0) = T(S_{n-1}, A, S_n)$$

Example: Simple Grid World [RN10]



(a) (b)

Simple 4x3 environment

- States $S = \{(i,j) | 1 \leq i \leq 4 \wedge 1 \leq j \leq 3\}$
- Actions $A = \{up, down, left, rigth\}$
- Reward function

$$R(s) = \begin{cases} -0.04 & \text{(small penalty) for nonterminal states} \\ \pm 1 & \text{for terminal states} \end{cases}$$

- Transition model $T((i,j), a, (i', j'))$ given by (b)

Outline

1. FSM Learning
   - FSM Learning Overview
   - Angluin's Algorithm
   - Example
2. Hidden Markov Model
   - A Brief Overview
3. Markov Decision Process
   - Introduction
   - Utility Function, Policy
   - Value Iteration
   - Policy Iteration
   - Conclusions

Utility Function [RN10, Jak10]

- Utility function captures agent's preferences
  - In sequential decison-making, utility is a function over sequences of states
- Utility function accumulates rewards:
  - Additive rewards (special case):

$$U_h([s_0, s_1, s_2, \dots]) = R(s_0) + R(s_1) + R(s_2) + \dots$$

  - Discounted rewards

$$U_h([s_0, s_1, s_2, \dots]) = R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \dots$$

    where $\gamma \in [0, 1]$ is the discount factor
- Discounted rewards for $\gamma < 1$ finite even for infinite horizons (see next slide)
- No other way of assigning utilities to state sequences is possible assuming stationary preferences between state sequences

Policy [RN10, Jak10]

- A stationary policy is a function

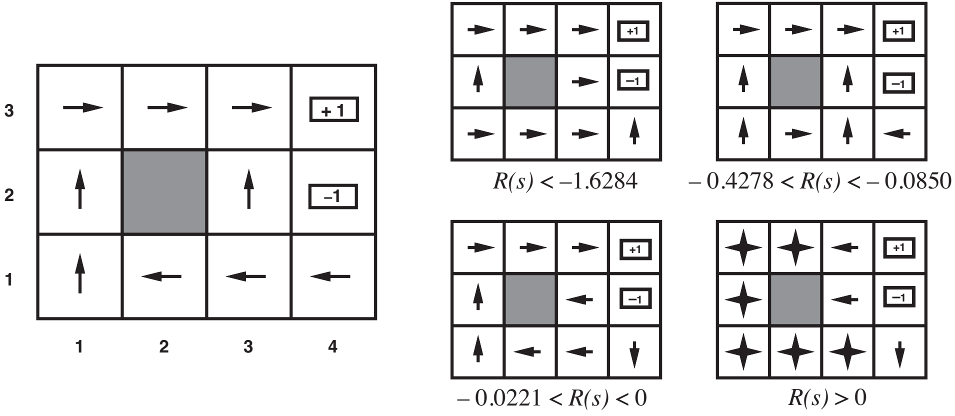
$$\pi : S \rightarrow A$$

- Optimal policy is a function maximizing expected utility

$$\pi^\star = \arg\max_{\pi} E[U([s_0, s_1, s_2, \ldots])|\pi]$$

- For an MDP with stationary dynamics and rewards with infinite horizon, there always exists an optimal stationary policy
  - no benefit to randomize even if environment is random

Example: Optimal Policies in the Grid World [RN10, Jak10]



(a) (b)

- (a) Optimal policy for state penalty $R(s) = -0.04$
- (b) Dependence on penalty

Decision-making Horizon [RN10, Jak10]

- A finite horizon means that there is a finite deadline N after which nothing matters (the game is over)
  - $\forall k \geq 1 \quad U_h([s_0, s_1, \ldots, s_{N+k}]) = U_h([s_0, s_1, \ldots, s_N])$
  - The optimal policy is non-stationary, i.e., it could change over time as the deadline approaches.
- An infinite horizon means that there is no deadline
  - The optimal policy is stationary $\Leftarrow$ there is no reason to behave differently in the same state at different times
  - Easier than the finite horizon case
- terminate / absorbing states – agents stay there forever receiving zero reward at each step

Outline

1 FSM Learning
- FSM Learning Overview
- Angluin's Algorithm
- Example

2 Hidden Markov Model
- A Brief Overview

3 Markov Decision Process
- Introduction
- Utility Function, Policy
- Value Iteration
- Policy Iteration
- Conclusions

Solving MDPs [RN10, Jak10]

- How do we find the optimum policy π^*?
- Two basic techniques:
  1. value iteration – compute utility $U(s)$ for each state and use is for selecting best action
  2. policy iteration – represent policy explicitly and update it in parallel to the utility function

Utility of State [RN10, Jak10]

- Utility of a state under a given policy π:

$$U^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) | \pi, s_0 = s\right]$$

- True utility $U(s)$ of a state is the utility assuming optimum policy π^*

$$U(s) := U^{\pi^*}(s)$$

  - Reward $R(s)$ is "short-term" reward for being in s;
    utility $U(s)$ is a "long-term" total reward from s onwards
- Selecting the optimum action according to the MEU (Maximum Expected Utility) principle

$$\pi^*(s) = \arg\max_a \sum_{s'} T(s, a, s')U(s')$$

Bellman Equation [RN10, Jak10]

- Definition of utility of states leads to a simple relationship among utilities of neighboring states
- The utility of a state is the immediate reward for the state plus the expected discounted utility of the next state, assuming the agent chooses the optimal action

Definition (Bellman equation (1957))

$$U(s) = R(s) + \gamma \max_a \sum_{s'} T(s, a, s')U(s') \quad \forall s \in S$$

- One equation per state $\Rightarrow$ n non-linear equations for n unknowns
  - The solution is unique

Iterative Solution [RN10, Jak10]

- Analytical solution is not possible $\Rightarrow$ iterative approach

Definition (Bellman update)

$$U_{i+1}(s) = R(s) + \gamma \max_a \sum_{s'} T(s,a,s') U_i(s') \quad \forall s \in S$$

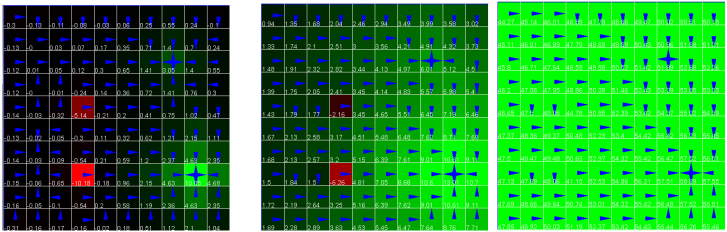
- Dynamic programming: given an estimate of the k-step lookahead value function, determine the $k+1$-step lookahead utility function.
- If applied infinitely often, guaranteed to reach an equilibrium and the final utility values are the solutions to the Bellman equations
- Value iteration propagates information through the state space by means of local updates.

Value Iteration Algorithm [RN10, Jak10]

Input: mdp, a MDP with states S, transition model T, reward function R, discount γ
Input: ϵ, the maximum error allowed in the utility of a state
Local variables: U, U', vectors of utilities for states in S, initially zero
Local variables: δ, the maximum change in the utility of any state in an iteration

repeat
    $U \leftarrow U'; \delta \leftarrow 0$;
    foreach *state* $s \in S$ **do**
        $U'[s] \leftarrow R[s] + \gamma \max_a \sum_{S'} T(s, a, s')U[s']$;
        if $|U'[s] - U[s]| > \delta$ **then**
            $\delta \leftarrow |U'[s] - U[s]|$;
        end
    end
until $\delta < \epsilon(1 - \gamma)/\gamma$;
return U

Value Iteration Example [RN10, PM10, Jak10]



(a) $\gamma = 0.6$ (b) $\gamma = 0.9$ (c) $\gamma = 0.99$

- 4 movement actions; 0.7 chance of moving in the desired direction, 0.1 in the others
- $R = -1$ for bumping into walls; four special rewarding states
  - +10 (at position (9,8); 9 across and 8 down),
  - one worth +3 (at position (8,3)),
  - one worth -5 (at position (4,5)) and
  - one -10 (at position (4,8))

Outline

1 FSM Learning
- FSM Learning Overview
- Angluin's Algorithm
- Example

2 Hidden Markov Model
- A Brief Overview

3 Markov Decision Process
- Introduction
- Utility Function, Policy
- Value Iteration
- Policy Iteration
- Conclusions

Policy Iteration [RN10, Jak10]

- Search for optimal policy and utility values simultaneously
- Alternates between two steps:
  1. policy evaluation – recalculates values of states $U_i = U^{\pi_i}$ given the current policy π_i
  2. policy improvement/iteration – calculates a new MEU policy π_{i+1} using one-step look-ahead based on U_i
- Terminates when the policy improvement step yields no change in the utilities.

Policy Iteration Algorithm [RN10, Jak10]

Input: mdp, a MDP with states S, transition model T
Local variables: U, a vector of utilities for states in S, initially zero
Local variables: π, a policy vector indexed by state, initially random
repeat
  $U \leftarrow \texttt{Policy-Evaluation}(\pi, U, mdp)$;
  $unchanged? \leftarrow$ true;
  foreach *state* $s \in S$ **do**
    if $\max_a \sum_{S'} T(s,a,s')U[s'] > \sum_{S'} T(s,\pi(s),s')U[s']$ **then**
      $\pi(s) \leftarrow \arg\max_a \sum_{S'} T(s,a,s')U[s']$;
      $unchanged? \leftarrow$ false;
    end
  end
until $unchanged?$;
return π

Policy Evaluation [RN10, Jak10]

- Simplified Bellman equations:

$$U_i(s) = R(s) + \gamma \sum_{S'} T(s, \pi_i(s), s')U_i(s') \quad \forall s \in S$$

- The equations are now linear $\Rightarrow$ can be solved in $O(n^3)$

Modified Policy Iteration [RN10, Jak10]

- Policy iteration often converges in few iterations but each iteration is expensive
    - $\Leftarrow$ has to solve large systems of linear equations
- Main idea: use iterative approximate policy evaluation
    - Simplified Bellman update:

$$U_{i+1}(s) \leftarrow R(s) + \gamma \sum_{S'} T(s, \pi_i(s), s') U_i(s') \quad \forall s \in S$$

    - Use a few steps of value iteration (with π fixed)
    - Start from the value function produced in the last iteration
- Often converges much faster than pure value iteration or policy iteration (combines the strength of both approaches)
- Enables much more general asynchronous algorithms
    - e.g. Prioritized sweeping

Choosing the Right Technique [RN10, Jak10]

- Many actions?$\Rightarrow$ policy iteration
- Already got a fair policy? $\Rightarrow$ policy iteration
- Few actions, acyclic? $\Rightarrow$ value iteration
- Modified policy iteration typically the best

Outline

1. FSM Learning
   - FSM Learning Overview
   - Angluin's Algorithm
   - Example
2. Hidden Markov Model
   - A Brief Overview
3. Markov Decision Process
   - Introduction
   - Utility Function, Policy
   - Value Iteration
   - Policy Iteration
   - Conclusions

Conclusions [RN10, Jak10]

- MDPs generalize deterministic state space search to stochastic environments
  - At the expense of computational complexity
- An optimum policy associates an optimal action with every state
- Iterative techniques used to calculate optimum policies
  - basic: value iteration and policy iteration
  - improved: modified policy iteration, asynchronous policy iteration
- Further issues
  - large state spaces – use state space approximation
  - partial observability (POMDPs) – need to consider information gathering; can be mapped to MDPs over continous belief space

Literatura I

[Ang86] Dana Angluin. Learning regular sets from queries and counter-examples. Technical Report YALEU/DCS/TR-464, Yale University, Department of Computer Science, March 1986.

[Ang87] Dana Angluin. Learning regular sets from queries and counterexamples. *Information and Computation*, 75(2):87–106, 1987.

[BP66] Leonard E. Baum and Ted Petrie. Statistical inference for probabilistic functions of finite state markov chains. *The Annals of Mathematical Statistics*, 37(6):1554–1563, 1966.

[Hon13] Marek Honzírek. Aktivní učení a automaty. Master's thesis, Katedra matematiky, Fakulta jaderná a fyzikálně inženýrská, ČVUT, Praha, 2013.

[Jak10] Michal Jakob. A3M33UI decision theory essentials, lecture notes. http://cw.felk.cvut.cz/doku.php/courses/a3m33ui/prednasky, February 2010.

[PM10] David Poole and Alan Mackworth. Artificial intelligence, foundations of computational agents. http://artint.info/slides/slides.html, 2010.

[RN10] Stuart J. Russell and Peter Norvig. *Artificial Intelligence, A Modern Approach*. Pre, third edition, 2010.

[Sha08] Muzammil Muhammad Shahbaz. *Reverse Engineering Enhanced State Models of Black Box Software Components to support Integration Testing*. PhD thesis, Institut Polytechnique de Grenoble, 2008.